Wave functions and compositeness for hadron resonances from the scattering amplitude

Takayasu SEKIHARA (RCNP, Osaka Univ.)

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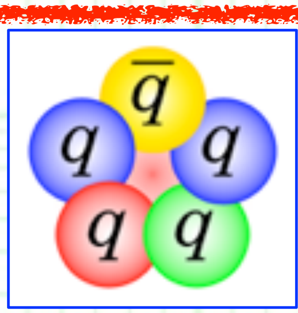
[1] T. S. , T. Hyodo and D. Jido, *Prog. Theor. Exp. Phys.* 2015, 063D04.
[2] T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, arXiv:1511.01200 [hep-ph].

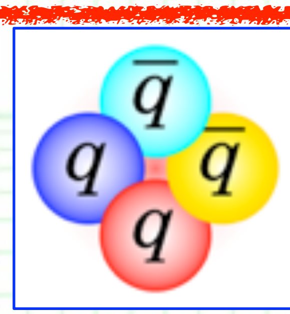
1. Introduction

++ Exotic hadrons and their structure ++

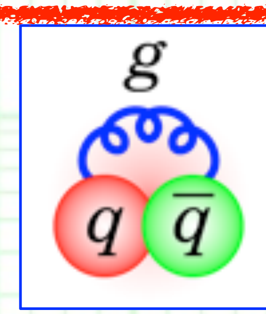
- **Exotic hadrons** --- not same quark component as ordinary hadrons
  = not qqq nor $q\bar{q}$.



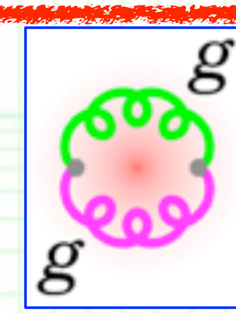
Penta-quarks



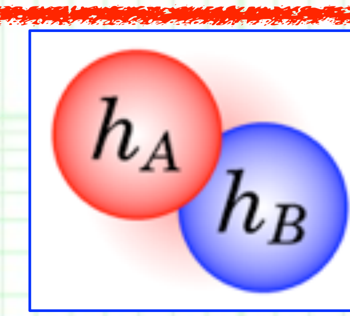
Tetra-quarks



Hybrids

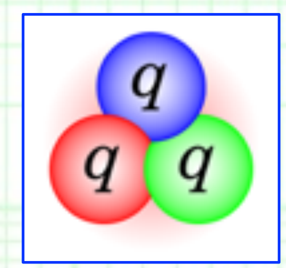


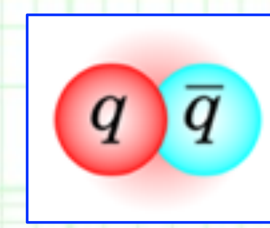
Glueballs



Hadronic molecules

...





Ordinary hadrons

--- Actually some hadrons cannot be described by the quark model.

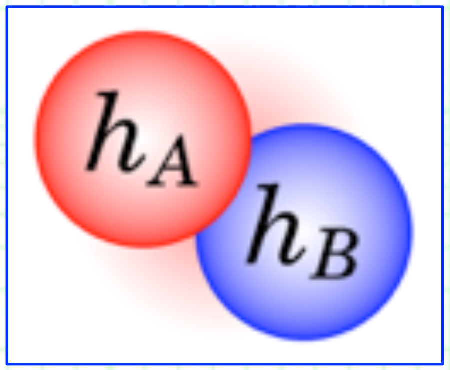
- □ Do exotic hadrons really exist ?
- □ If they do exist, how are their properties ?
  --- Re-confirmation of quark models.
  --- Constituent quarks in multi-quarks ? "Constituent" gluons ?
- □ If they do not exist, what mechanism forbids their existence ?

<-- We know very few about hadrons (and dynamics of QCD).

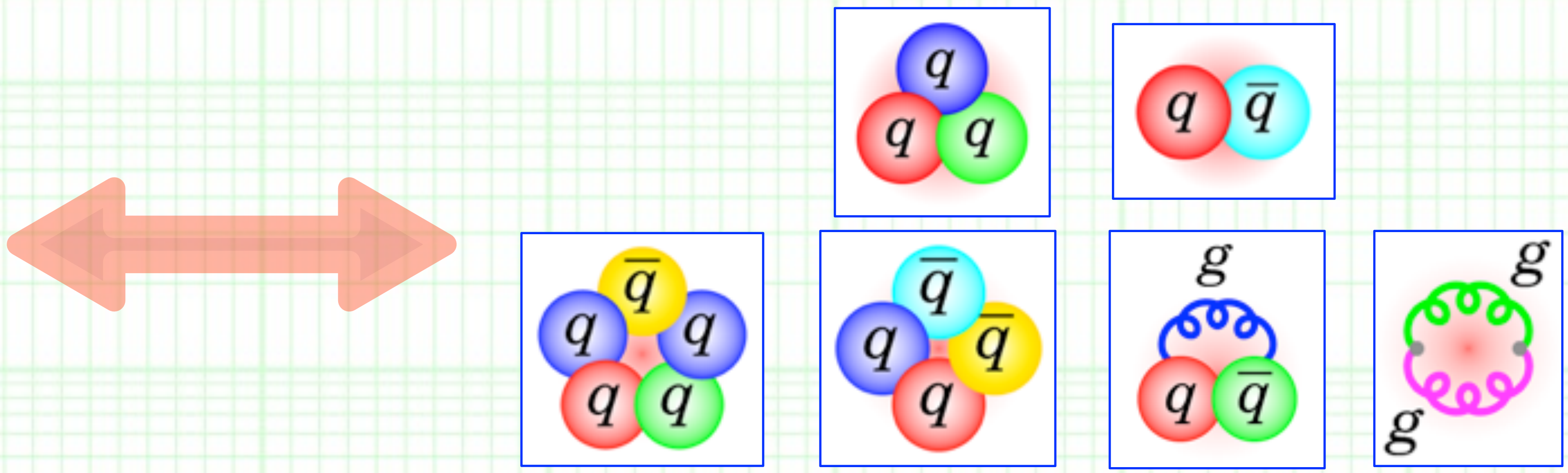
1. Introduction

++ Uniqueness of hadronic molecules ++

- **Hadronic molecules should be unique, because they are composed of hadrons themselves, which are color singlet.**



Hadronic molecules
(cf. deuteron)



...

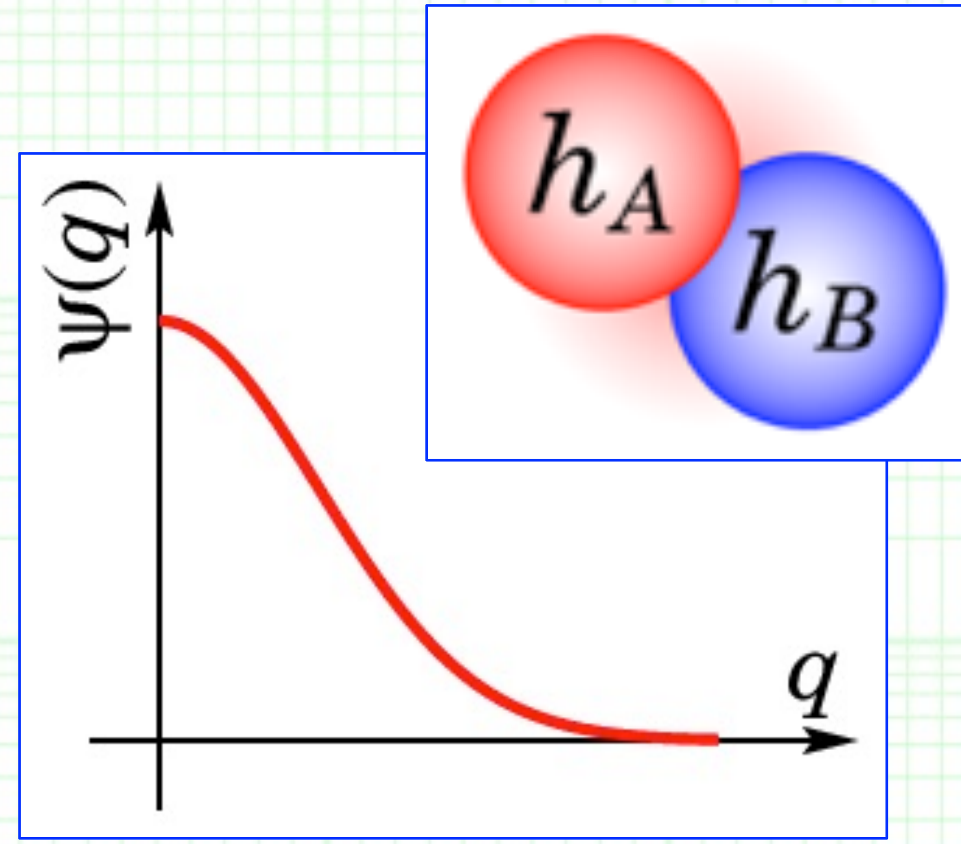
--> Various quantitative/qualitative diff. from other compact hadrons.

- □ **Large spatial size due to the absence of strong confining force.**
- □ **Hadron masses are "observable", in contrast to quark masses.**
  --> Expectation of the existence around two-body threshold.
- □ **Treat them without complicated calculations of QCD.**
  --- We can use quantum mechanics with appropriate interactions.

1. Introduction

++ How to clarify their structure ? ++

- How can we <u>use quantum mechanics</u> to clarify the structure of hadronic molecule candidates ?



- We evaluate the wave function of hadron-hadron composite contribution.

--- Cf. Wave function for relative motion of <u>two nucleons inside deuteron</u>.

- How to evaluate the wave function ?

<-- We employ a fact that <u>the two-body wave function appears in the residue of the scattering amplitude</u> of the two particles <u>at the resonance pole</u>.

--- The wave function from the residue is <u>automatically normalized</u> !

--> Calculating the norm of the two-body wave function = compositeness, we may <u>measure the fraction of the composite component</u> and conclude the composite structure !

1. Introduction

++ Purpose and strategy of this study ++

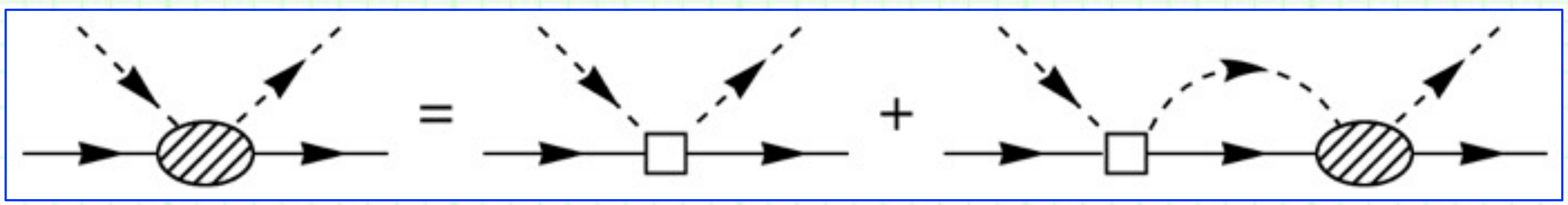
- In this study we **evaluate the hadron-hadron two-body wave functions** and **their norms = compositeness** for hadron resonances from the hadron-hadron scattering amplitudes.

- We have to use **precise scattering amplitudes** for the evaluation.

--> Employ **the chiral unitary approach**.

Kaiser-Siegel-Weise ('95); Oset-Ramos ('98); Oller-Meissner ('01); Lutz-Kolomeitsev ('02); Oset-Ramos-Bennhold ('02); Jido-Oller-Oset-Ramos-Meissner ('03); ...

$$\boxed{T = V + VGT}$$



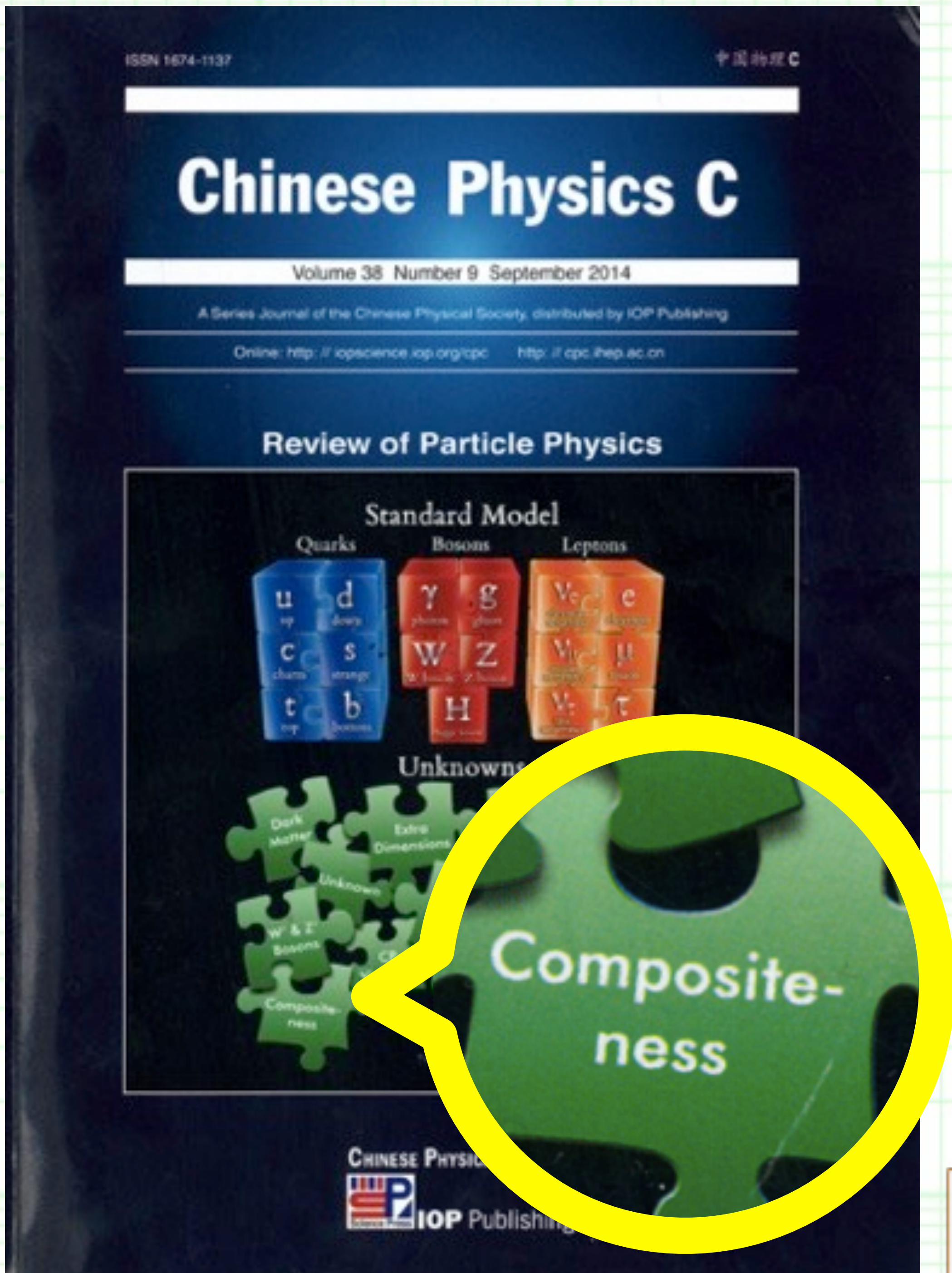
- □ Interaction kernel V from the chiral perturbation theory: Leading order (LO) + next-to-leading order (NLO) (+ bare Δ).

