Effects of ΛNN three-body force in B_Λ values of hypernuclei

Masahiro ISAKA (RIKEN)

Collaborators: Y. Yamamoto, Th.A. Rijken

Grand challenges of hypernuclear physics

Interaction: To understand baryon-baryon interaction

- **2 body interaction between baryons (nucleon, hyperon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

A major issue in hypernuclear physics

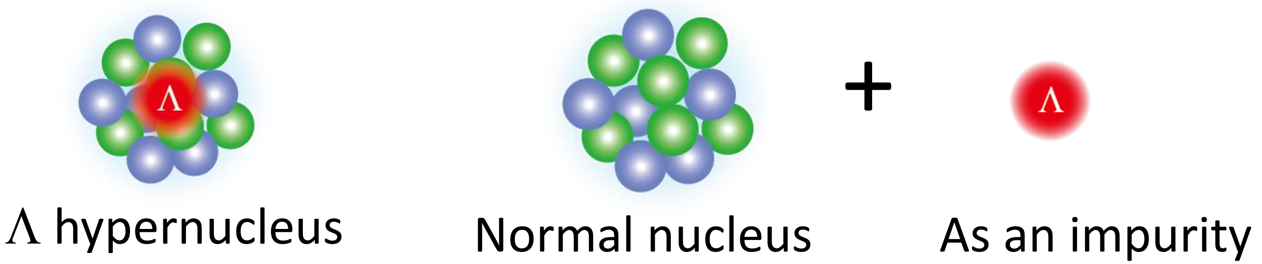
Structure: To understand many-body system of nucleons and hyperon

- **Addition of hyperon(s) shows us new features of nuclear structure**

Ex.) Structure change by hyperon(s)

- No Pauli exclusion between N and Y
- YN interaction is different from NN

"Hyperon as an impurity in nuclei"

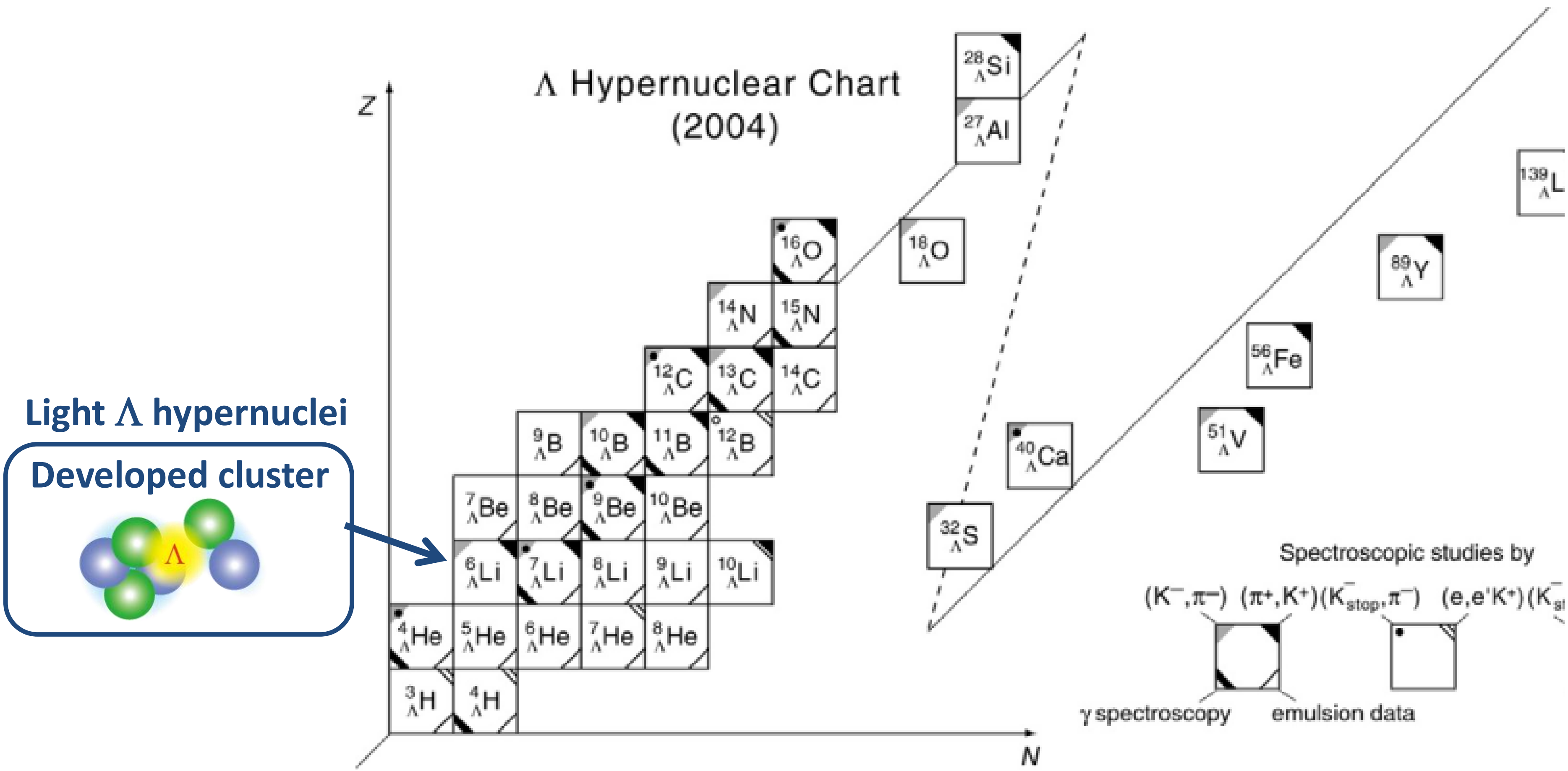


Today's talk: "structure of Λ hypernuclei" and "Λ binding energy"

Structure of Λ hypernuclei

Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei**
- **Most of them have well pronounced cluster structure**



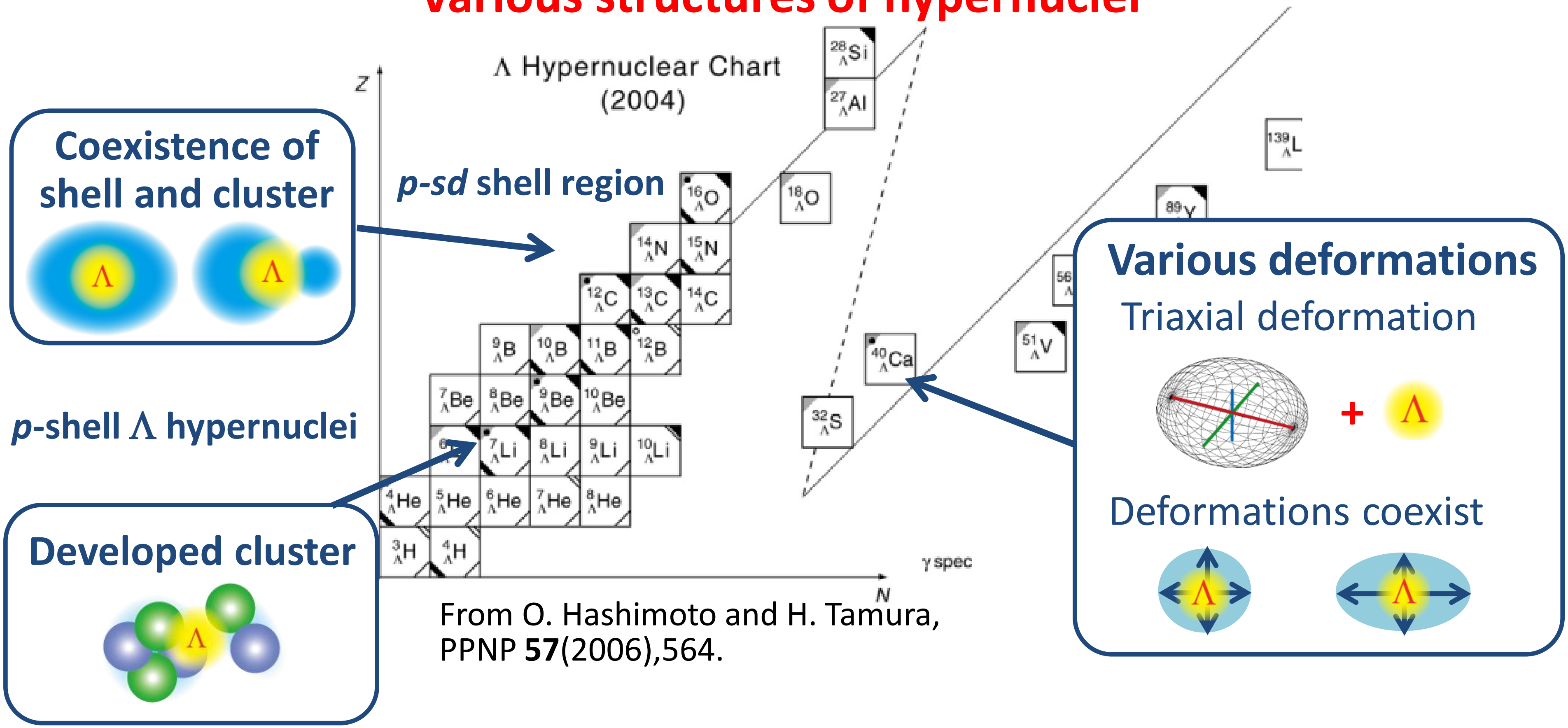
Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Toward heavier and exotic Λ hypernuclei

<u>Experiments at J-PARC, JLab and Mainz etc.</u>

- **Hypernuclear chart will be extended to heavier regions**

"Various structures of hypernuclei"



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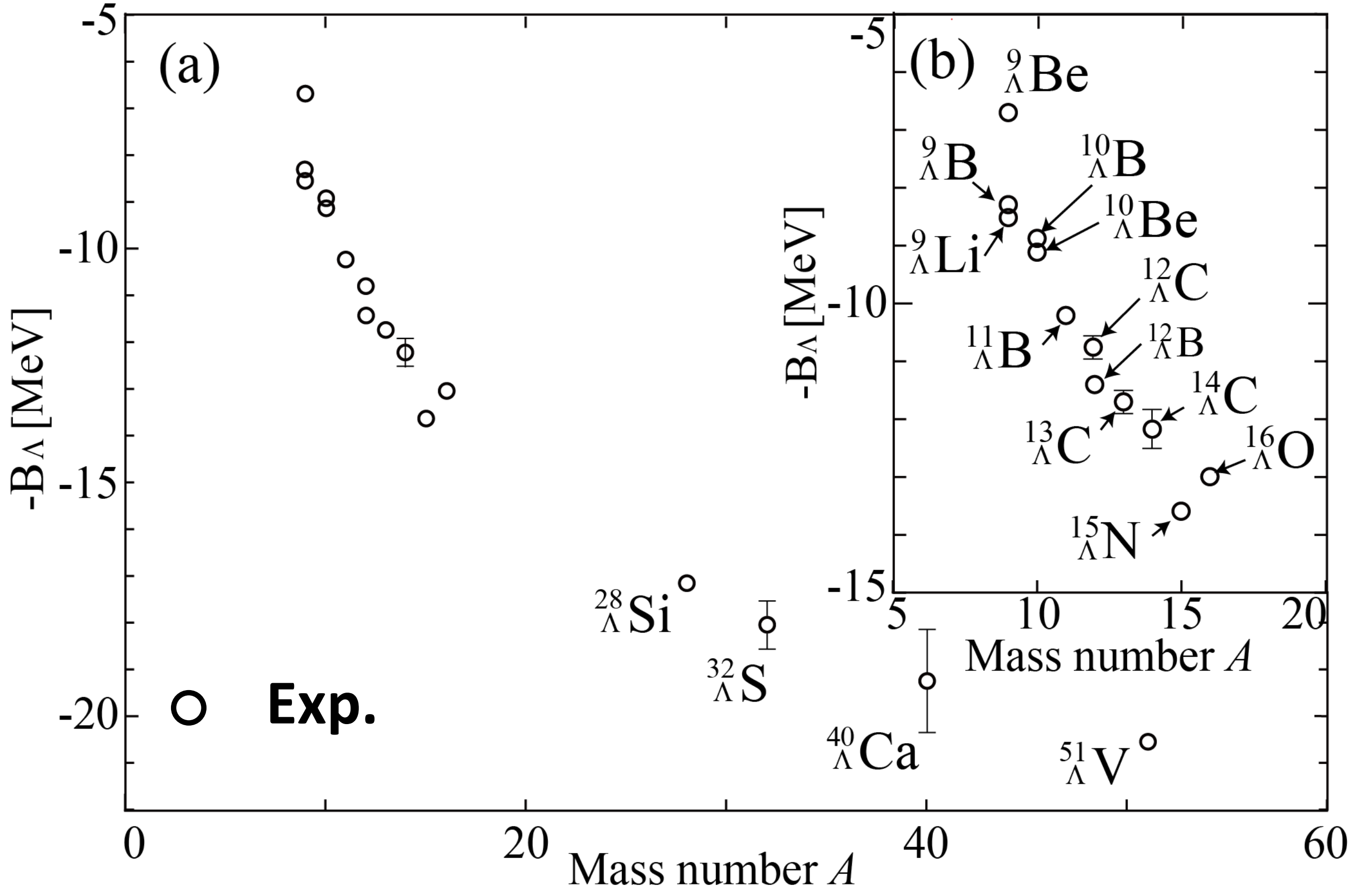
How do core nuclei affect the mass dependence of B_{Λ}?

