Recent Studies on Heavy-quark Hadrons

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Confinement
&
Heavy-quark potential

Nonperturbative QCD

QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}$$

$$D_\mu = \partial_\mu - iA^a_\mu t^a, \ a = 1, \cdots 8$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$$

Gauge invariance

$$\psi \to S\psi$$

$$A_\mu \to SA_\mu S^{-1} + iS\partial_\mu S^{-1}$$

Nonperturbative QCD

QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}$$

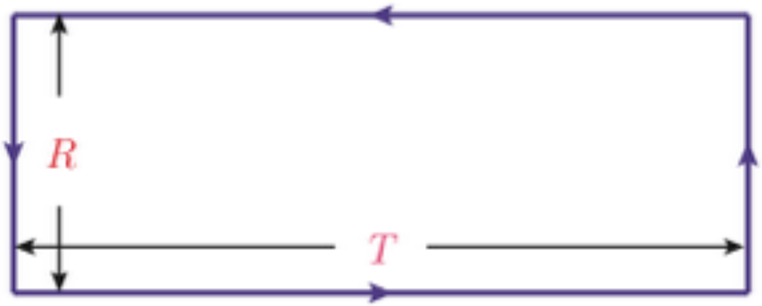
This classical Lagrangian looks simple
but has profound nonperturbative nature.

1.Confinement (Understood only qualitatively)

2.Chiral symmetry and its spontaneous breakdown

A clue about Quark Confinement

Wilson's criteria of the quark confinement



Heavy-quark propagator

$$W_J = \mathrm{Tr}\left[P \exp i \oint dx^\mu A^a_\mu t^a_J\right]$$

$$\langle W_J \rangle = \exp\left[-V(R)T\right] \quad \text{at } T \to \infty$$

$Q\bar{Q}$ potential at separation R

Wilson's Area Law

$$W \sim \exp(-\sigma \mathrm{Area})$$

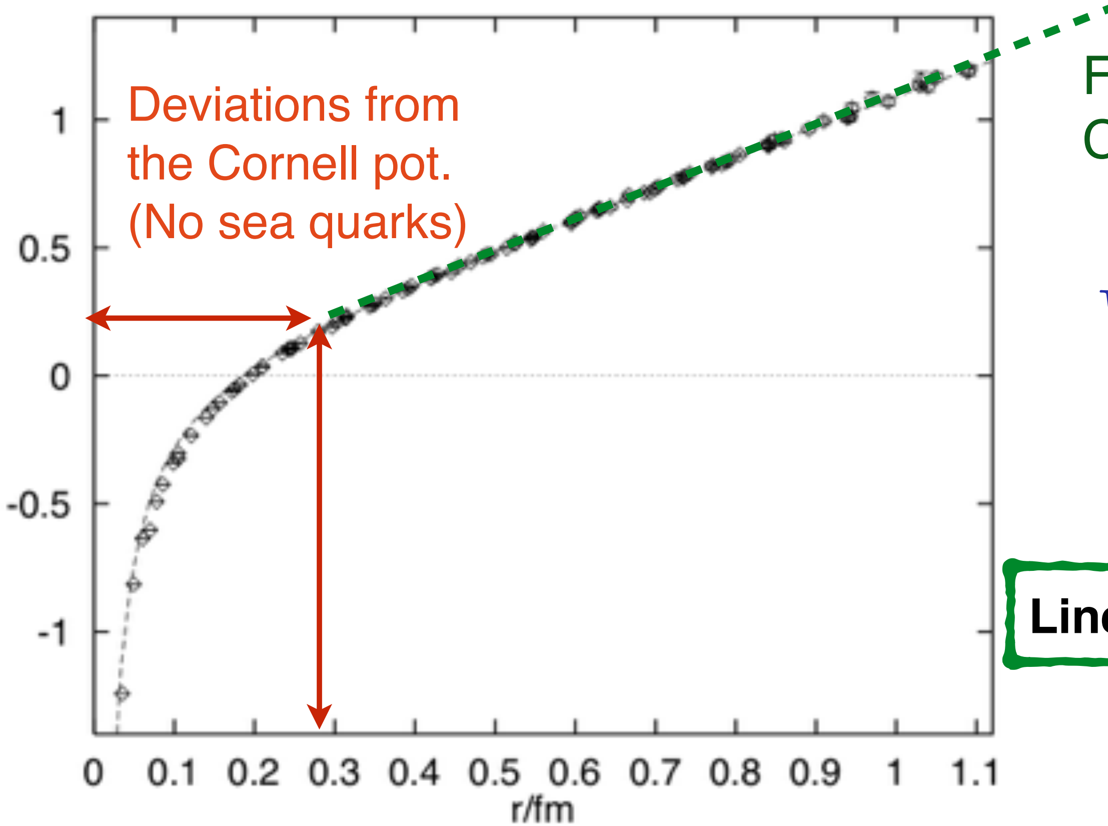
⇒ a linearly rising potential

$$V(R) \sim \sigma R$$

String tension $\sqrt{\sigma} = 420\,\mathrm{MeV}$

Central Q-Qbar potential

In the limit of infinitely heavy quark mass



Lattice results

Fitted to the phenomenological Cornell potential $r \geq 3\,\mathrm{fm}$

$$V_{\mathrm{Cornell}} = \sigma r + \frac{\kappa}{r}$$

$\sqrt{\sigma} = 420\,\mathrm{MeV}$

Linear confining potential

Coulomb-type potential from one-gluon exchange

G. S. Bali, K. Schilling, A. Wachter, hep-lat/9506017 (RCNP, Confinement 1995)

Motivation 1

The ground state of bottomonia: $\eta_b(1S)$

$$m_{\eta_b} = 9394.2^{+4.8}_{-4.9}(\text{stat}) \pm 2.0(\text{syst})\,\text{MeV}/c^2$$

It was first found by the BABAR collaboration in 2009 and was confirmed by the CLEO collaboration.

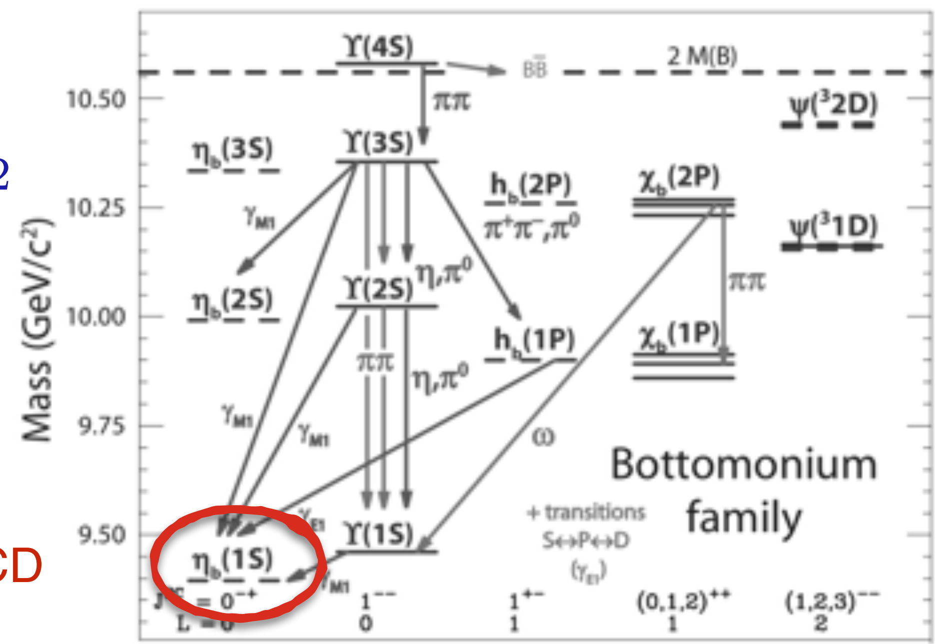
BABAR, PRL 103 (2009) 161801, CLEO, PRD 81 (2010) 031104

pQCD prediction

$$m_{\eta_b} = 9419 \pm 11(\text{th})^{+9}_{-8}\,\text{MeV}/c^2$$

B. A. Kniehl et al., PRL 92 (2004) 242001

Discrepancy between experiments and pQCD



Motivation 1

The ground state of bottomonia: $\eta_b(1S)$

$$m_{\Upsilon}(1S) - m_{\eta_b} = 66.1^{+4.9}_{-4.8} \pm 2.0\,\mathrm{MeV}/c^2$$

It was first found by the BABAR collaboration in 2009 and was confirmed by the CLEO collaboration.

BABAR, PRL 103 (2009) 161801, CLEO, PRD 81 (2010) 031104

Full Lattice prediction (including light-quark vacuum polarizations)

$$m_{\Upsilon}(1S) - m_{\eta_b} = 61 \pm 14\,\mathrm{MeV}/c^2$$ Consistent with experiments

Gray etal. PRD72 (2005) 094507

Certain nonperturbative effects should come into play!

(They may be more important than confinement for low-lying charmonia.)

Motivation 2

Many exotic heavy-light quark hadrons were newly found (XYZ mesons) and many new states will be measured.

We will present in this talk a recent preliminary result for the heavy quark potential from the instanton vacuum as a step toward constructing an effective action for heavy-light quark systems.

Light-quark sector

Instantons
&
SχSB

Effective Partition function

QCD partition function

$$\mathcal{Z}_{\rm QCD} = \int DA_\mu D\psi D\psi^\dagger \exp\left[\sum_{f=1}^{N_f}\int d^4x \psi_f^\dagger (i\not{D} + im_f)\psi_f - \frac{1}{4g^2}\int d^4x G^2\right]$$

$$= \int DA_\mu \exp\left[-\frac{1}{4g^2}\int d^4x G^2\right] \mathrm{Det}(i\not{D} + im_f)$$

Integrating over gluons means averaging the partition function over (anti-)instantons

$$\Longrightarrow \quad \mathcal{Z}_{\rm eff} = \overline{\mathrm{Det}(i\not{D} + im_f)}$$

Instanton fields

$$A_\mu^a = 2\bar{\eta}^a_{\mu\nu}(x-z)_\nu \frac{\rho^2}{(x-z)^2[(x-z)^2+\rho^2]}$$

Zero-mode solution

Zero-mode equation

$$i\not{D}\Phi_n = \lambda_n \Phi_n$$

⇒ Zero modes $\lambda_0 = 0,\ \Phi_0$

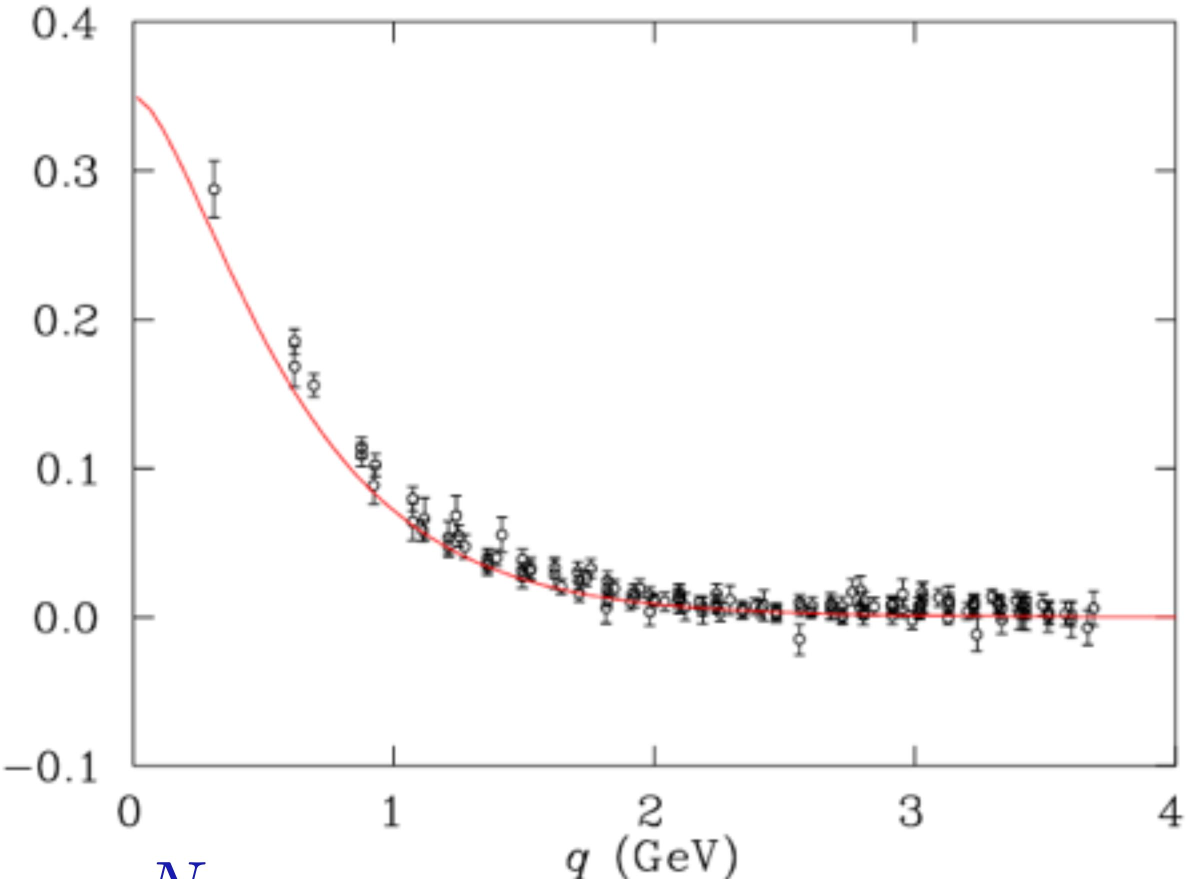
Fourier transform of the zero mode will bring about the momentum dependent quark mass.

