Lattice QCD study of $Z_c(3900)$

based on Y. Ikeda et al. [HAL QCD], PoS(Lattice2015)091.

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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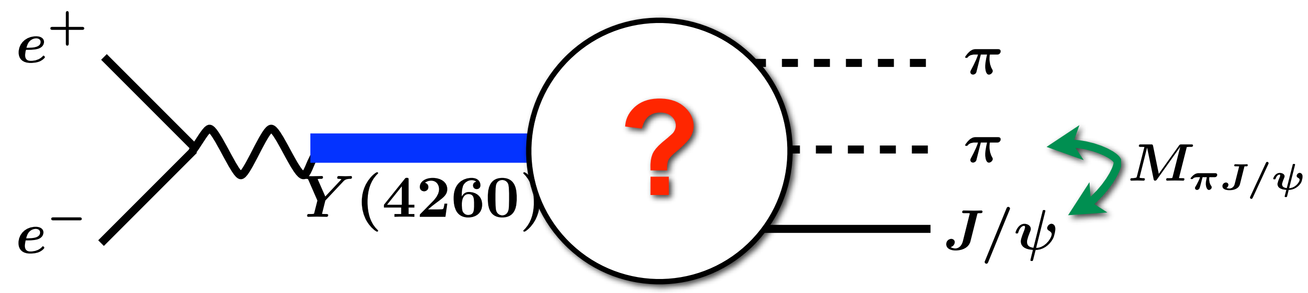
Hadrons to Atomic nuclei HAL from Lattice QCD

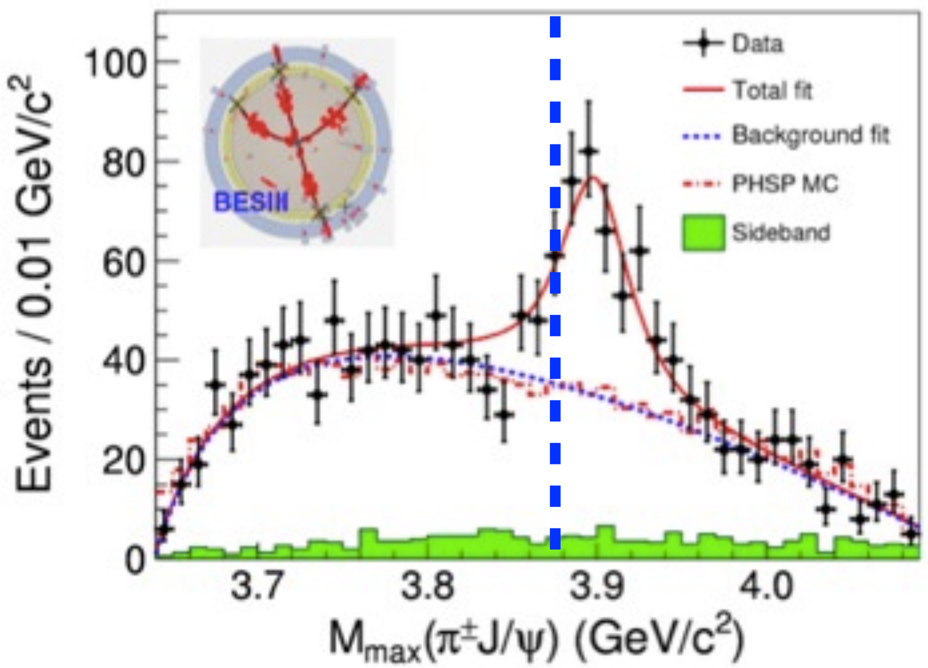
Reimei Workshop, Jan. 18--20.

@JAEA, Tokai, Japan

What is $Z_c(3900)$?



BESIII Coll., PRL110, 252001, (2013).



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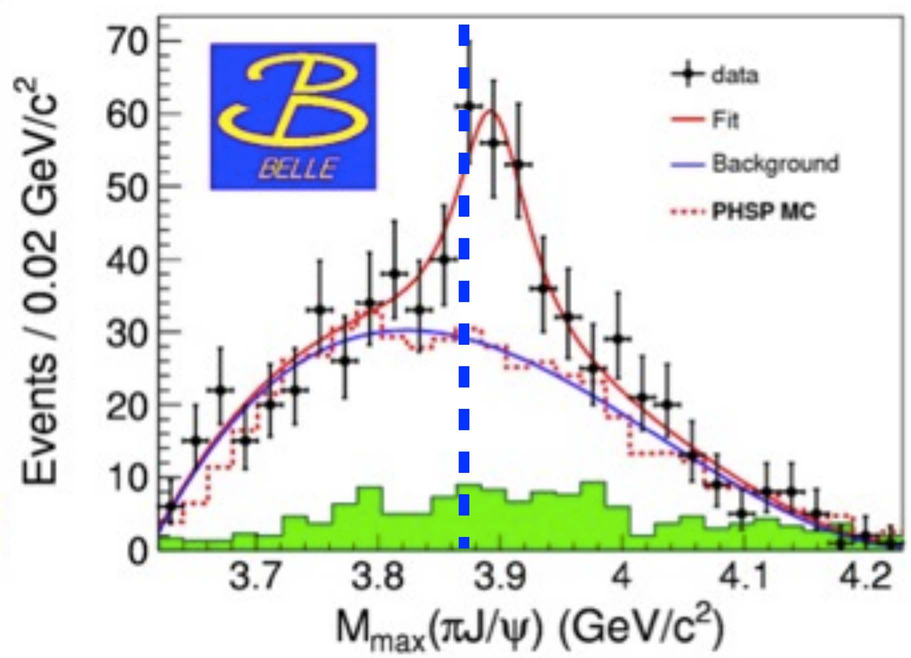
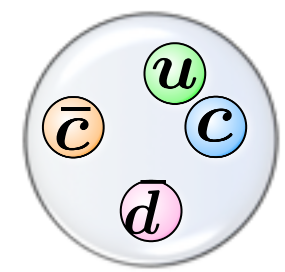
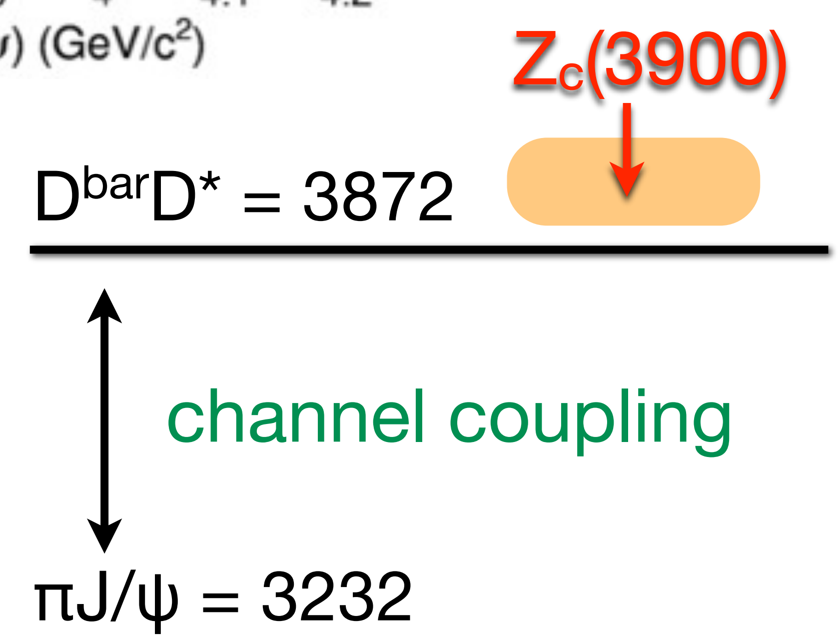




- $Z_c(3900)$ is observed in $\pi^\pm J/\psi$ invariant mass
- Minimal quark content : 4 quarks ($c^{bar}c\ u^{bar}d$)
- M ~ 3900, Γ ~ 60 MeV from BW line shape
- $J^P=1^+$ is most probable

BESIII Coll., PRL112 (2014).



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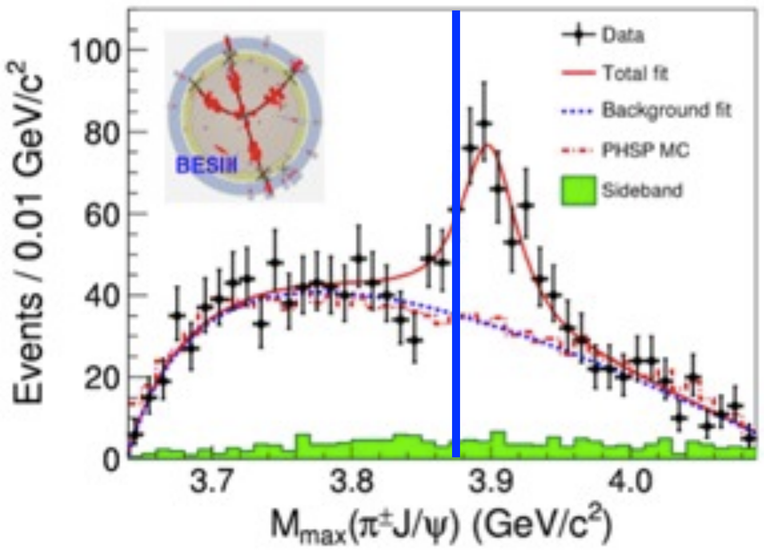

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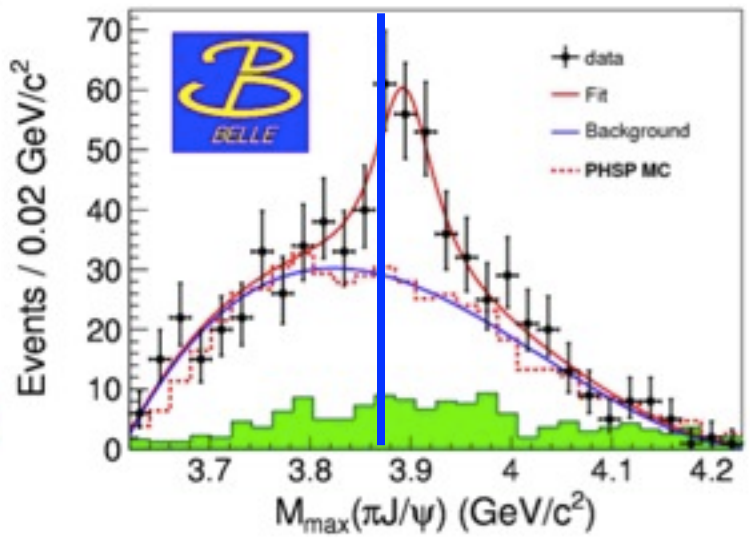


Detailed study by BESIII Coll.