- □ Loop function G calculated **with the dispersion relation** in a covariant way.

- We discuss **the structure of $\Lambda(1405)$, $N(1535)$, $N(1650)$, and $\Delta(1232)$**.

2. Wave functions and compositeness

++ Wave function for hadron ++



Particle Data Group (2014).
(similar but not same as our compositeness)

- **Wave function of a hadronic molecule $|\Psi>$ should be unique, since it should contain dominant two-body component.**
  - **This can be measured with the decomposition of unity:**

$$\mathbb{1} = \int \frac{d^3 q}{(2\pi)^3} |\vec{q}\rangle\langle\vec{q}| + |\psi_0\rangle\langle\psi_0|$$

--- $|q>$: two-body state,
$|\psi_0>$: bare state.

  - **Compositeness (X) can be defined as the norm of the two-body wave function in the normalization of the total wave function $|\Psi>$.**

$$\langle\Psi^*|\Psi\rangle = \sum_j X_j + Z = 1$$

$$X_j = \int \frac{d^3 q}{(2\pi)^3} \langle\Psi^*|\vec{q}_j\rangle\langle\vec{q}_j|\Psi\rangle$$

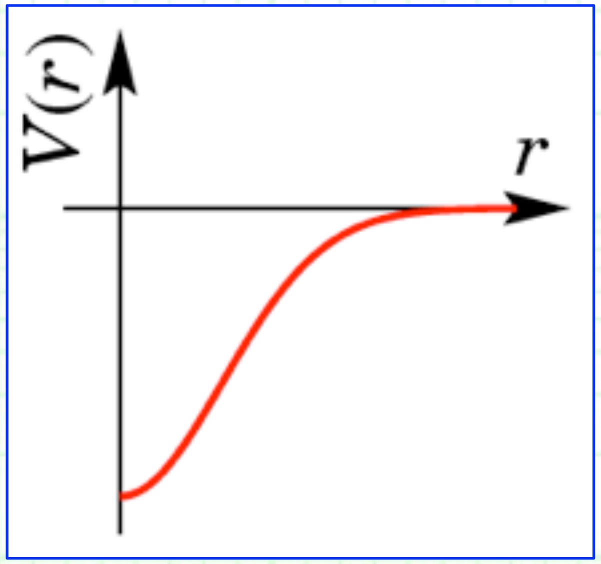
T. S. , Hyodo and Jido, *PTEP* 2015, 063D04; ...

2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function.

Ex.) A bound state in a NR single-channel problem.



- Usual approach: Solve the Schrödinger equation.

$$\hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\mathrm{pole}}|\Psi\rangle$$

--- Wave function in coordinate / momentum space:

$$\langle \vec{r}|\Psi\rangle = \psi(r) \qquad \langle \vec{q}|\Psi\rangle = \tilde{\psi}(q)$$

$$\left[M_{\mathrm{th}} - \frac{\nabla^2}{2\mu} + V(r) \right] \psi(r) = E_{\mathrm{pole}} \psi(r)$$

--> After solving the Schrödinger equation,
we have to normalize the wave function by hand.

$$\int d^3 r \, [\psi(r)]^2 = 1 \quad \text{or} \quad \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(q) \right]^2 = 1$$ <-- We require !

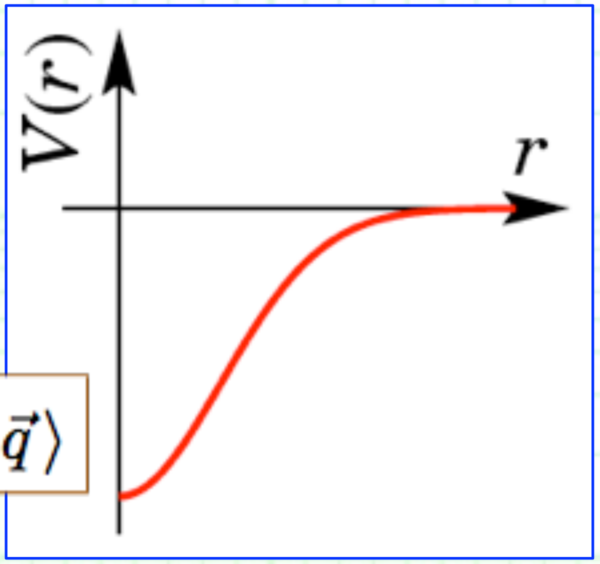
2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are **several approaches to calculate the wave function**.

  Ex.) A bound state in a NR single-channel problem.

  - <u>Our approach: Solve the Lippmann-Schwinger equation</u> at **the pole position** of the bound state.



$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E - \hat{H}_0}\hat{T} = \hat{V} + \hat{V}\frac{1}{E - \hat{H}}\hat{V}$$

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle$$

--- Near **the resonance pole position E_{pole}**, amplitude is dominated by the pole term in <u>the expansion by the eigenstates of H</u> as

$$\langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \langle \vec{q}\,' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\mathrm{pole}}} \langle \Psi^{*} | \hat{V} | \vec{q} \rangle$$

$|\Psi\rangle, |\vec{q}_{\mathrm{full}}\rangle, \ldots$ $\langle \Psi^{*}|, \langle \vec{q}_{\mathrm{full}}|, \ldots$

$$\Rightarrow \quad 1 = |\Psi\rangle\langle\Psi^{*}| + \cdots$$

--- **The residue of the amplitude at the pole position has information on the wave function !**

$$\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \vec{q} | (\hat{H} - \hat{H}_0) | \Psi \rangle = [E_{\mathrm{pole}} - E(q)] \tilde{\psi}(q)$$

$$\langle \Psi^{*} | \hat{V} | \vec{q} \rangle = [E_{\mathrm{pole}} - E(q)] \tilde{\psi}(q)$$

$$E(q) = M_{\mathrm{th}} + \frac{q^2}{2\mu}$$

2. Wave functions and compositeness

++ How to calculate the wave function ++

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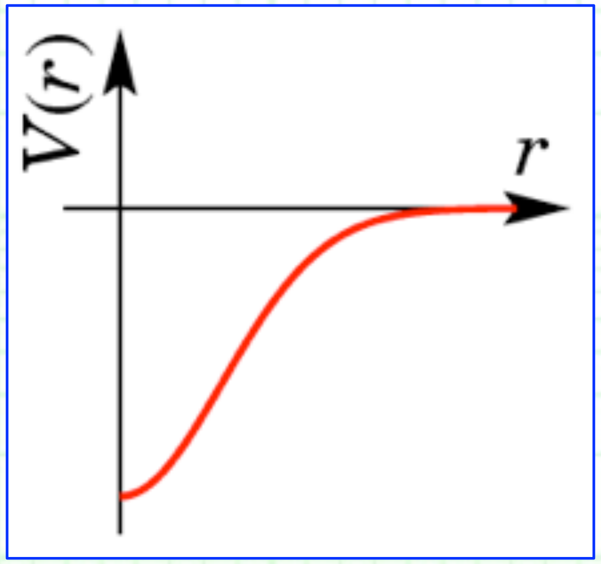
Ex.) A bound state in a NR single-channel problem.



□ Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.

--- The wave function can be extracted from the residue of the amplitude at the pole position:

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

$$\gamma(q) \equiv \langle \vec{q} | \hat{V} | \Psi \rangle = [E_{\text{pole}} - E(q)] \tilde{\psi}(q)$$

--> Because the scattering amplitude cannot be freely scaled due to the optical theorem, the wave function from the residue of the amplitude is automatically normalized !

Purely molecule --> $$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\text{pole}} - E(q)} \right]^2 = 1$$ <-- We obtain !

E. Hernandez and A. Mondragon, *Phys. Rev.* C 29 (1984) 722.

--> Therefore, from precise hadron-hadron scattering amplitudes with resonance poles, we can calculate their two-body WF.

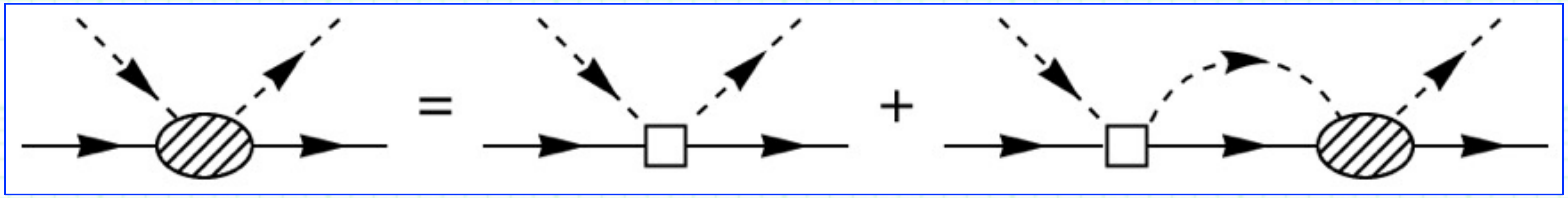
2. Wave functions and compositeness

++ Our strategy ++

- In this study we **investigate the structure of hadronic molecule candidates** in the following strategy.

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, arXiv:1511.01200.

1. Construct precise hadron-hadron scattering amplitude, which contains **resonance poles for hadronic molecule candidates**, in appropriate effective models (in a covariant version).



$$T_{jk}(\vec{q}\,', \vec{q}; s) = V_{jk}(\vec{q}\,', \vec{q}; s) + i \sum_{l} \int \frac{d^4 q''}{(2\pi)^4} \frac{V_{jl}(\vec{q}\,', \vec{q}\,''; s) T_{lk}(\vec{q}\,'', \vec{q}; s)}{(q^2 - m_l^2)[(P-q)^2 - M_l^2]}$$

2. Extract the two-body wave function from **the residue** of the amplitude at the resonance pole.

$$T_{jk}(\vec{q}\,', \vec{q}; s) = \frac{\gamma_j(q')\gamma_k(q)}{s - s_{\text{pole}}} + (\text{regular at } s = s_{\text{pole}})$$

$$\gamma_j(q) \equiv \langle \vec{q}_j | \hat{V} | \Psi \rangle = [s_{\text{pole}} - s_j(q)] \tilde{\psi}_j(q)$$

2. Wave functions and compositeness

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$$\gamma_j(q) \equiv \langle \vec{q}_j | \hat{V} | \Psi \rangle = [s_{\text{pole}} - s_j(q)] \tilde{\psi}_j(q)$$

3. Calculate the compositeness X_j = **norm of the two-body wave function in channel j**, from Amp. and compare it with unity.

$$X_j = \int \frac{d^3q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[\tilde{\psi}(q)\right]^2 = \int \frac{d^3q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[\frac{\gamma_j(q)}{s_{\text{pole}} - s_j(q)}\right]^2$$

- **The sum of X_j will exactly unity** for a purely molecular state.
  <= The interaction does not have energy dependence.

E. Hernandez and A. Mondragon (1984).

- On the other hand, if the interaction has energy dependence, which can be interpreted as **the contribution from missing channels**, the sum of X_j deviates from unity.

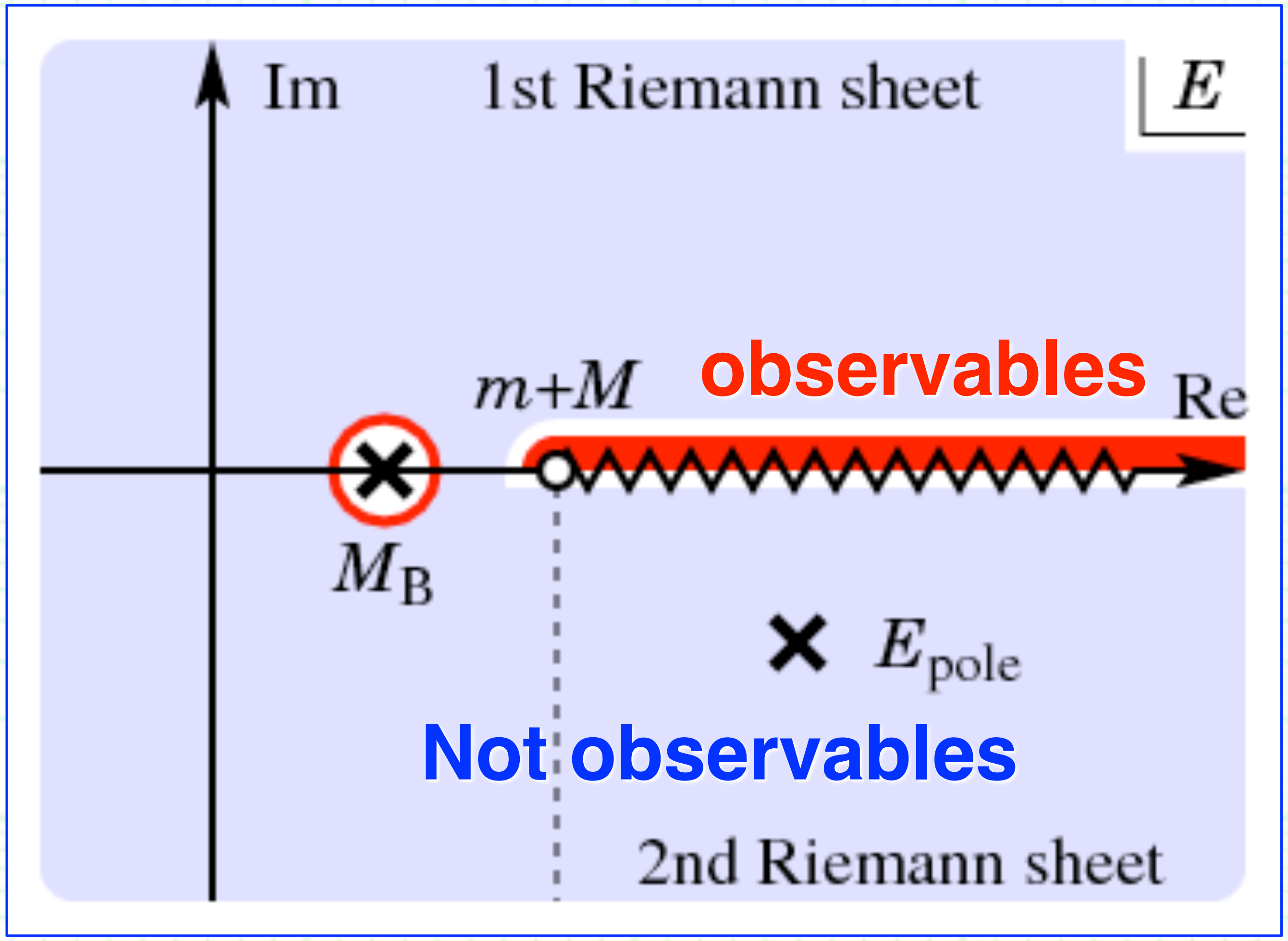
--> **Fraction of missing channels** is expressed by Z:

$$\sum_j X_j = 1 - Z$$

2. Wave functions and compositeness

++ Observable and model (in)dependence ++

- Here we comment on the observables and non-observables.
  - Observables:
    Cross section.
    Its partial-wave decomposition.
    --> On-shell Scatt. amplitude
    via the optical theorem.
    Mass of bound states.
  - NOT observables:
    Wave function and potential.
    Resonance pole position.
    Residue at pole.
    Off-shell amplitude.