"clustering/deformations", "Density dependence of interactions"

B_Λ as a function of mass number A

Observed data of Λ binding energy B_Λ ($9 \leq A \leq 51$)



Do core nuclei affect the mass dependence of B_Λ?

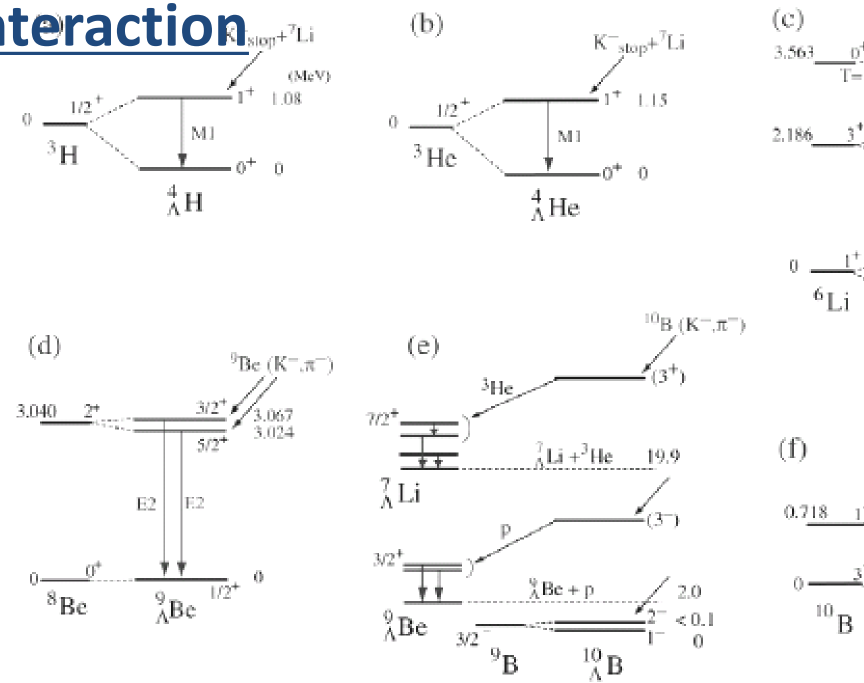
"clustering/deformations", "Density dependence of interactions"

Bertini *et al.*, NPA**83**,306(1979), Davis, Juric , *et al.*, NPB**52**(1973), Davis, NPA**547**,369(1992);NPA**754**,3c(2005), Ajimura *et. al.*, NPA**639**(1998)93c, Pile *et al.*, PRL**66**,2585(1991), Hotchi *et al.*, PRC**64**, 044302(2001), Hashimoto and Tamura, PPNP**57**,564(2006), Tang, *et. al.*, PRC**90**,034320(2014).

Background of our research

Knowledge of ΛN two-body effective interaction

- **Study of light (*s*, *p*-shell) Λ hypernuclei**
  - Accurate solution of few-body problems [1]
  - ΛN G-matrix effective interactions [2]
  - Increases of experimental information [3]



Development of theoretical models

- **Through the study of unstable nuclei**

  Ex.: Antisymmetrized Molecular Dynamics (AMD)[4]
  - AMD can describe dynamical changes of various structure
  - No assumption on clustering and deformation

AMD calculation by using YNG-ESC

[1] E. Hiyama, NPA **805** (2008), 190c, [2] Y. Yamamoto, *et al.*, PTP Suppl. **117** (1994), 361.,
[3] O. Hashimoto and H. Tamura, PPNP **57** (2006), 564., [4] Y. Kanada-En'yo *et al.*, PTP **93** (1995), 115.

Theoretical framework: HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N}$$

NN : Gogny D1S
ΛN : YNG interaction

◆Wave function

- **Nucleon part : Slater determinant**
  Spatial part of single particle w.f. is described as Gaussian packet

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i \qquad \chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single particle w.f. of Λ hyperon:**
  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu \nu_\sigma (r - z_m)_\sigma^2\right] \chi_m \qquad \chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

ΛN interaction used

YNG interaction derived from …

ESC08c + MPP + TBA

2-body including ΛN-ΣN coupling

Many-body force (MPP + TBA)

MPP: repulsion which is essential with high dens.

TBA: phenomenological 3 body attraction

$$V_{\Lambda N}(r;k_F) = \sum_{i=1}^{3} (a_i + b_i k_F + c_i k_F^2) \exp(-r^2/\beta_i^2)$$

NNN part of MPP + TBA: scattering data of ^{16}O + ^{16}O, and saturation property

TBA with Λ: to reproduce observed spectra of $^{89}_{\Lambda}Y$

⟶ Enough stiff EoS to give $2M_{\odot}$ maximum mass of neutron star

Yamamoto, Furumoto, Yasutake and Rijken, PRC**88**,022801(2013); PRC**90**,045805(2014).

$^{89}_{\Lambda}Y$

	s	p	d	f
ESC	-26.5	-19.0	-11.4	-4.3
MPa	-24.0	-17.3	-10.5	-3.9
exp	-23.7	-17.6	-10.9	-3.7

exp : O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

◆ k_F determined by density

- **Averaged density approximation(ADA):**

$$\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \qquad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$$

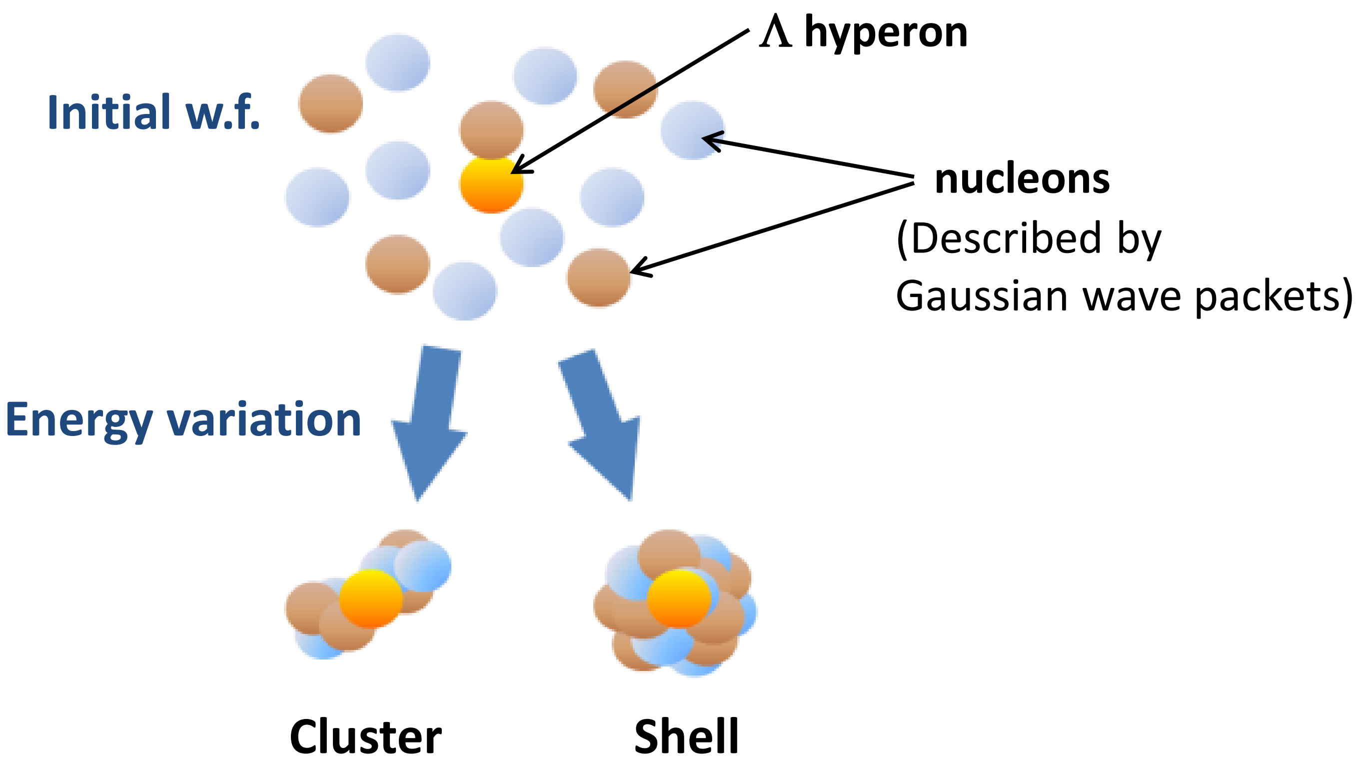
No free parameter except for TBA

Theoretical framework: HyperAMD

◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method $$\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*} \qquad \kappa < 0$$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$



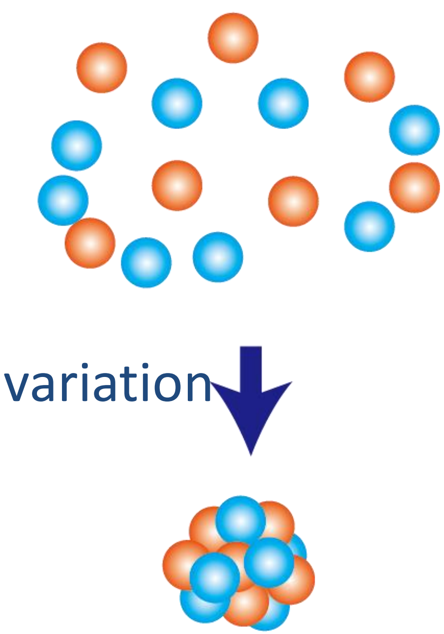
Actual calculation of HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