- Decay rate of $Z_c(3900)$

$$\frac{\Gamma(Z_c(3900) \to \bar{D}D^*)}{\Gamma(Z_c(3900) \to \pi J/\psi)} \simeq 6.2$$

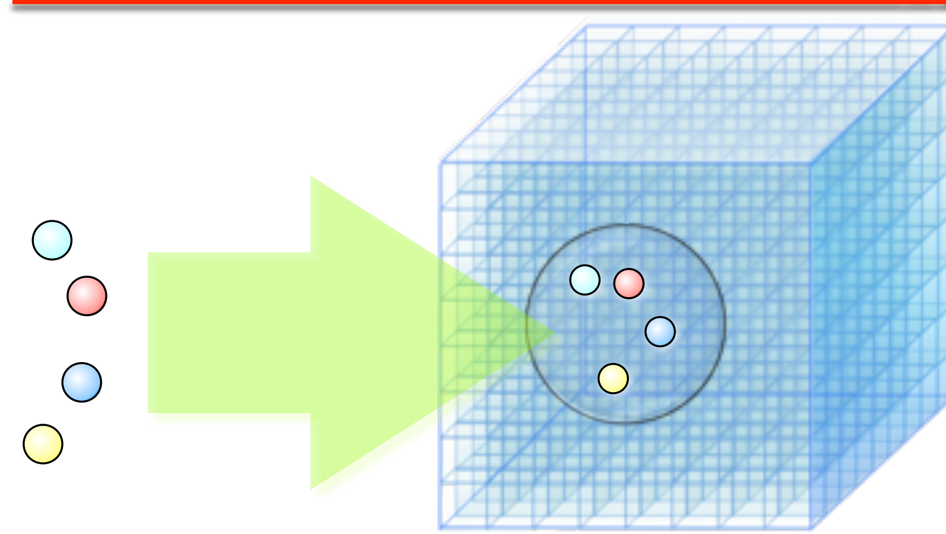
BESIII Coll., PRL112 (2014).

★ Structure of $Z_c(3900)$ from models

- Tetraquark? Maiani et al. (2013).
- $D^{bar}D^*$ molecule? Nieves et al. (2011) + many others
- J/ψ + π-cloud? Voloshin (2008).
- threshold kinematical effect? Chen et al. (2013), Swanson (2015).

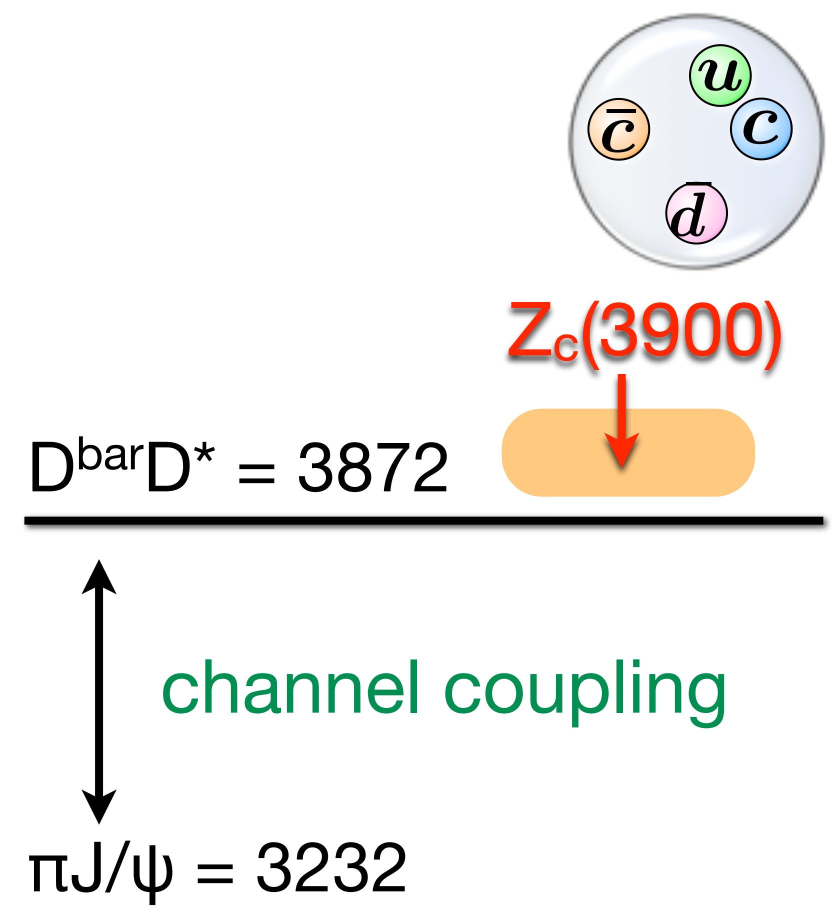
➡ poor information on interactions

★ LQCD simulations for $Z_c(3900)$



Contents

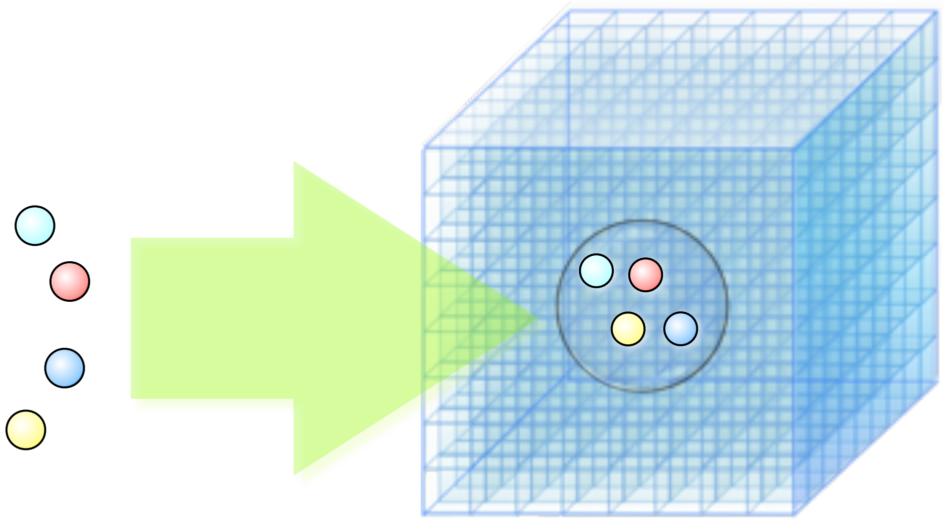
- Brief introduction to $Z_c(3900)$
- How to study $Z_c(3900)$ on the lattice?
- Coupled-channel interactions for $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$
- Numerical results for observables
- Discussion about structure of $Z_c(3900)$
- Summary



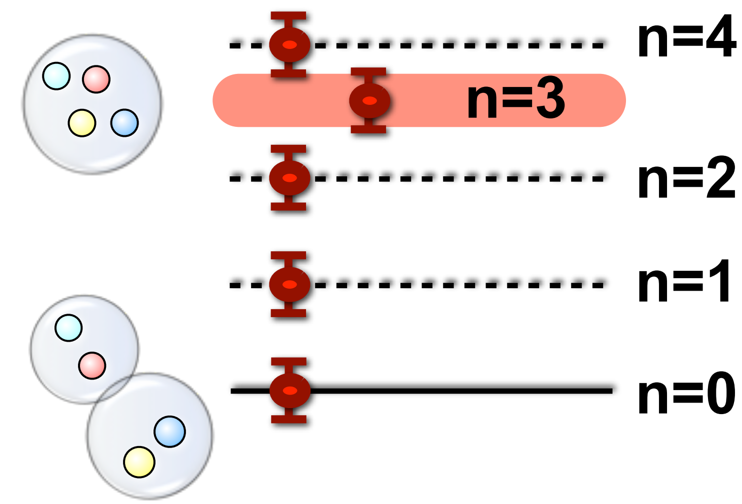
How to study $Z_c(3900)$ on the lattice?

✦ Conventional approach: LQCD spectrum

➡ identify all relevant $W_n(L)$ (n=0,1,2,3,...)



$$\langle 0|\Phi[c\bar{c}u\bar{d}](\tau)|W_n\rangle = e^{-W_n\tau}$$



✓ No positive evidence for $Z_c(3900)$ in $J^P=1^+$

S. Prelovsek et al., PLB 727, 172 (2013).
S.-H. Lee et al., PoS Lattice2014 (2014).
S. Prelovsek et al., PRD91, 014504 (2015).

★ Why is the peak observed in expt.?

- Broad resonance? Kinematical effect?
- Key is S-matrix elements w/ coupled-channel

➡ Lüscher's finite size formula in coupled-channel system

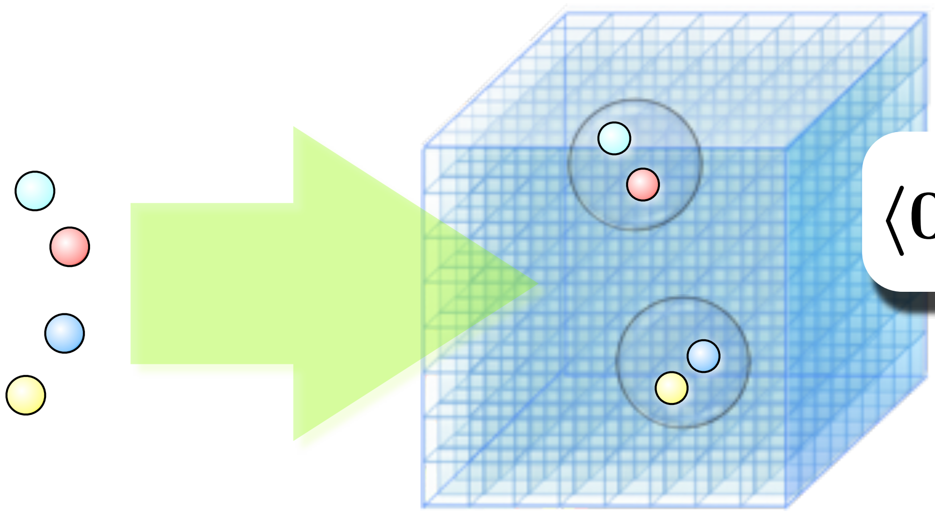
in practice, assumption about interaction kernels or K-matrices necessary

HAL QCD "potential" approach

✦ HAL QCD approach: energy-independent interaction kernel

➡ measure not only temporal but also spatial correlation

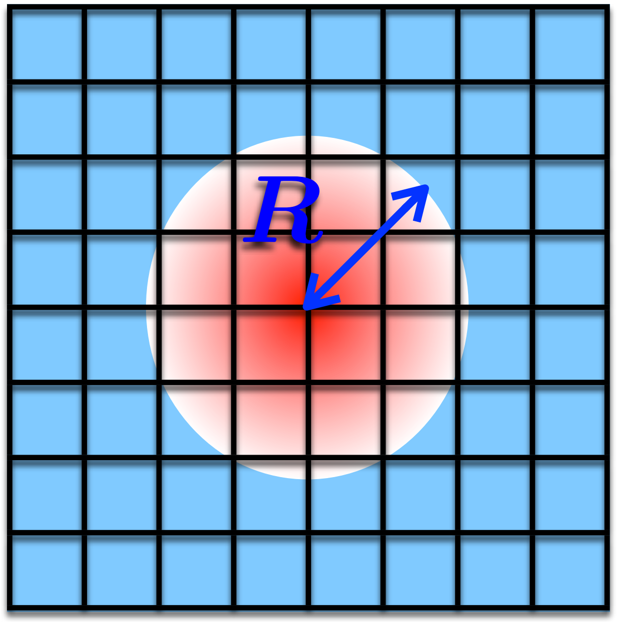
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al.(HAL QCD), PLB712, 437(2012).