Momentum-dependent quark mass *M(k)*

$$F(k\rho) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right]\bigg|_{t=k\rho/2}$$

Momentum-dependent quark mass

P. Bowman, U. Heller, D. Leinweber and A. Williams, hep-lat/0209129

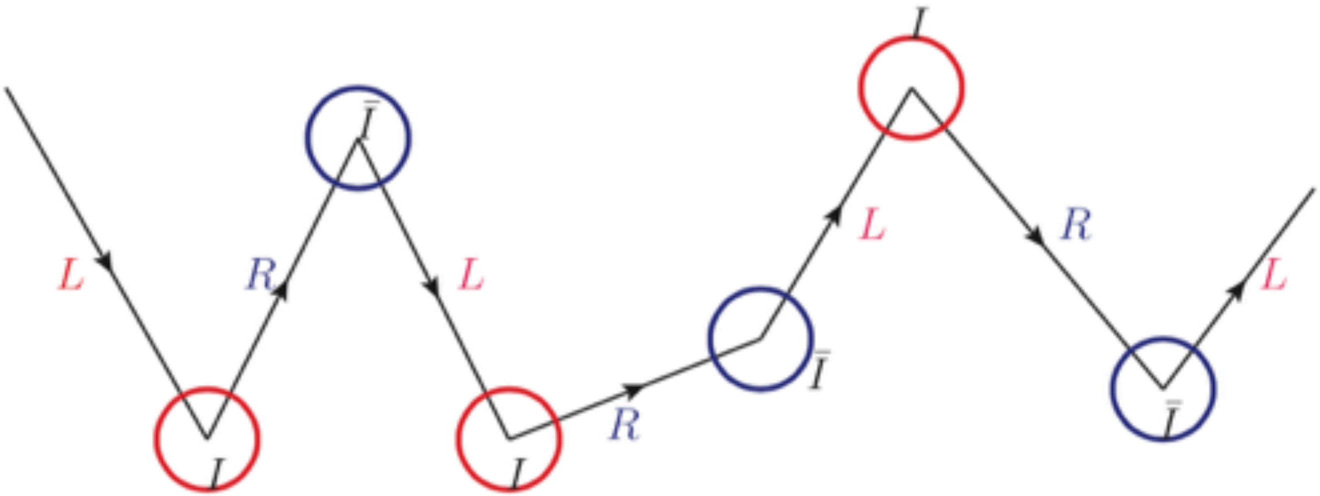


$$\frac{N}{V} \approx 1\,\mathrm{fm}^{-1}$$

$$\rho \approx 0.3\,\mathrm{fm}$$

⟹ $M(0) = 345\,\mathrm{MeV}$

Spontaneous Chiral Symmetry Breaking



Helicity of a light quark is flipped by hoping from instants to anti-instantons and vice versa. By doing that, the quark acquires the dynamical quark mass *M(p).*

$$S(p) = \frac{i}{\not{p} + iM(p^2)}$$

Nonzero quark condensate: $-i\langle\psi^\dagger\psi\rangle = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)} = -(253\,\mathrm{MeV})^3$

Eff. Chiral Action from the instanton vacuum

Effective QCD action from the instanton vacuum

$$\mathcal{Z} = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_{f=1}^{N_f} \bar{\psi}_f i\partial\!\!\!/ \psi^f\right) \left(\frac{Y_{N_f}^+}{VM_1^{N_f}}\right)^{N_+} \left(\frac{Y_{N_f}^-}{VM_1^{N_f}}\right)^{N_-}$$

$$Y_{N_f}^+ = \int d\rho\, d(\rho) \int dU \prod_{f=1}^{N_f} \left\{ \int \frac{d^4 k_f}{(2\pi)^4} \left[2\pi\rho F(k_f\rho)\right] \int \frac{d^4 l_f}{(2\pi)^4} \left[2\pi\rho F(l_f\rho)\right] \right.$$

$$\left. \cdot (2\pi)^4 \delta(k_1 + \ldots + k_{N_f} - l_1 - \ldots - l_{N_f}) \cdot U_{i'_f}^{\alpha_f} U_{\beta_f}^{\dagger j'_f} \epsilon^{i_f i'_f} \epsilon_{j_f j'_f} \left[i\psi_{L f \alpha_f i_f}^\dagger (k_f) \psi_L^{f\beta_f j_f}(l_f) \right] \right\}.$$

$d(\rho)$: instanton distribution, U: Color orientation

After integrating over zero modes and bosonizing, we get the effective chiral action:

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \log \left[i\partial\!\!\!/ + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

Heavy-quark sector

&

Instantons

Heavy-quark propagator

Decompose the QCD Lagrangian

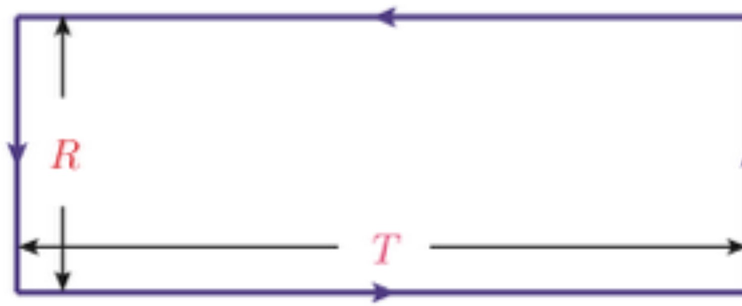
$$\mathcal{L}_{\rm QCD} = q^\dagger (i\not{D} + im) q + Q^\dagger (i\not{D} + iM) Q - \frac{1}{4g^2} G^2$$

Foldy-Wouthuysen transformation (Heavy-quark expansion)

$$\Rightarrow \quad \mathcal{L}_{\rm eff} = \bar{Q}_v \left[iv\cdot D - i\not{D}_\perp \frac{1}{2m_Q + iv\cdot D} i\not{D}_\perp \right] Q_v$$

Wilson-loop as a heavy-quark propagator

$$W = \mathrm{Tr}\left[P \exp i \oint dx_\mu \sum_{I\bar{I}} A_\mu^I \right]$$



Heavy-quark propagator

$$W(C) = \langle T| \left(\frac{d}{dt} - \sum_I a_I \right)^{-1} |0\rangle \qquad a_I = iA_{I\mu}[x(t)]\dot{x}_\mu(t)$$

Average over instanton ensemble (average over all positions and orientations of instantons)

$$w = \left\langle \left\langle \left(\theta^{-1} - \sum_I a_I \right)^{-1} \right\rangle \right\rangle, \qquad \theta^{-1} = \frac{d}{dt}$$

⇓

$$w^{-1} = \theta^{-1} - \frac{N}{2VN_c} \mathrm{Tr}_c \left[\int d^4 z_I \theta^{-1}(w_I - \theta)\theta^{-1} + (I \to \bar{I}) \right] + \mathcal{O}((N/VN_c)^2)$$

$$w_I = (\theta^{-1} - a_I)^{-1}$$

D. Diakonov, V. Petrov, P. Pobylitsa, PLB **226**, 372 (1989)

Corrections to the heavy quark mass

Taking a limit $T \to \infty$

$$\mathrm{Tr} P \exp\left[i \int_0^T A_4 dx_4 \right] \sim \exp[-\Delta M T]$$

$$\Delta M = -\frac{N}{2VN_c} \int dt \int dt' \int d^3 z_I \mathrm{Tr}_c \langle t | \theta^{-1} (w_I - \theta) \theta^{-1} | t' \rangle \Big|_{z_{I4}=0} + (I \to \bar{I})$$

$$\Delta M = \frac{N}{2VN_c} \int d^3 z_I \mathrm{Tr}_c \left[1 - P \exp\left(i \int_{-\infty}^{\infty} dx_4 A_{I4} \right) \Big|_{z_{I4}=0} \right] + (I \to \bar{I})$$

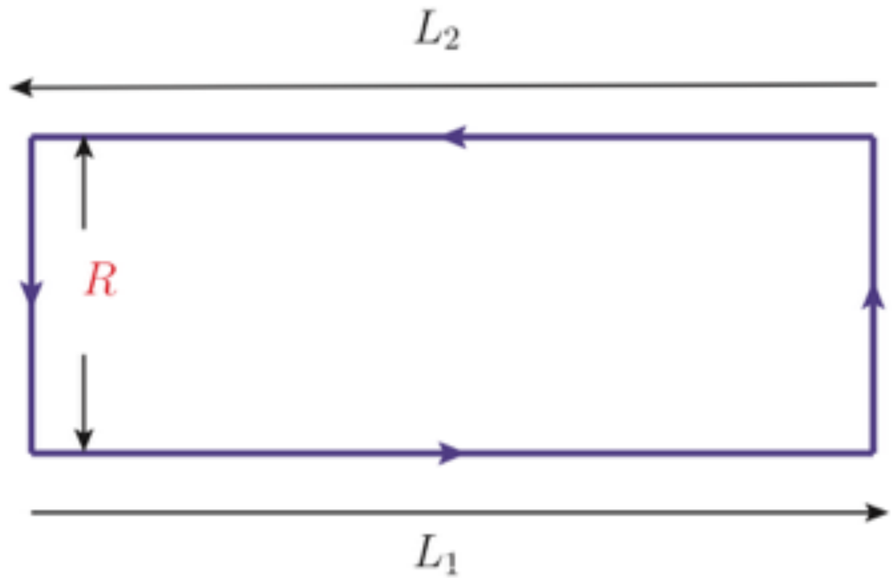
Put here the instanton solution

$\Delta M \simeq 70\,\mathrm{MeV}$: Spin-independent

D. Diakonov, V. Petrov, P. Pobylitsa, PLB **226**, 372 (1989)

Heavy-quark potential

$$W(L_1 L_2) \sim \exp(-V(R)T)$$

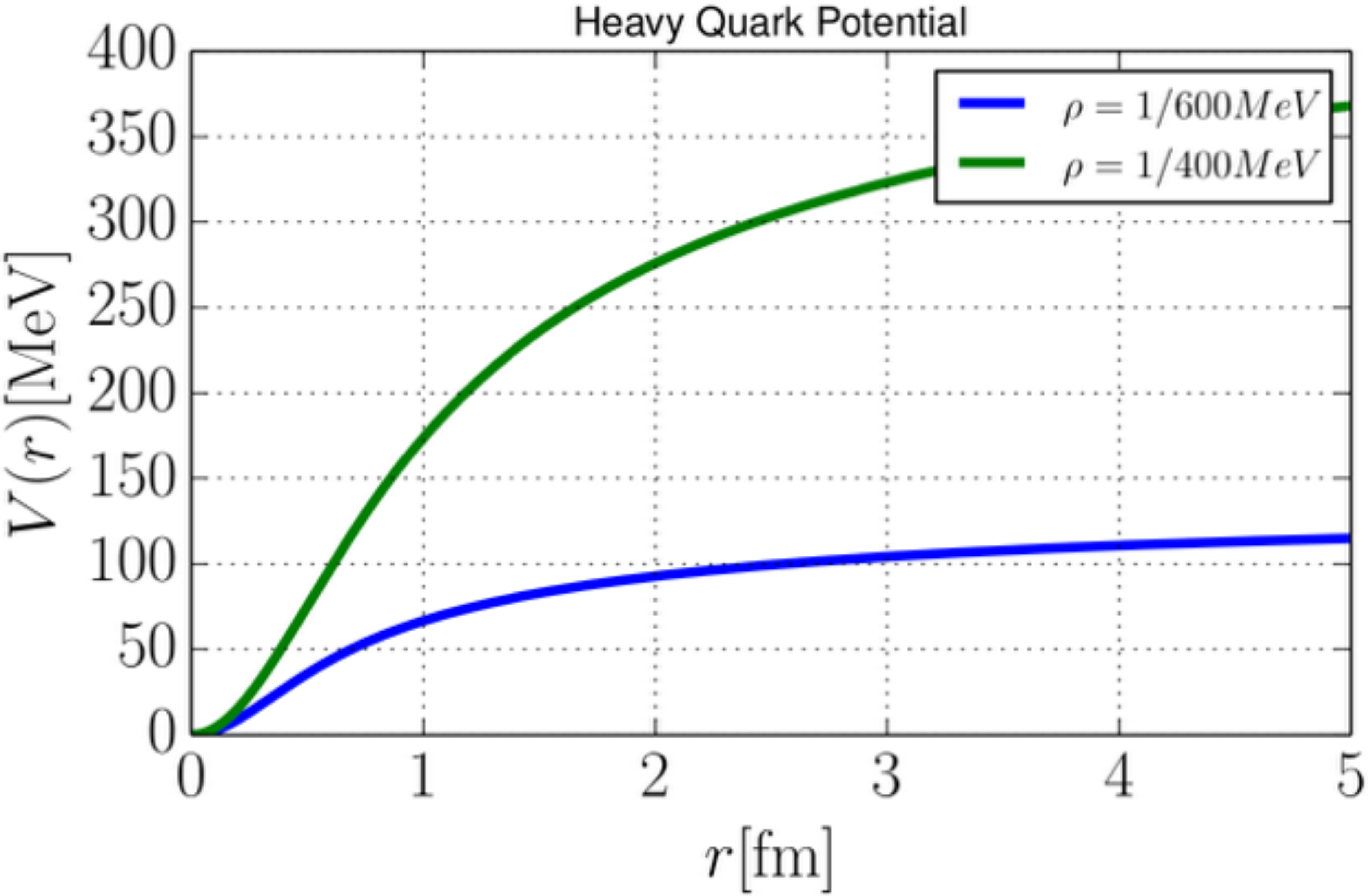


$$V(R) = \frac{N}{2VN_c} \int d^3 z_I \mathrm{Tr}_c \left[1 - P\exp\left(i\int_{L_1} dx_4 A_{I4}\right) P\exp\left(-i\int_{L_2} dx_4 A_{I4}\right)\right] + (I \to \bar{I})$$

$$V(0) = 0,\ V(\infty) = 2\Delta M$$

D. Diakonov, V. Petrov, P. Pobylitsa, PLB **226**, 372 (1989)

Instanton effects on heavy quark potential



Heavy-quark propagator

Decompose the QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}(i\not{D} - m)q + \boxed{\bar{\Psi}(i\not{D} - M_Q)\Psi} - \frac{1}{4g^2}G^2$$

$$\Psi(x) = e^{-iM_Q v\cdot x}[Q_v(x) + h_v(x)], \qquad \not{D} = \not{v}(v\cdot D) + \not{D}_\perp$$