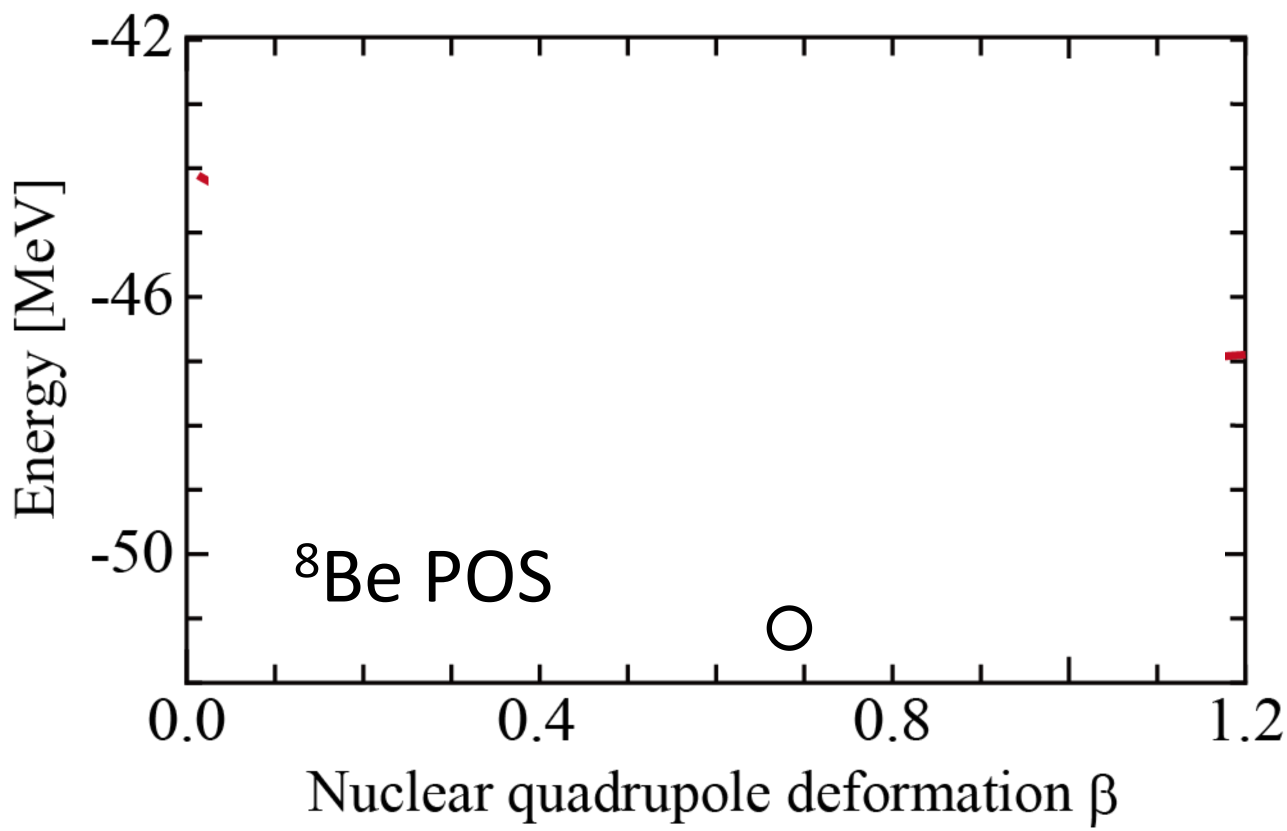
Energy variation with constraint on nuclear quadrupole deformation

Ex.) ^{8}Be

Initial w.f.



w/o constraint on β



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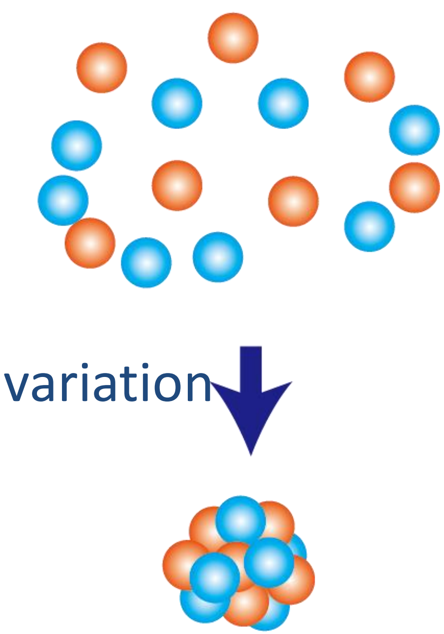
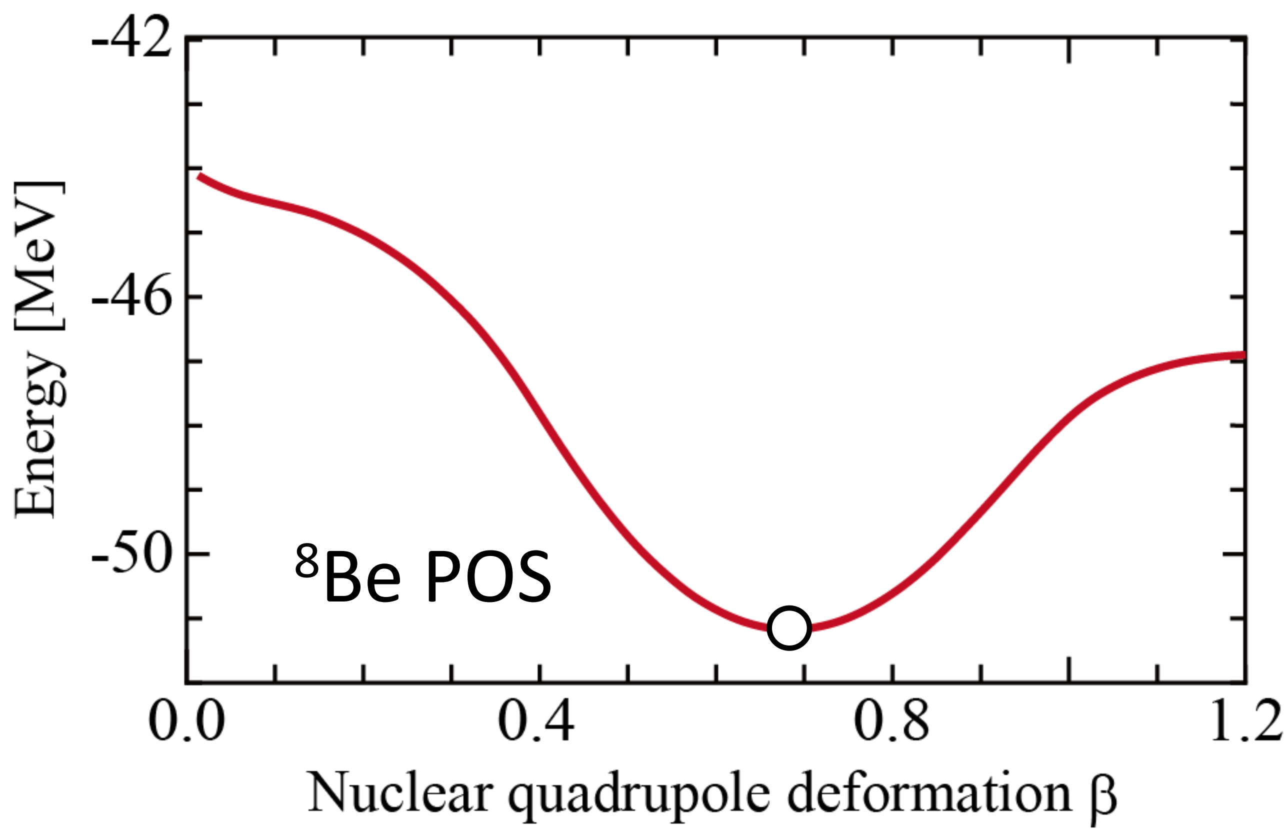


with constraint on β



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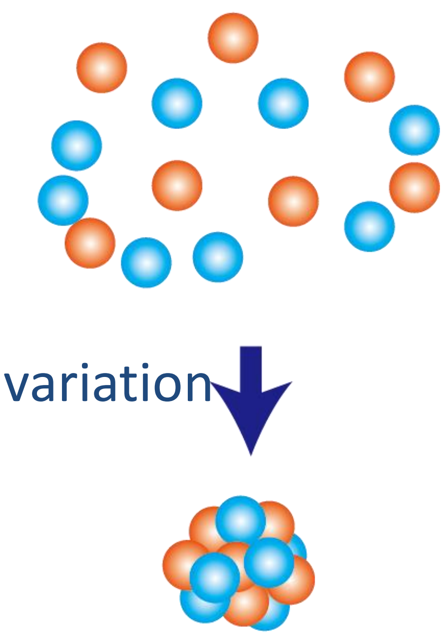