$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)|W_n\rangle = \sqrt{Z_1 Z_2}\psi_n(\vec{r})e^{-W_n\tau}$$

★ Nambu-Bethe-Salpeter wave functions: $\psi_n(r)$

- Equal-time choice of NBS amplitudes



✔ NBS wave functions satisfy Helmholtz equation in asymptotic region

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

➡ Nonrelativistic approx. is NOT required

HAL QCD "potential" approach

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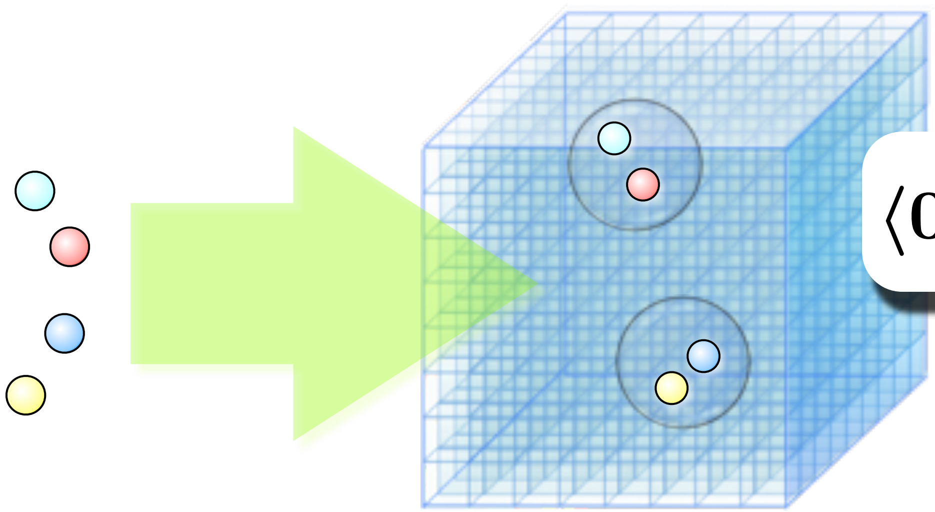
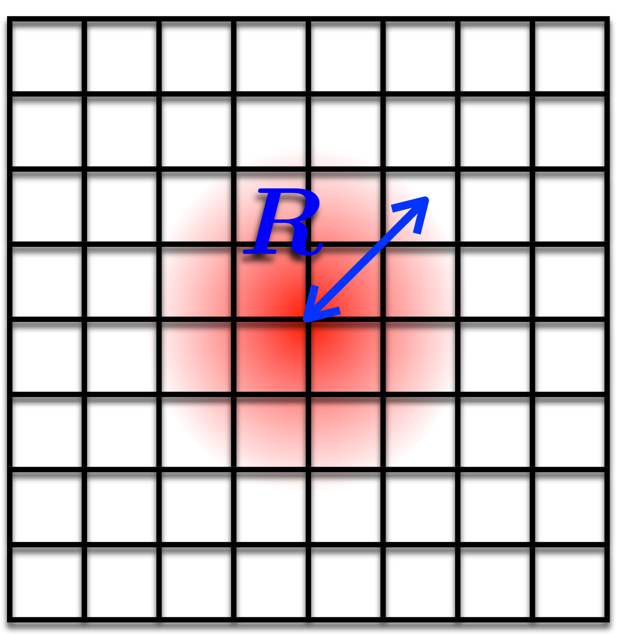


$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)|W_n\rangle = \sqrt{Z_1 Z_2}\psi_n(\vec{r})e^{-W_n\tau}$$

★ Nambu-Bethe-Salpeter wave functions: $\psi_n(r)$

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu\int d\vec{r}'U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$



✓ Potentials for infinite volume calc. --> S-matrix : S(k)

HAL QCD "potential" approach

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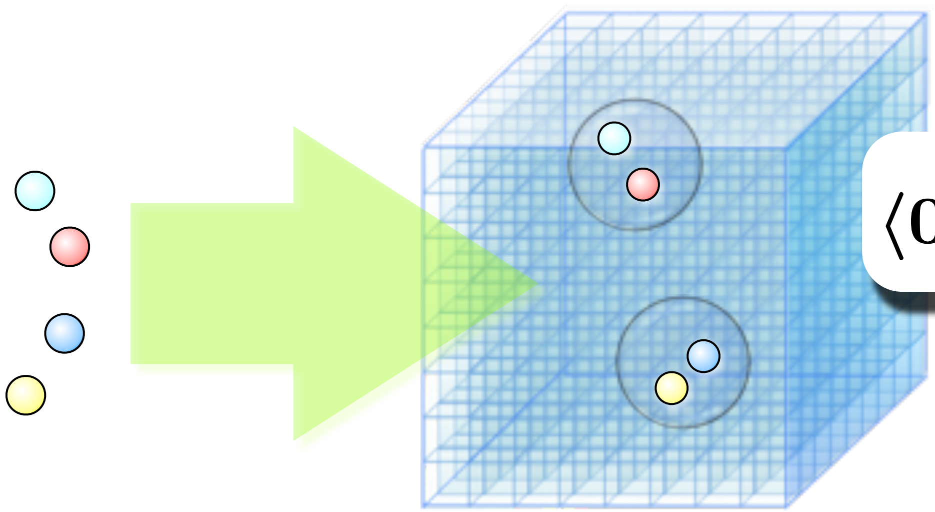


$$\langle 0|\phi_1^a(\vec{x}+\vec{r},\tau)\phi_2^a(\vec{x},\tau)|W_n\rangle = \sqrt{Z_1^a Z_2^a}\,\psi_n^a(\vec{r})e^{-W_n\tau}$$

★ Nambu-Bethe-Salpeter wave functions: $\psi_n(r)$

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}'U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

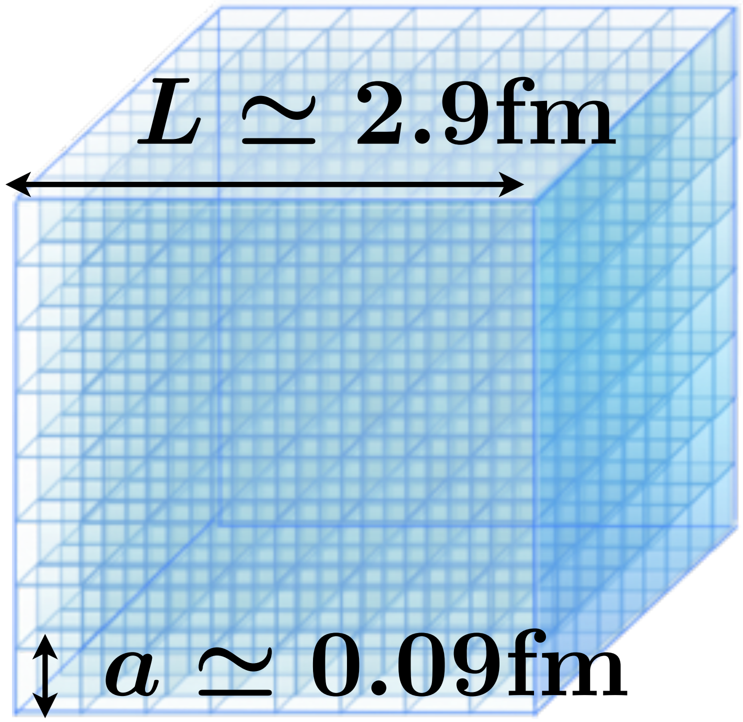
✓ Potentials for infinite volume calc. --> S-matrix : $S^{ab}(k)$

➡ Extension to coupled-channel system is straightforward

- measure wave functions in each channel
- extract potential matrix
- calculate observables (mass spectrum, pole position, ...)

Aoki et al. [HAL QCD Coll.], Proc. Jpn. Acad., Ser. B, 87 (2011); PTEP 2012, 01A105 (2012).

Lattice QCD setup



★N_f=2+1 full QCD

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- a=0.0907(13) fm --> L~2.9 fm (32^3 x 64)

★Relativistic Heavy Quark action for charm

S. Aoki et al., PTP109, 383 (2003).
Y. Namekawa et al., PRD84, 074505 (2011).

- remove leading cutoff errors $O((m_c\, a)^n)$, $O(\Lambda_{QCD}\, a)$, ...

➡ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

Light meson mass (MeV)

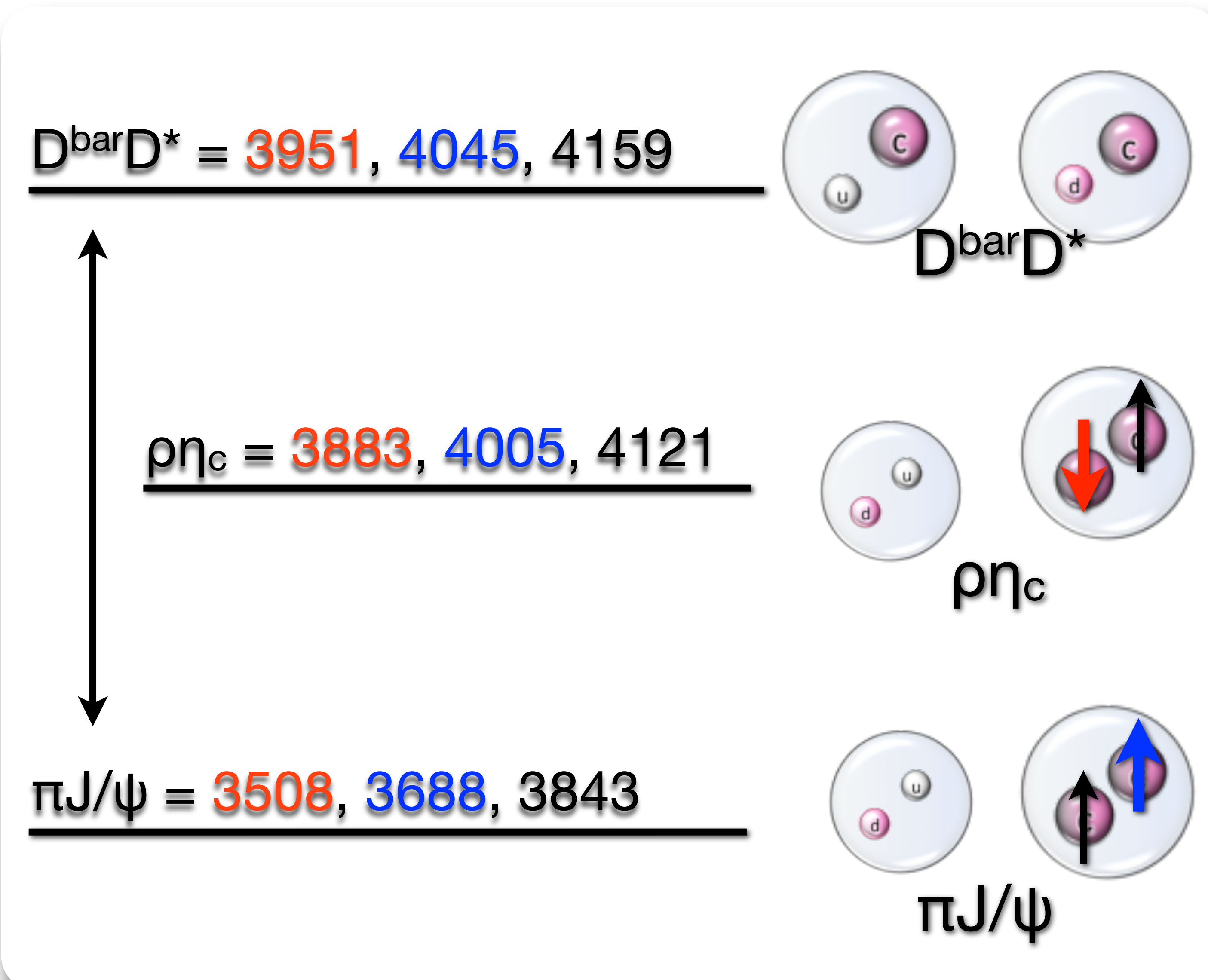
m_π= 411(1), 572(1), 701(1)
m_K= 635(2), 714(1), 787(1)
m_ρ= 896(8), 1000(5), 1097(4)