Foldy-Wouthuysen transformation (Heavy-quark mass expansion)

$$\mathcal{L}_{\mathrm{HQET}} = \bar{Q}_v(x)\boxed{\left(iv\cdot D - i\not{D}_\perp \frac{1}{2M_Q + iv\cdot D} i\not{D}_\perp\right)} Q_v(x)$$

Inverse of the heavy quark propagator

Heavy-quark propagator

$$\left(iv\cdot D - i\not{D}_\perp \frac{1}{2M_Q + iv\cdot D} i\not{D}_\perp\right) S(x,y,A) = \delta^4(x-y)$$

Leading heavy-quark propagator

$$(iv\cdot D) S_0(x,y,A) = \delta^4(x-y) \qquad v_\mu = (1,\mathbf{0})$$

$$S_0(x,y,A) = i\theta(x_0 - y_0) P \exp\left(i\int_{y_0}^{x_0} dz_4 A_4\right)\delta^3(\mathbf{x}-\mathbf{y})$$

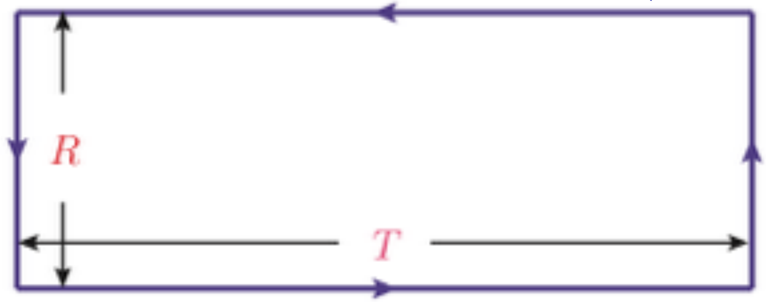
Effective full propagator as an integral equation

$$S(x,y,A) = S_0(x,y,A) - \int d^4z S_0(x,z,A)\left[i\not{D}_\perp \frac{1}{2M_Q + iv\cdot D} i\not{D}_\perp\right] S(z,y,A)$$

Heavy-quark propagator

Wilson-loop as a heavy-quark propagator

$$W = \mathrm{Tr}\left[P \exp i \oint dx_\mu \sum_{I\bar{I}} A_\mu^I\right]$$



$$W = \mathrm{Tr}\left[S\left(x_2, y_2, -i\frac{\delta}{\delta J}\right) P(y_1, y_2)\bar{\Gamma} S\left(y_1, x_1, -i\frac{\delta}{\delta J}\right) P(x_2, x_1)\Gamma\right] Z[J]\Big|_{J=0}$$

$$P(x, y) = P \exp\left(i \int_x^y \mathrm{d}z^\mu A_\mu\right)$$

$$\mathcal{Z}[J] = \int DA_\mu \exp i \int d^4x \left[-\frac{1}{4}(G_{\mu\nu}^a)^2 + J_a^\mu A_\mu^a\right]$$

E. Eichten and F. Feinberg, PRD 23, 2724 (1981).

Heavy-quark potential

As $m_Q \to \infty$

$$W = \mathrm{Tr}[\Gamma\bar{\Gamma}w]\delta(\mathbf{x}_1 - \mathbf{y}_1)\delta(\mathbf{x}_2 - \mathbf{y}_2)$$

$$\begin{aligned} w = \langle 1\rangle &- \frac{1}{4m_Q^2}\int_{-T/2}^{T/2} \mathrm{d}z\, \epsilon^{ijk}\sigma_1^k \big\langle E^i(z,\mathbf{x}_1)D^j(z,\mathbf{x}_1)\big\rangle + (1 \longrightarrow 2) \\ &- \frac{1}{4m_Q^2}\int_{-T/2}^{T/2}\mathrm{d}z\int_{-T/2}^{T/2}\mathrm{d}z'\sigma_1^i\big\langle B^i(z,\mathbf{x}_1)\mathbf{D}^2(z',\mathbf{x}_1)\big\rangle + (1 \longrightarrow 2) \\ &- \frac{1}{4m_Q^2}\int_{-T/2}^{T/2}\mathrm{d}z\int_{-T/2}^{T/2}\mathrm{d}z'\,\Big[\sigma_1^i\big\langle B^i(z,\mathbf{x}_1)\mathbf{D}^2(z',\mathbf{x}_2)\big\rangle \\ &+ \sigma_2^i\big\langle \mathbf{D}^2(z,\mathbf{x}_1)B^i(z',\mathbf{x}_2)\big\rangle + \sigma_1^i\sigma_2^j\big\langle B^i(z,\mathbf{x}_1)B^j(z',\mathbf{x}_2)\big\rangle\Big] \end{aligned}$$

$$\langle\mathcal{O}\rangle = \int DA_\mu \mathrm{Tr}_c P\left[\mathcal{O}\exp\left(i\oint_C \mathrm{d}z_\mu A_\mu(z)\right)\right]e^{iS_{YM}}$$

Heavy-quark potential with $1/M_Q^2$

$$V_{SD}(r) = \left(\frac{\sigma_1\cdot\mathbf{L}_1}{4m_Q^2} - \frac{\sigma_2\cdot\mathbf{L}_2}{4m_{\bar{Q}}^2}\right)\left(\frac{1}{r}\frac{dV(r)}{dr} + \frac{2}{r}\frac{dV_1(r)}{dr}\right)$$
$$+\left(\frac{\sigma_1\cdot\mathbf{L}_1}{2m_Q m_{\bar{Q}}} - \frac{\sigma_2\cdot\mathbf{L}_2}{2m_Q m_{\bar{Q}}}\right)\frac{1}{r}\frac{dV_2(r)}{dr}$$
$$+\frac{1}{6m_Q m_{\bar{Q}}}\sigma_1\cdot\sigma_2\nabla^2 V_2(r)$$
$$+\frac{1}{12m_Q m_{\bar{Q}}}(3\sigma_1\cdot\mathbf{n}\,\sigma_2\cdot\mathbf{n} - \sigma_1\cdot\sigma_2)V_3(r)$$

$$V_1(r) = -\frac{1}{2}V(r),$$
$$V_2(r) = \frac{1}{2}V(r),$$
$$V_3(r) = \left(\frac{1}{r}\frac{d}{dr} - \frac{d^2}{dr^2}\right)V(r)$$

$$V(r) = \frac{4\pi}{N_c}\frac{1}{R^4}\int_0^\infty \mathrm{d}z z^2 \int_{-1}^{1}\mathrm{d}t\left\{1 - \cos\left(\pi\sqrt{\frac{z^2+r^2/4+zrt}{z^2+r^2/4+zrt+\rho^2}}\right)\cos\left(\pi\sqrt{\frac{z^2+r^2/4-zrt}{z^2+r^2/4-zrt+\rho^2}}\right)\right.$$
$$\left.-\frac{z^2-r^2/4}{\sqrt{(z^2+r^2/4)^2-(zrt)^2}}\sin\left(\pi\sqrt{\frac{z^2+r^2/4+zrt}{z^2+r^2/4+zrt+\rho^2}}\right)\sin\left(\pi\sqrt{\frac{z^2+r^2/4-zrt}{z^2+r^2/4-zrt+\rho^2}}\right)\right\}$$

Heavy-quark potential with $1/M_Q^2$

$$V_{Q\bar{Q}}(r)=V(r)+V_{SS}(r)(\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}})+V_{LS}(r)(\mathbf{L}\cdot\mathbf{S})$$
$$+V_T(r)\left[3(\mathbf{S}_Q\cdot\mathbf{n})(\mathbf{S}_{\bar{Q}}\cdot\mathbf{n})-\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right].$$

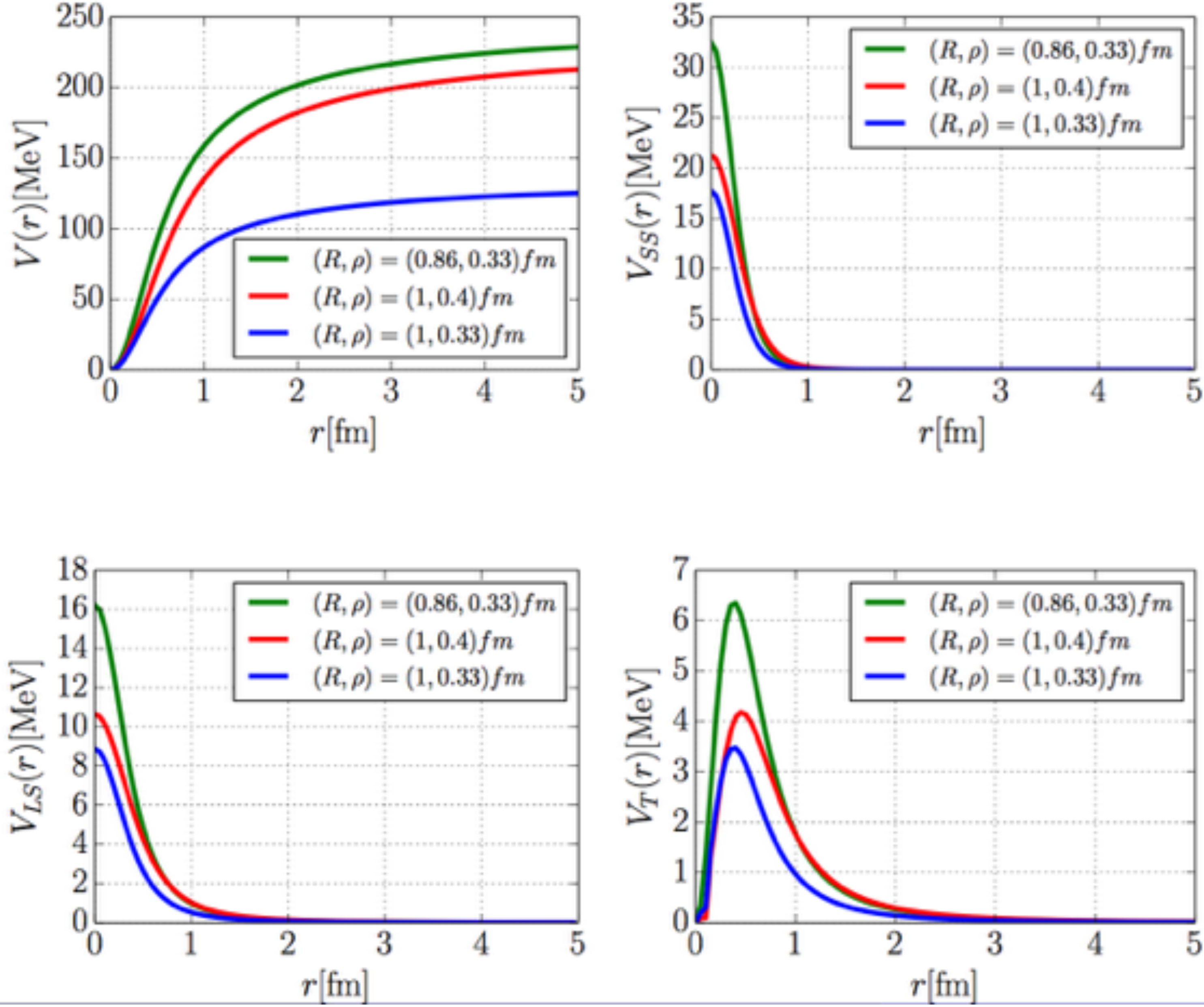
Spin-Spin Interaction $$V_{SS}(r)=\frac{1}{3m_Q^2}\nabla^2V(r),$$

Spin-Orbit Interaction $$V_{LS}(r)=\frac{1}{2m_Q^2 r}\frac{dV(r)}{dr},$$

Tensor Interaction $$V_T(r)=\frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV(r)}{dr}-\frac{d^2V(r)}{dr^2}\right).$$

B. Turimov, HChK,U. Yakhshiev, M.M. Musakhanov, E. Hiyama, in preparation

Instanton effects on heavy quark potential



Instanton effects on heavy quark potential

We first estimate the instanton effects on charmonium and bottomonium mass splittings with some approximations considered.