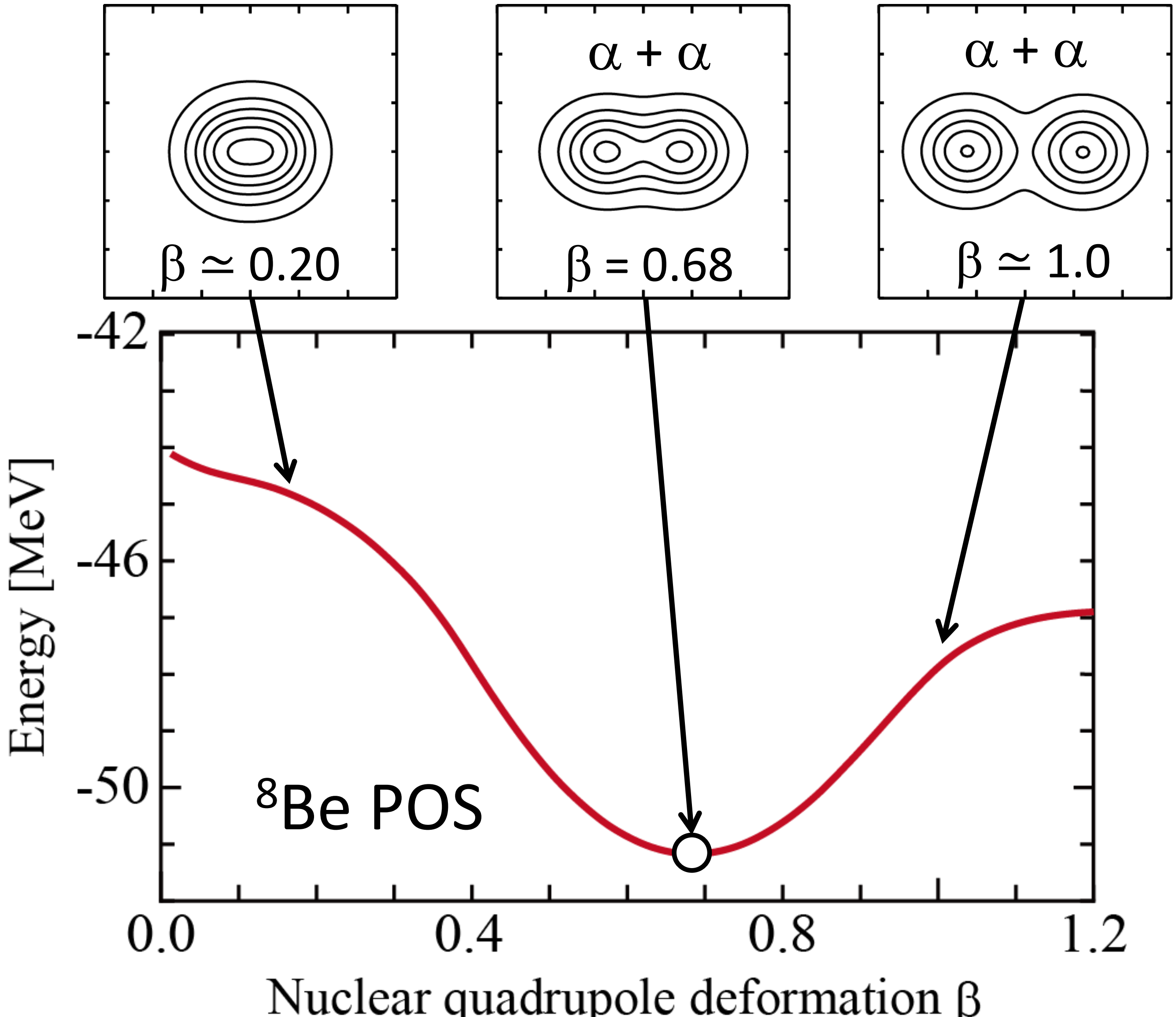

Actual calculation of HyperAMD

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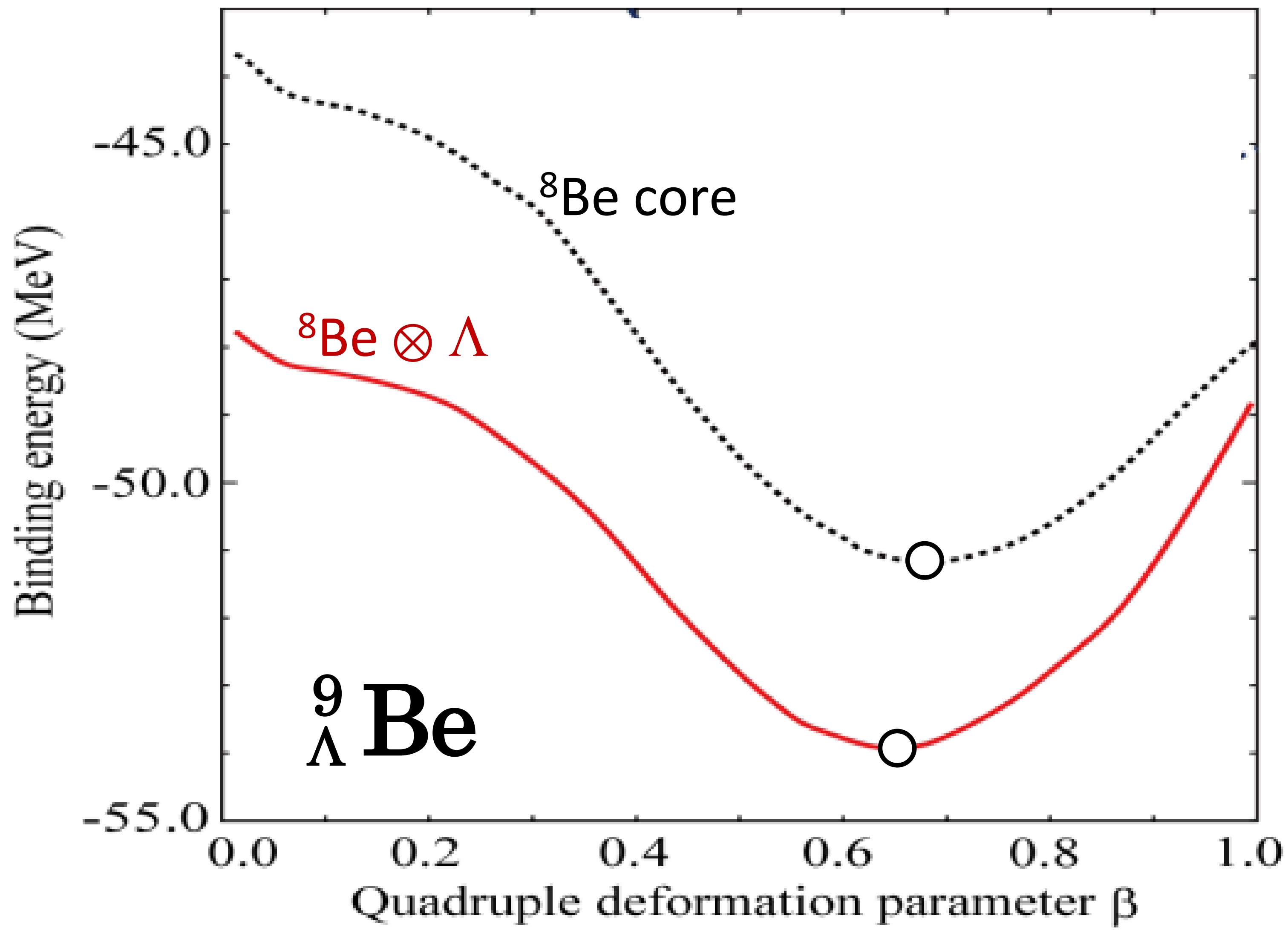
◆ **For hypernuclei**



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Angular Momentum Projection

$$\left|\Phi_K^s ; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

Generator Coordinate Method(GCM)

- Superposition of the w.f. with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s ; J^{\pm}M \left| \hat{H} \right| \Phi_{K'}^{s'} ; J^{\pm}M \right\rangle$$

$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s ; J^{\pm}M \middle| \Phi_{K'}^{s'} ; J^{\pm}M \right\rangle$$

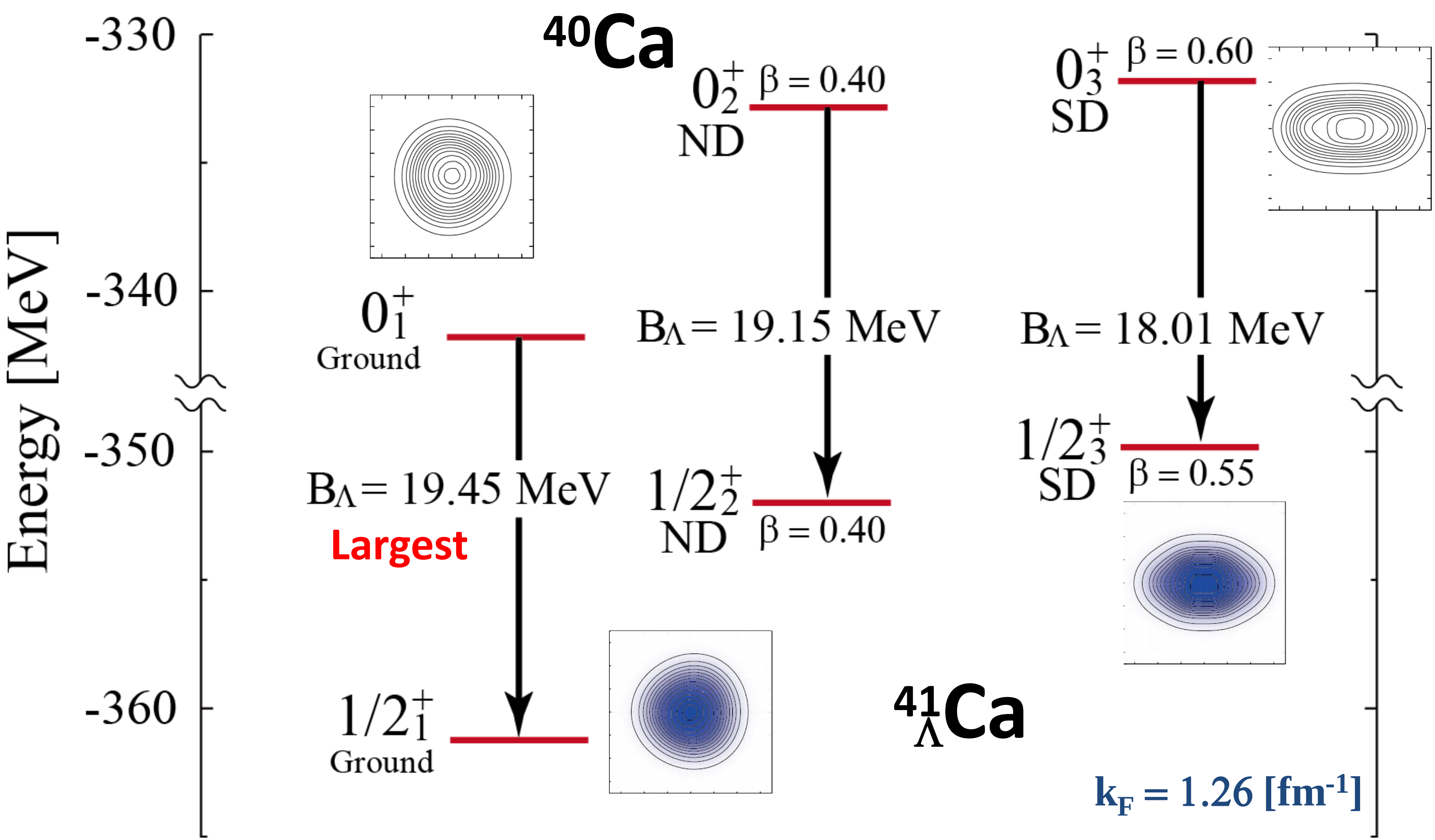
$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s ; J^{\pm}M\right\rangle$$

Applications of HyperAMD ($^{41}{}_{\Lambda}$Ca)

Example: $^{41}{}_{\Lambda}$Ca

B_{Λ} is dependent on nuclear deformation

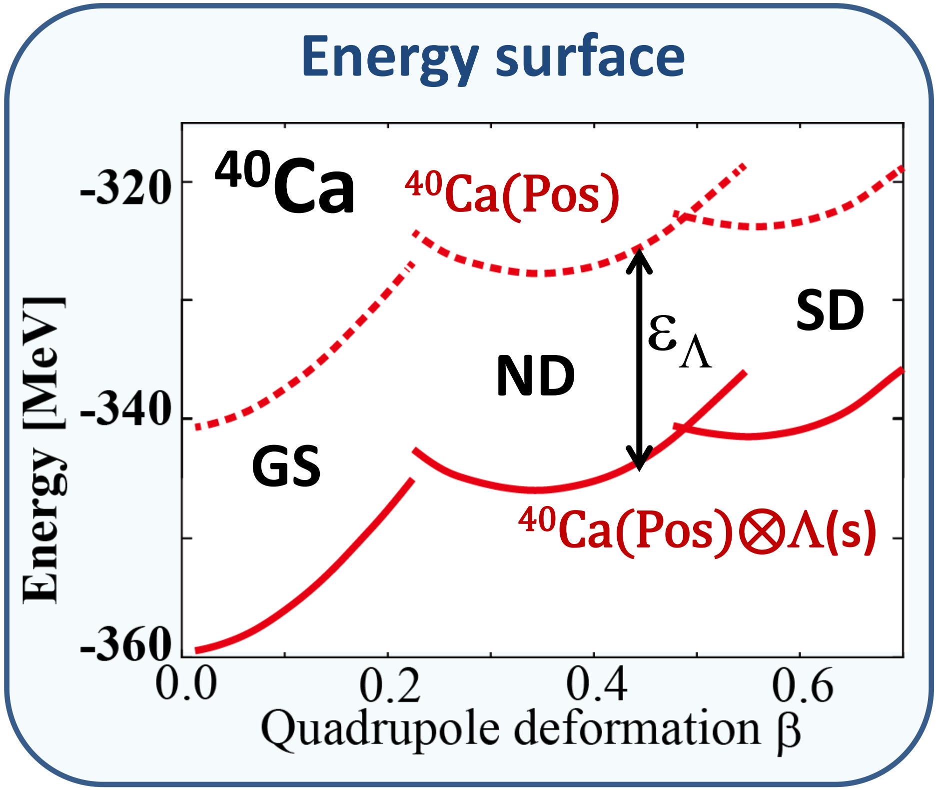
M. Isaka, *et al.*, PRC89, 024310(2014)



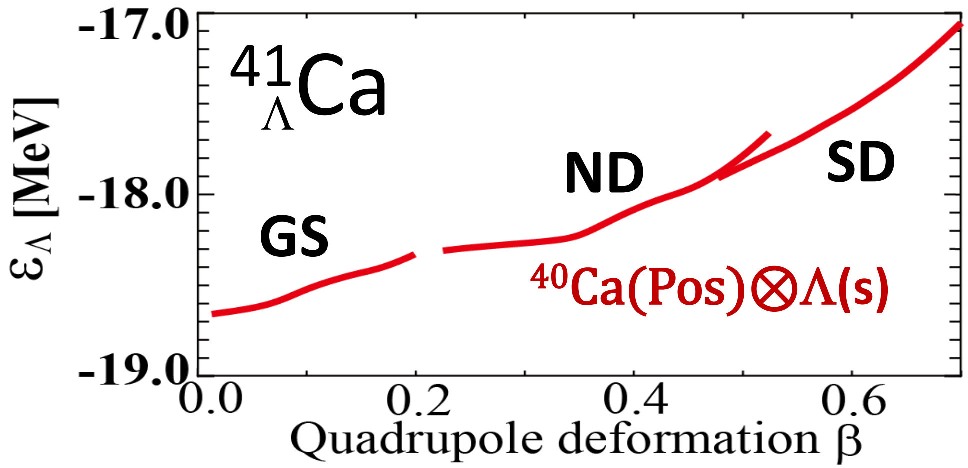
M. Isaka, *et al.*, PRC89, 024310(2014)

Applications of HyperAMD ($^{41}{}_{\Lambda}$Ca)

- **Definition:** $\epsilon_\Lambda(\beta) = E({}^{41}_{\Lambda}\mathrm{Ca})(\beta) - E({}^{40}\mathrm{Ca})(\beta)$
- **ε_Λ changes within 1 - 2 MeV as β increases**