Charmed meson mass (MeV)

$m_{\eta c}$= 2988(1), 3005(1), 3024(1)
$m_{J/\psi}$= 3097(1), 3118(1), 3143(1)
m_D= 1903(1), 1947(1), 2000(1)
m_{D^*}= 2056(3), 2101(2), 2159(2)

➡ This talk : LQCD results at m_π=410MeV will be shown

Potential matrix : $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ in $I^G(J^P)=1^+(1^+)$



✓ Velocity expansion:

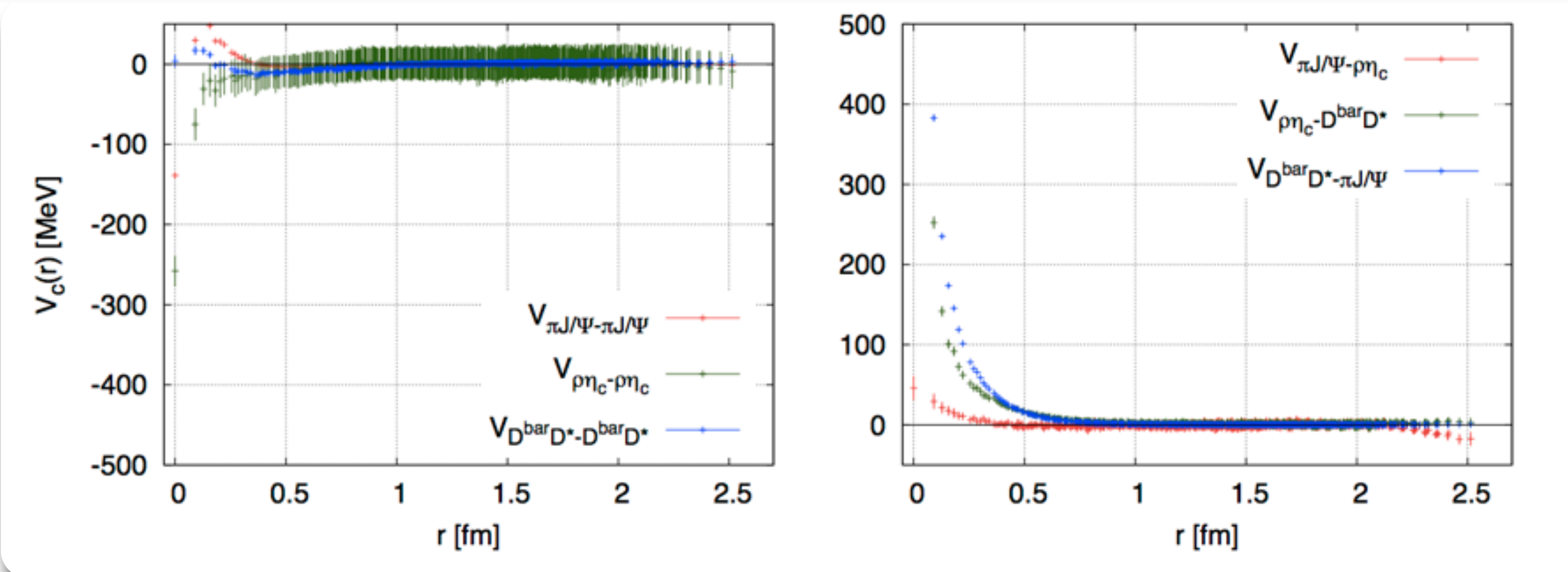
$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

$$V(\vec{r},\nabla) = V_{\rm LO}(\vec{r}) + \mathcal{O}(\nabla)$$

Extract (effective) LO potential :

$$\left(\nabla^2 + (\vec{k}^a_n)^2\right)\psi^a_n(\vec{r}) = 2\mu^a \sum_b V^{ab}(\vec{r})\psi^b_n(\vec{r})$$

Potential matrix($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

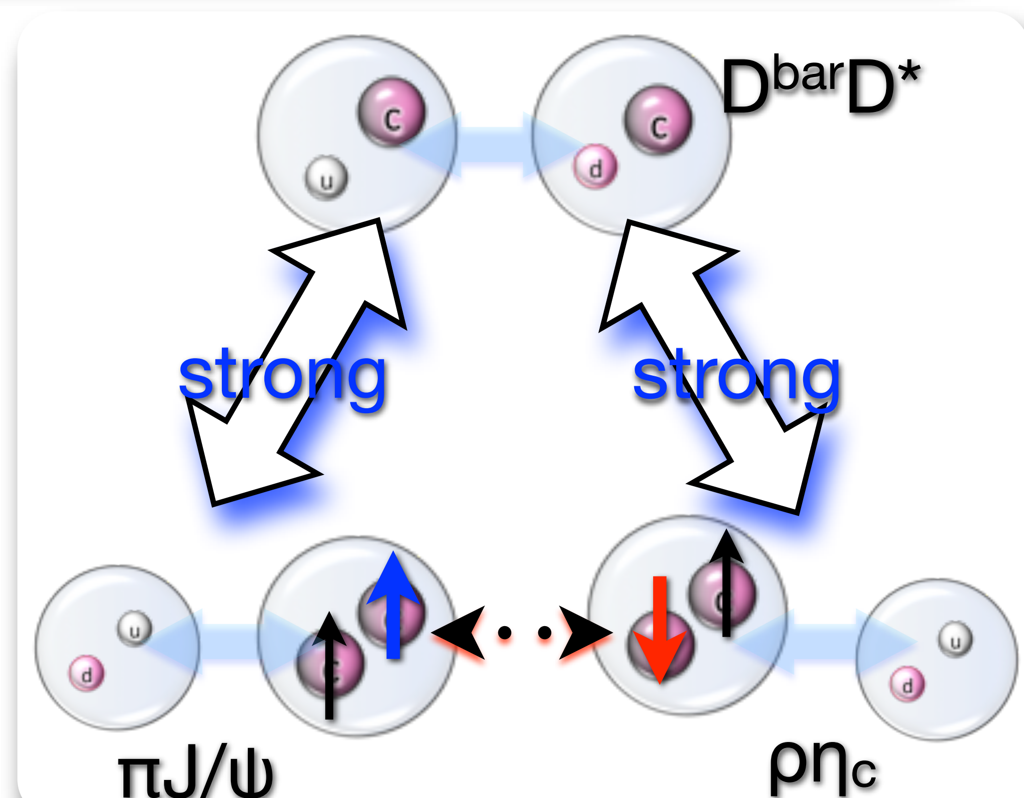


- **Weak** diagonal potentials
- **Weak** $\pi J/\psi$-$\rho\eta_c$ potential

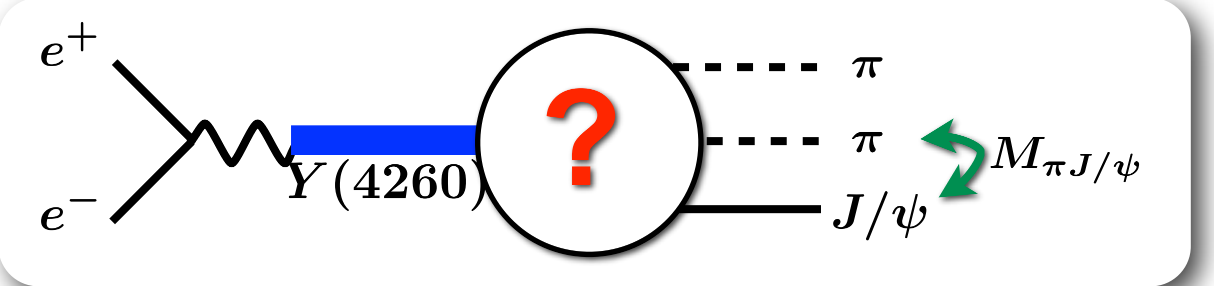
➡ Heavy quark spin symmetry

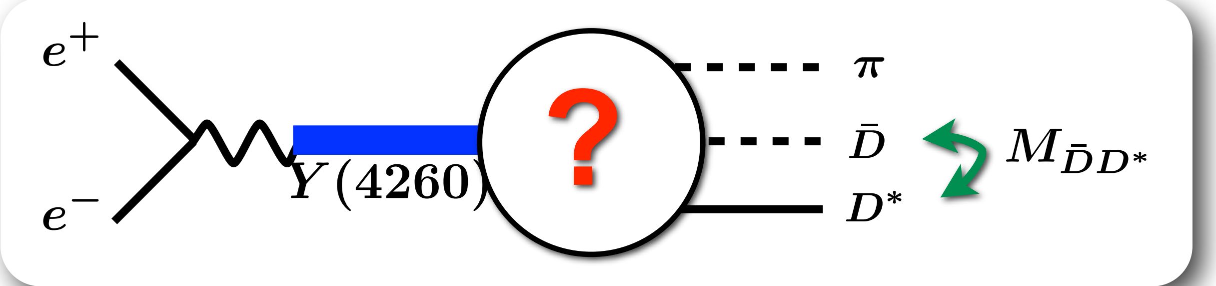
- **Strong** off-diagonal $D^{bar}D^*$ potentials

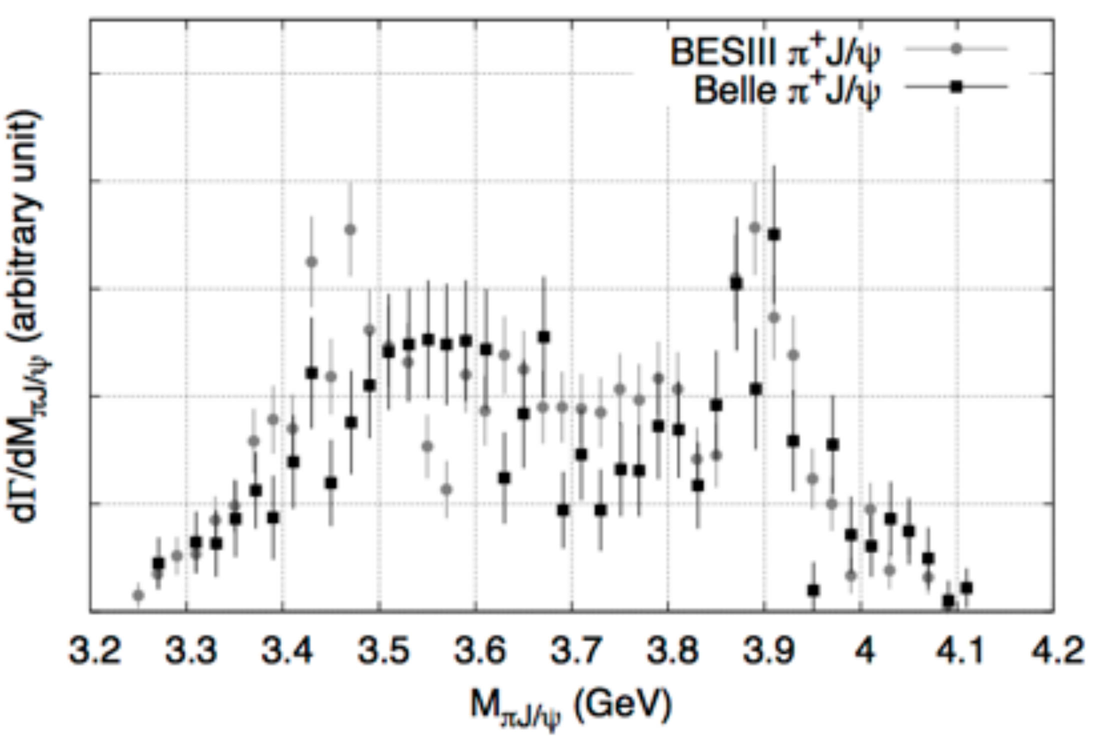
(Weak quark mass dependence)