--> Since the residue of the amplitude at the resonance pole is NOT observable, the wave function and its norm = compositeness are also not observable and model dependent.

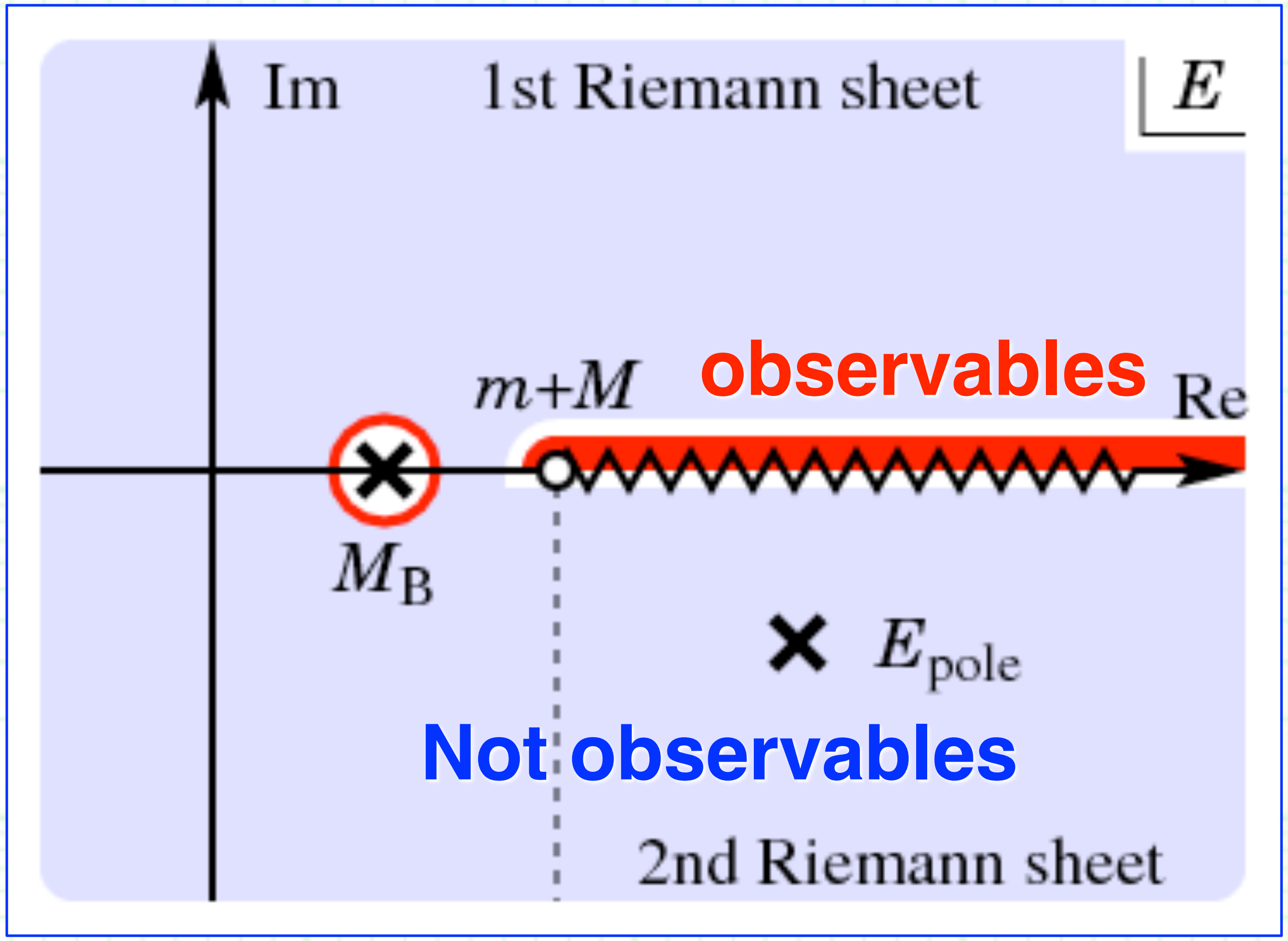
--- Exception: Compositeness for near-threshold poles.

2. Wave functions and compositeness

++ Observable and model (in)dependence ++

- Special case: Compositeness for near-threshold poles.

--- Compositeness can be expressed with threshold parameters such as scattering length and effective range.



- Deuteron.
  Weinberg ('65).
- $f_0(980)$ and $a_0(980)$.
  Baru et al. ('04),
  Kamiya-Hyodo, arXiv:1509.00146.
- Λ(1405).
  Kamiya-Hyodo, arXiv:1509.00146.
- ...

- General case: Compositeness are model dependent quantity.

--> Therefore, we have to employ appropriate effective models (V) to construct precise hadron-hadron scattering amplitude in order to discuss the structure of hadronic molecule candidate !

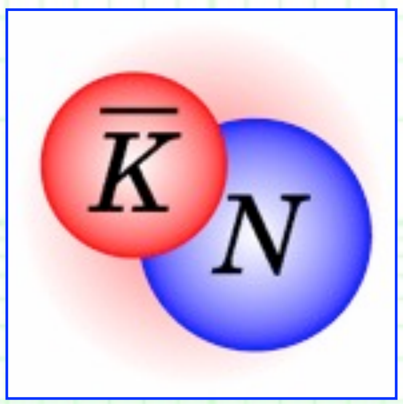
3. Applications

++ List of hadron resonances in our analysis ++

- In this talk, we **discuss the structure of candidates of hadronic molecules** listed as follows **in terms of the compositeness**:

1. $\Lambda(1405)$.

--- One of classical examples of the exotic hadron candidates.



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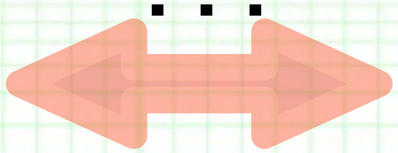
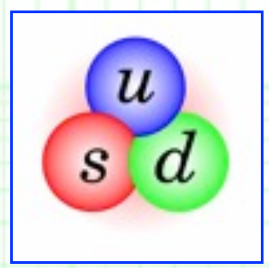
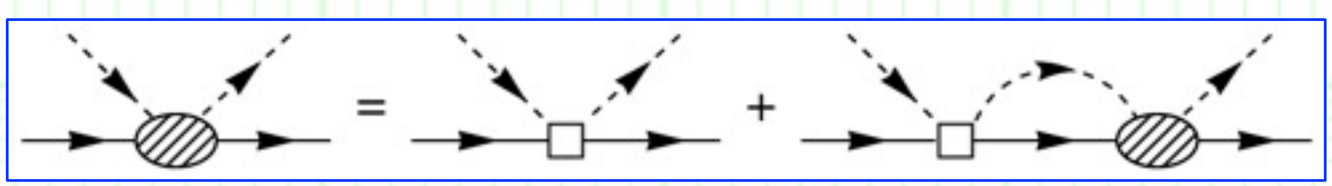




2. $N(1535)$ and $N(1650)$.

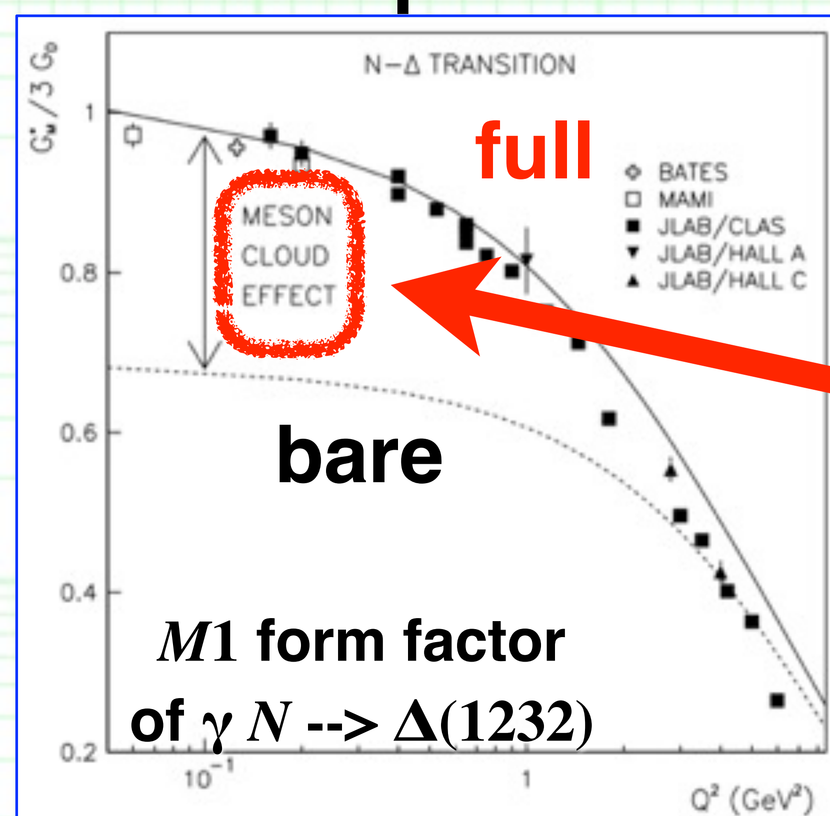
--- Expected to be usual qqq states, but can be described also in meson-baryon d.o.f.



Kaiser-Siegel-Weise ('95), Bruns-Mai-Meissner ('11), ...

3. $\Delta(1232)$.

--- Established as a member of the decuplet in the flavor $SU(3)$ symmetry, together with $\Sigma(1385)$, $\Xi(1530)$, and Ω, in the quark model, but ...



Meson cloud effect
~ 30 % !

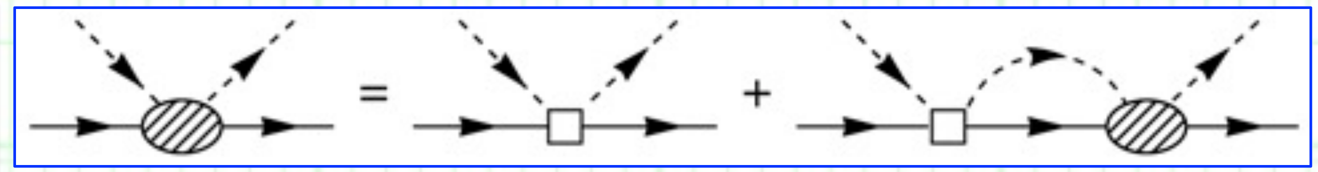
Sato and Lee ('09).

3. Applications

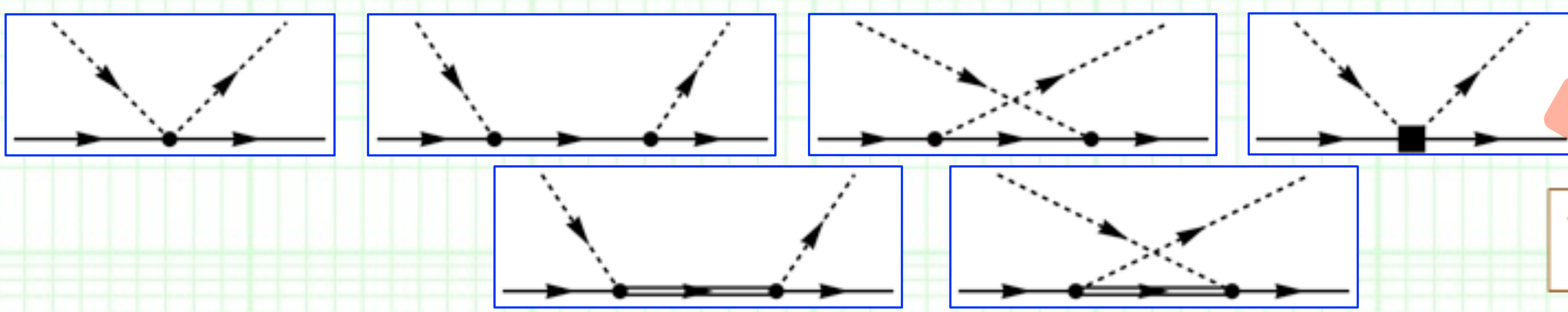
++ Chiral unitary approach ++

- We employ **chiral unitary approach for meson-baryon scatterings**.

$$T'_{\alpha L,jk}(s) = V'_{\alpha L,jk}(s) + \sum_{l} V'_{\alpha L,jl}(s) G_{L,l}(s) T'_{\alpha L,lk}(s)$$



- For the interaction kernel V we take LO + NLO (+ bare Δ) of chiral perturbation theory and project it to partial wave L and quantum number α to construct a separable interaction. --> V^{prime}.



$$V_{\alpha L,jk} = |\vec{q}\,|^{2L} \times V'_{\alpha L,jk}(s)$$

- The loop function G_L is obtained with the dispersion relation:

$$G_{L,j}(s) = \int_{s_{\mathrm{th},j}}^{\infty} \frac{ds'}{2\pi} \frac{\rho_j(s') q_j(s')^{2L}}{s' - s - i0} = i \int \frac{d^4 q}{(2\pi)^4} \frac{|\vec{q}\,|^{2L}}{[(P-q)^2 - m_j^2](q^2 - M_j^2)}$$

$\rho_j(s)$: phase space in channel j.

--- We need one subtraction for s wave / two subtractions for p wave which are fixed as discussed below.

