Hypernuclei in halo/cluster EFT

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Outline

- Singular potentials:
  Limit cycle and Efimov states in three-body systems
- ${}^{4}_{\Lambda\Lambda}$H as $\Lambda\Lambda d$ system in halo EFT
- ${}^{6}_{\Lambda\Lambda}$He as $\Lambda\Lambda\alpha$ system in cluster EFT
- Summary

Limit cycle

- Three-body systems in unitary (asymptotic) limit
  - If an interaction is singular, the system exhibits cyclic singularities, so called limit cycle.
  - It is necessary to introduce a counter term for renormalization.
- Efimov-like bound states
  Infinitely many three-body bound states (whose energies $B^{(n)}$) appear, for three-boson case,

$$B^{(n)} = \left(e^{-2\pi/s_0}\right)^{n-n*} \kappa_*^2/m\,,$$

where $s_0 \simeq 1.00624$ and $e^{\pi/s_0} \simeq 22.7$.

Halo/Cluster EFT

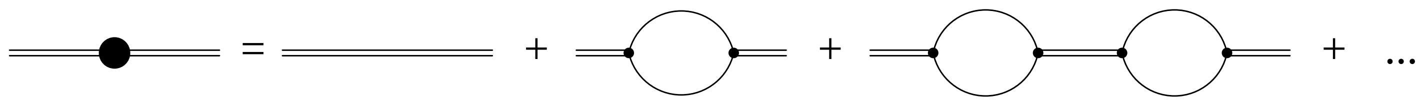
- Effective Field Theories (EFTs)
  - Model independent approach
  - Separation scale
  - Counting rules
  - Parameters should be fixed by experiments
  - For the study of three-body systems the unitary limit can be chosen as a first approximation.

${}_{\Lambda\Lambda}^{4}$H at LO

- $\Lambda\Lambda d$
- ${}^{3}_{\Lambda}$H, $B_\Lambda = 0.13$ MeV
- d, $B_2 = 2.22$ MeV
- S-waves are considered at LO.
- $S = 0$: no limit cycle, one parameter $\gamma_{\Lambda d}$, and we find no bound state for ${}_{\Lambda\Lambda}^{4}$H and $a_0 = 16.0 \pm 3.0$ fm for Λ-${}^{3}_{\Lambda}$H scattering.
- $S = 1$: ${}_{\Lambda\Lambda}^{4}$H shows a limit cycle, three parameters, $a_{\Lambda\Lambda}$, $\gamma_{\Lambda d}$, $g_1(\Lambda_c)$, and the three-body interaction is fixed by using the results of the potential models.

Two-body part: $\Lambda\Lambda$ *in* $^1S_0$ *state*

- Dressed dibaryon propagator



- Renormalized dressed dibaryon propagator

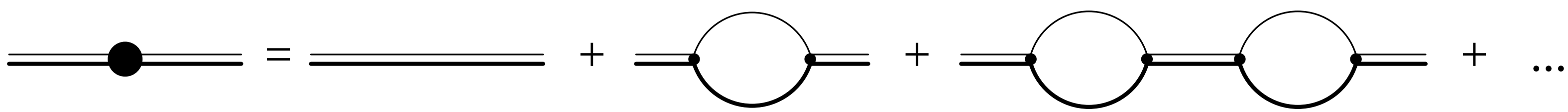
$$D_s(p_0, \vec{p}) \quad = \quad \frac{4\pi}{m_\Lambda y_s^2} \frac{1}{\frac{1}{a_{\Lambda\Lambda}} - \sqrt{-m_\Lambda p_0 + \frac{1}{4}\vec{p}^2 - i\epsilon}} \, .$$

$$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \text{ fm} \, ,$$

from ^{12}C(K^-,$K^+\Lambda\Lambda X$) reaction [Gasparyan *et al.*, PRC85(2012)015204].