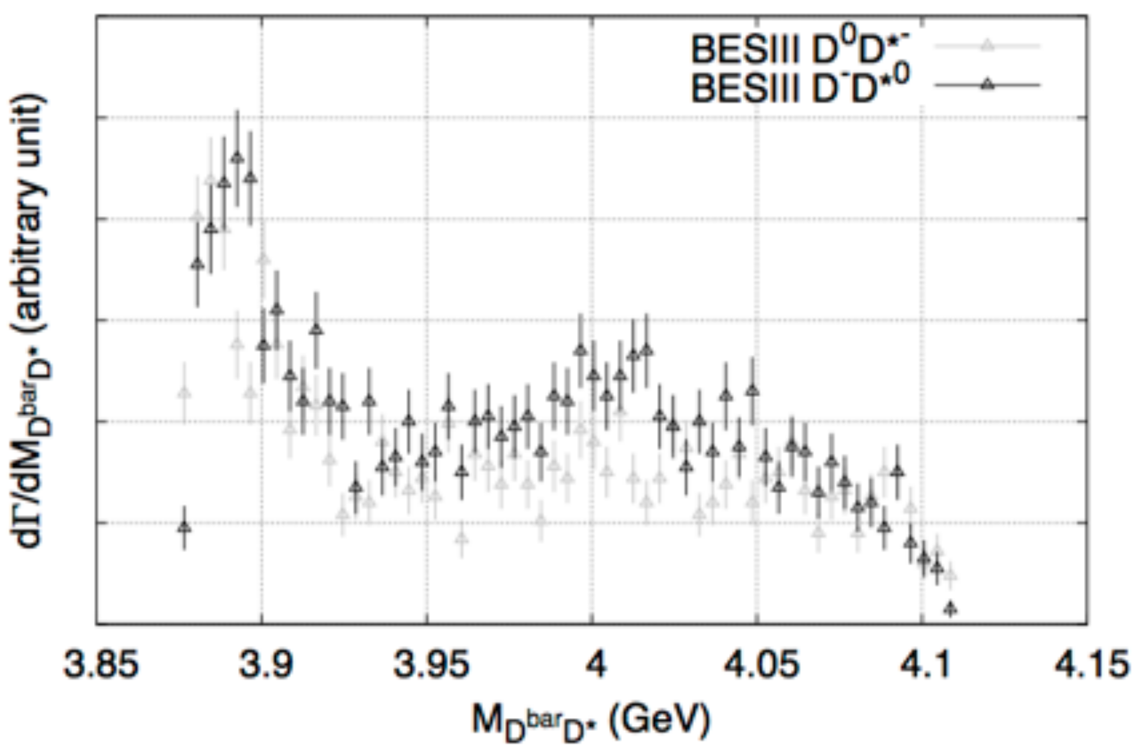


Y(4260) three-body decay : comparison with expt. data









BESIII Coll., PRL110, 252001, (2013).

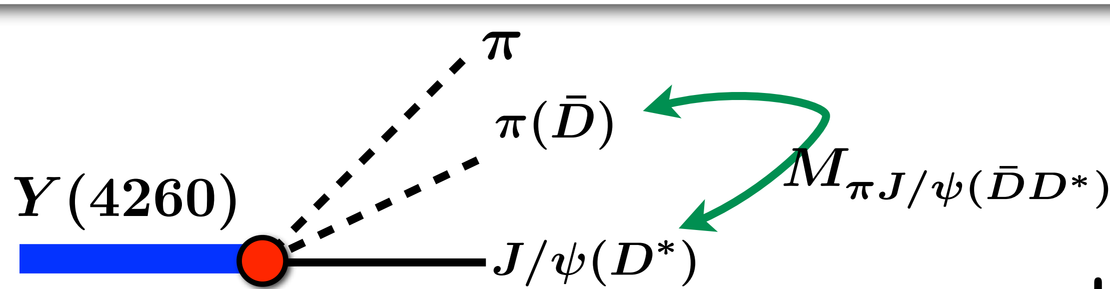
Belle Coll., PRL110, 252002, (2013).

BESIII Coll., PRL112, 022001, (2014).

✓ check whether expt. data of Y(4260) decay can be reproduced w/ LQCD potential

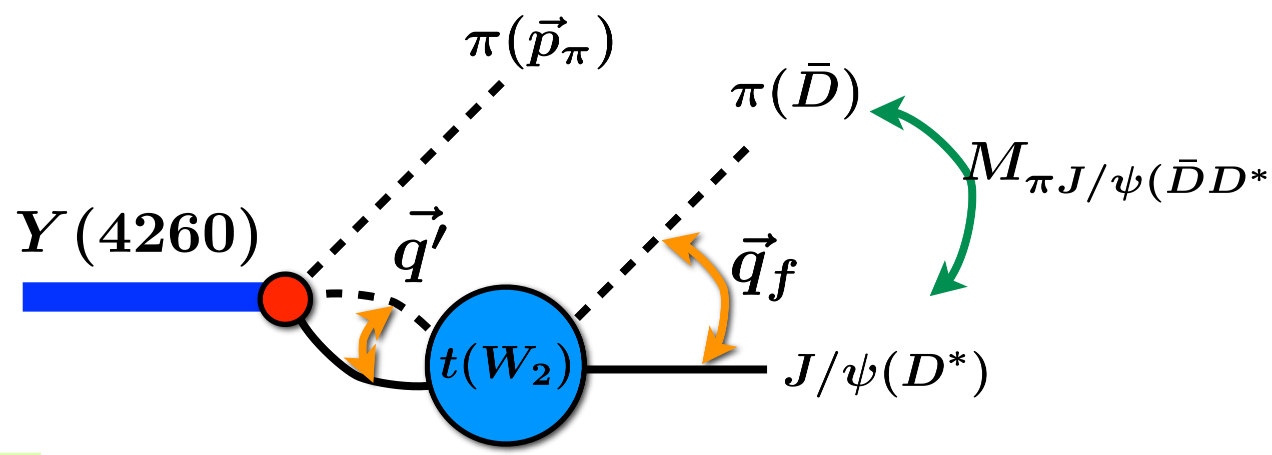
Three-body decay of Y(4260)

$$d\Gamma_f \propto (2\pi)^4 \delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f)) d^3p_\pi d^3q_f |T_f(\vec{p}_\pi, \vec{q}_f; W_3)|^2$$



➡ constant Y(4260) decay vertex is assumed

+



★ input $V^{LQCD}(r)$ --> *t*-matrix

➡ No free parameter

✔ Three-body amplitudes

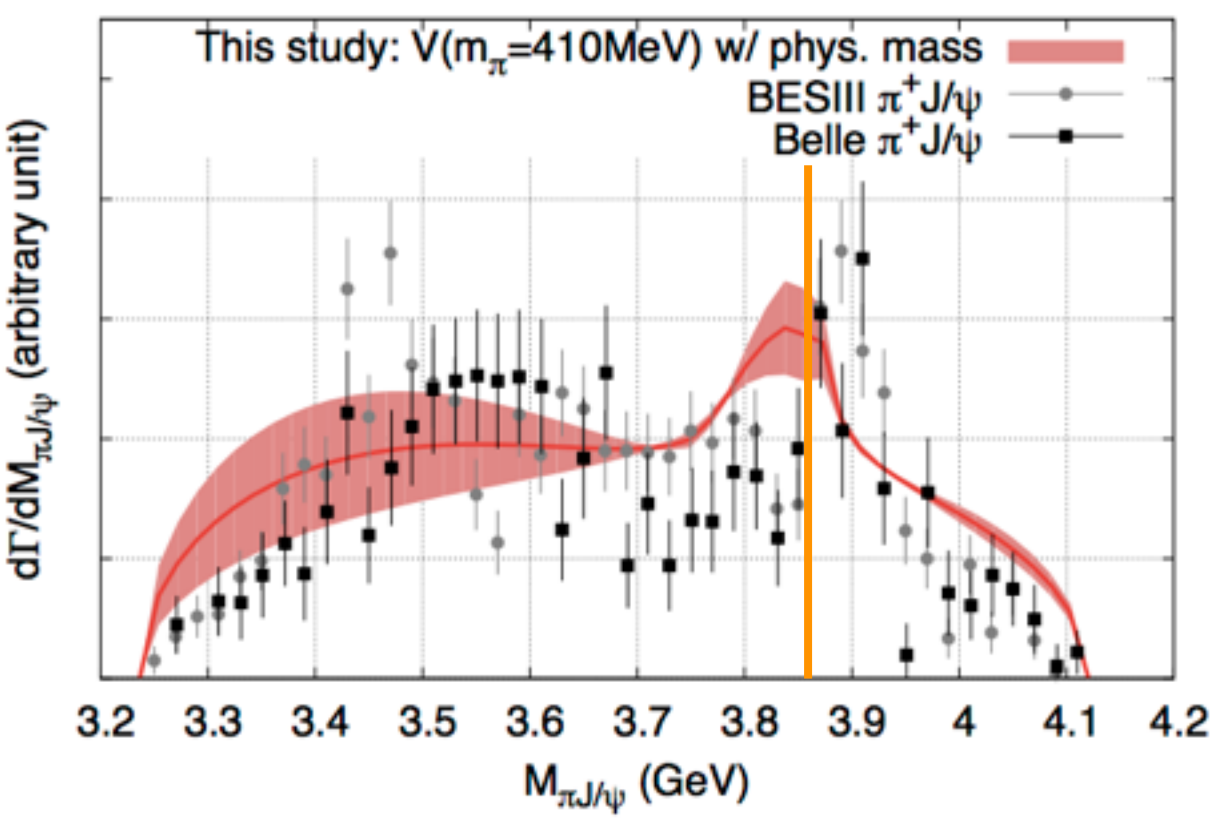
$$T_f(\vec{p}_\pi, \vec{q}_f; W_3) = \sum_{n=\pi\pi J/\psi, \pi\bar{D}D^*} C_n^{Y(4260)} \left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}', \vec{q}_f, \vec{p}_\pi; W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}', \vec{p}_\pi) + i\epsilon} \right]$$

<u>physical hadron masses employed</u> to compare w/ expt. data

✔ fix decay vertex by Y(4260) --> ππJ/ψ expt. data

✔ predict Y(4260) --> $\pi D^{bar} D^*$ decay spectrum

Mass spectra (πJ/ψ & $D^{bar}D^*$)



parameters: $C_{D^{bar}D^*}/C_{\pi J/\psi} = Re^{i\theta}$

--> R=0.95(18), θ=-58(44) deg. (+overall factor)

★ Zc(3900) production rate in expt.