3. Applications

++ Compositeness with separable interaction ++

- **For the separable interaction**, which we employ in this study, we can calculate the residue at the resonance pole as:

$$\langle \vec{q}\,' | \hat{T}(s) | \vec{q}\,\rangle \approx \langle \vec{q}\,' | \hat{V}(s_{\rm pole}) | \Psi \rangle \frac{1}{s - s_{\rm pole}} \langle \Psi^* | \hat{V}(s_{\rm pole}) | \vec{q}\,\rangle$$

$$\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \Psi^* | \hat{V} | \vec{q}\,\rangle = g \times |\vec{q}\,|^L$$

Aceti and Oset, *Phys. Rev.* D86 (2012) 014012;

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, arXiv:1511.01200.

  - For resonances in L wave, g is the coupling constant.
  - This form is necessary for the correct behavior of the wave function at small q region: $\tilde{\psi}(q) = \mathcal{O}(q^L) \quad \text{for small } q$

- As a result, the norm of the two-body wave function is written as

$$X_j = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \langle \Psi^* | \vec{q}_j \rangle \langle \vec{q}_j | \Psi \rangle = -g_j^2 \left[\frac{dG_{L,j}}{ds} \right]_{s = s_{\rm pole}}$$

--- G_L is the loop function in L wave.

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, arXiv:1511.01200.

<=> Elementariness Z with separable interaction:

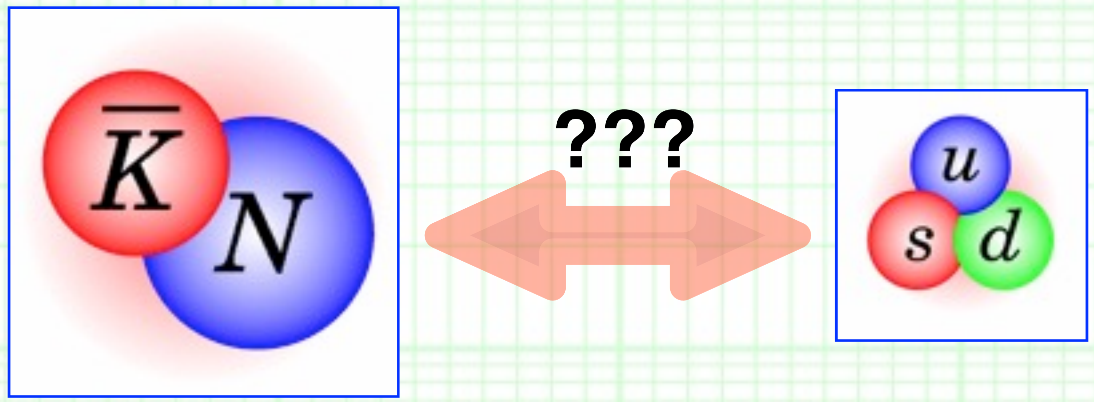
$$Z = -\sum_{j,k} g_k g_j \left[G_{L,j} \frac{dV'_{\alpha L,jk}}{ds} G_{L,k} \right]_{s = s_{\rm pole}}$$

3. Applications

++ Compositeness for Λ(1405) ++

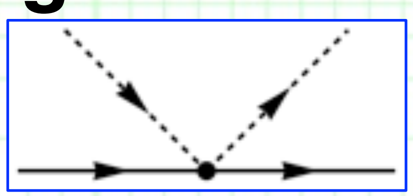
- Λ(1405) --- <u>The lightest excited baryon with $J^P = 1/2^-$, Why ??</u>
  - Strongly attractive $\bar{K}N$ interaction in the $I = 0$ channel.

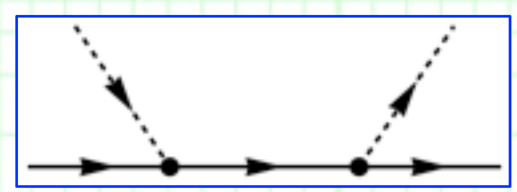
--> Λ(1405) is a $\bar{K}N$ quasi-bound state ??? Dalitz and Tuan ('60), ...

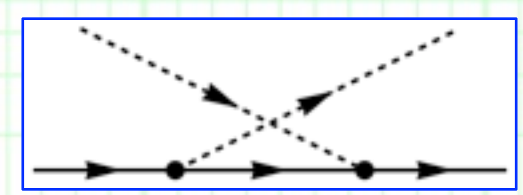


- We use the Ikeda-Hyodo-Weise amplitude for Λ(1405) in chiral unitary approach, which was <u>constrained by the recent data of the 1s shift and width of kaonic hydrogen</u>. Ikeda, Hyodo, and Weise ('11), ('12).

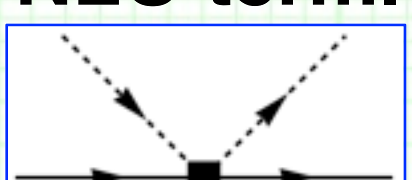
--- V: Weinberg-Tomozawa term + s- and u-channel Born term







+ NLO term.

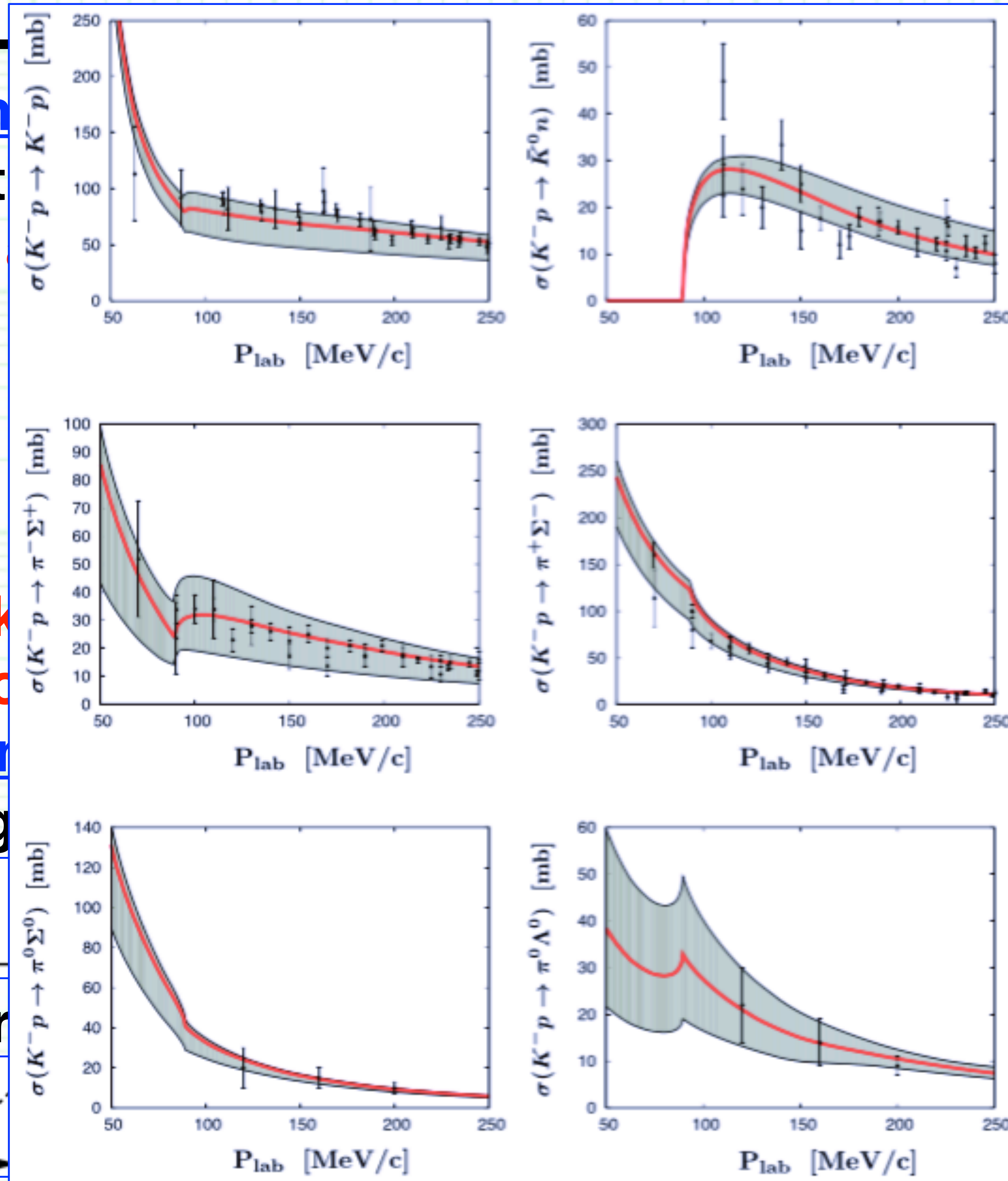


3. Applications

++ ++

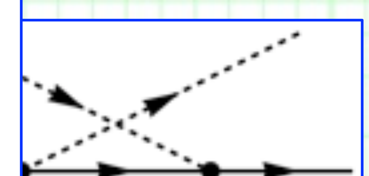
- **Λ(1405)** --- Th , Why ??
  - □ Strongly att nel.

--> Λ(1405) is Yuan ('60), ...

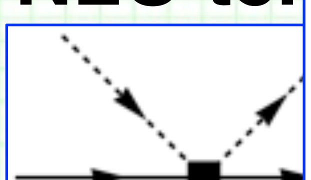


- We use the Ik 5) in chiral unitary appro cent data of the 1s shift ar , and Weise ('11), ('12).

--- V: Weinberg orn term



+ NLO ter

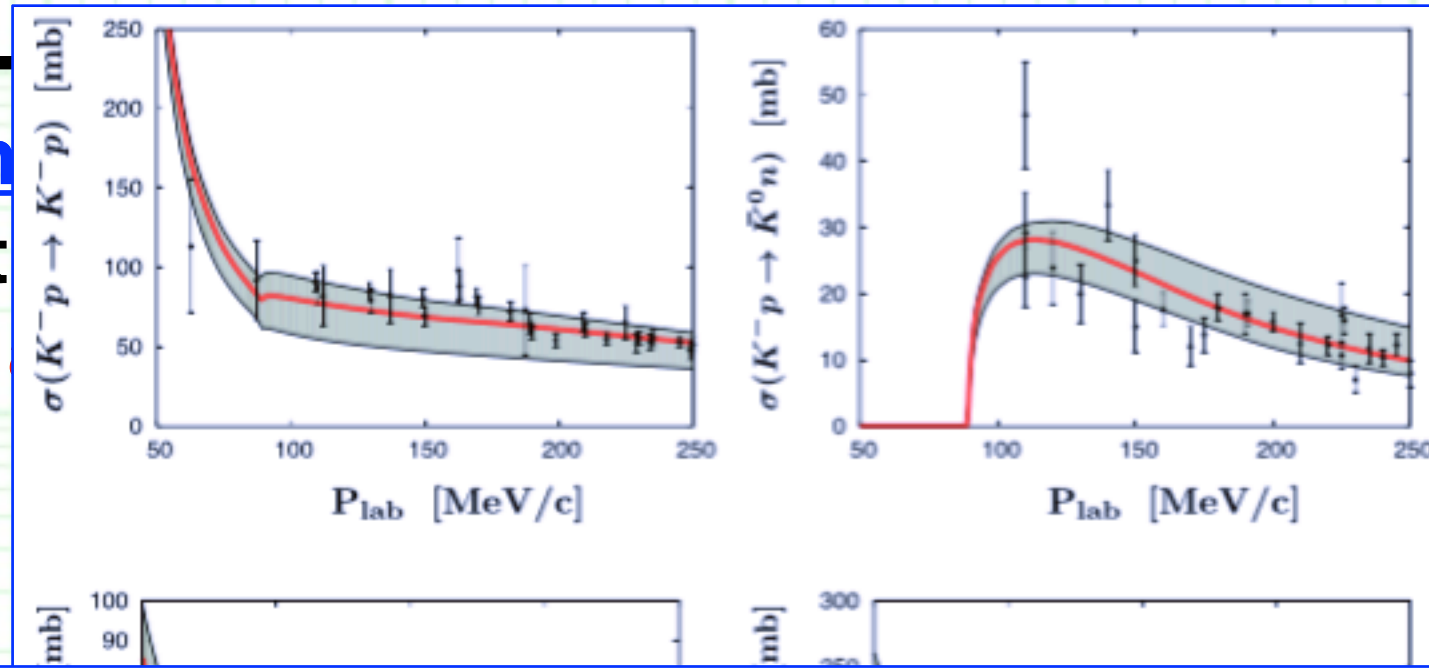


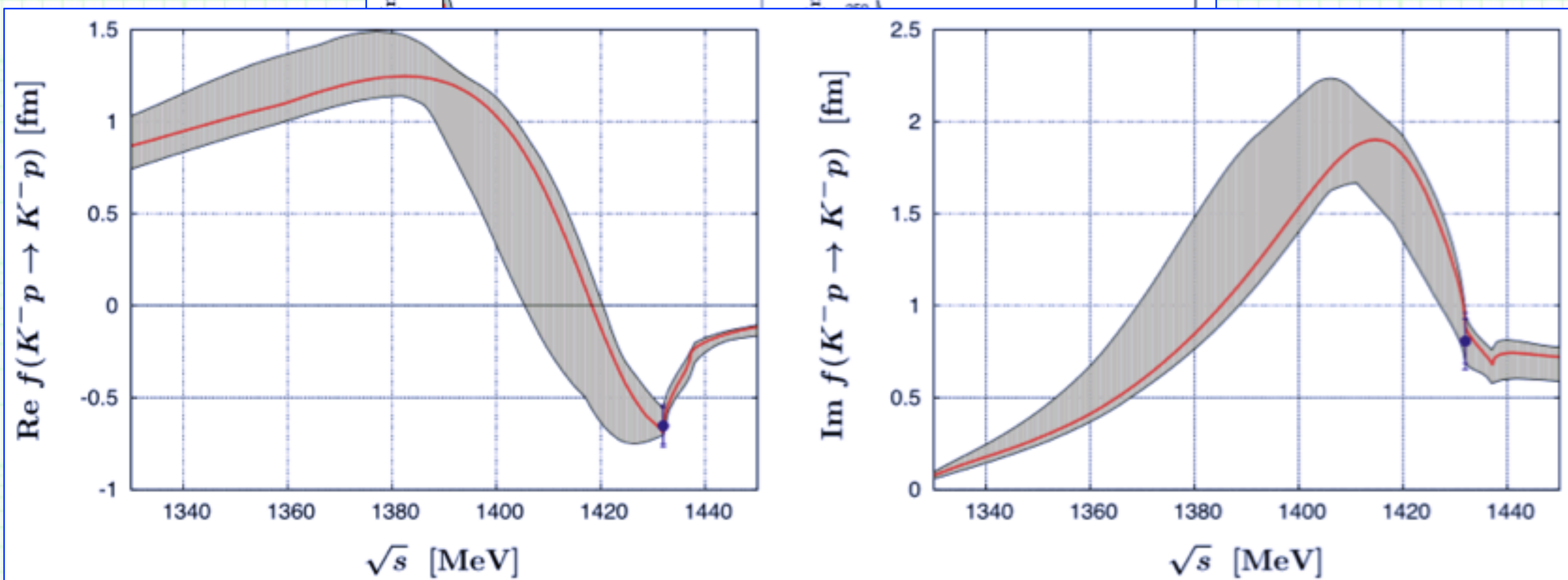
3. Applications

++ ... ++

- Λ(1405) --- Th... , Why ??
  - Strongly att... nel.