With large *r*

$$V(r) = 2M_Q - \frac{\pi^3}{N_c}\left(\frac{\rho}{R}\right)^4 \frac{1}{r}$$

With small *r*

$$V(r) = \frac{\rho\eta}{R^4} r^2 - \frac{\xi}{\rho R^4} r^4$$

$$V_{SS}(r) = \frac{1}{m_Q^2}\left[\frac{2\rho\eta}{R^4} - \frac{20\xi}{3\rho R^4} r^2\right]$$

$$V_{LS}(r) = \frac{1}{m_Q^2}\left[\frac{\rho\eta}{R^4} - \frac{2\zeta}{\rho R^4} r^2\right]$$

$$V_T(r) = \frac{8\zeta}{3m_Q^2 \rho R^4} r^2$$

$$\eta \simeq 5.6, \quad \xi \simeq 2.1$$

Instanton effects on quarkonia masses

These are the estimates. The results from the full calculation are to come!

	This work [MeV]	Experiment [MeV][12]
Charmonium states		
$\Delta M_{J/\psi-\eta_c}$	18.48	113.32 ± 0.689
$\Delta M_{\chi_{c2}-\chi_{c1}}$	18.48	45.54 ± 0.02
$\Delta M_{\chi_{c2}-\chi_{c0}}$	27.72	141.45 ± 0.22
$\Delta M_{\chi_{c1}-\chi_{c0}}$	9.24	95.91 ± 0.24
$\Delta M_{\chi_{c2}-h_c}$	27.72	30.82 ± 0.02
$\Delta M_{\chi_{c1}-h_c}$	9.24	-14.72 ± 0.04
Bottomonium states		
$\Delta M_{\Upsilon-\eta_b}$	1.70	62.30 ± 2.94
$\Delta M_{\chi_{b2}-\chi_{b1}}$	1.70	19.43
$\Delta M_{\chi_{b2}-\chi_{b0}}$	2.55	52.77 ± 0.16
$\Delta M_{\chi_{b1}-\chi_{b0}}$	0.85	33.34 ± 0.16
$\Delta M_{\chi_{b2}-h_b}$	2.55	12.91 ± 0.43
$\Delta M_{\chi_{b1}-h_b}$	0.85	-6.52 ± 0.43

small but non negligible

Tiny effects

$\rho = 0.33\,\mathrm{fm}$

$R = 1\,\mathrm{fm}$

Outlook

Outlook

Things to do

• Compute the mass splittings of the low-lying quarkonia, using the potential together with instanton effects.

• Compute the light-quark corrections to the heavy-quark potential and to the quarkonia mass.

• Construct the effective partition function for heavy-light-quark systems.

Charm & Bottom baryons

Motivation 1

The masses of bottom baryons:

$$M_{\Sigma_b^+} = 5811.3^{+0.9}_{-0.8} \pm 1.7\,\mathrm{MeV} \qquad M_{\Sigma_b^-} = 5815.5^{+0.6}_{-0.5} \pm 1.7\,\mathrm{MeV}$$

$$M_{\Sigma_b^{*+}} = 5832.1 \pm 0.7^{+1.7}_{-1.8}\,\mathrm{MeV} \qquad M_{\Sigma_b^{*-}} = 5835.1 \pm 0.7^{+1.7}_{-1.8}\,\mathrm{MeV}$$

CDF, PRD85, 092011 (2012)

$$M_{\Xi_b} = 5948.9 \pm 0.8 \pm 1.2\,\mathrm{MeV}$$

CMS, PRL 108, 252002 (2012)

$$M_{\Xi_b'} = 5935.02 \pm 0.02 \pm 0.05\,\mathrm{MeV}$$

$$M_{\Xi_b^*} = 5955.33 \pm 0.12 \pm 0.05\,\mathrm{MeV}$$

LHCb, PRL 114 062004 (2015)

The masses of the low-lying bottom baryons are now much known with the help of LHC.

Motivation 2

The nucleon can be considered as a chiral soliton in the large Nc limit.

- The model was successful in describing the structure of the nucleon.
- Will this mean-field approach (large Nc limit) work also for excited as well as heavy baryons?

→ **The answer is YES!**

Chiral Quark-Soliton Approach
(Quarks in the pion mean fields)

Chiral quark-soliton model

$$S_{\text{eff}} = -N_c \text{Tr} \ln(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m})$$

Nucleon consisting of Nc quarks

$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma^{\{f\}}_{JJ_3Y'TT_3Y} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t)$$

$$\lim_{T \to \infty} \Pi_N(T) \simeq e^{-M_N T}$$

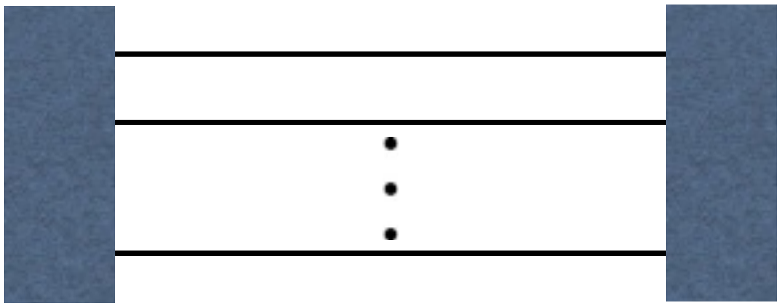
$$\Pi_N(\vec{x}, t) = \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle_{f,g} e^{-S_{\text{eff}}}$$

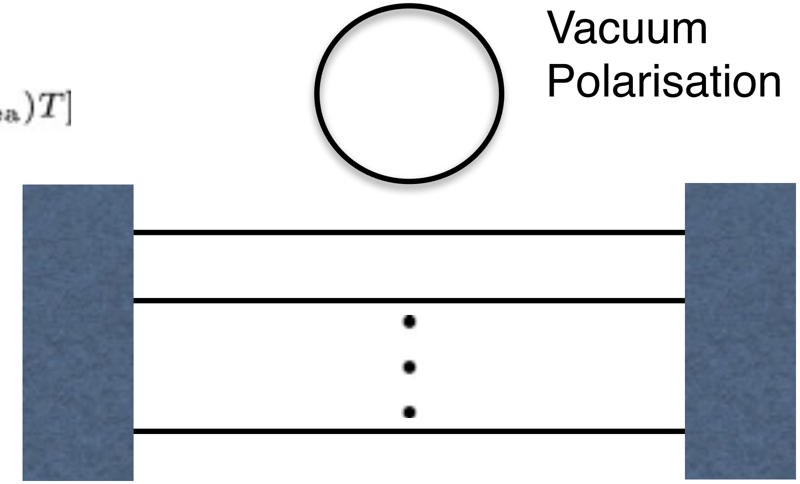
$$\lim_{T \to \infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle \sim e^{-(N_c E_{\text{val}}(U) + E_{\text{sea}}(U))T}$$

Chiral quark-soliton model

Classical solitons

$$\langle J_N(\vec{x},T)J_N^\dagger(\vec{y},-T)\rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\rm val}+E_{\rm sea})T]}$$

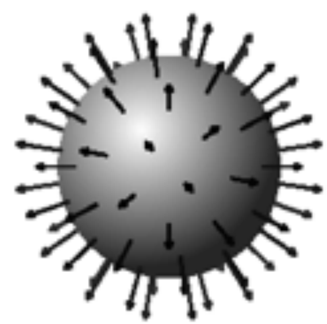




$$\frac{\delta}{\delta U}(N_c E_{\rm val} + E_{\rm sea}) = 0 \rightarrow M_{\rm cl} = N_c E_{\rm val}(U_c) + E_{\rm sea}(U_c)$$

Hedgehog Ansatz:

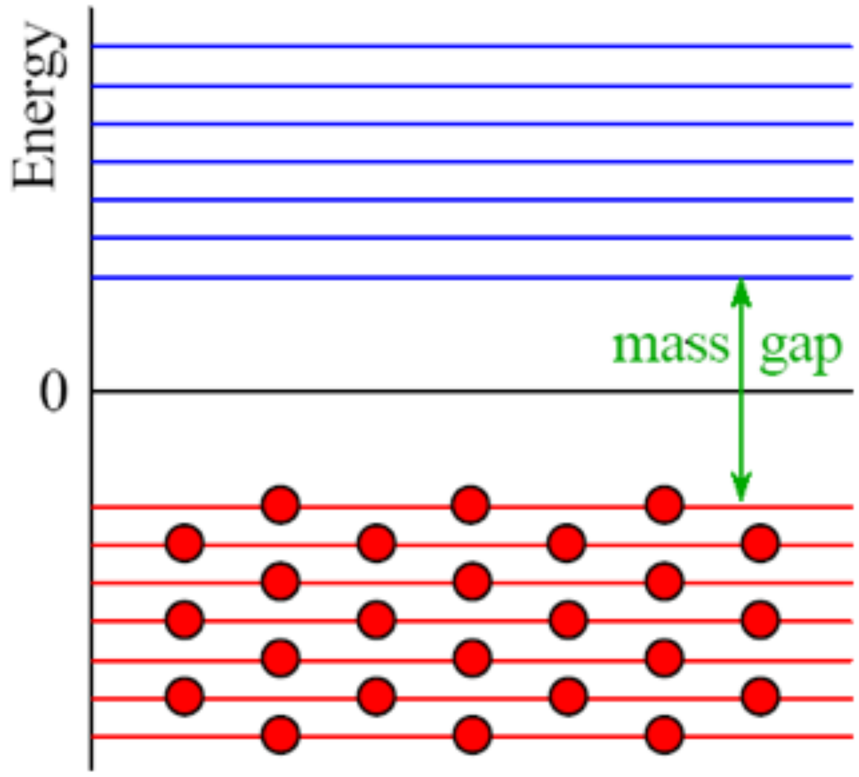
$$U_{\rm SU(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$



hedgehog

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

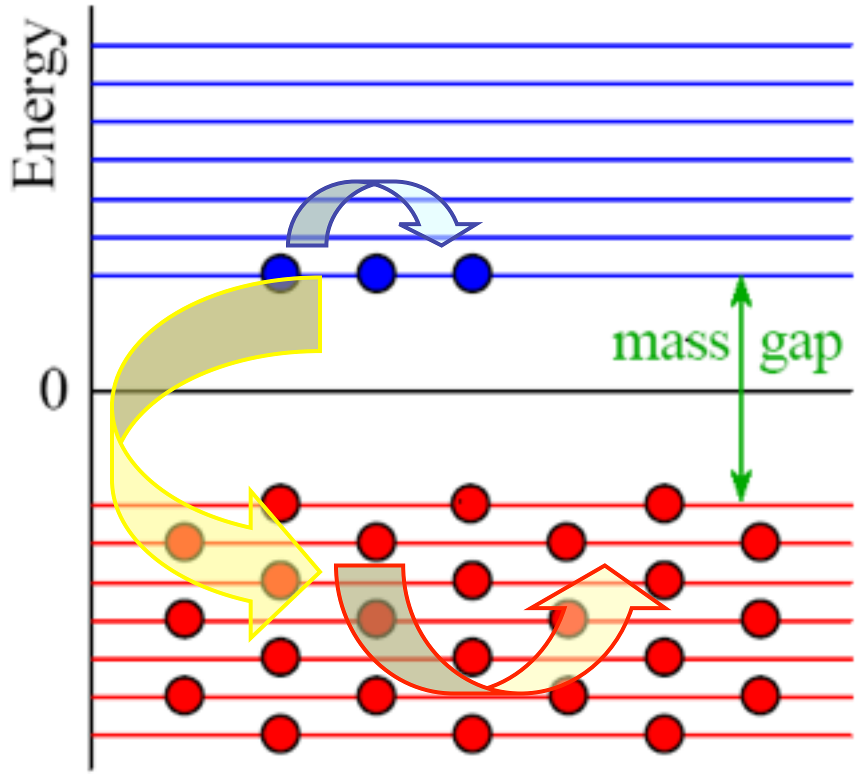
Chiral quark-soliton picture



Spontaneous chiral symmetry breaking

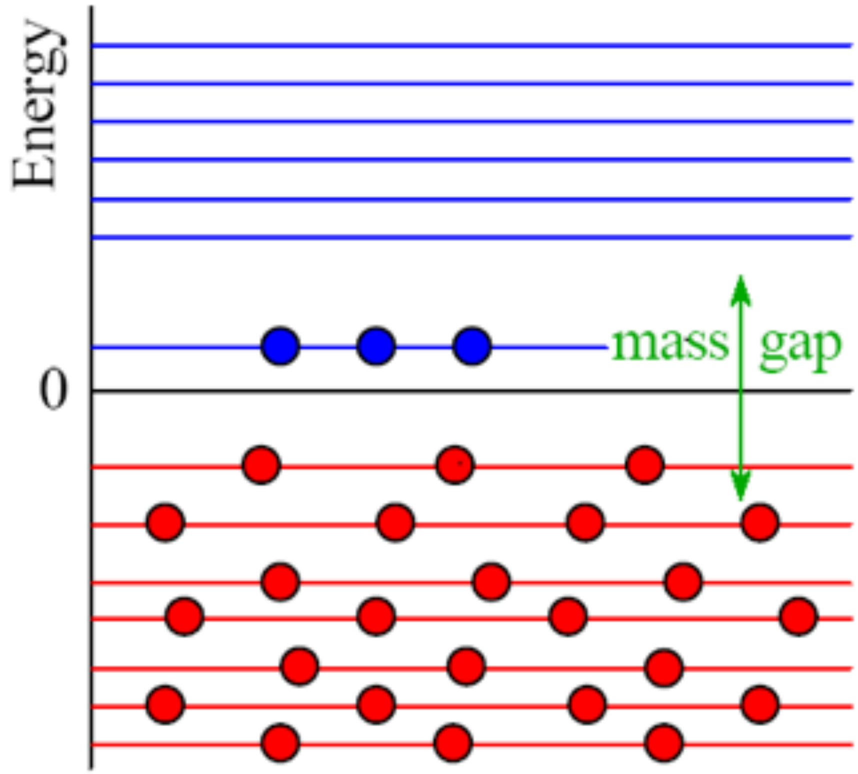
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture

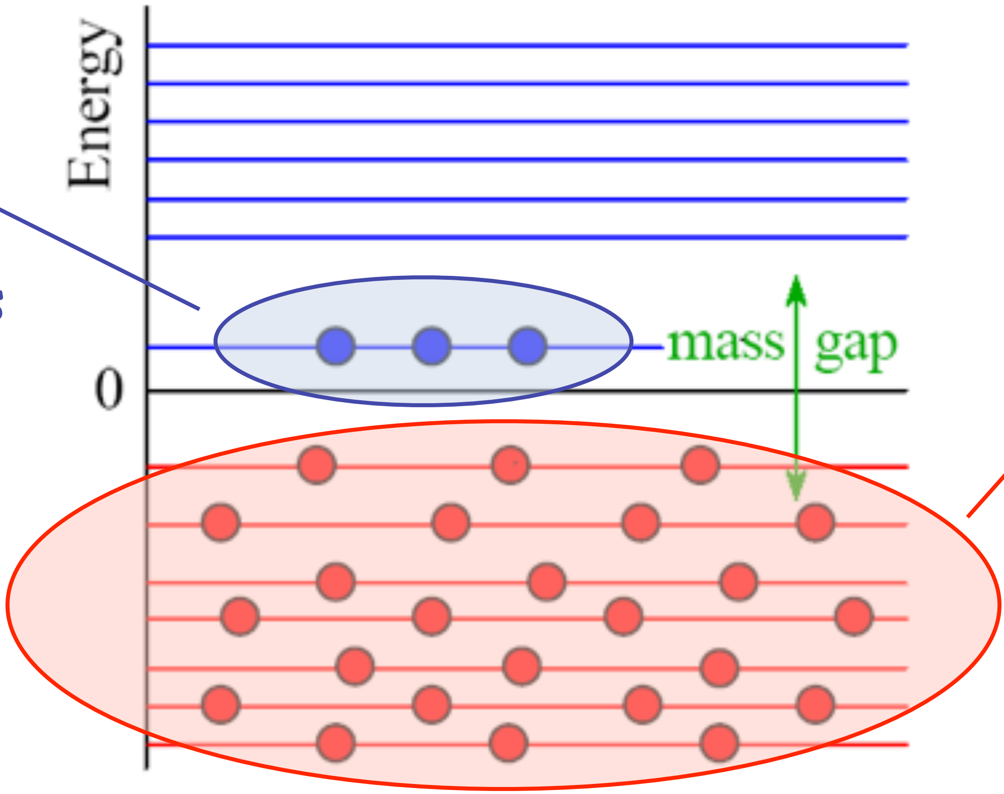


HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture

valence level:

energy decreases



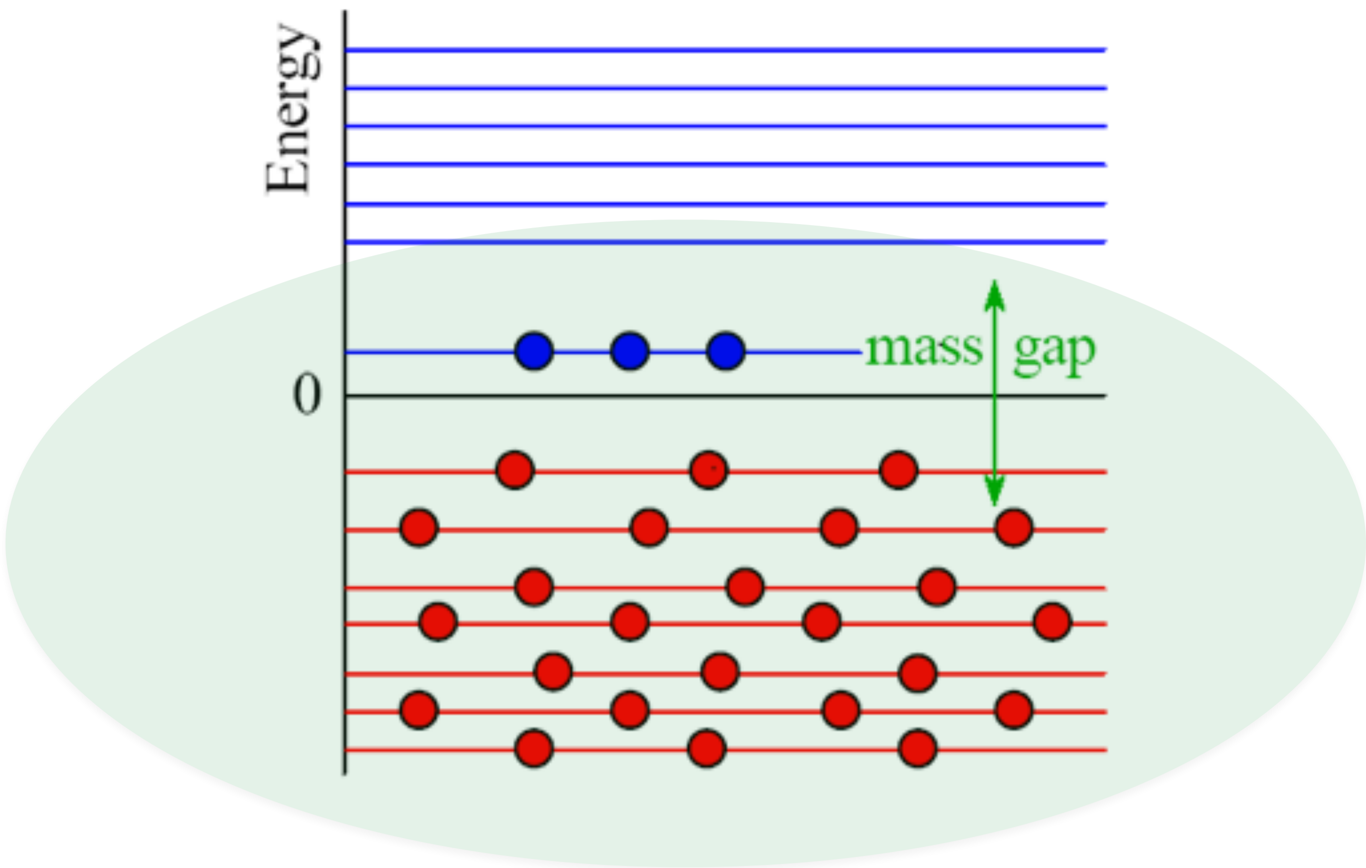
sea levels:

energy increases

system is stabilized.

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture



The valence quarks produce the mean field in the large Nc limit!

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral Quark-Soliton model

- Effective and relativistic low energy theory
- Large N_c limit : meson mean field
  → soliton
- Quantizing SU(3) rotated-meson fields
  → Collective Hamiltonian, model baryon states

Hedgehog Ansatz: $U_{\mathrm{SU(2)}} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$

Collective quantization

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

SU(2) E. Witten's imbedding into **SU(3)**: SU(2) X U(1)

Chiral Quark-Soliton model

Collective Hamiltonian for flavor symmetry breakings

$$H_{\text{Hadronic}} = H_{\text{cl}} + H_{\text{rot}} + H_{\text{sb}}$$

$$H_{\text{rot}} = \frac{1}{2I_1}\sum_{i=1}^{3} J_i^2 + \frac{1}{2I_2}\sum_{p=1}^{3} J_p^2$$

$$\begin{aligned} H_{\text{sb}} = & (m_s - \hat{m})\left(\alpha D_{88}^{(8)}(\mathcal{R}) + \beta \hat{Y} + \frac{1}{\sqrt{3}}\gamma \sum_{i=1}^{3} D_{8i}^{(8)}(\mathcal{R})\hat{J}_i\right) \\ & + (m_d - m_u)\left(\frac{\sqrt{3}}{2}\alpha D_{38}^{(8)}(\mathcal{R}) + \beta \hat{T}_3 + \frac{1}{2}\gamma \sum_{i=1}^{3} D_{3i}^{(8)}(\mathcal{R})\hat{J}_i\right) \\ & + (m_u + m_d + m_s)\sigma \end{aligned}$$

$$\alpha = -\left(\frac{2}{3}\frac{\Sigma_{\pi N}}{m_{\text{u}} + m_{\text{d}}} - \frac{K_2}{I_2}\right), \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)$$

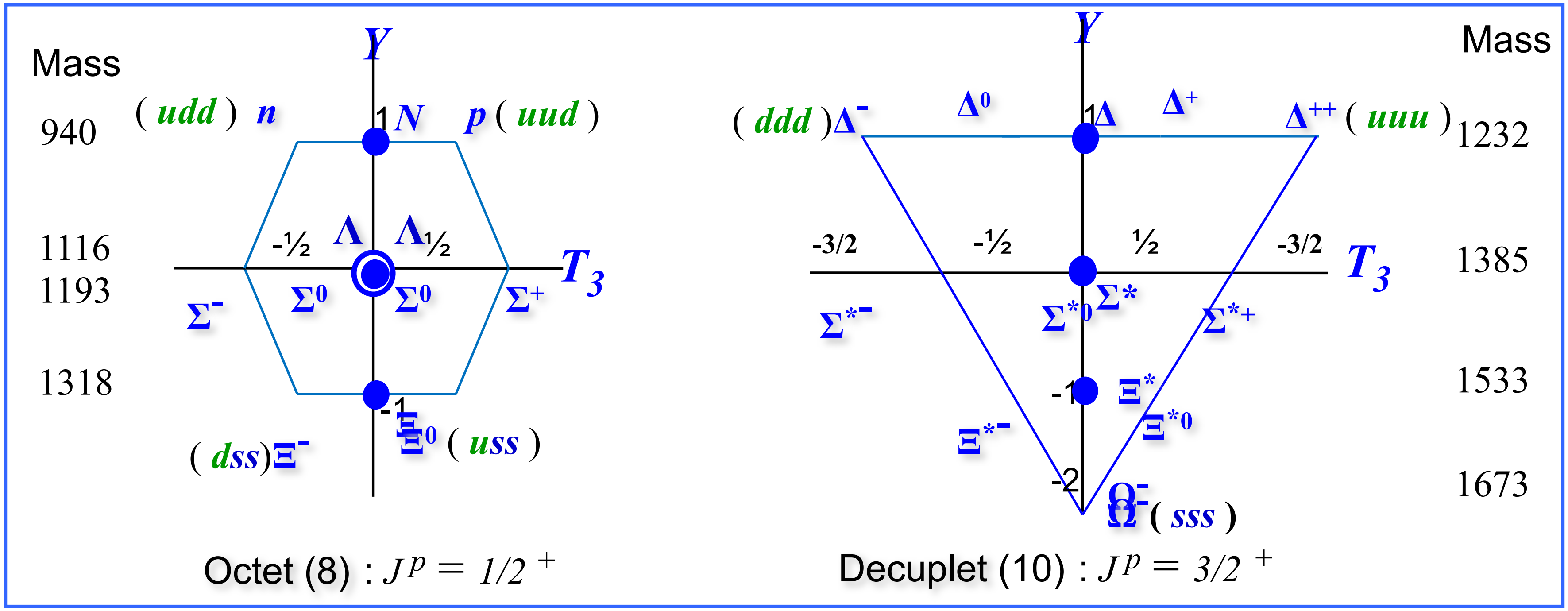
$$\sigma = -(\alpha + \beta) = \frac{2}{3}\frac{\Sigma_{\pi N}}{m_{\text{u}} + m_{\text{d}}}$$

Details will be explained by Gh.S. Yang on Wednesday.