Λ single particle energy



	$-B_\Lambda$	T_Λ	$V_{\Lambda N}$
GS	−18.7	5.5	−24.3
ND	−18.2	5.5	−23.9
SD	−17.7	5.6	−23.4

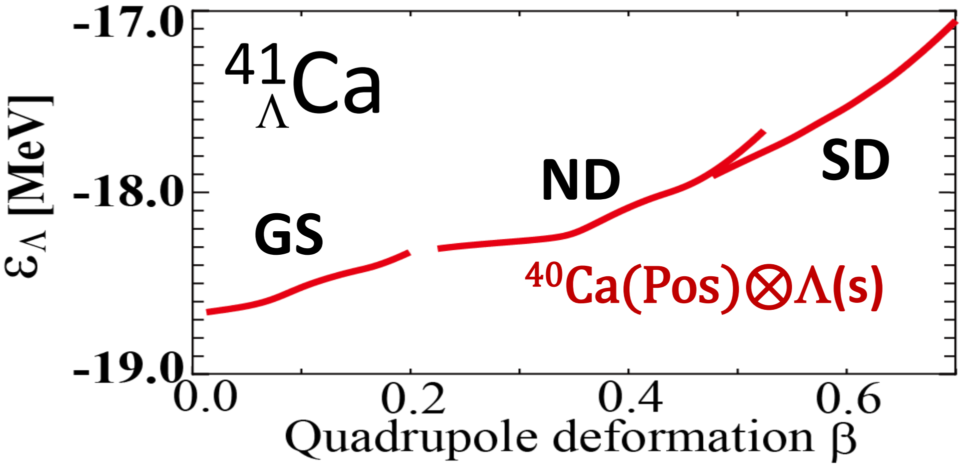
Difference of ε_Λ is mainly coming from ΛN potential energy

Why? —— "overlap between Λ and core"

M. Isaka, *et al.*, PRC89, 024310(2014)

Applications of HyperAMD ($^{41}{}_{\Lambda}$Ca)

- **ε_{Λ} varies due to changes of overlap between Λ and N**
  - **Deformation of Λ distribution is small, while nuclear part is deformed**



Overlap between the Λ and nucleons

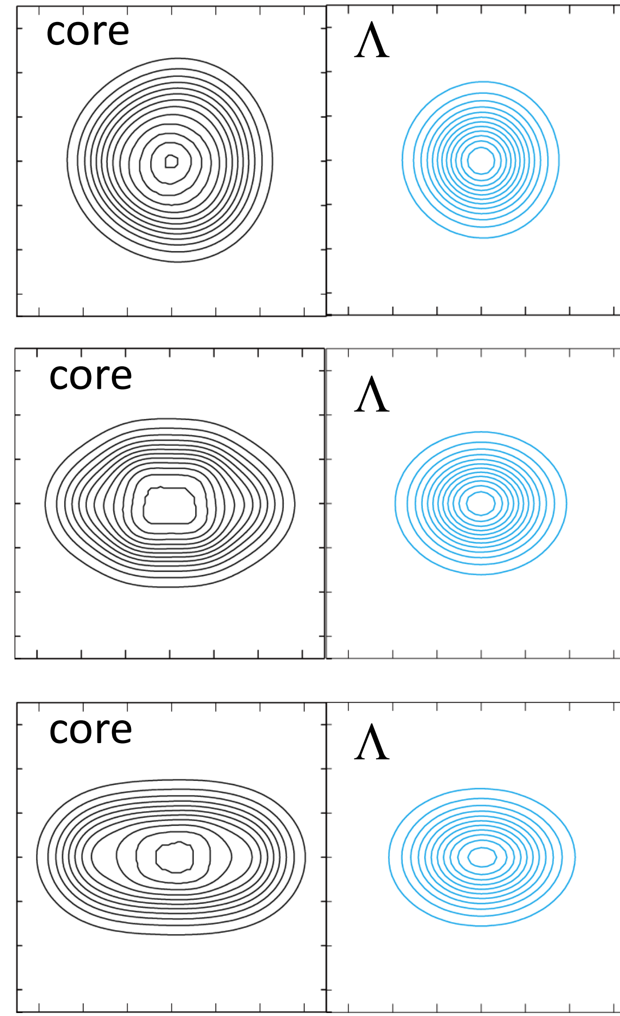
$$I = \int d^3 r \rho_N(\mathbf{r}) \rho_{\Lambda}(\mathbf{r})$$

GS
$\beta = 0.01$
$I = 0.1364$

ND
$\beta = 0.35$
$I = 0.1356$

SD
$\beta = 0.55$
$I = 0.1336$

I [fm^{-3}]



Decrease of overlap makes $V_{\Lambda N}$ shallower

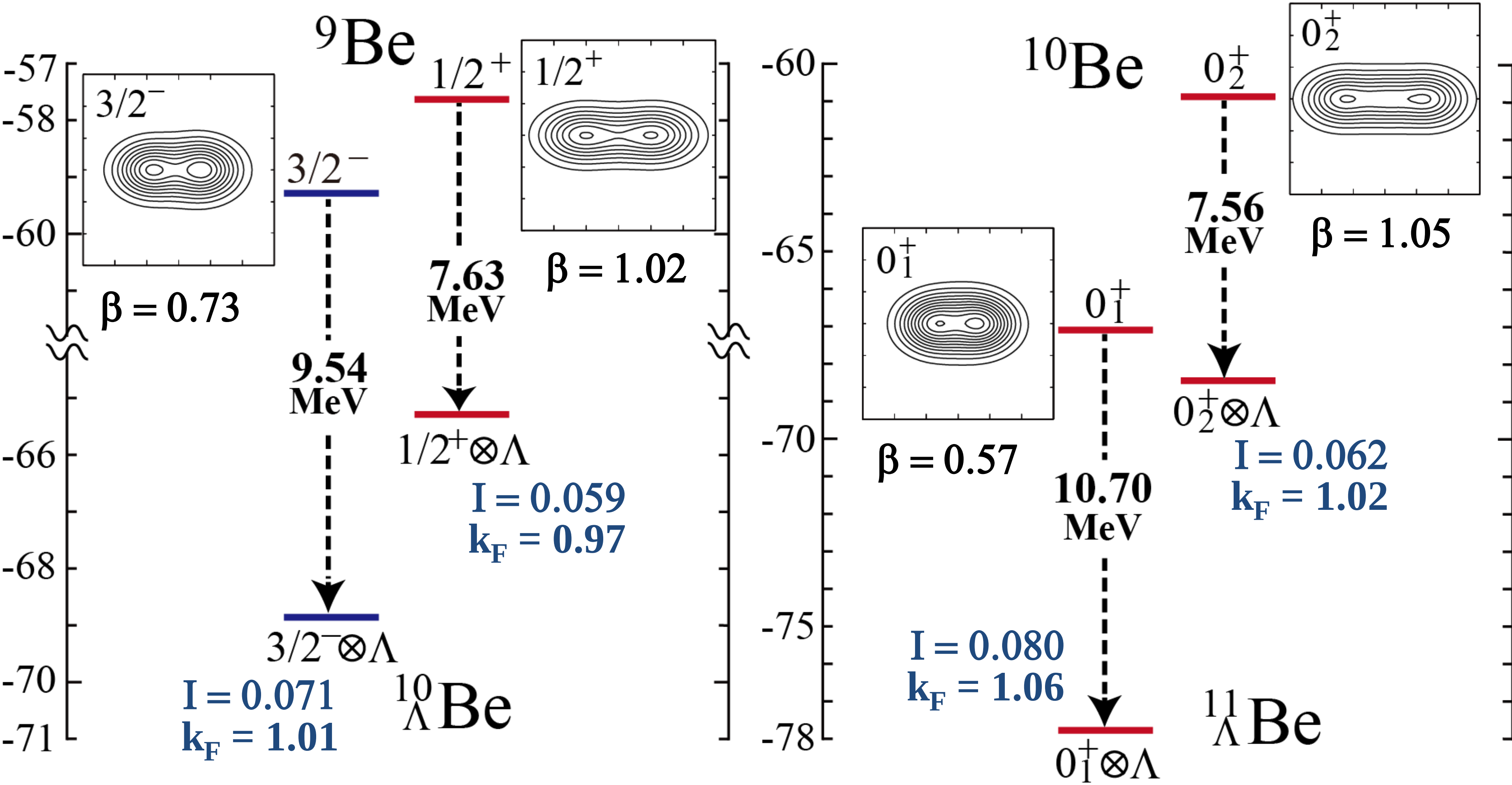
M. Isaka, M. Kimura, PRC**92**, 044326(2015)

Applications of HyperAMD ($_\Lambda$Be)

Example: $^{10}_{\Lambda}$Be, $^{11}_{\Lambda}$Be

Dependence of B_Λ on cluster structures

I [fm^{-3}], k_F [fm^{-1}]



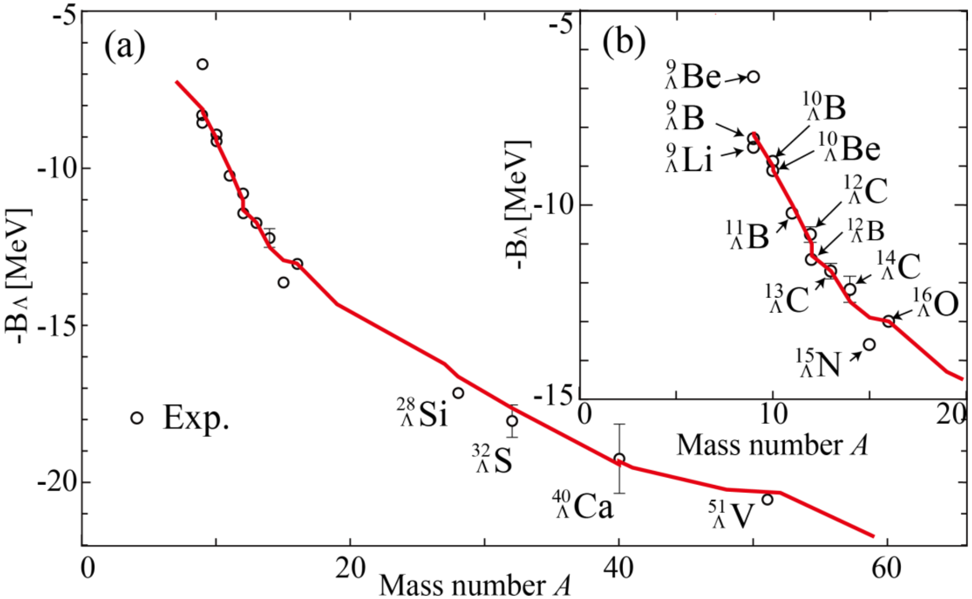
Same trend as in $^{41}_{\Lambda}$Ca, whereas smaller k_F is used in excited states

Results and Discussions

“mass dependence of B_{Λ}”