--> Λ(1405) is ... uan ('60), ...





3. Applications

++ Compositeness for Λ(1405) ++

- Compositeness X and elementariness Z for hadrons in the model.

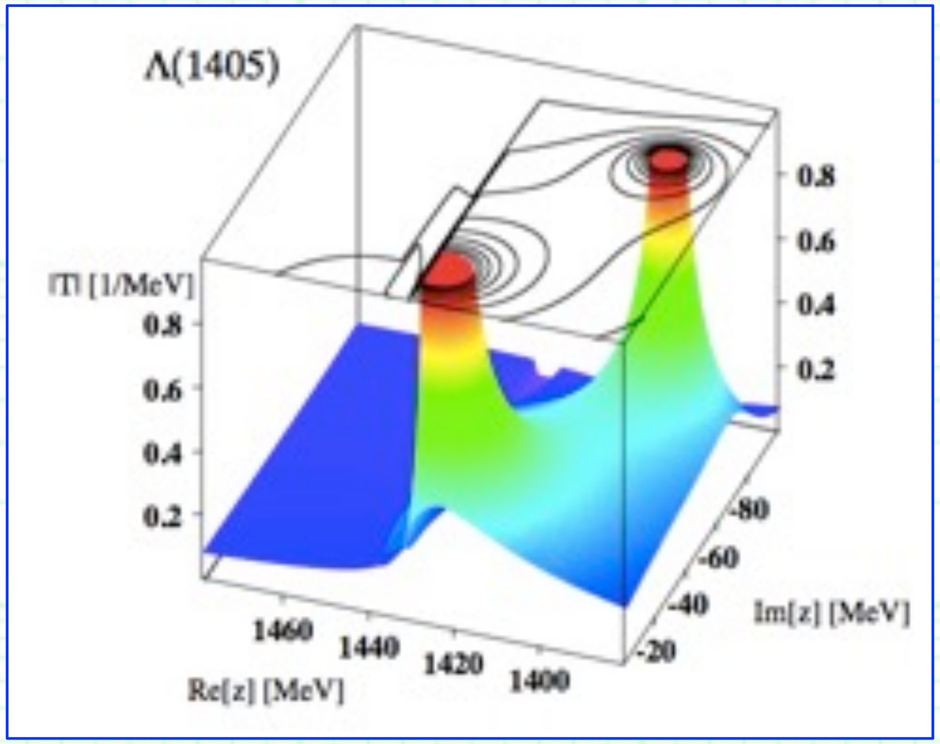
T. S. , Hyodo and Jido, *PTEP* 2015, 063D04.

$$X_j = -g_j^2 \left[\frac{dG_j}{ds} \right]_{s = s_{\rm pole}}$$

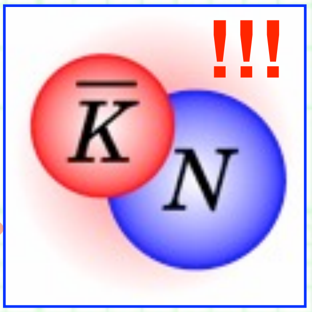
$$Z = - \sum_{j,k} g_k g_j \left[G_j \frac{dV_{jk}}{ds} G_k \right]_{s = s_{\rm pole}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_j X_j + Z = 1$$

- □ Λ(1405) (two poles!).



Hyodo and Jido ('12).



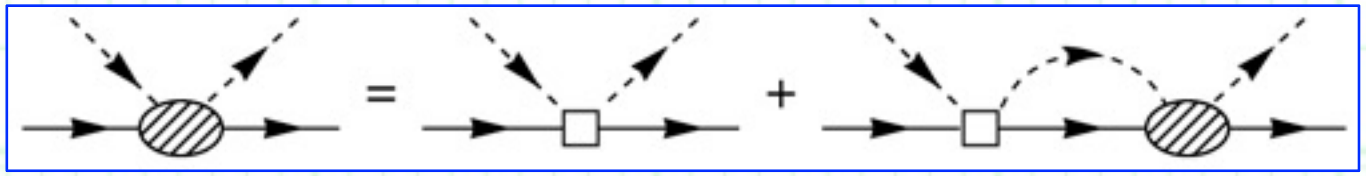
	Λ(1405), higher pole	Λ(1405), lower pole
$\sqrt{s_{\rm pole}}$	$1424 - 26i$ MeV	$1381 - 81i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$	$-0.39 - 0.07i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$	$0.66 + 0.52i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$	$-0.04 + 0.01i$
$X_{K\Xi}$	$0.00 + 0.00i$	$-0.00 + 0.00i$
Z	$-0.08 + 0.19i$	$0.77 - 0.46i$

--- Large $\bar{K}N$ component for (higher) Λ(1405), since $X_{\bar{K}N}$ is almost unity with small imaginary parts.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

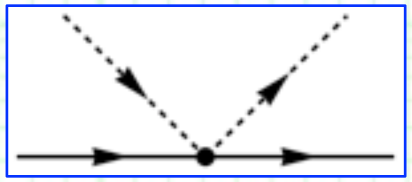
- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^-$.
  - We naively expect that they are conventional qqq states, but there are several studies that they can be dynamically generated from the meson-baryon degrees of freedom without explicit resonance poles, especially in the chiral unitary approach.



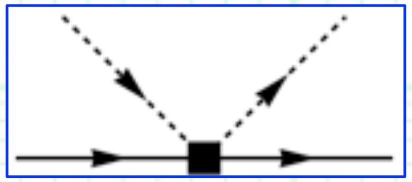
Kaiser-Siegel-Weise ('95); Nieves-Ruiz Ariola ('01); Inoue-Oset-Vicente Vacas ('02); Bruns-Mai-Meissner ('11); ...

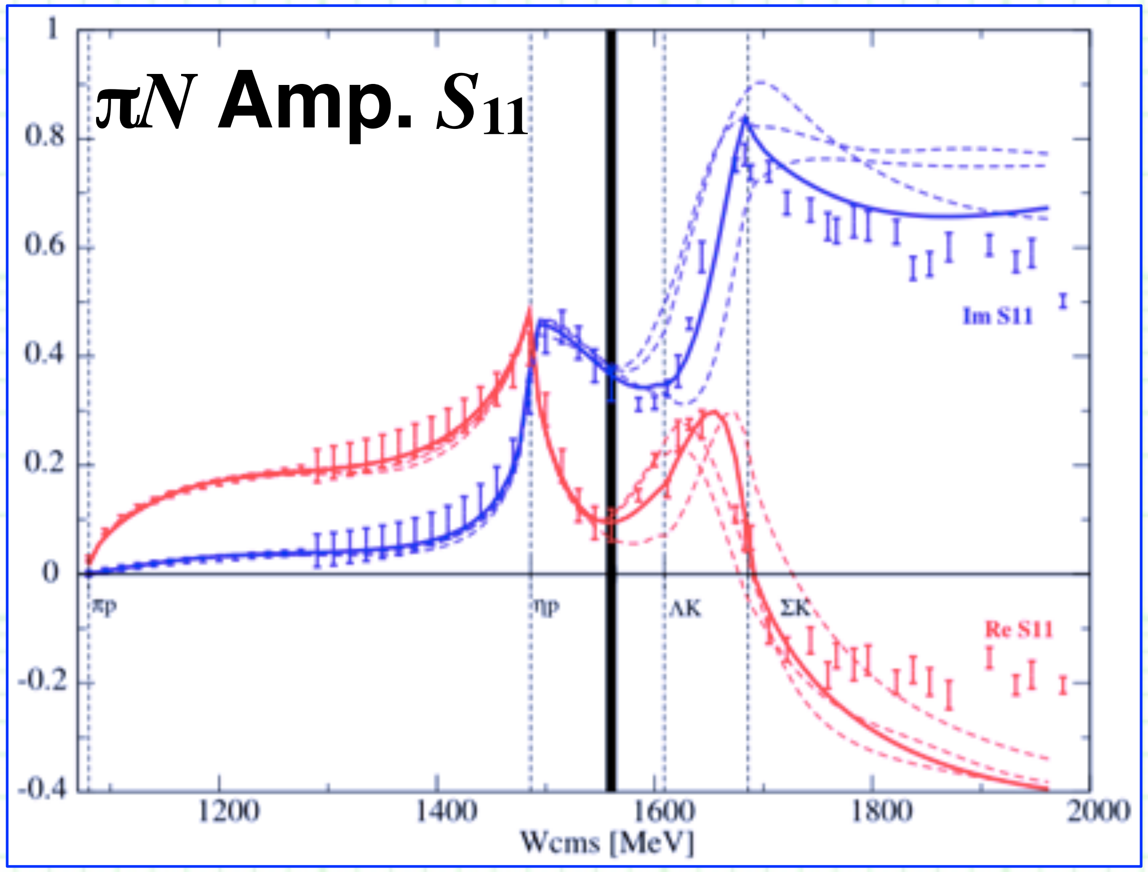
  - For example:

--- V: Weinberg-Tomozawa term



+ NLO term.





Bruns, Mai and Meissner, *Phys. Lett.* B 697 (2011) 254.

3. Applications

++ Compositeness for *N*(1535) and *N*(1650) ++

- *N*(1535) and *N*(1650) --- Nucleon resonances with $J^P = 1/2^-$.
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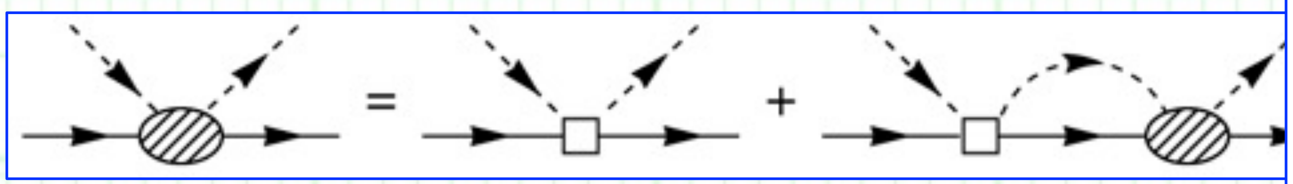
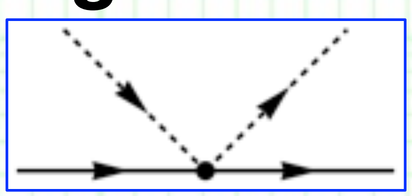
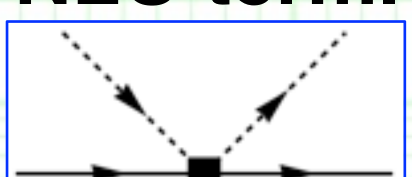


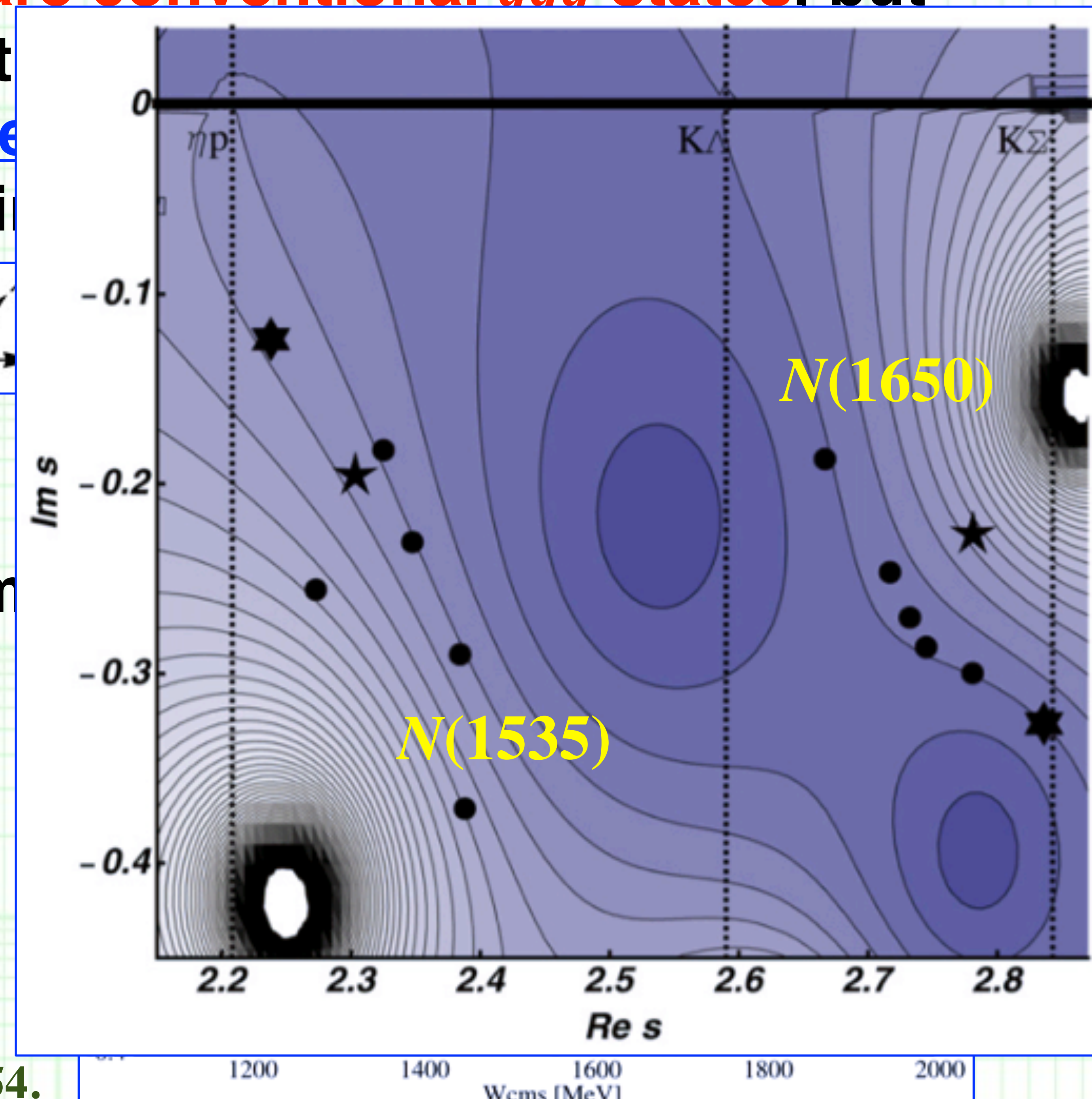
  - For example:

--- *V*: Weinberg-Tomozawa term



+ NLO term.