Chiral Quark-Soliton model

Collective Hamiltonian for flavor symmetry breakings

$$H_{\rm sb} = (m_s - \hat{m})\left(\alpha D^{(8)}_{88}(\mathcal{R}) + \beta \hat{Y} + \frac{1}{\sqrt{3}}\gamma \sum_{i=1}^{3} D^{(8)}_{8i}(\mathcal{R})\,\hat{J}_i\right)$$
$$+ (m_d - m_u)\left(\frac{\sqrt{3}}{2}\alpha D^{(8)}_{38}(\mathcal{R}) + \beta \hat{T}_3 + \frac{1}{2}\gamma \sum_{i=1}^{3} D^{(8)}_{3i}(\mathcal{R})\,\hat{J}_i\right)$$



SU(3) flavor symmetry breaking + Isospin symmetry breaking

SU(3) Baryon Mass differences

Mass [MeV]		T_3	Y	Exp Input	Numerical results
M_N	p	$1/2$	1	938.27203 ± 0.00008	938.76 ± 3.65
	n	$-1/2$		939.56536 ± 0.00008	940.27 ± 3.64
M_Λ	Λ	0	0	1115.683 ± 0.006	1109.61 ± 0.70
M_Σ	Σ^+	1	0	1189.37 ± 0.07	1188.75 ± 0.70
	Σ^0	0		1192.642 ± 0.024	1190.20 ± 0.77
	Σ^-	-1		1197.449 ± 0.030	1195.48 ± 0.71
M_Ξ	Ξ^0	$1/2$	-1	1314.83 ± 0.20	1319.30 ± 3.43
	Ξ^-	$-1/2$		1321.31 ± 0.13	1324.52 ± 3.44

$$R = \frac{m_s - \hat{m}}{m_d - m_u} = \frac{M_p - M_{\Sigma^+} + M_{\Sigma^0} - M_{\Xi^-}}{2\,(M_{\Sigma^+} - M_{\Sigma^0})},$$

$$R = 58.1 \pm 1.3.$$

$$(m_d - m_u)\,\alpha = -4.390 \pm 0.004, \qquad (m_s - \hat{m})\,\alpha = -255.029 \pm 5.821,$$
$$(m_d - m_u)\,\beta = -2.411 \pm 0.001, \qquad (m_s - \hat{m})\,\beta = -140.040 \pm 3.195,$$
$$(m_d - m_u)\,\gamma = -1.740 \pm 0.006, \qquad (m_s - \hat{m})\,\gamma = -101.081 \pm 2.332,$$

SU(3) Baryon Mass differences

Employing the value of the ratio $(m_d - m_u)/(m_d + m_u) = 0.28 \pm 0.03$,

$$\Sigma_{\pi N} = (36.4 \pm 3.9)\,\mathrm{MeV}.$$

Numerical results of Decuplet mass

Mass [MeV]		T_3	Y	Experiment[41]	Predictions
M_Δ	Δ^{++}	$3/2$	1	$1231 - 1233$	1248.54 ± 3.39
	Δ^{+}	$1/2$			1249.36 ± 3.37
	Δ^{0}	$-1/2$			1251.53 ± 3.38
	Δ^{-}	$-3/2$			1255.08 ± 3.37
M_{Σ^*}	Σ^{*+}	1	0	1382.8 ± 0.4	1388.48 ± 0.34
	Σ^{*0}	0		1383.7 ± 1.0	1390.66 ± 0.37
	Σ^{*-}	-1		1387.2 ± 0.5	1394.20 ± 0.34
$M_{\Xi^{*0}}$	Ξ^{*0}	$1/2$	-1	1531.80 ± 0.32	1529.78 ± 3.38
	Ξ^{*-}	$-1/2$		1535.0 ± 0.6	1533.33 ± 3.37
$M^{\star}_{\Omega^-}$	Ω^{-}	0	-2	1672.45 ± 0.29	Input

G.S. Yang & HChK, Prog. Theor. Phys. 128,397(2012) ; Prog. Theor. Exp. Phys. 2013, 013D01

SU(3) Baryon Mass differences

- **Physical mass differences of baryon decuplet**

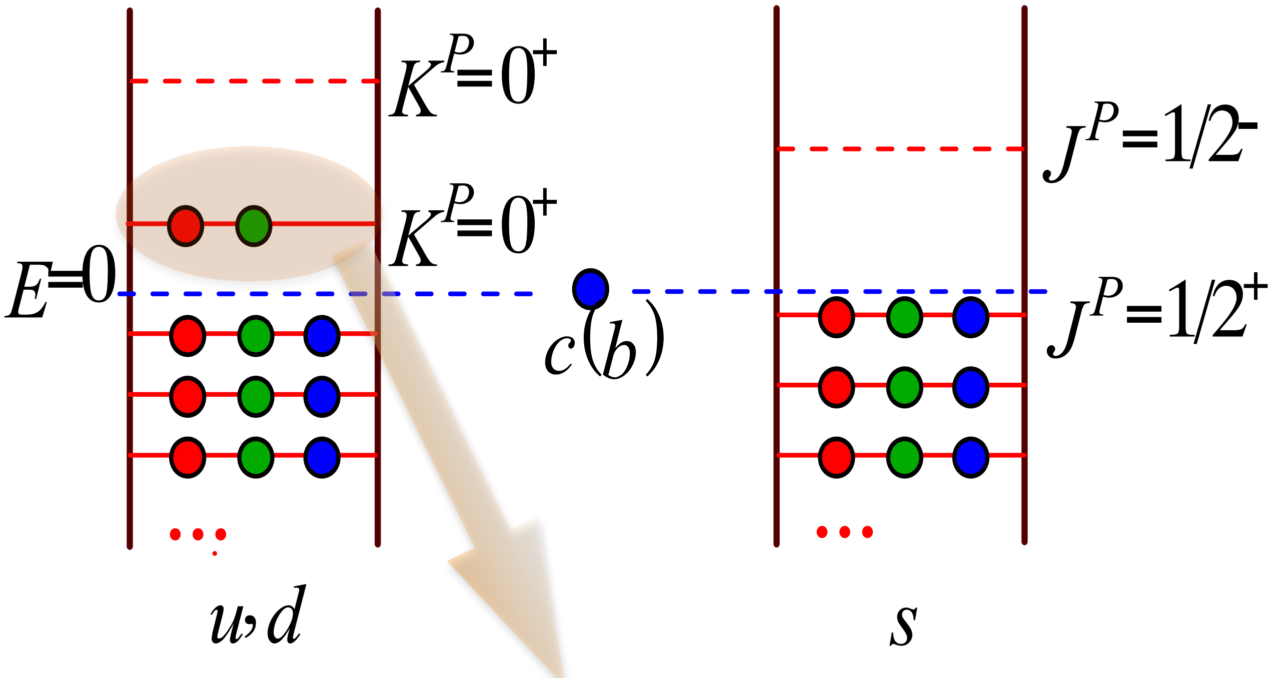
$(\Delta M_{B_{10}})$	This work	Experimental data	
$(M_{\Delta^{++}} - M_{\Delta^{+}})$	-0.59 ± 0.47		
$(M_{\Delta^{+}} - M_{\Delta^{0}})$	-1.95 ± 0.13		
$(M_{\Delta^{0}} - M_{\Delta^{-}})$	-3.32 ± 0.32		
$(M_{\Sigma^{*+}} - M_{\Sigma^{*0}})$	-1.95 ± 0.13		
$(M_{\Sigma^{*0}} - M_{\Sigma^{*-}})$	-3.32 ± 0.32	-3.1 ± 0.6	[D.W.Thomas *et al.*]
$(M_{\Xi^{*0}} - M_{\Xi^{*-}})$	-3.32 ± 0.32	-2.9 ± 0.9	[PDG, 2010]
$(M_{\Delta^{++}} - M_{\Delta^{0}})$	-2.54 ± 0.57	-2.86 ± 0.30	[GW, 2006]
$(M_{\Delta^{+}} - M_{\Delta^{-}})$	-5.28 ± 0.30		
$(M_{\Delta^{++}} - M_{\Delta^{-}})$	-5.86 ± 0.38	-5.9 ± 3.1	[Gatchina, 1981]
$(M_{\Sigma^{*+}} - M_{\Sigma^{*-}})$	-5.28 ± 0.30		

$$(M_{\Sigma^{*0}} - M_{\Sigma^{*-}}) = (M_{\Xi^{*0}} - M_{\Xi^{*-}})$$

Charmed baryons in the mean fields

Charmed baryons

- Valence quarks are bound by the pion mean field.
- Light quarks govern a heavy-light quark system.
- Heavy quarks can be considered as merely static color sources.



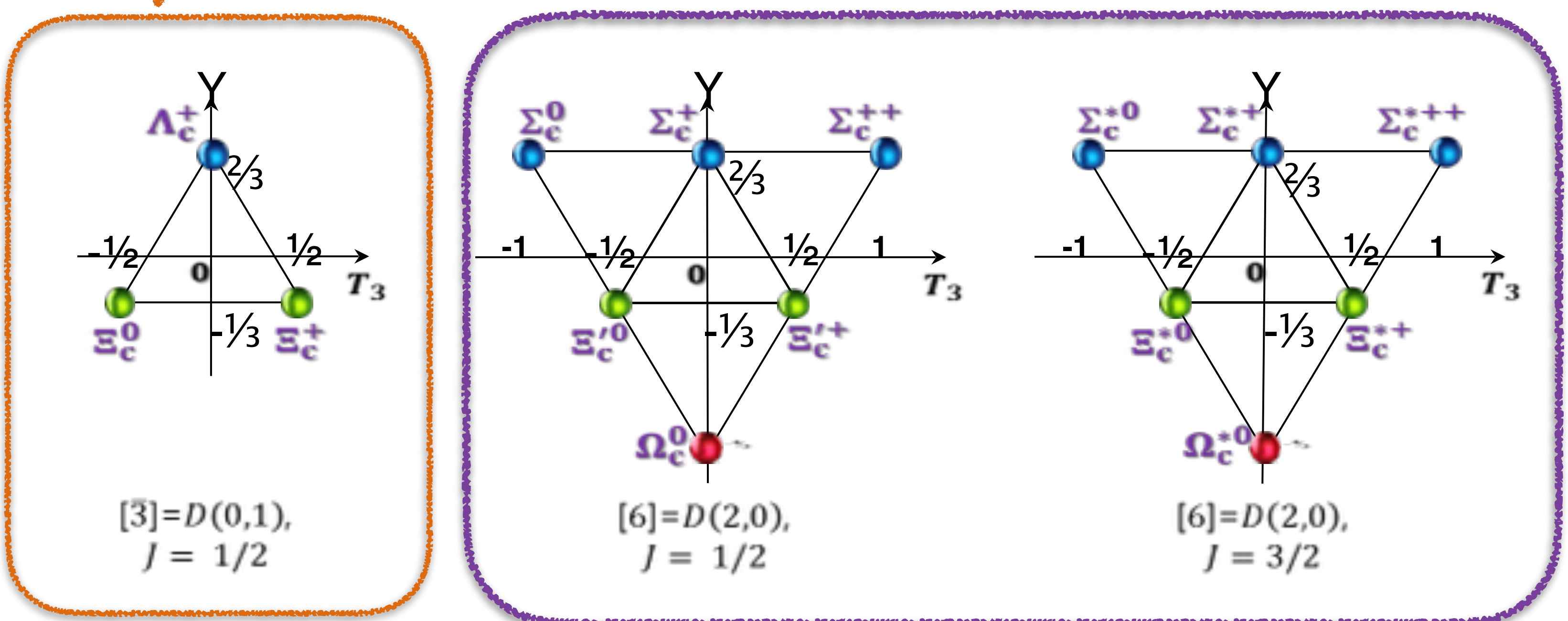
Meson mean field by ***N_c*-1** valence quarks

Suggested by the late D. Diakonov

Charmed baryons

Weight diagram for charmed baryons without heavy quark *c*

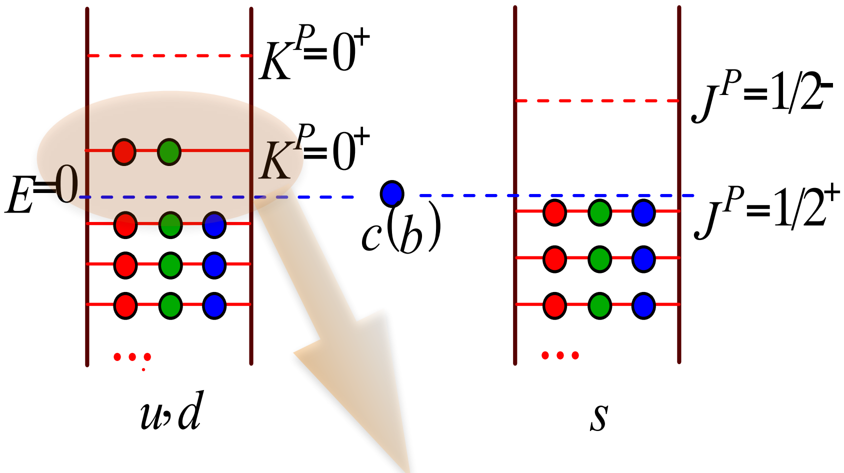
$$3 \otimes 3 = \bar{3} + 6$$



Anti-triplet

Sextet

Modification of the Hamiltonian



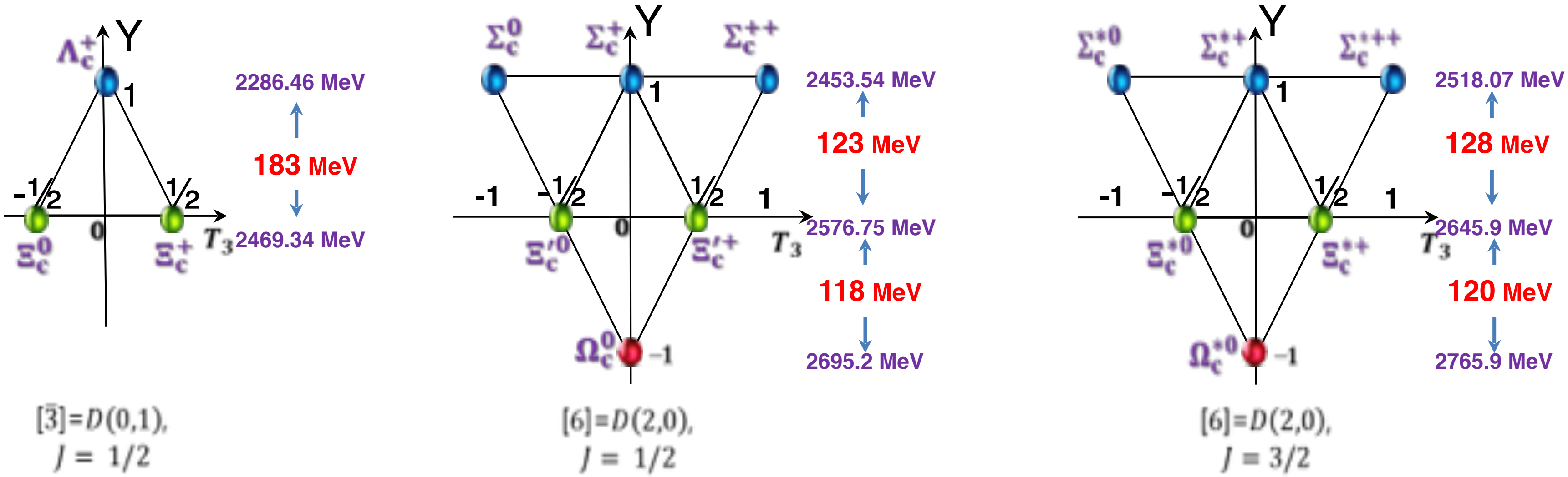
Moments of Inertia and Sigma pi-N term: sum over valence quark states:

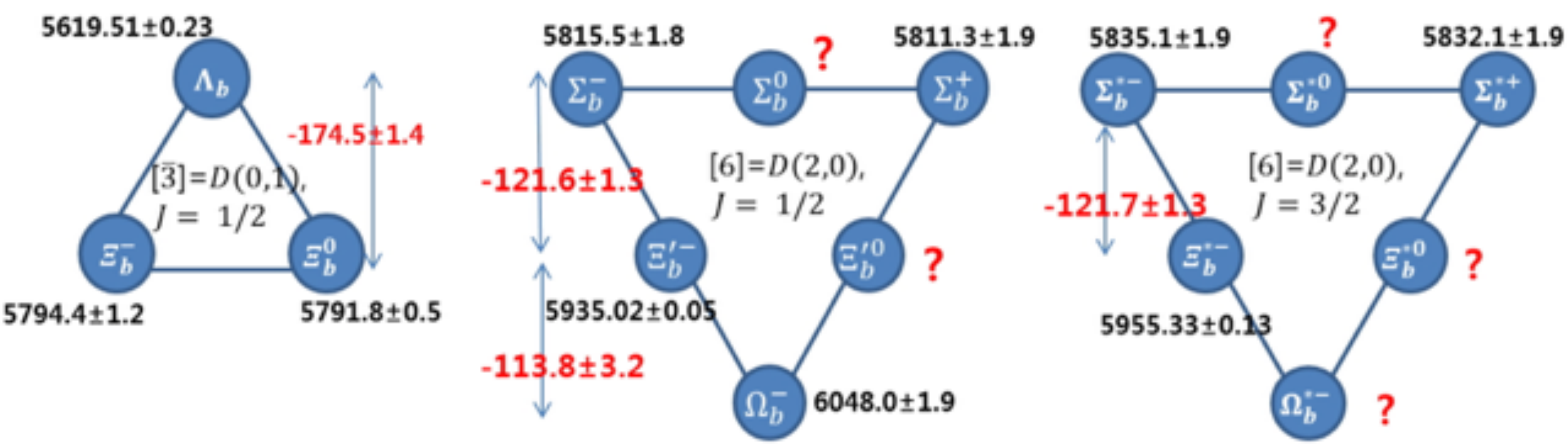
$$I_{1,2},\quad K_{1,2},\quad \Sigma_{\pi N} \longrightarrow \left(\frac{N_c-1}{N_c}\right) I_{1,2},\quad \left(\frac{N_c-1}{N_c}\right) K_{1,2},\quad \left(\frac{N_c-1}{N_c}\right) \Sigma_{\pi N},$$

Collective Hamiltonian for flavor symmetry breakings

$$H_{\rm sb}^{m_s} = \left(\frac{N_c-1}{N_c}\right) \alpha\, D_{88}^{(8)}(\mathcal{R}) + \beta\, \hat{Y} + \frac{1}{\sqrt{3}}\gamma \sum_{i=1}^{3} D_{8i}^{(8)}(\mathcal{R})\, \hat{J}_i$$

Heavy baryons





Heavy baryon Mass Formulae

$$M^{\overline{\mathbf{3}}}_{B-Q} = \mathcal{M}_{\text{soliton}} + \frac{3}{4I_2} + \delta_{\overline{3}} Y$$

$$M^{\mathbf{6}}_{B-Q} = \mathcal{M}_{\text{soliton}} + \frac{3}{4I_2} + \frac{3}{2I_1} + \delta_6 Y$$

$$\delta_{\overline{3}} = \frac{1}{4}\alpha + \beta \quad = \quad (-203.80 \pm 3.51)\ \text{MeV},$$

$$\delta_6 = \frac{1}{10}\alpha + \beta - \frac{3}{10}\gamma \quad = \quad (-135.22 \pm 3.32)\ \text{MeV}$$

$\alpha,\ \beta,\ \gamma$ are determined from the baryon octet mass!!

G.S. Yang, HChK, M. Polyakov, M. Praszalowicz, in preparation

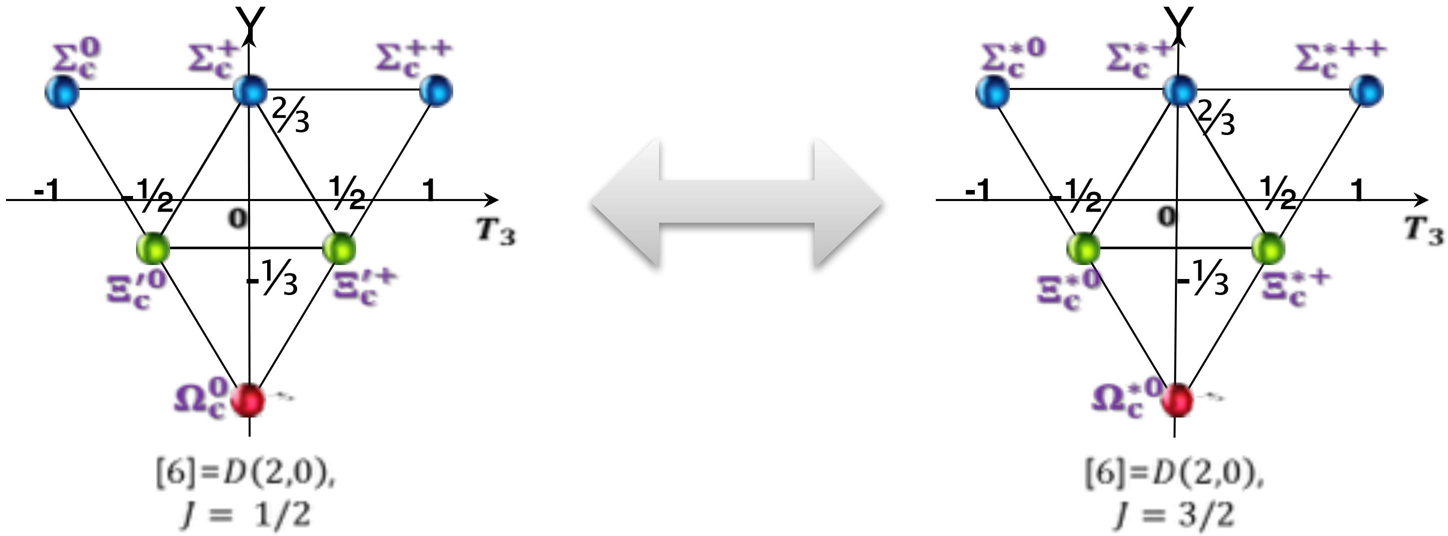
Heavy baryon Mass splitting

Rep.	ΔM	This work	Exp. [MeV]
$[\overline{3},\ J=\frac{1}{2}]$	$\Lambda_c - \Xi_c$ $\Lambda_b - \Xi_b$	-203.80 ± 3.51	-182.88 ± 0.38 -174.50 ± 1.39
$[6,\ J=\frac{1}{2}]$	$\Sigma_c - \Xi_c'$ $\Sigma_b - \Xi_b'^{-}$	-135.22 ± 3.32	-123.21 ± 2.13 -121.62 ± 1.31
	$\Xi_c' - \Omega_c$ $\Xi_b'^{-} - \Omega_b$	-135.22 ± 3.32	-118.45 ± 2.72 -113.78 ± 3.20
$[6,\ J=\frac{3}{2}]$	$\Sigma_c^* - \Xi_c^*$ $\Sigma_b^* - \Xi_b^{*-}$	-135.22 ± 3.32	-127.83 ± 0.89 -121.73 ± 1.34
	$\Xi_c^* - \Omega_c^*$ $\Xi_b^* - \Omega_b^*$	-135.22 ± 3.32	-120.0 ± 2.03 .

G.S. Yang, HChK, M. Polyakov, M. Praszalowicz, in preparation

Chrmomagnetic splitting

Splitting between spin 1/2 and 3/2 in the Baryon Sextet



$$\Delta H_Q\left(J_Q,\ J_S\right) = \frac{2}{3}\frac{\kappa}{m_Q\, M_{B-Q}}\vec{J}_S\cdot\vec{J}_Q$$

$$= \begin{cases} 0 & \text{for } \left[\overline{3}\right] \text{ with } J = 1/2, \\ -\frac{2}{3}\frac{\kappa}{m_Q\, M^{\mathbf{6}}_{B-Q}} & \text{for } [6] \text{ with } J = 1/2, \\ \frac{1}{3}\frac{\kappa}{m_Q\, M^{\mathbf{6}}_{B-Q}} & \text{for } [6] \text{ with } J = 3/2, \end{cases}$$

G.S. Yang, HChK, M. Polyakov, M. Praszalowicz, in preparation

Results

Preliminary Results

$\mathcal{R}_Q^J$	B	Mass Prediction [MeV]	Theor. - Exp.[MeV]: %	Exp. [MeV]
$\overline{3}_c^{J=\frac{1}{2}}$	Λ_c	Input		2283.46 ± 0.14
	Ξ_c	2490.23 ± 1.29	21.33 ± 2.17 : $(0.86)\,\%$	2469.34 ± 0.35
$6_c^{J=\frac{1}{2}}$	Σ_c	2424.76 ± 2.26	-28.78 ± 2.22 : $-(1.17)\,\%$	2453.54 ± 0.15
	Ξ_c'	2559.98 ± 1.13	-16.77 ± 2.39 : $-(0.65)\,\%$	2576.75 ± 2.12
	Ω_c	Input		2695.2 ± 1.7
$6_c^{J=\frac{3}{2}}$	Σ_c^*	2495.46 ± 2.26	-22.60 ± 2.36 : $-(0.90)\,\%$	2518.07 ± 0.82
	Ξ_c^*	2630.68 ± 1.13	-15.22 ± 1.17 : $-(0.58)\,\%$	2645.9 ± 0.35
	Ω_c^*	Input		2765.9 ± 2.0

G.S. Yang, HChK, M. Polyakov, M. Praszalowicz, in preparation

Summary

- Assuming that the valence quarks are bound by the pion mean fields, we can regard the nucleon as a chiral soliton.
- Formulating the most general expressions of the collective Hamiltonian and determining all dynamical parameters by using the experimental data unequivocally, we are able to find the the collective baryon wavefunctions.
- Then we predicted the mass splittings of the baryon decuplet.
- We also presented preliminary results of the masses for the charmed and bottom baryons **(light quarks govern their structure!)**.

Outlook

- Coupling constants & Form factors of Heavy baryons
- Decays of heavy Baryons (strong, radiative, semileptonic, nonleptonic,……)
- strange quark mass dependence of charmed & bottom baryon properties.
- Heavy Pentaquarks (In fact, Pc belongs to the baryon octet according to the light quarks). In this case, the mean-field approach in the large Nc limit seems even more plausible!
- Doubly charmed & bottom baryons

Thanks to My collaborators

- B. Turimov (Inha Univ.)
- U. Yakhshiev (Inha Univ.)
- E. Hiyama (RIKEN)
- M.M. Musakhanov (Uzbekistan Nat'l Univ.)
- Gh.-S. Yang (Soong-Sil Univ.)
- M.V. Polyakov (Ruhr-Uni Bochum)
- M. Praszalowicz (Jagiellonian Univ.)

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!