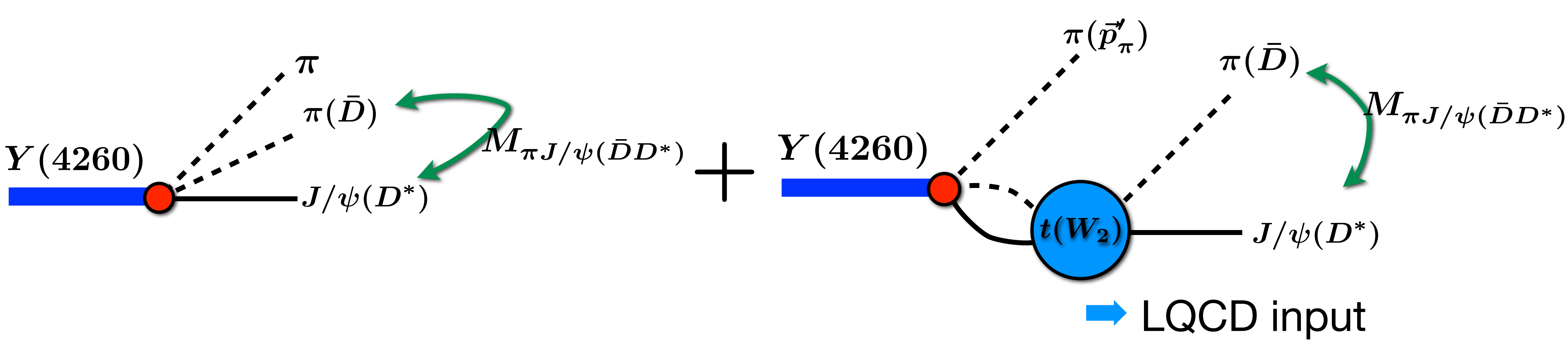
$$21.5 \pm 3.3\% \text{ (BESIII)}$$

$$29.0 \pm 8.9\% \text{ (Belle)}$$

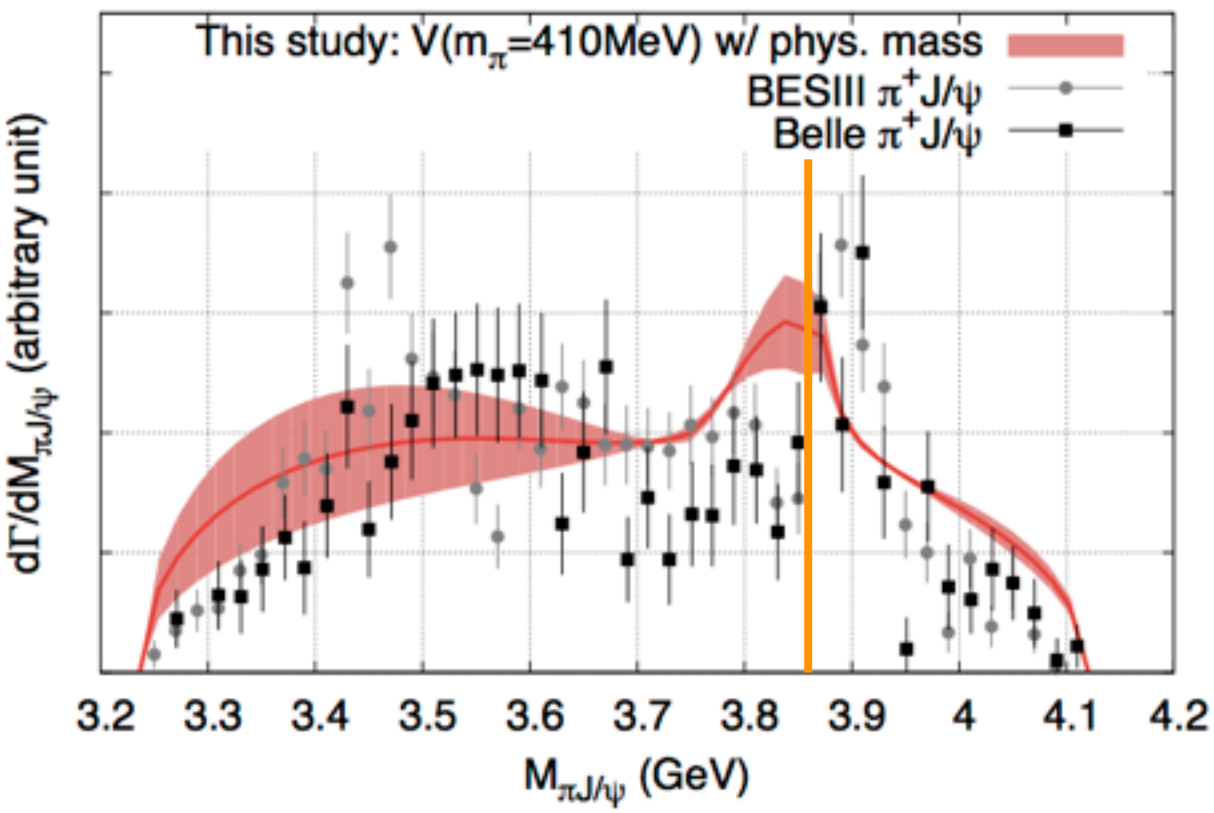
★ Calculation w/ LQCD potential

$$32 \pm 1\%$$

➡ fix Y(4260)-->ππJ/ψ, π$D^{bar}D^*$ decay vertex from πJ/ψ mass spectrum



Mass spectra (πJ/ψ & D^bar D*)



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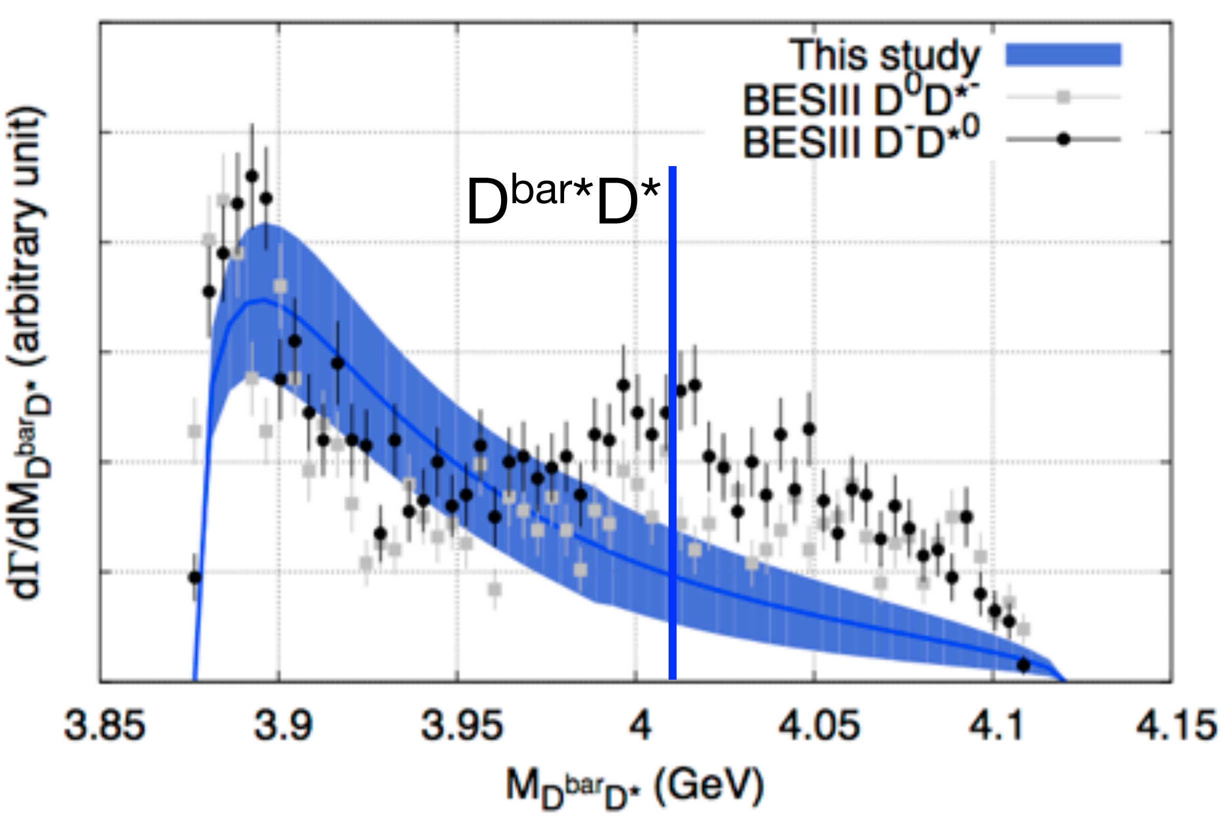
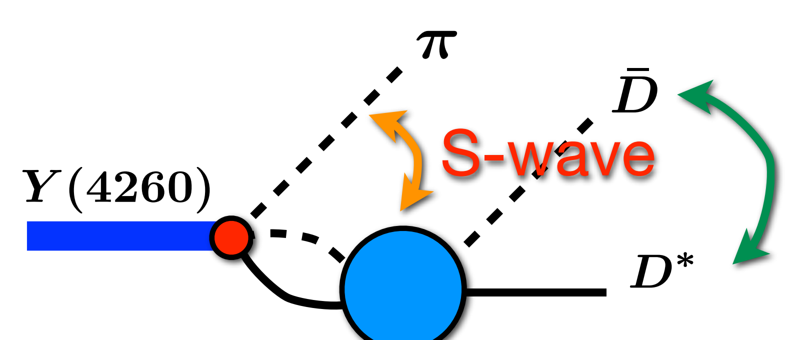


➡ Similar line shape

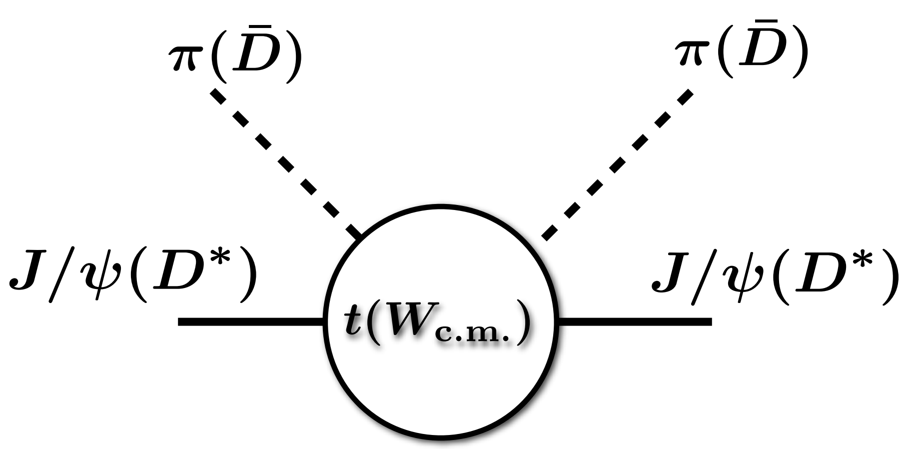
- Deviation from expt. data at high energies
  - explicit $D^{bar*}D^*$ channel coupling?
  - higher partial wave?