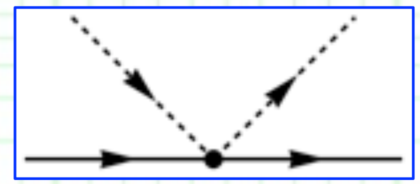
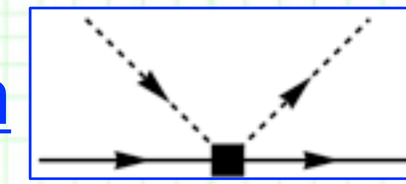
Bruns, Mai and Meissner,
Phys. Lett. B 697 (2011) 254.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

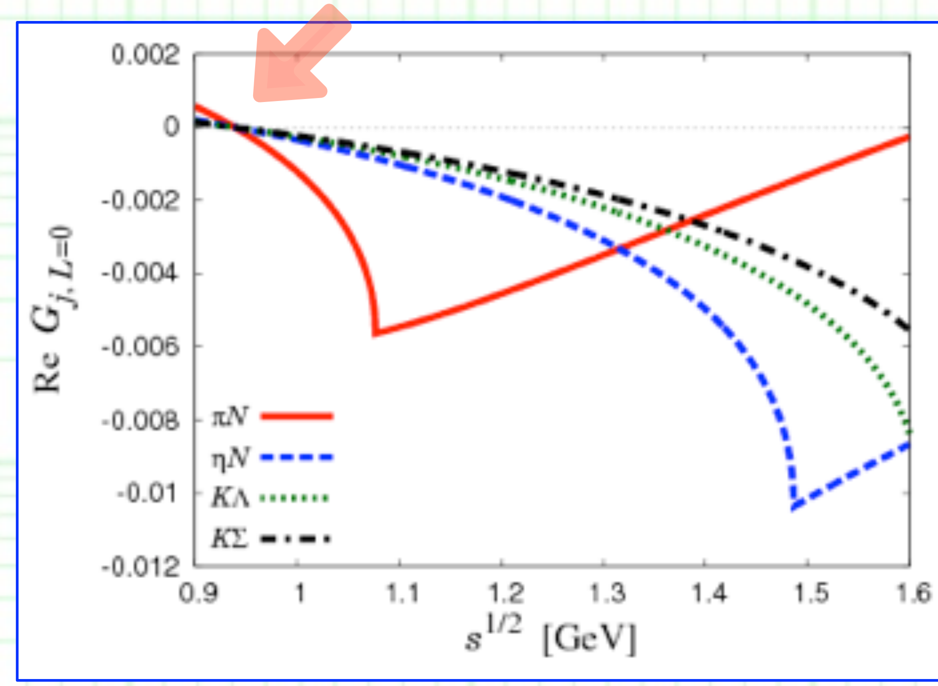
- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^-$.

- We construct our own s-wave πN-ηN-$K\Lambda$-$K\Sigma$ scattering amplitude in the chiral unitary approach.

  - V: Weinberg-Tomozawa term  + NLO term  .
  - G: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in G.

$$G_{j,L=0}(s = M_N^2) = 0$$

Hyodo, Jido and Hosaka, *Phys. Rev.* C78 (2008) 025203.



  - Parameters: The low-energy constants in NLO term.

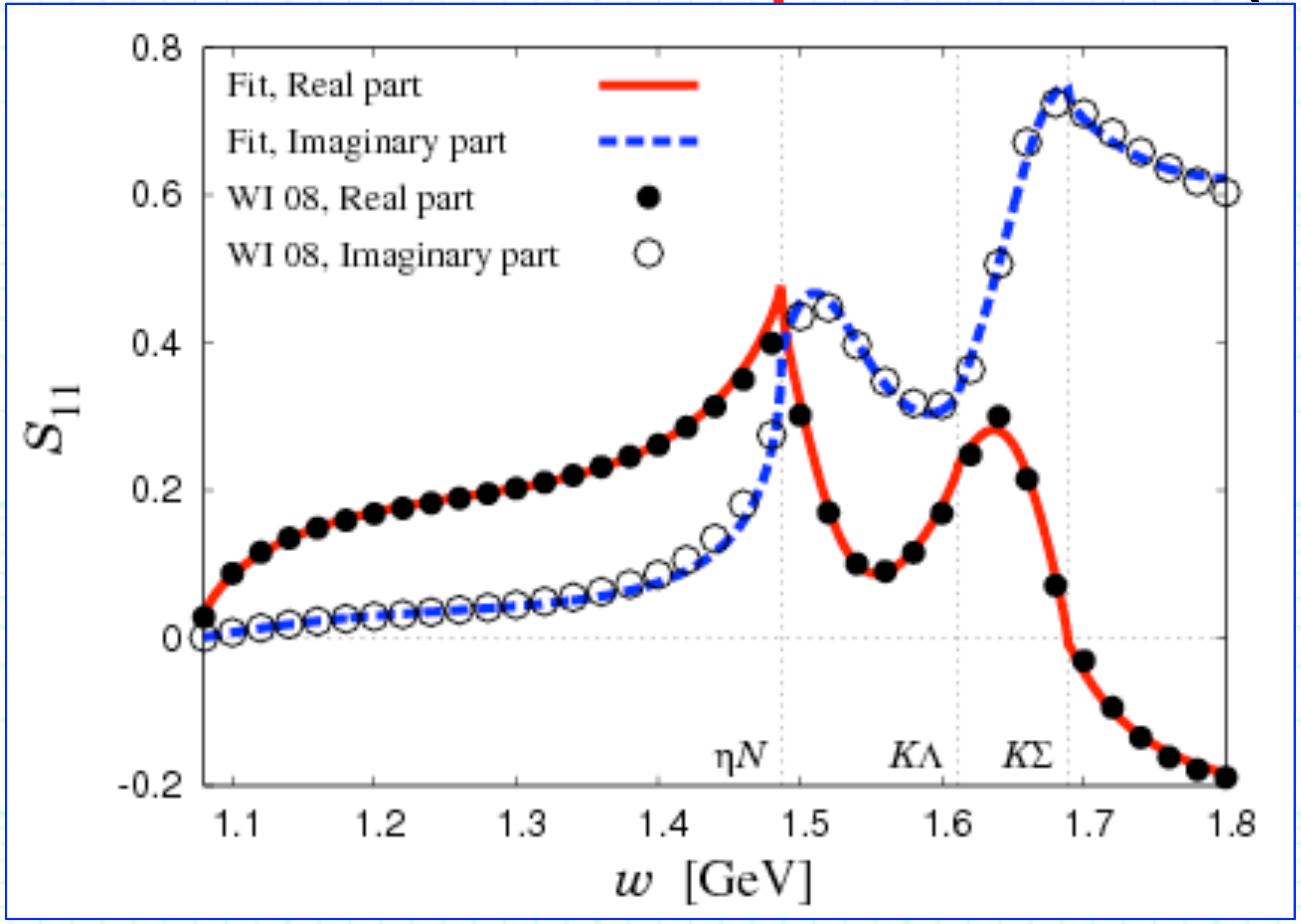
--> Parameters are fixed so as to reproduce the πN scattering amplitude S_{11} as a PWA solution "WI 08" up to $\sqrt{s} = 1.8$ GeV.

Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- **Fitted to the πN amplitude** WI 08 (S_{11}).



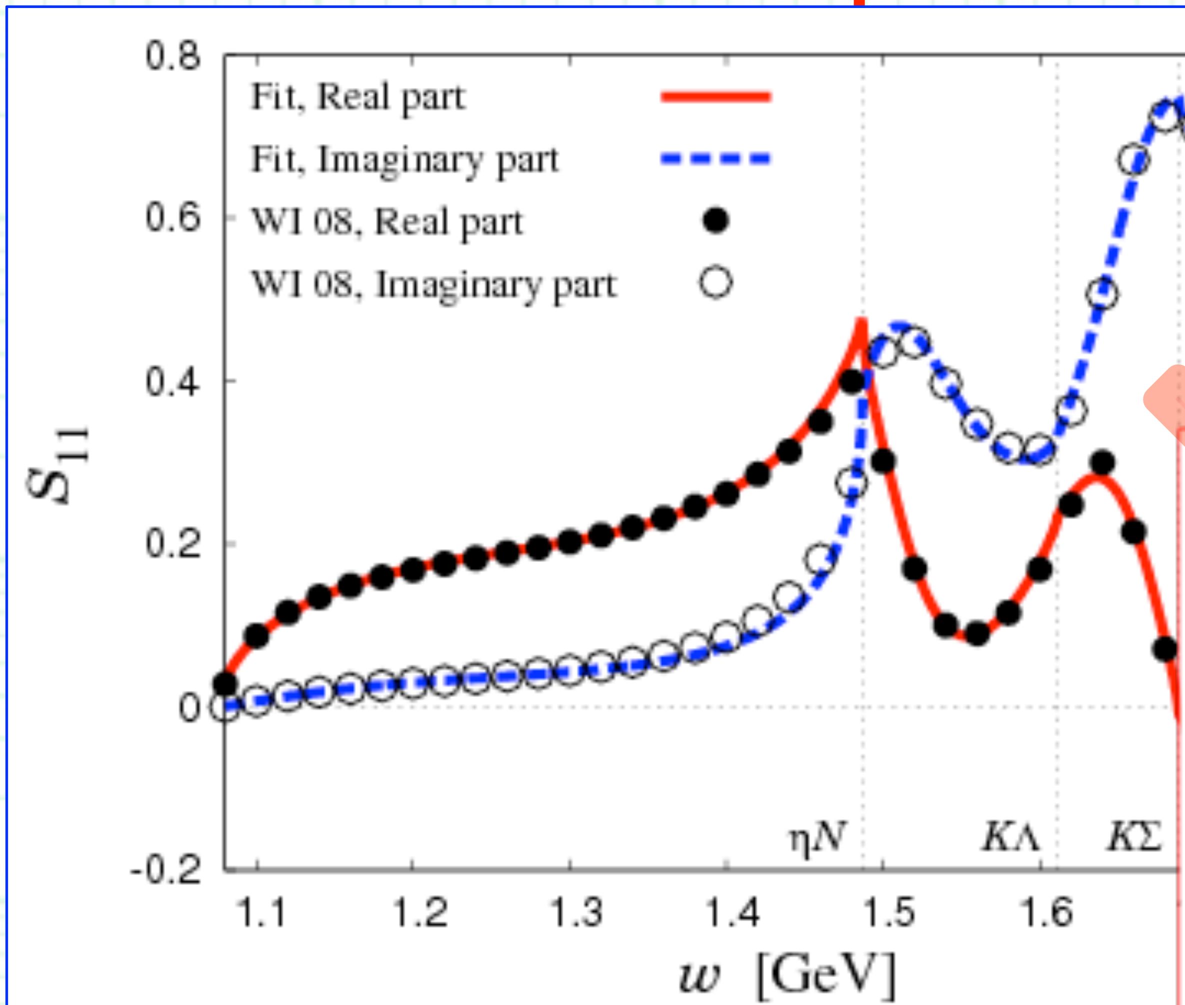
--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6.$

- **Chiral unitary approach reproduces the amplitude of PWA very well.**

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- Fitted to the πN amplitude WI 08 (S_{11}).



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6$.

- Chiral unitary approach reproduces the amplitude of PWA very well.

	$N(1535)$	$N(1650)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1496.4 - 58.7i$	$1660.7 - 70.0i$
$X_{\pi N}$	$-0.02 + 0.03i$	$0.00 + 0.04i$
$X_{\eta N}$	$0.04 + 0.37i$	$0.00 + 0.01i$
$X_{K\Lambda}$	$0.14 + 0.00i$	$0.08 + 0.05i$
$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$

- The pole positions of both $N(1535)$ and $N(1650)$ are consistent with the PDG value.

N(1535) 1/2⁻ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

Breit-Wigner mass = 1525 to 1545 (≈ 1535) MeV
Breit-Wigner full width = 125 to 175 (≈ 150) MeV
Re(pole position) = 1490 to 1530 (≈ 1510) MeV
−2Im(pole position) = 90 to 250 (≈ 170) MeV

N(1650) 1/2⁻ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

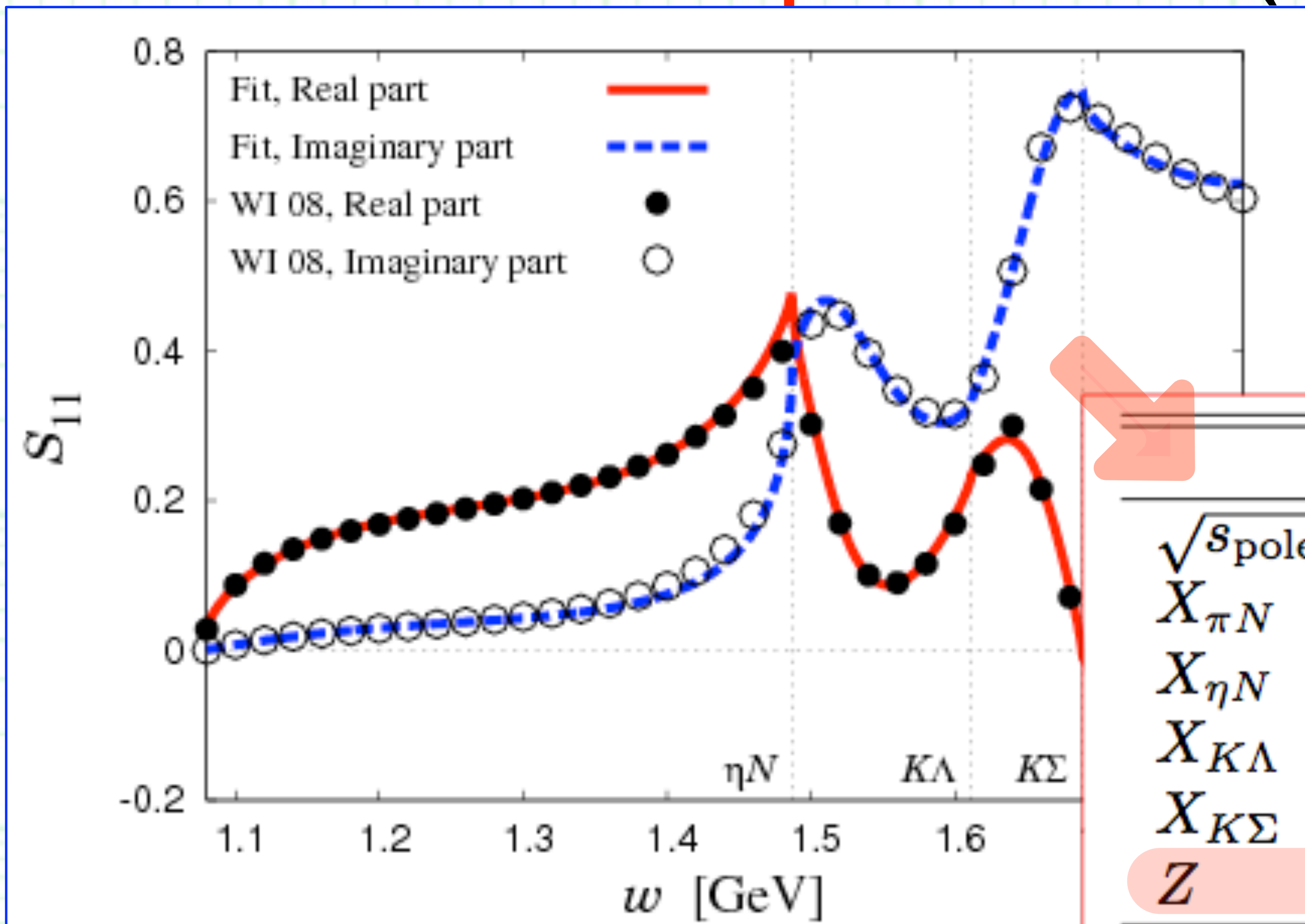
Breit-Wigner mass = 1645 to 1670 (≈ 1655) MeV
Breit-Wigner full width = 110 to 170 (≈ 140) MeV
Re(pole position) = 1640 to 1670 (≈ 1655) MeV
−2Im(pole position) = 100 to 170 (≈ 135) MeV

Particle Data Group.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- **Fitted to the πN amplitude** WI 08 (S_{11}).