B_Λ as a function of mass number *A*

ESC08c + MPP + TBA
repulsive attraction



	β	γ	$\langle\rho\rangle$	k_F	$-B_\Lambda^{\rm calc}$	$-B_\Lambda^{\rm exp}$
$^{9}_{\Lambda}$Li	0.50	2°	0.072	1.02	−8.1	-8.50 ± 0.12 [2
$^{9}_{\Lambda}$Be	0.87	1°	0.060	0.96	−8.1	-6.71 ± 0.04 [2
$^{9}_{\Lambda}$B	0.45	2°	0.072	1.02	−8.2	-8.29 ± 0.18 [2
$^{10}_{\Lambda}$Be	0.57	1°	0.077	1.04	−9.0	-9.11 ± 0.22 [2
						-8.55 ± 0.18 [3
$^{10}_{\Lambda}$B	0.58	1°	0.075	1.04	−9.1	-8.89 ± 0.12 [2
$^{11}_{\Lambda}$B	0.50	29°	0.081	1.06	−10.0	-10.24 ± 0.05 [
$^{12}_{\Lambda}$B	0.39	48°	0.083	1.07	−11.3	-11.37 ± 0.06 [2
						-11.38 ± 0.02 [
$^{12}_{\Lambda}$C	0.41	34°	0.086	1.08	−11.0	-10.76 ± 0.19 [
$^{13}_{\Lambda}$C	0.45	60°	0.090	1.10	−11.7	-11.69 ± 0.19 [
$^{14}_{\Lambda}$C	0.45	31°	0.093	1.11	−12.5	-12.17 ± 0.33 [
$^{15}_{\Lambda}$N	0.28	60°	0.098	1.13	−12.9	-13.59 ± 0.15 [
$^{16}_{\Lambda}$O	0.02	–	0.105	1.16	−13.0	-12.96 ± 0.05 [2
$^{19}_{\Lambda}$O	0.30	3°	0.110	1.18	−14.3	–
$^{27}_{\Lambda}$Mg	0.36	36°	0.125	1.23	−16.2	–
$^{28}_{\Lambda}$Si	0.32	53°	0.125	1.23	−16.6	-17.1 ± 0.2 [9
$^{32}_{\Lambda}$S	0.28	0°	0.130	1.24	−17.6	-18.0 ± 0.5 [23
$^{40}_{\Lambda}$K	0.01	–	0.136	1.26	−19.4	–
$^{40}_{\Lambda}$Ca	0.03	–	0.136	1.26	−19.3	-19.24 ± 1.1 [2
$^{41}_{\Lambda}$Ca	0.13	12°	0.136	1.26	−19.5	–
$^{48}_{\Lambda}$K	0.01	–	0.141	1.28	−20.2	–
$^{51}_{\Lambda}$V	0.18	2°	0.151	1.31	−20.3	-20.51 ± 0.13 [2
$^{59}_{\Lambda}$Fe	0.26	23°	0.142	1.28	−21.7	–

B_Λ as a function of mass number *A*

ESC08c + MPP + TBA

repulsive attraction

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						–
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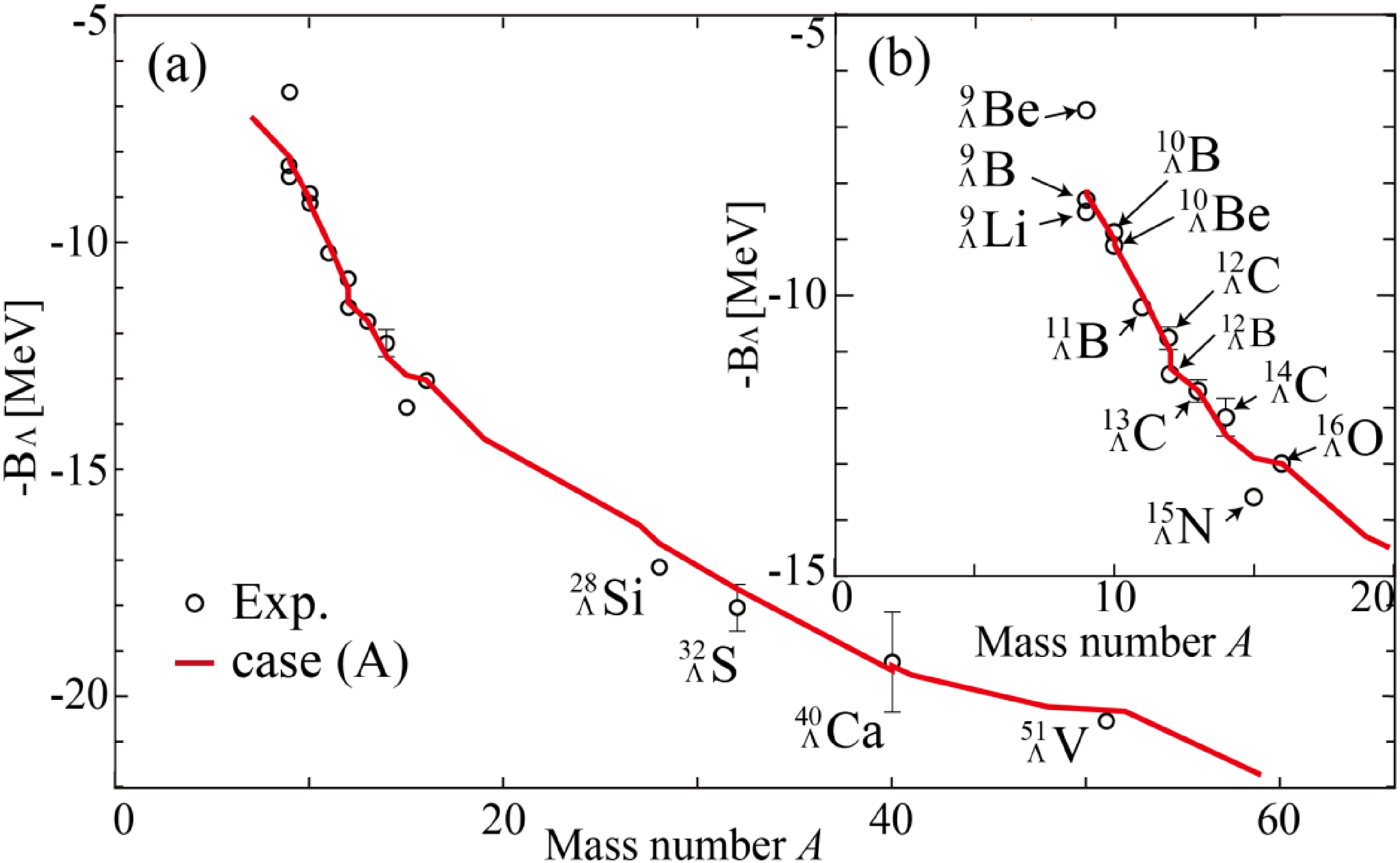
						–



HyperAMD w/ ESC08c + MPP + TBA successfully reproduces B_Λ

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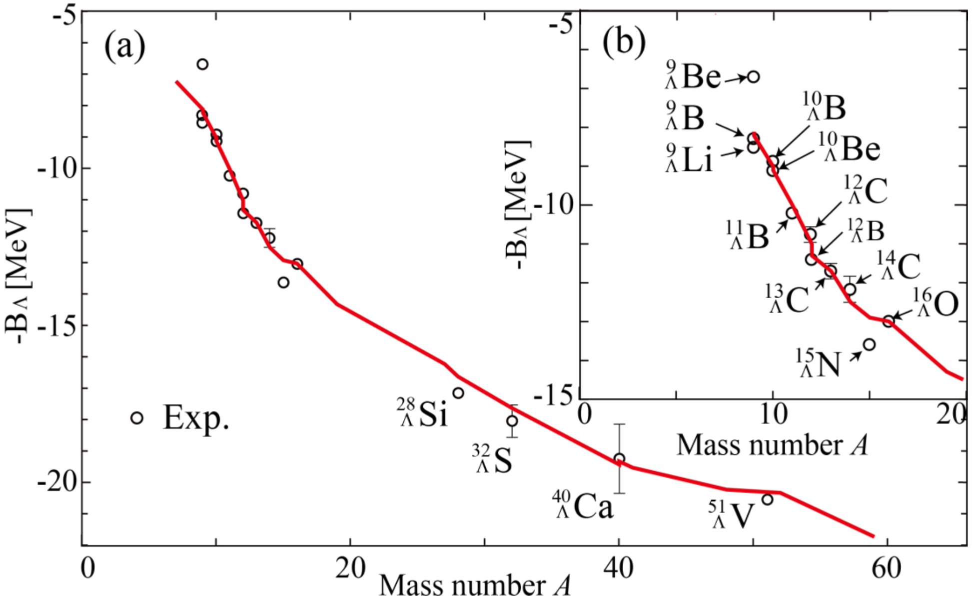

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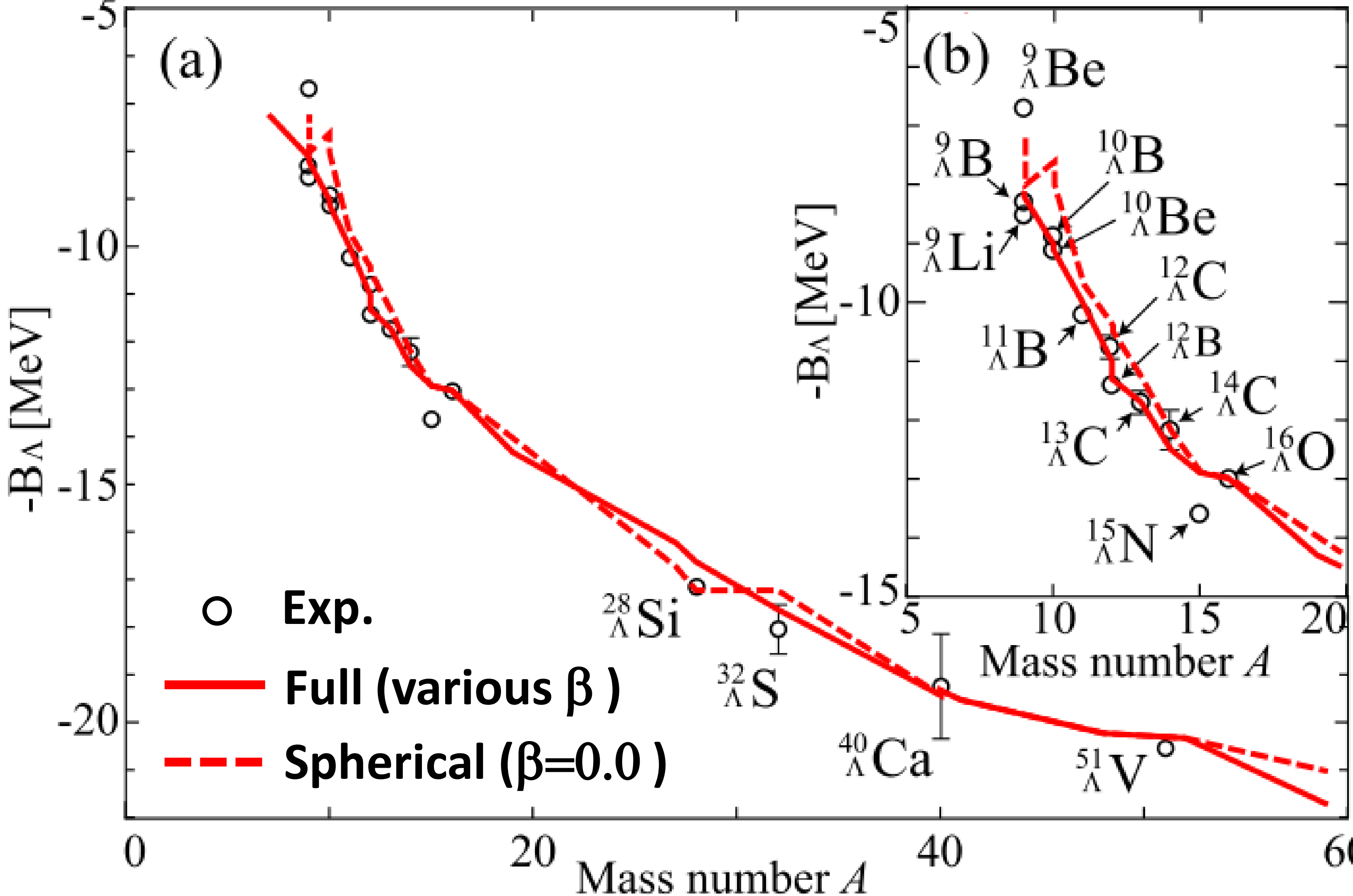
What is essential to reproduce B_Λ