Two-body observables : structure of $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$

★Two-body $\pi J/\psi$ & $D^{bar}D^*$ s-wave scattering

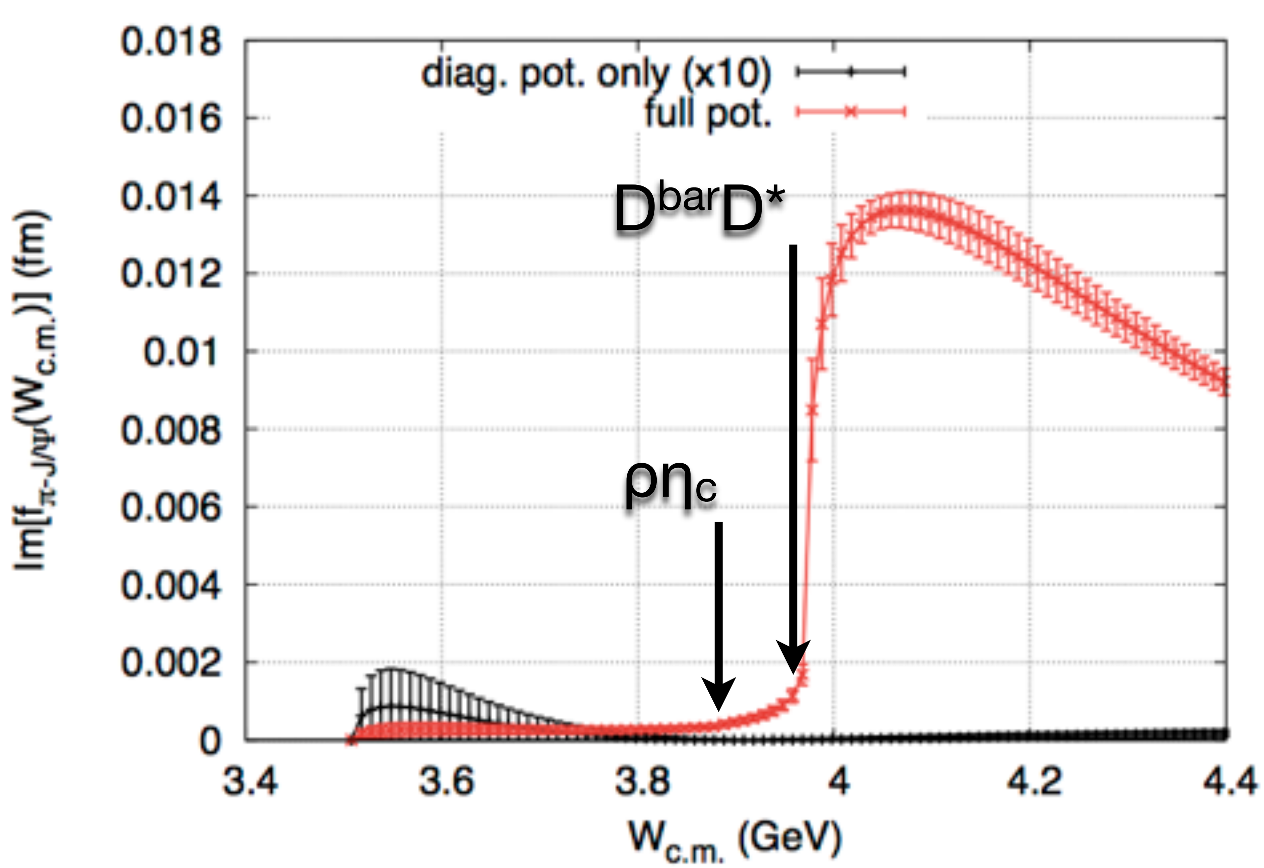
➡ideal setting for production reactions of $Z_c(3900)$



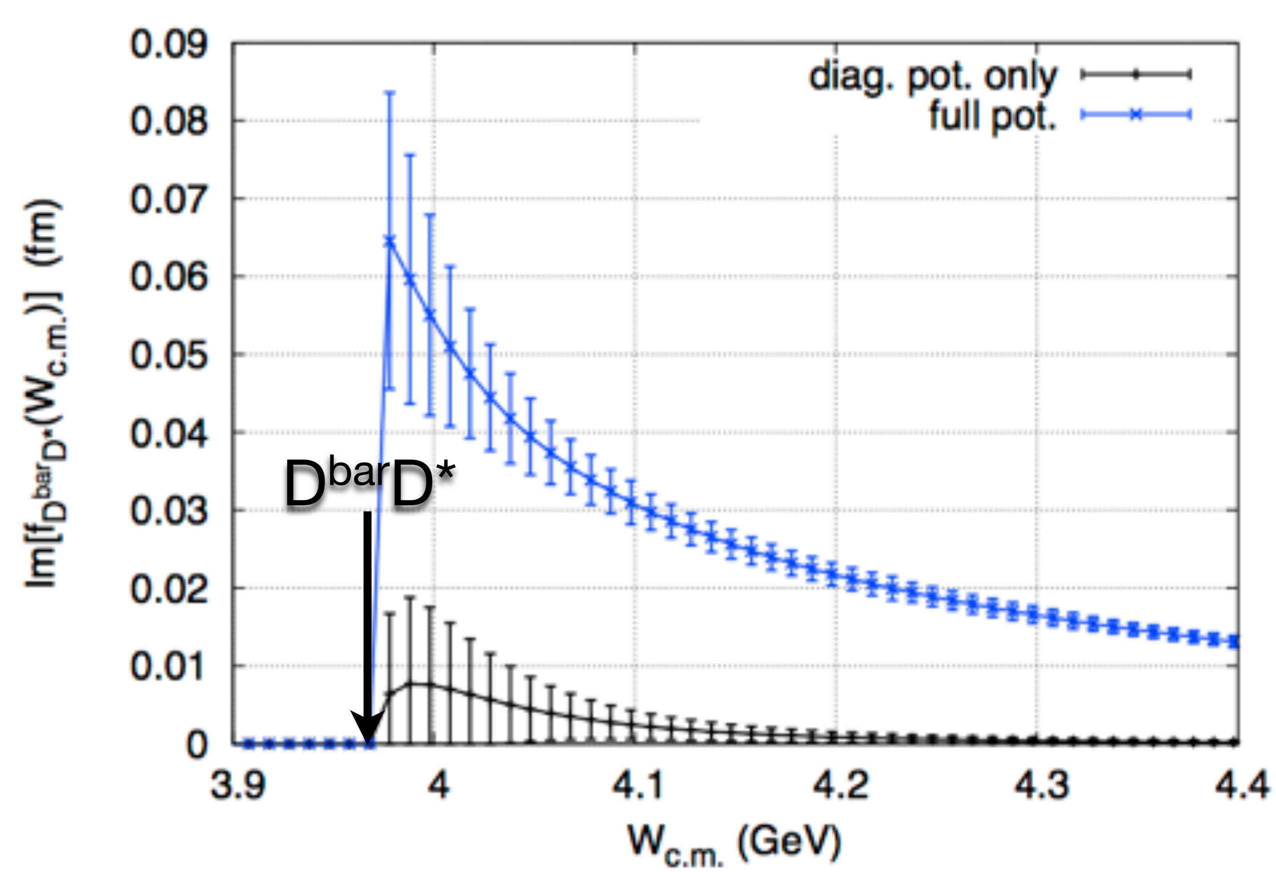
- Invariant mass distributions of $\pi J/\psi$ & $D^{bar}D^*$
- Analytic structure of amplitudes : pole positions of $Z_c(3900)$

Invariant mass spectra of $\pi J/\psi$ & $D^{bar}D^*$

- $\pi J/\psi$ invariant mass (m_π=410MeV)



- $D^{bar}D^*$ invariant mass (m_π=410MeV)



$$\frac{\Gamma(Zc(3900)\to\bar{D}D^*)}{\Gamma(Zc(3900)\to\pi J/\psi)} = 6.2(1.1)(2.7)\ (\mathbf{BESIII})$$

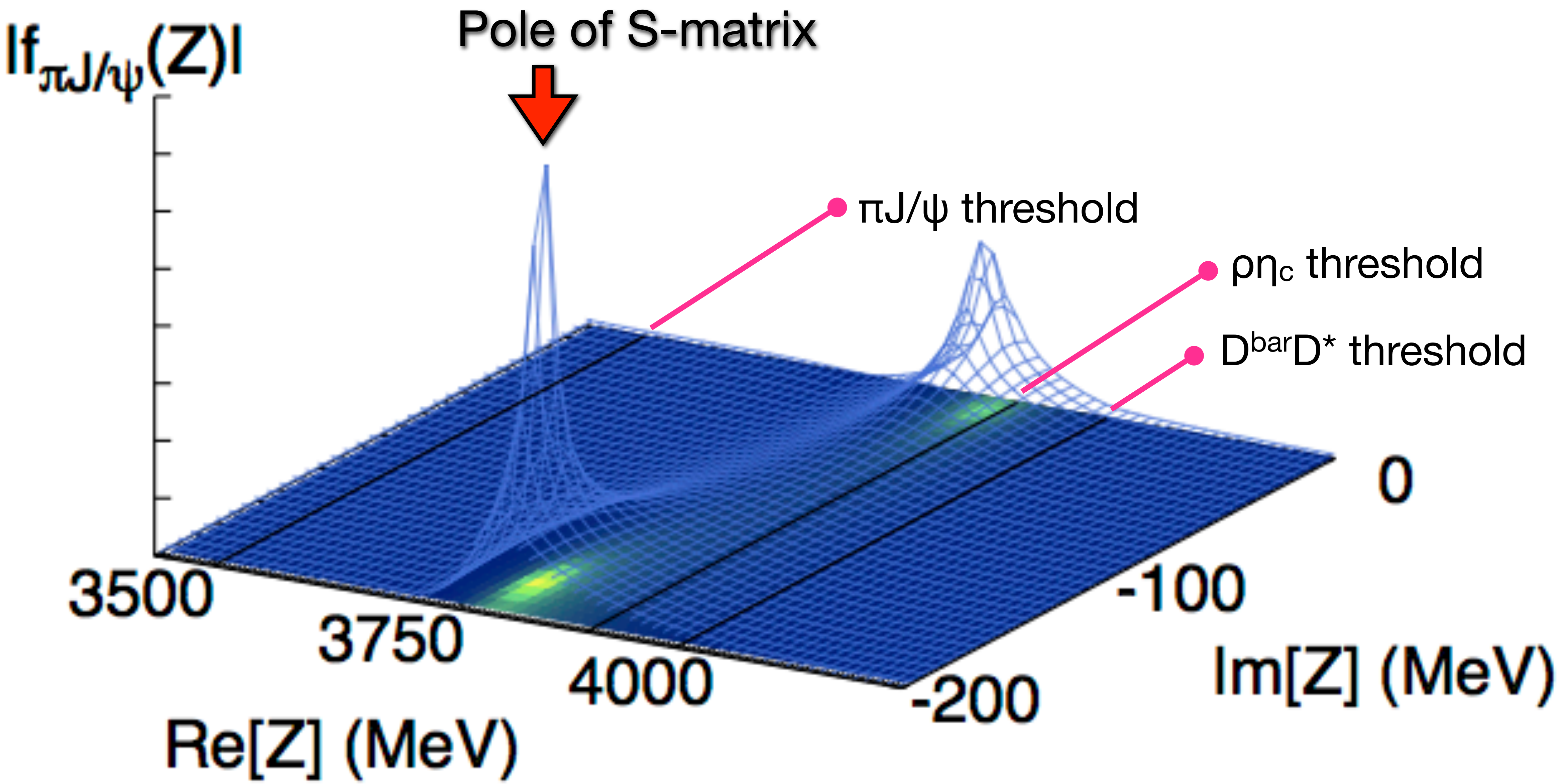
✓ Enhancement near $D^{bar}D^*$ threshold due to large $\pi J/\psi$-$D^{bar}D^*$ coupling