Chiral Quark-Soliton model

Model baryon state

$$|B\rangle = \sqrt{\dim(\mathcal{R})}(-1)^{J_3+Y'/2} D^{(\mathcal{R})*}_{(Y,T,T_3)(-Y',J,-J_3)}$$

Constraint for the collective quantization : $Y' = -\frac{N_c B}{3}$

Mixings of baryon states

$$|B_8\rangle = |8_{1/2}, B\rangle + c^B_{\overline{10}} |\overline{10}_{1/2}, B\rangle + c^B_{27} |27_{1/2}, B\rangle ,$$
$$|B_{10}\rangle = |10_{3/2}, B\rangle + a^B_{27} |27_{3/2}, B\rangle + a^B_{35} |35_{3/2}, B\rangle ,$$
$$|B_{\overline{10}}\rangle = |\overline{10}_{1/2}, B\rangle + d^B_8 |8_{1/2}, B\rangle + d^B_{27} |27_{1/2}, B\rangle + d^B_{\overline{35}} |\overline{35}_{1/2}, B\rangle$$

$$|B, S\rangle = |R, B, S\rangle - \sum_{R' \neq R} |R', B, S\rangle \frac{\langle R', B, S| H' |R, B, S\rangle}{M^{(0)}(R') - M^{(0)}(R)}.$$

Chiral Quark-Soliton model

Mixing coefficients

$$c^B_{\overline{10}} = c_{\overline{10}} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad c^B_{27} = c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix}, \quad a^B_{27} = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, \quad a^B_{35} = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix},$$

$$d^B_{8} = d_8 \begin{bmatrix} 0 \\ \sqrt{5} \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad d^B_{\overline{27}} = d_{\overline{27}} \begin{bmatrix} 0 \\ \sqrt{3/10} \\ 2/\sqrt{5} \\ \sqrt{3/2} \end{bmatrix}, \quad d^B_{\overline{35}} = d_{\overline{35}} \begin{bmatrix} 1/\sqrt{7} \\ 3/(2\sqrt{14}) \\ 1/\sqrt{7} \\ \sqrt{5/56} \end{bmatrix}$$

respectively in the basis $[N, \Lambda, \Sigma, \Xi]$, $[\Delta, \Sigma^*, \Xi^*, \Omega]$, $[\Theta^+, N_{\overline{10}}, \Sigma_{\overline{10}}, \Xi_{\overline{10}}]$

$$c_{\overline{10}} = -\frac{I_2}{15}(m_s - \hat{m})\left(\alpha + \frac{1}{2}\gamma\right), \quad c_{27} = -\frac{I_2}{25}(m_s - \hat{m})\left(\alpha - \frac{1}{6}\gamma\right),$$

$$a_{27} = -\frac{I_2}{8}(m_s - \hat{m})\left(\alpha + \frac{5}{6}\gamma\right), \quad a_{35} = -\frac{I_2}{24}(m_s - \hat{m})\left(\alpha - \frac{1}{2}\gamma\right),$$

$$d_8 = \frac{I_2}{15}(m_s - \hat{m})\left(\alpha + \frac{1}{2}\gamma\right), \quad d_{\overline{27}} = -\frac{I_2}{8}(m_s - \hat{m})\left(\alpha - \frac{7}{6}\gamma\right),$$

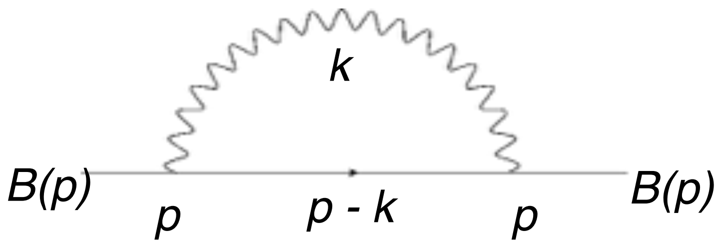
$$d_{\overline{35}} = -\frac{I_2}{4}(m_s - \hat{m})\left(\alpha + \frac{1}{6}\gamma\right)$$

$$\Delta \overline{M}_{10-8} = \frac{3}{2\,I_1}$$

$$\Delta \overline{M}_{\overline{10}-8} = \frac{3}{2\,I_2}$$

EM Mass differences

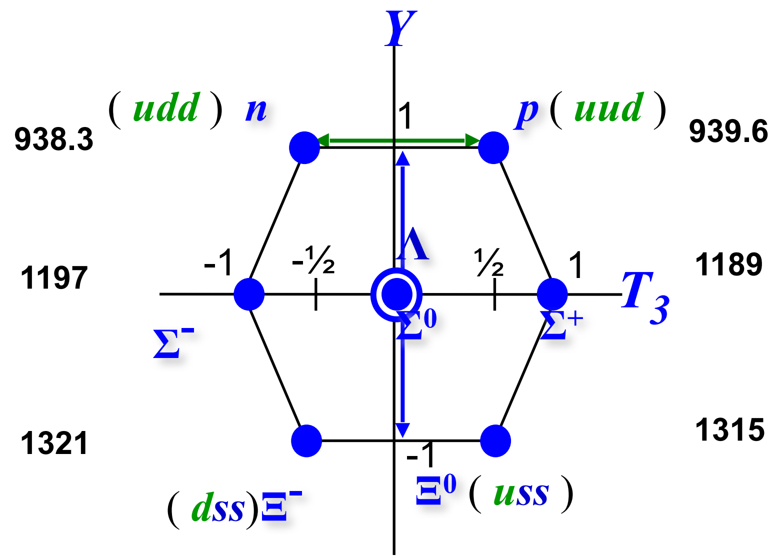
- **EM mass corrections**



Electromagnetic (*EM*) self-energy

EM [MeV]	Exp.
$(p - n)_{EM}$	**0.76**±0.30
$(\Sigma^+ - \Sigma^-)_{EM}$	-0.17±0.30
$(\Xi^0 - \Xi^-)$	-0.86±0.30

Gasser, Leutwyler, **Phys.Rep 87, 77** *"Quark Masses"*



$$\Delta M_B = M_{B_1} - M_{B_2} = (\Delta M_B)_H + (\Delta M_B)_{EM}$$

$(p - n)_{exp} \sim$ **− 1.293** MeV

$(p - n)_{EM} \sim$ **0.76** MeV

EM Mass differences

In the ChSM, $(\Delta M_B)_{\mathrm{EM}} = \langle B | J_\mu(x) J^\mu(0) | B\rangle = \langle B | \mathcal{O}_{\mathrm{EM}} | B \rangle$

$$\mathcal{O}_{\mathrm{EM}} = -\frac{e^2}{2}\int d^3x\, d^3y D_\gamma(\boldsymbol{x},\boldsymbol{y}) \int \frac{d\omega}{2\pi}\mathrm{tr}\left\langle \boldsymbol{x}\left|\frac{1}{\omega+iH}\gamma_\mu\lambda^a\right|\boldsymbol{y}\right\rangle\left\langle \boldsymbol{y}\left|\frac{1}{\omega+iH}\gamma_\mu\lambda^b\right|\boldsymbol{x}\right\rangle D^{(8)}_{Qa}D^{(8)}_{Qb}$$
$$= \alpha_1 \sum_{i=1}^{3} D^{(8)}_{Qi}D^{(8)}_{Qi} + \alpha_2 \sum_{p=4}^{7} D^{(8)}_{Qp}D^{(8)}_{Qp} + \alpha_3 D^{(8)}_{Q8}D^{(8)}_{Q8}$$

It can be further reduced to

$$\mathcal{O}^{\mathrm{EM}} = c^{(27)}\left(\sqrt{5}D^{(27)}_{\Sigma_2^0\Lambda_{27}} + \sqrt{3}D^{(27)}_{\Sigma_1^0\Lambda_{27}} + D^{(27)}_{\Lambda_{27}\Lambda_{27}}\right) + c^{(8)}\left(\sqrt{3}D^{(8)}_{\Sigma^0\Lambda} + D^{(8)}_{\Lambda\Lambda}\right) + c^{(1)}D^{(1)}_{\Lambda\Lambda}$$

$$c^{(27)} = \frac{1}{40}(\alpha_1 - 4\alpha_2 + 3\alpha_3),$$
$$c^{(8)} = \frac{1}{10}\left(\alpha_1 - \frac{2}{3}\alpha_2 - \frac{1}{3}\alpha_3\right),$$
$$c^{(1)} = \frac{1}{8}\left(\alpha_1 + \frac{4}{3}\alpha_2 + \frac{1}{3}\alpha_3\right)$$

$$\mathbf{8}\otimes\mathbf{8} = \mathbf{1}\oplus\mathbf{8}_s\oplus\mathbf{8}_a\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

Because of Bose symmetry

G. S. Yang, H.-Ch. Kim and M. V. Polyakov, Phys. Lett. B 695, 214 (2011)

EM Mass differences

$$(M_p - M_n)_{\rm EM} = \frac{1}{5}\left(c^{(8)} + \frac{4}{9}c^{(27)}\right)$$
$$(M_{\Sigma^+} - M_{\Sigma^-})_{\rm EM} = c^{(8)}$$
$$(M_{\Xi^0} - M_{\Xi^-})_{\rm EM} = \frac{4}{5}\left(c^{(8)} - \frac{1}{9}c^{(27)}\right)$$

Coleman-Glashow relation

$$(M_p - M_n)_{\rm EM} = (M_{\Sigma^+} - M_{\Sigma^-})_{\rm EM} - (M_{\Xi^0} - M_{\Xi^-})_{\rm EM}$$

EM [MeV]	**Exp.** [input]
$(M_p - M_n)_{EM}$	**0.76±0.30**
$(M_{\Sigma+} - M_{\Sigma-})_{EM}$	**-0.17±0.30**
$(M_{\Xi 0} - M_{\Xi-})_{EM}$	**-0.86±0.30**

$$c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39$$

EM Mass differences

χ^2 fit

$$(M_p - M_n)_{\rm EM} = \frac{1}{5}\left(c^{(8)} + \frac{4}{9}c^{(27)}\right)$$
$$(M_{\Sigma^+} - M_{\Sigma^-})_{\rm EM} = c^{(8)}$$
$$(M_{\Xi^0} - M_{\Xi^-})_{\rm EM} = \frac{4}{5}\left(c^{(8)} - \frac{1}{9}c^{(27)}\right)$$

Coleman-Glashow relation

$$(M_p - M_n)_{\rm EM} = (M_{\Sigma^+} - M_{\Sigma^-})_{\rm EM} - (M_{\Xi^0} - M_{\Xi^-})_{\rm EM}$$

EM [MeV]	**Exp.** [input]	**reproduced**
$(M_p - M_n)_{EM}$	**0.76±0.30**	**0.74±0.22**
$(M_{\Sigma^+} - M_{\Sigma^-})_{EM}$	**-0.17±0.30**	**-0.15±0.23**
$(M_{\Xi^0} - M_{\Xi^-})_{EM}$	**-0.86±0.30**	**-0.88±0.28**

$$c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39$$

EM Mass differences

$(\Delta M_{B_{10}})_{\rm EM}$	Numerical results	$(\Delta M_{B_{10}})_{\rm EM}$	Numerical results
$(M_{\Delta^{++}} - M_{\Delta^{+}})_{\rm EM}$	1.60 ± 0.46	$(M_{\Delta^{++}} - M_{\Delta^{0}})_{\rm EM}$	1.84 ± 0.54
$(M_{\Delta^{+}} - M_{\Delta^{0}})_{\rm EM}$	0.24 ± 0.10	$(M_{\Delta^{+}} - M_{\Delta^{-}})_{\rm EM}$	-0.89 ± 0.26
$(M_{\Delta^{0}} - M_{\Delta^{-}})_{\rm EM}$	-1.13 ± 0.30	$(M_{\Delta^{++}} - M_{\Delta^{-}})_{\rm EM}$	0.71 ± 0.29
$(M_{\Sigma^{*+}} - M_{\Sigma^{*0}})_{\rm EM}$	0.24 ± 0.10	$(M_{\Sigma^{*+}} - M_{\Sigma^{*-}})_{\rm EM}$	-0.89 ± 0.26
$(M_{\Sigma^{*0}} - M_{\Sigma^{*-}})_{\rm EM}$	-1.13 ± 0.30		
$(M_{\Xi^{*0}} - M_{\Xi^{*-}})_{\rm EM}$	-1.13 ± 0.30		

SU(3) Baryon Mass differences

Present analysis reproduces all kind of well-known mass relations

- ***Coleman-Glashow*** relation is still satisfied

$$M_p - M_n = (M_{\Sigma^+} - M_{\Sigma^-}) - (M_{\Xi^0} - M_{\Xi^-})$$

- ***Generalized Gell-Mann-Okubo*** relation

$$2\,(M_p + M_{\Xi^0}) = 3M_\Lambda + \overline{M}_\Sigma + (M_{\Sigma^+} - M_{\Sigma^-}) + \frac{2}{3}\Delta M_\Sigma,$$
$$2\,(M_n + M_{\Xi^-}) = 3M_\Lambda + \overline{M}_\Sigma - (M_{\Sigma^+} - M_{\Sigma^-}) + \frac{2}{3}\Delta M_\Sigma,$$

$$\text{where } \Delta M_\Sigma = M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0}.$$

When the effect of the ***isospin sym. br*** is turned off,

$$2\,(\overline{M}_N + \overline{M}_\Xi) = 3M_\Lambda + \overline{M}_\Sigma$$

★ **Generalized *Guadagnini* formulae**

$$8\,(\overline{M}_N + \overline{M}_{\Xi^*}) + 3\,\overline{M}_\Sigma = 11\,\overline{M}_\Lambda + 8\,\overline{M}_{\Sigma^*}$$