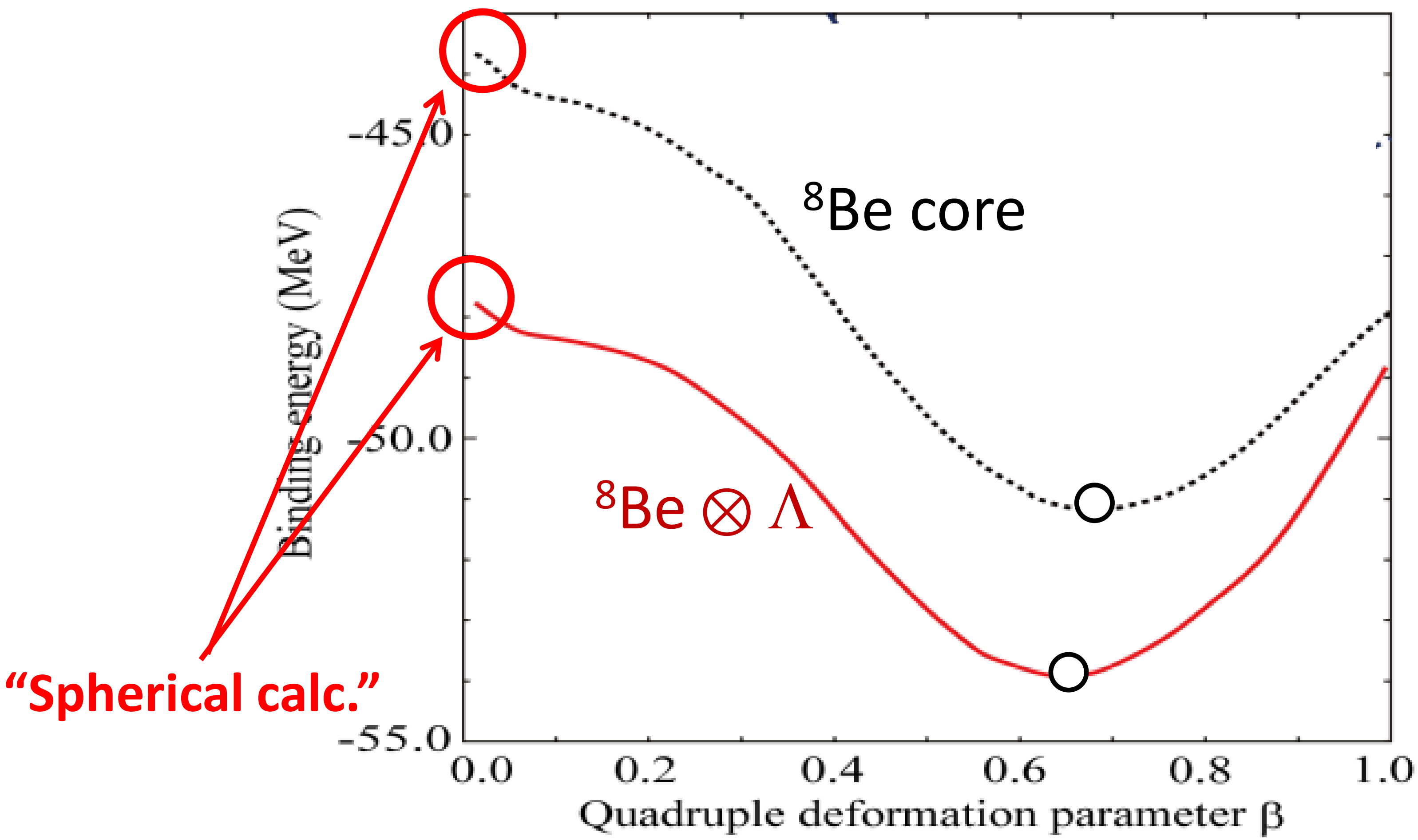
"Description of the core structure"

"Full calc." vs. "Spherical calc."



What is essential to reproduce B_Λ

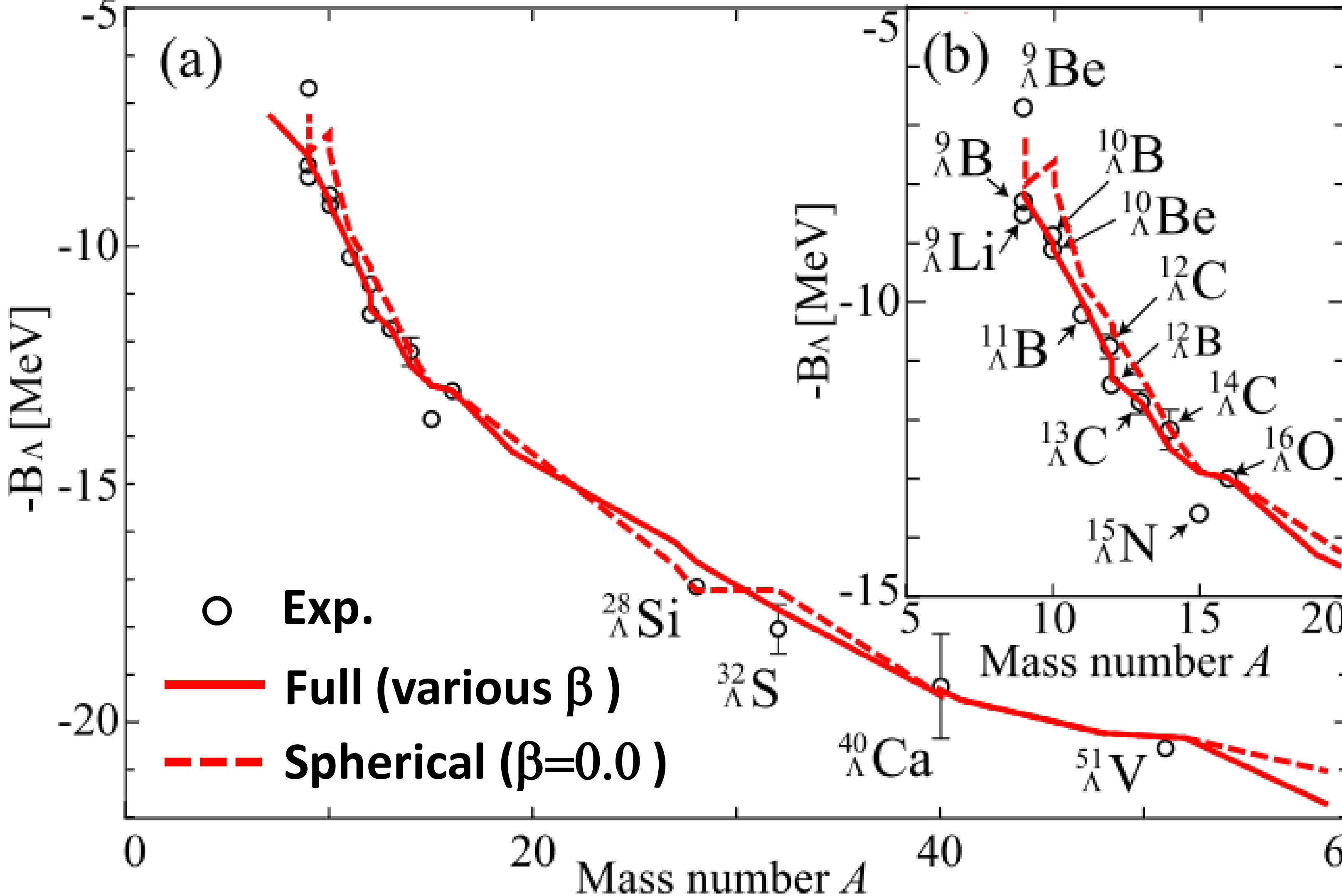
Ex. $^{9}_{\Lambda}$Be **“Full calc.” vs. “Spherical calc.”**



“Full calc.”: all of w.f. on energy curve in GCM calc.

What is essential to reproduce B_Λ

"Description of the core structure"



Deformation of the ground states is essential to reproduce B_Λ

What is essential to reproduce B_Λ

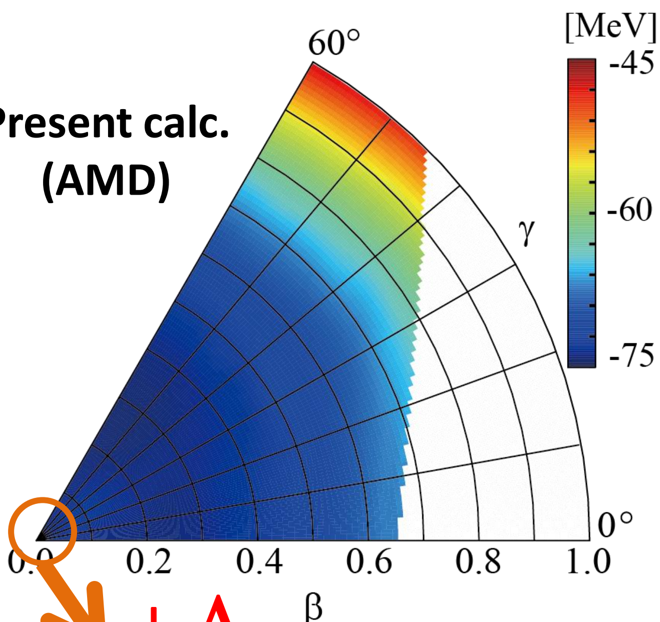
"Description of the core structure"

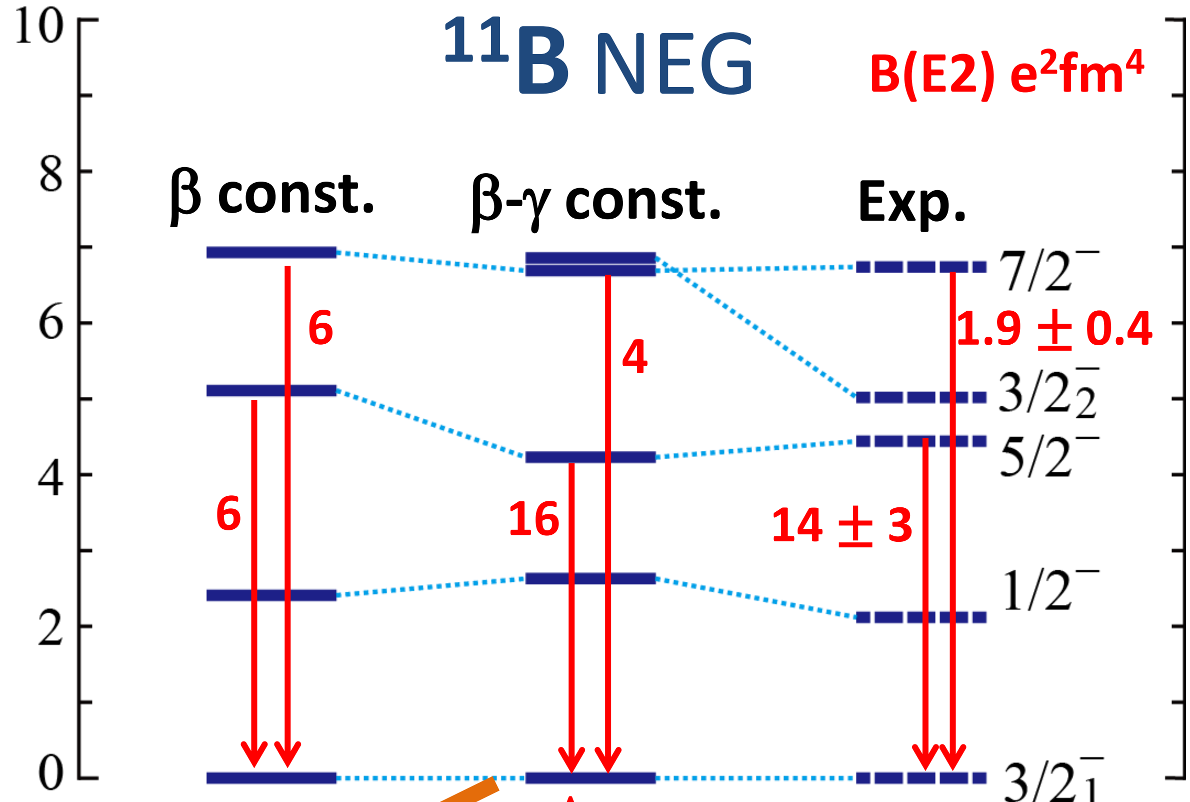
Ex.: ^{11}B

More sophisticated treatment: GCM calc. on (β, γ) plane

T. Suhara and Y. Kanada-En'yo, PTP**123**,303(2010)

Present calc. (AMD)





+ Λ

$^{12}_{\Lambda}B$ (Spherical)

B_Λ = 9.5 MeV

(k_F = 1.16 fm^{-1})

+ Λ

$^{12}_{\Lambda}B$ (β-γ)