- Peak in $\pi J/\psi$ invariant mass (Not Breit-Wigner line shape)
- Threshold enhancement in $D^{bar}D^*$ invariant mass

✓ Is Z_c(3900) a conventional resonance? --> pole positions

Pole search ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)

✤ input : LQCD potential matrix @ m_π=410MeV



✓ "Virtual" pole on 2nd sheets is found (far from $D^{bar}D^*$ threshold)

✓ The pole position is 150MeV below $D^{bar}D^*$ threshold w/ large widths

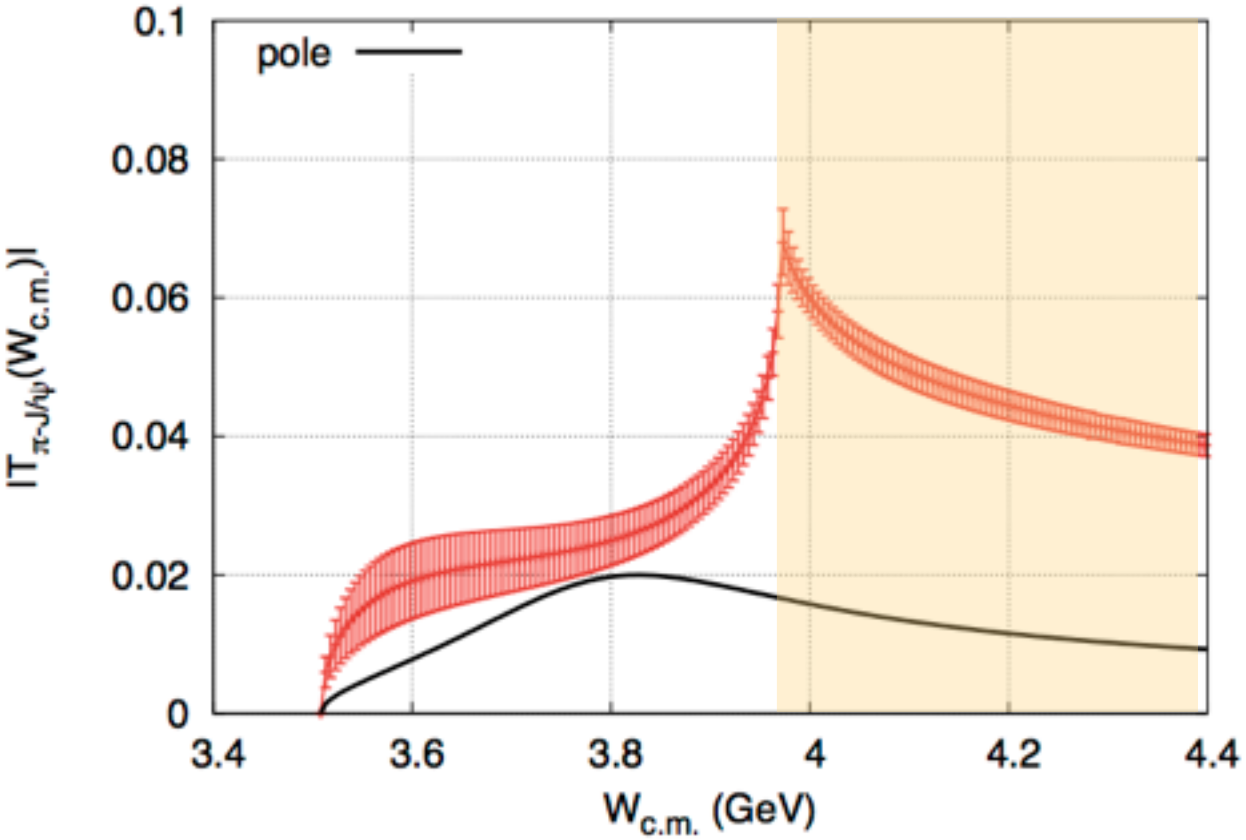
✓ How large is pole contribution to scattering amplitudes?

T-matrix of $\pi J/\psi$ & $D^{bar}D^*$

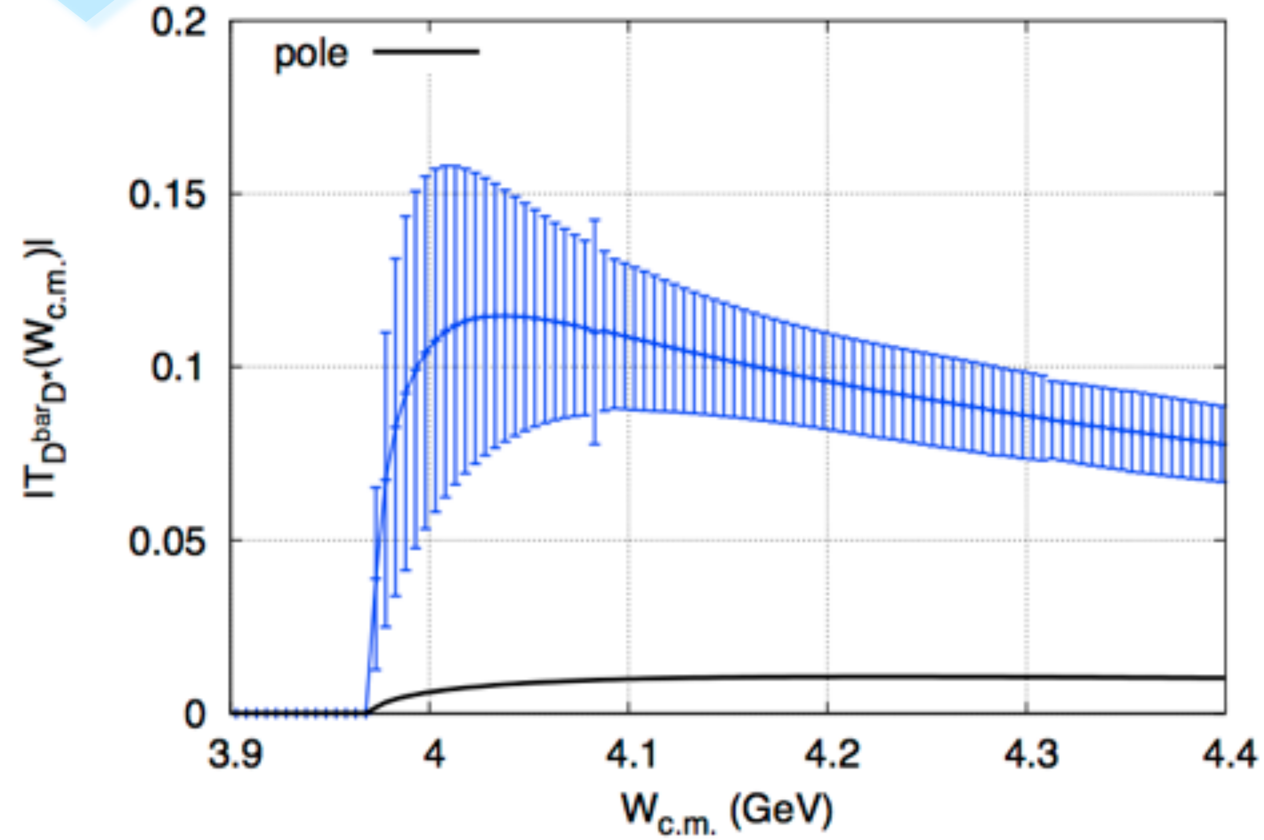
- calculate residues of T-matrices in $\pi J/\psi$ & $D^{bar}D^*$ channels

$$S(k) = 1 + 2iT(k)$$

- $\pi J/\psi$-$\pi J/\psi$ T-matrix (m_π=410MeV)



- $D^{bar}D^*$-$D^{bar}D^*$ T-matrix (m_π=410MeV)



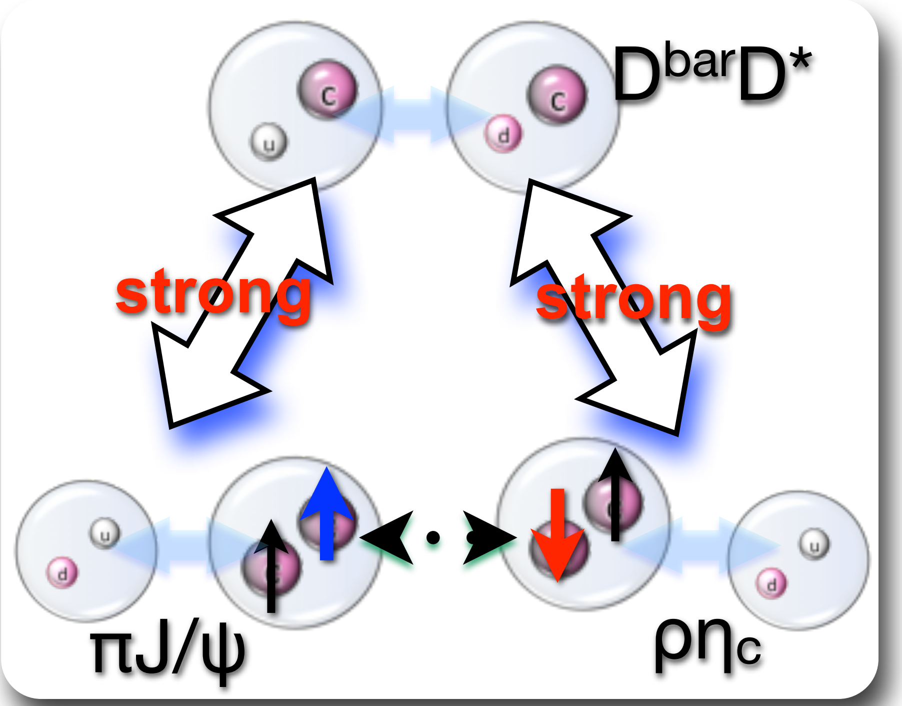
- Cusp at $D^{bar}D^*$ threshold in $\pi J/\psi$ T-matrix
- Pole contributions less than 10% in $D^{bar}D^*$ T-matrix

➡ Cusp scenario is favored (for m_π>400MeV)

Summary

✦ $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$ channel on the lattice for $m_\pi>400$MeV

- ★ Large channel coupling between $\pi J/\psi$ and $D^{bar}D^*$ is a key
- ★ Enhancement at $D^{bar}D^*$ threshold in 2-body amplitudes
- ★ Heavy quark spin symmetry is observed in c.c. potentials
  - $Z_c(3900)$ is neither simple $D^{bar}D^*$ molecule nor J/ψ + π-cloud
  - Shadow poles are found (very far from $D^{bar}D^*$ threshold...)
  - Cusp scenario is favored



✤ Physical point simulation is the next step

✤ Future targets

- other systems : X(3872), P_c^+ ($J/\psi N$ - $\Lambda_c D^*$ - $\Sigma_c D^*$ - $\Sigma_c^* D^*$)
- extension to bottom systems