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6$.

□ Chiral unitary approach <u>reproduces the amplitude of PWA very well</u>.

	$N(1535)$	$N(1650)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1496.4 - 58.7i$	$1660.7 - 70.0i$
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$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$

□ For both N^* resonances, <u>the elementariness Z is dominant</u>.

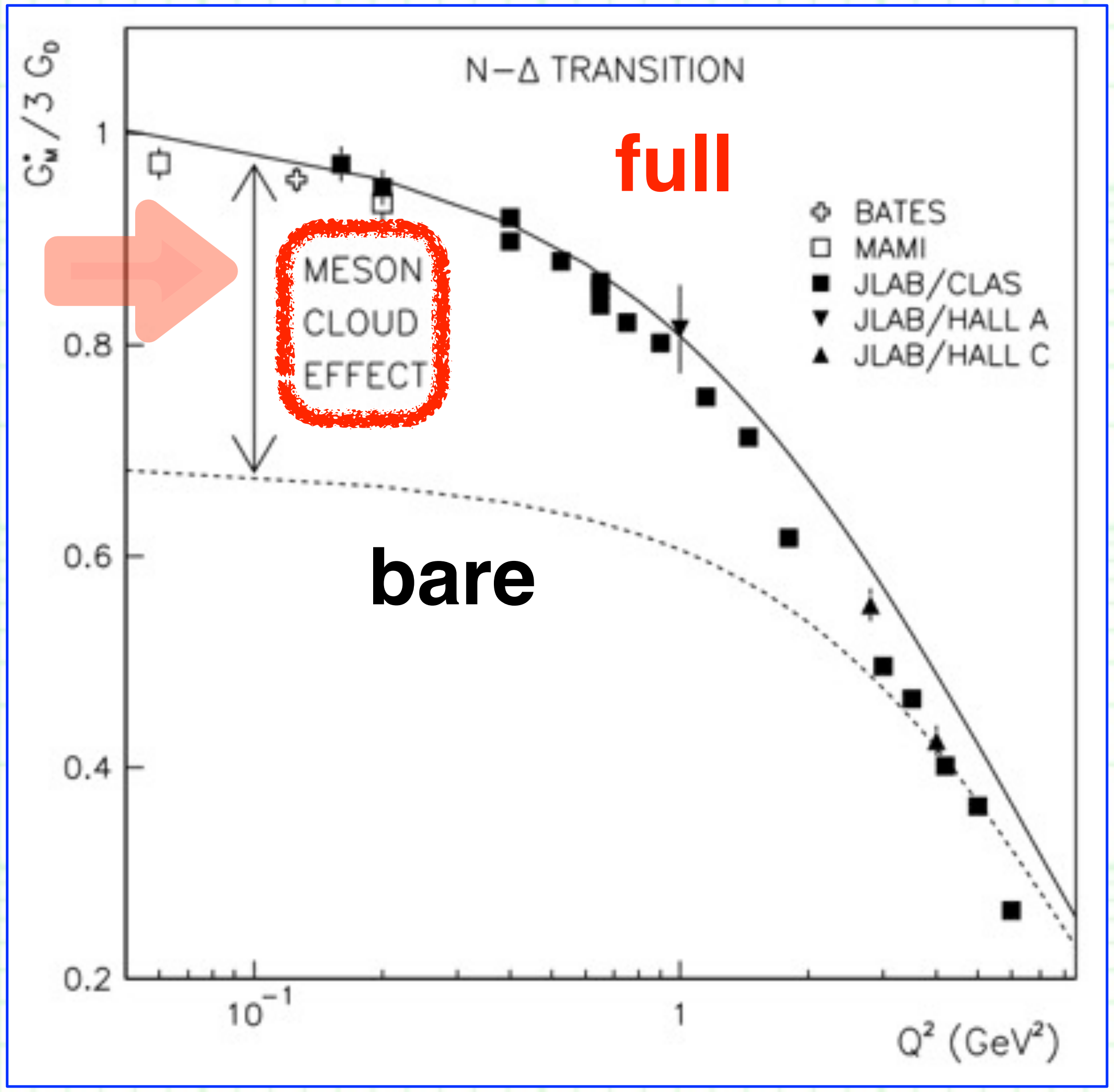
--> **$N(1535)$ and $N(1650)$ have large components** originating from contributions **other than πN, ηN, $K\Lambda$, and $K\Sigma$**. The missing channels should be <u>encoded in the energy dep. of V and LEC</u>.

3. Applications

++ Compositeness for Δ(1232) ++

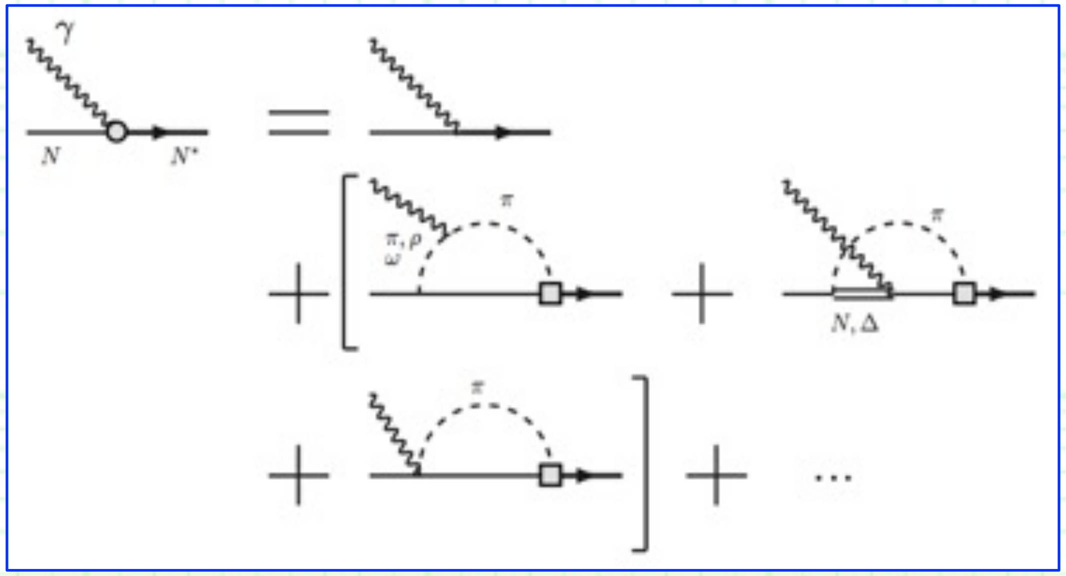
- **Δ(1232) --- The excellent successes of the quark model strongly indicate that Δ(1232) is described as genuine *qqq* states very well.**

- **However, effect of the meson-nucleon cloud for Δ(1232) seems to be "large".**



□ **The magnetic $M1$ form factor of γN --> Δ(1232) shows that the meson cloud effect brings ~ 30 % of the form factor at $Q^2 = 0$.**

Sato and Lee, *J. Phys.* G36 (2009) 073001.

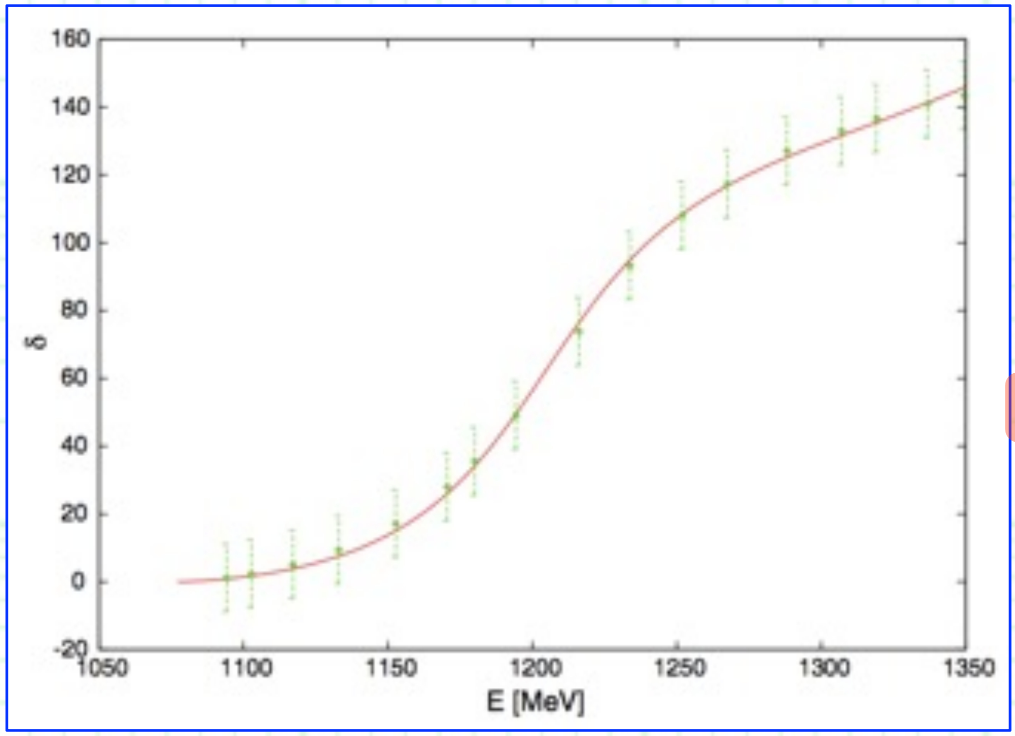


3. Applications

++ Compositeness for $\Delta(1232)$ ++

- $\Delta(1232)$ --- The excellent successes of the quark model strongly indicate that $\Delta(1232)$ is described as genuine qqq states very well.

- However, effect of the meson-nucleon cloud for $\Delta(1232)$ seems to be "large".



□ The πN compositeness for $\Delta(1232)$ is evaluated in a very simple model.

Aceti *et al.*, *Eur. Phys. J.* A50 (2014) 57.

$$-\tilde{g}_{\Delta}^{2}\left[\frac{\mathrm{d}G^{II}(s)}{\mathrm{d}\sqrt{s}}\right]_{\sqrt{s}=\sqrt{s_0}} = (0.62 - i0.41),$$

□ Large real part of the πN compositeness, but imaginary part is non-negligible.

--- The result implies large πN contribution to, *e.g.*, the transition form factor.

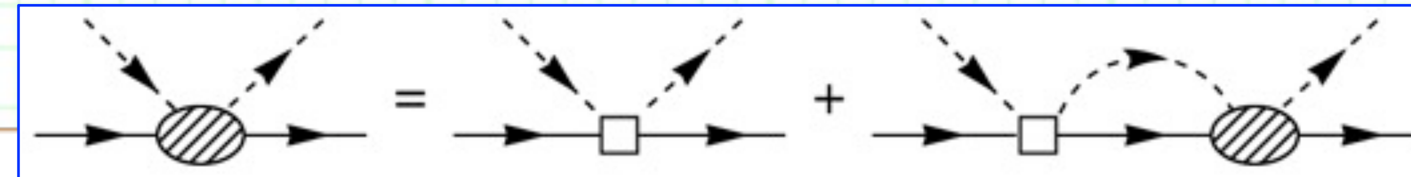
- However, this result was obtained in a very simple model.

--> Need a more refined model !

3. Applications

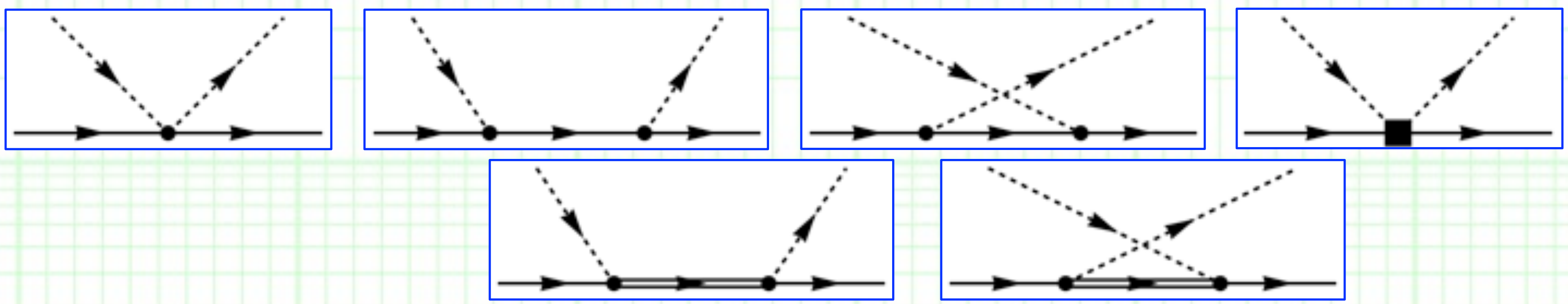
++ Compositeness for Δ(1232) ++

- We **construct our own πN elastic scattering amplitude** in the chiral unitary approach.



$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$

□ V:



--- We include an explicit Δ(1232) pole term.

□ G: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in G.

$$G_{j,L}(s = M_N^2) = 0$$

Hyodo, Jido and Hosaka, *Phys. Rev.* C78 (2008) 025203.

--- This makes the physical N(940) mass in the full Amp. unchanged.