B_Λ = 11.3 MeV

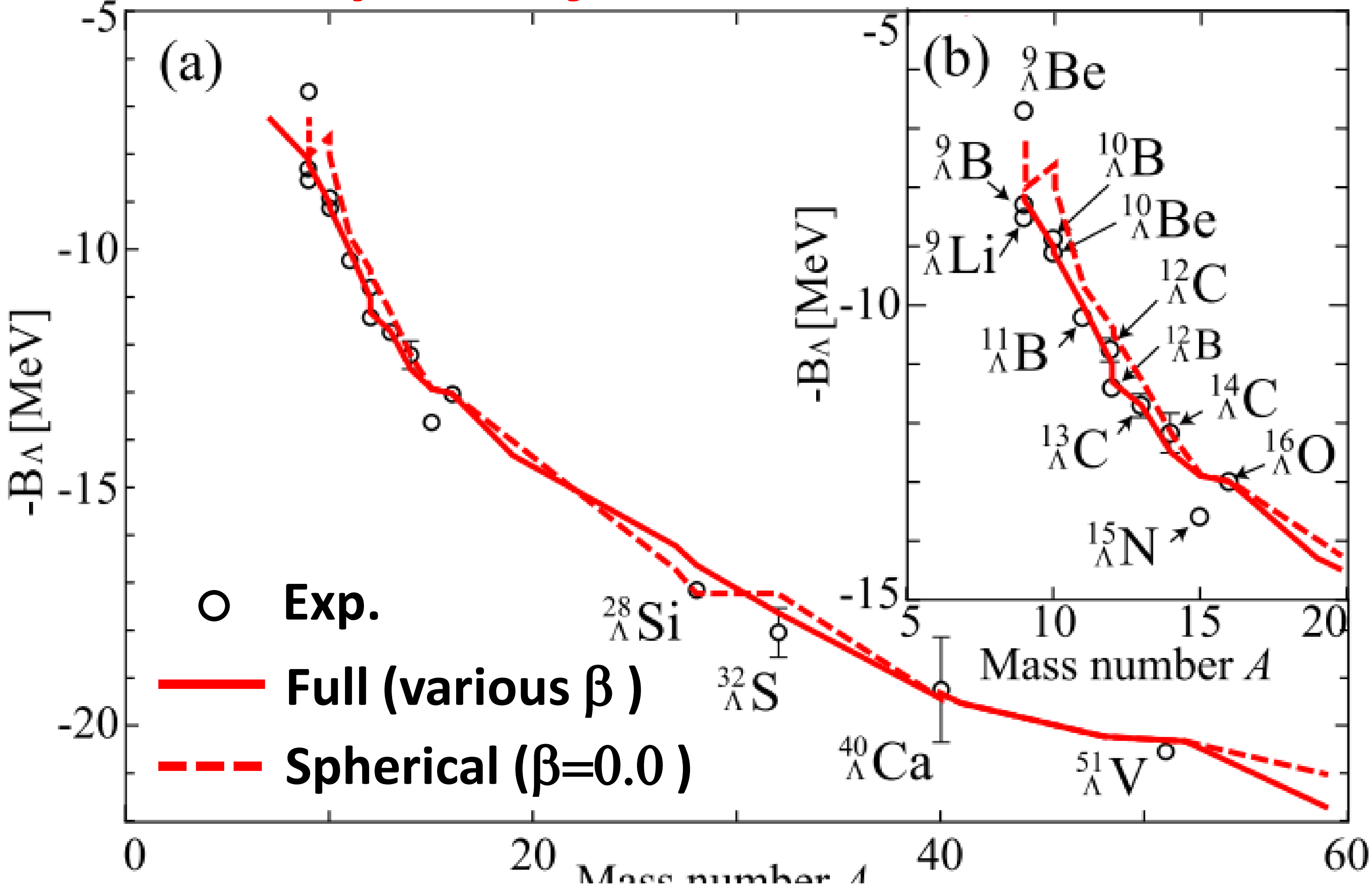
(k_F = 1.07 fm^{-1})

$^{12}_{\Lambda}B$ (EXP)

B_Λ = 11.4 ±0.02 MeV

What is essential to reproduce B_Λ

"Description of the core structure"



Deformations affect *A* dep. of B_Λ through k_F dep. of interaction and overlap between Λ and nucleons

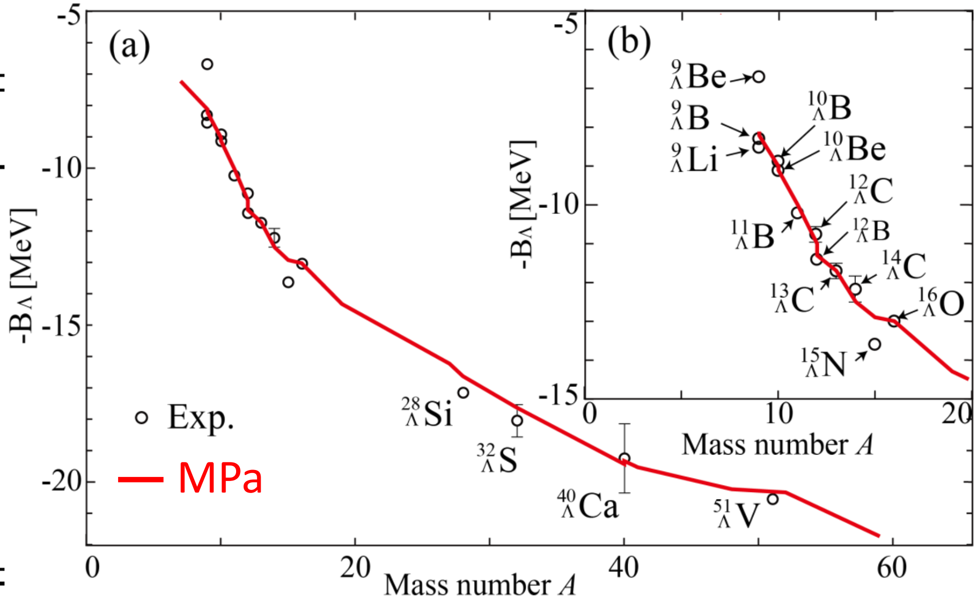
Comparison with the results with ESC08c only

◆ Effects of many-body force

ESC: ESC08c only
MPa: ESC08c + MPP + TBA

Over-binding with ESC08c only

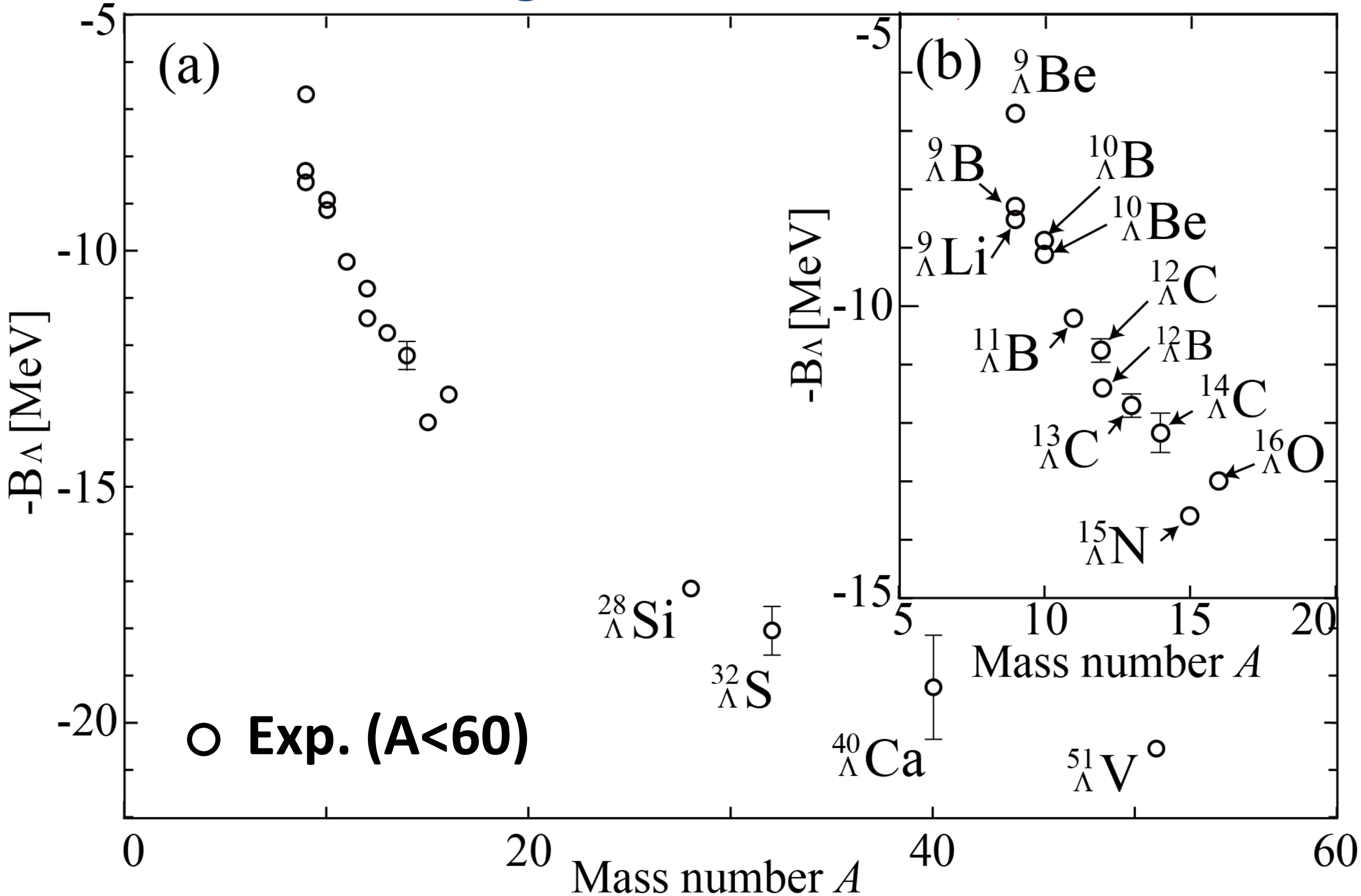
[MeV]	ESC	MPa	Exp.
$^{13}_{\Lambda}$C	-11.5	-11.7	-11.69 ± 0.19
$^{16}_{\Lambda}$O	-13.3	-13.0	-12.96 ± 0.05
$^{28}_{\Lambda}$Si	-17.7	-16.6	-17.1 ± 0.2
$^{40}_{\Lambda}$K	-21.5	-19.4	–
$^{48}_{\Lambda}$K	-22.6	-20.2	–
$^{51}_{\Lambda}$V	-23.5	-20.3	-20.51 ± 0.13



- **Observed B$_\Lambda$ is well reproduced with MPP + phenomenological TBA**
- **Systematic data of B$_\Lambda$ will provide a new insight to many-body force**

Current status of observed B_Λ

Observations are not enough with A > 16



Systematic and accurate data of observed B_Λ are desired

Bertini *et al.*, NPA**83**,306(1979), Davis, Juric , *et al.*, NPB**52**(1973), Davis, NPA**547**,369(1992);NPA**754**,3c(2005), Ajimura *et. al.*, NPA**639**(1998)93c, Pile *et al.*, PRL**66**,2585(1991), Hotchi *et al.*, PRC**64**, 044302(2001), Hashimoto and Tamura, PPNP**57**,564(2006), Tang, *et. al.*, PRC**90**,034320(2014).

Summary

◆ Summary

- HyperAMD + GCM was applied with ESC08c + MPP + TBA interaction

Observed B_Λ are successfully reproduced in wide mass regions

- **Structure of the core nuclei**
  - Spherical shape: deviate from observed B_Λ
  - Description of core deformation is essential

Density dependence of ΛN interaction

- **Many-body (MPP + TBA) force effects**
  - Input: experimental data of $^{89}_{\Lambda}$Y
  - MPP + TBA force could be determined by systematic data of B_Λ

◆ Future plan

- To reveal reasons for deviation of B_Λ with A < 9 (e.g. $^{9}_{\Lambda}$Be)
- Many-body force effects: from systematics of B_Λ, Λ in excited orbit?