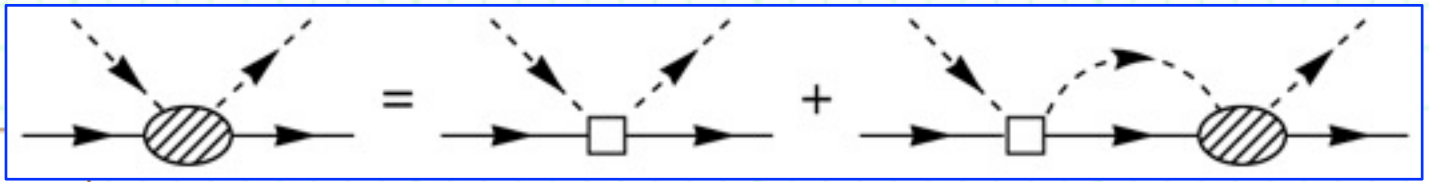
--- In addition, we constrain G so as to exclude unphysical bare-state contributions to N(940):

$$\frac{dG_L}{ds}(s = M_N^2) \le 0$$

3. Applications

++ Compositeness for $\Delta(1232)$ ++

- We construct our own πN elastic scattering amplitude in the chiral unitary approach.



$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$

- We have model parameters of: LECs, bare Δ mass and coupling constant to πN, and a subtraction const.

--> Fitted to six πN scattering amplitudes ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$) obtained as a PWA solution "WI 08" up to √s = 1.35 GeV.

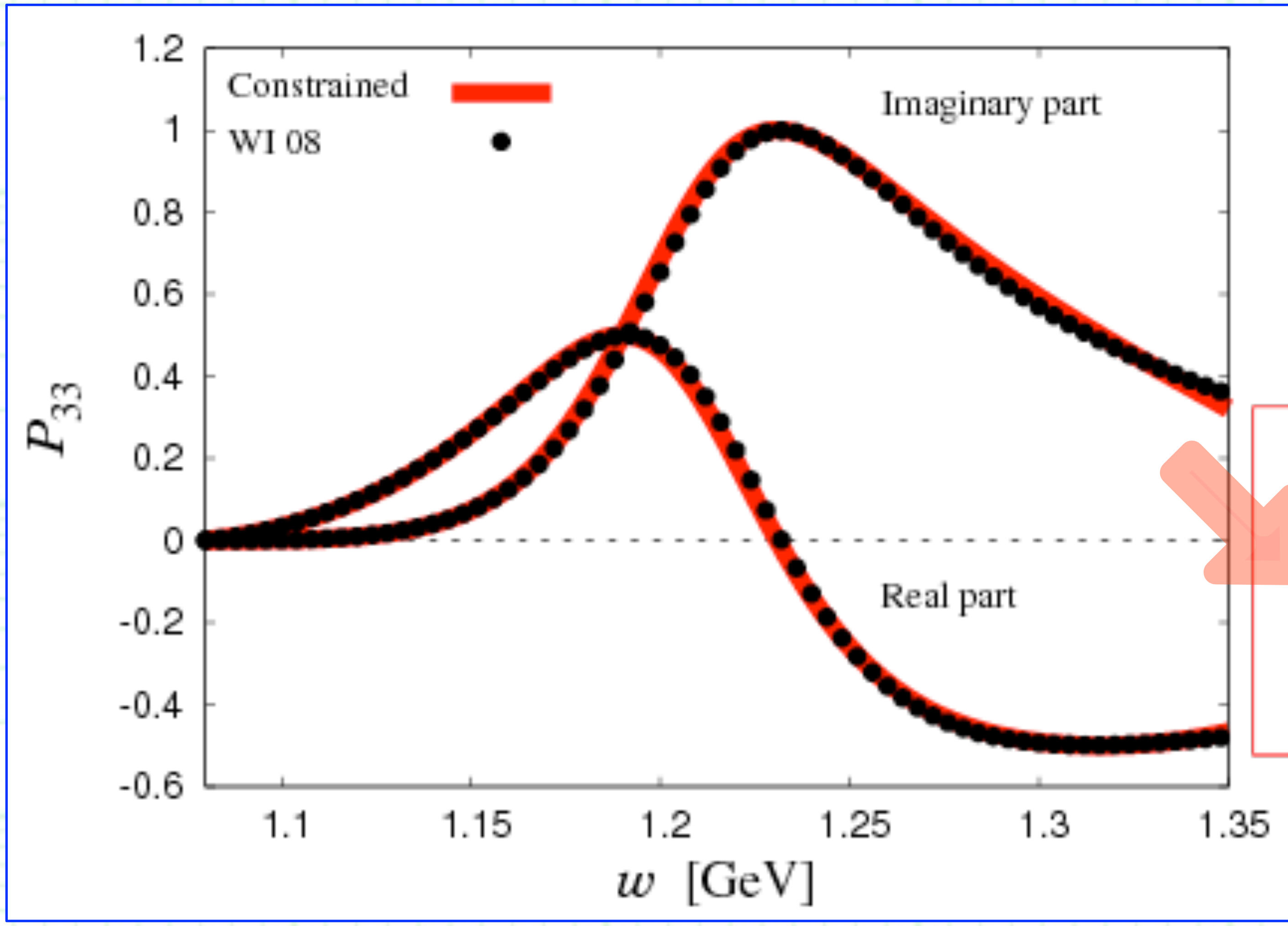
Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

- The P_{11} and P_{33} amplitude contain poles corresponding to the physical $N(940)$ and $\Delta(1232)$, respectively:

3. Applications

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).**



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 1240 / 809 \approx 1.5$.

□ **Chiral unitary approach reproduces the amplitude of PWA well.**

Constrained	$\Delta(1232)$	$N(940)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1206.9 - 49.6i$	938.9
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

□ **For $\Delta(1232)$, its pole position is very similar to the PDG value.**

Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV
−2Im(pole position) = 98 to 102 ($\approx$ 100) MeV

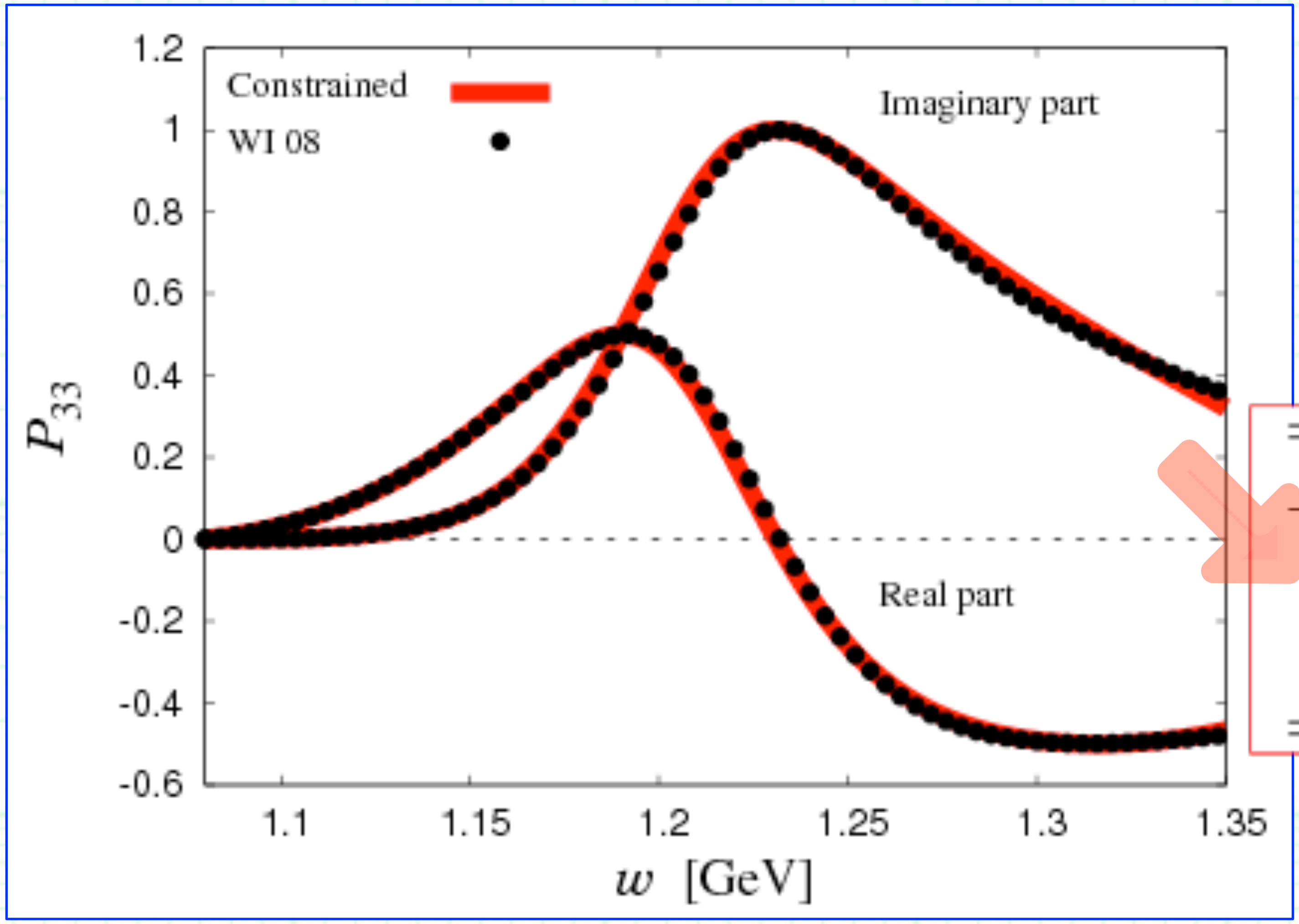
□ **The πN compositeness $X_{\pi N}$ takes large real part ! But non-negligible imaginary part as well.**

--> Our refined model reconfirms the result in the previous study.

3. Applications

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude** WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).



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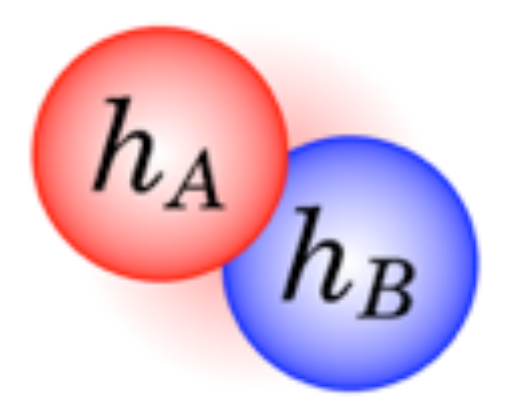
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

- For $N(940)$, $X_{\pi N}$ is non-negative and zero.

--> Implies that $N(940)$ is not described by the πN molecular picture.

4. Summary

- **Hadronic molecules are unique**, because they are composed of color singlet states, which can be observed as asymptotic states.
  - We can use quantum mechanics in a usual manner.
  - In particular, we can investigate their structure of composites by the two-body wave functions and their norms = compositeness.



- The two-body wave functions can be extracted from the hadron-hadron scattering amplitude, although they are model dependent.

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}}$$

$$\gamma(q) \equiv \langle \vec{q} | \hat{V} | \Psi \rangle = [E_{\rm pole} - E(q)] \tilde{\psi}(q)$$

--- The residue at the pole position contains information on the two-body wave function, which is automatically normalized.

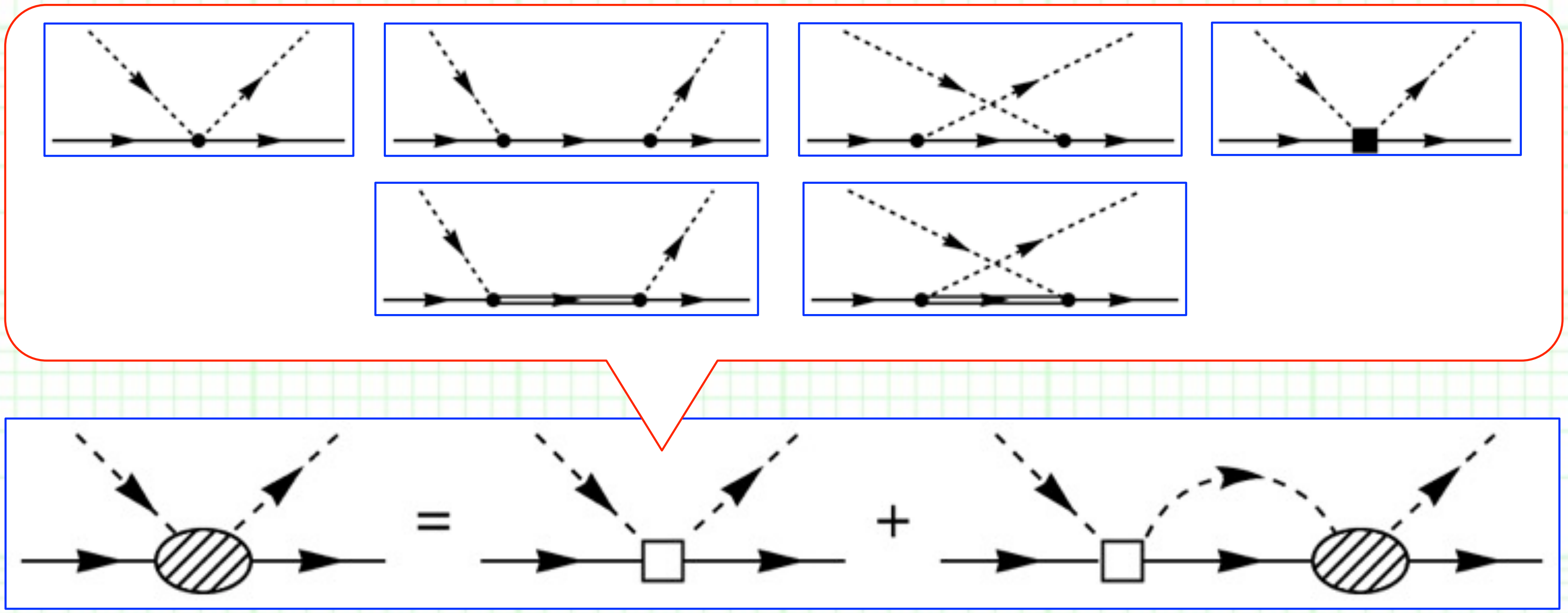
$$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\rm pole} - E(q)} \right]^2 = 1$$

<-- If the state is purely molecule.

--> Comparing the norm = compositeness with unity, we may able to conclude the structure of hadronic molecule candidates.

4. Summary

- We apply this scheme to $\Lambda(1405)$, $N(1535)$, $N(1650)$, and $\Delta(1232)$ in an effective model, chiral unitary approach, with a separable interaction of LO + NLO (+ bare Δ) taken from chiral perturbation theory.



--- In this model, we find that ...

- $\Lambda(1405)$ (higher pole) is indeed a $\bar{K}N$ molecule.
- $N(1535)$ and $N(1650)$ have small πN, ηN, $K\Lambda$ and $K\Sigma$ components.
- $\Delta(1232)$ has a non-negligible πN component.

Thank you very much for your kind attention !