Two-body part: Λd *in* ${}^3_\Lambda$*H channel*

- Dressed ${}^3_\Lambda$H propagator



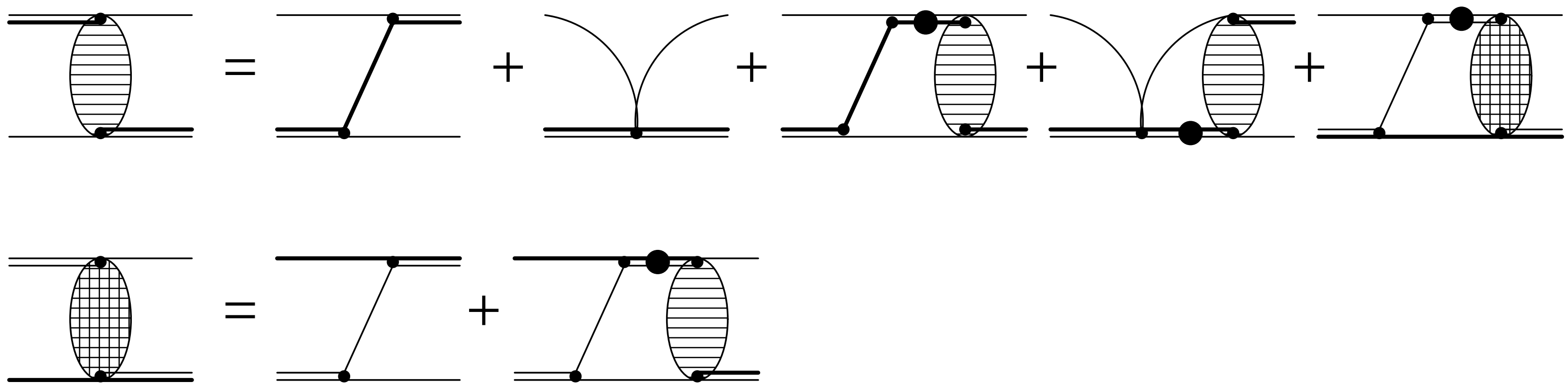
- Renormalized dressed ${}^3_\Lambda$H propagator

$$D_t(p_0, \vec{p}) = \frac{2\pi}{\mu_{\Lambda d} y_t^2} \frac{1}{\gamma_{\Lambda d} - \sqrt{-2\mu_{\Lambda d}\left(p_0 - \frac{1}{2(m_\Lambda + m_d)}\vec{p}^2\right)}} \,,$$

with

$$\gamma_{\Lambda d} = \sqrt{2\mu_{\Lambda d} B_\Lambda} \,.$$

Three-body part: $S = 1$ ***channel***



$$
\begin{aligned}
a(p,k;E) &= K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 \left[K_{(a)}(p,l;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b1)}(p,l;E) D_s\left(E - \frac{1}{2m_d}l^2, \vec{l}\right) b(l,k;E)\,, \\
b(p,k;E) &= K_{(b2)}(p,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b2)}(p,l;E) D_t\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E)\,,
\end{aligned}
$$

where

$$K_{(a)}(p,l;E) = \frac{m_d y_t^2}{6pl} \ln\left(\frac{p^2+l^2+\frac{2\mu_{\Lambda d}}{m_d}-2\mu_{\Lambda d}E}{p^2+l^2-\frac{2\mu_{\Lambda d}}{m_d}-2\mu_{\Lambda d}E}\right),$$

$$K_{(b1)}(p,l;E) = -\sqrt{\frac{2}{3}}\frac{m_\Lambda y_s y_t}{2pl} \ln\left(\frac{p^2+\frac{m_\Lambda}{2\mu_{\Lambda d}}l^2+pl-m_\Lambda E}{p^2+\frac{m_\Lambda}{2\mu_{\Lambda d}}l^2-pl-m_\Lambda E}\right),$$

$$K_{(b2)}(p,l;E) = -\sqrt{\frac{2}{3}}\frac{m_\Lambda y_s y_t}{2pl} \ln\left(\frac{\frac{m_\Lambda}{2\mu_{\Lambda d}}p^2+l^2+pl-m_\Lambda E}{\frac{m_\Lambda}{2\mu_{\Lambda d}}p^2+l^2-pl-m_\Lambda E}\right).$$

Evaluation formula for the limit cycle

- In the asymmetric limit, there is no scale in the integral equations. The scale invariance suggests that the power-law behavior for the amplitude

$$a(p) \sim p^{-1+s} \, .$$

- After Mellin transformations we have

$$1 = C_1 I_1(s) + C_2 I_2(s) I_3(s) \, .$$

- It has imaginary solutions for s, $s = \pm i s_0$,

$$s_0 = 0.4492 \cdots \ ,$$

and thus $e^{\pi/s_0} \simeq 1.09 \times 10^3$.

- Evaluation formula for the limit cycle (for the $\Lambda\Lambda d$ system)

$$1 = C_1 I_1(s) + C_2 I_2(s) I_3(s)\,,$$

with

$$C_1 = \frac{1}{6\pi}\frac{m_d}{\mu_{\Lambda d}}\sqrt{\frac{\mu_{\Lambda(\Lambda d)}}{\mu_{\Lambda d}}}\,,\quad C_2 = \frac{\sqrt{2}}{3\pi^2}\frac{\sqrt{m_\Lambda \mu_{d(\Lambda\Lambda)}\mu_{\Lambda(\Lambda d)}}}{\mu_{\Lambda d}^{3/2}}\,,$$

where $\mu_{d(\Lambda\Lambda)} = 2m_\Lambda m_d/(2m_\Lambda + m_d)$, and

$$I_1(s) = \frac{2\pi}{s}\frac{\sin[s\sin^{-1}\left(\frac{1}{2}a\right)]}{\cos\left(\frac{\pi}{2}s\right)},$$

$$I_2(s) = \frac{2\pi}{s}\frac{1}{b^{s/2}}\frac{\sin[s\cot^{-1}\left(\sqrt{4b-1}\right)]}{\cos\left(\frac{\pi}{2}s\right)},$$

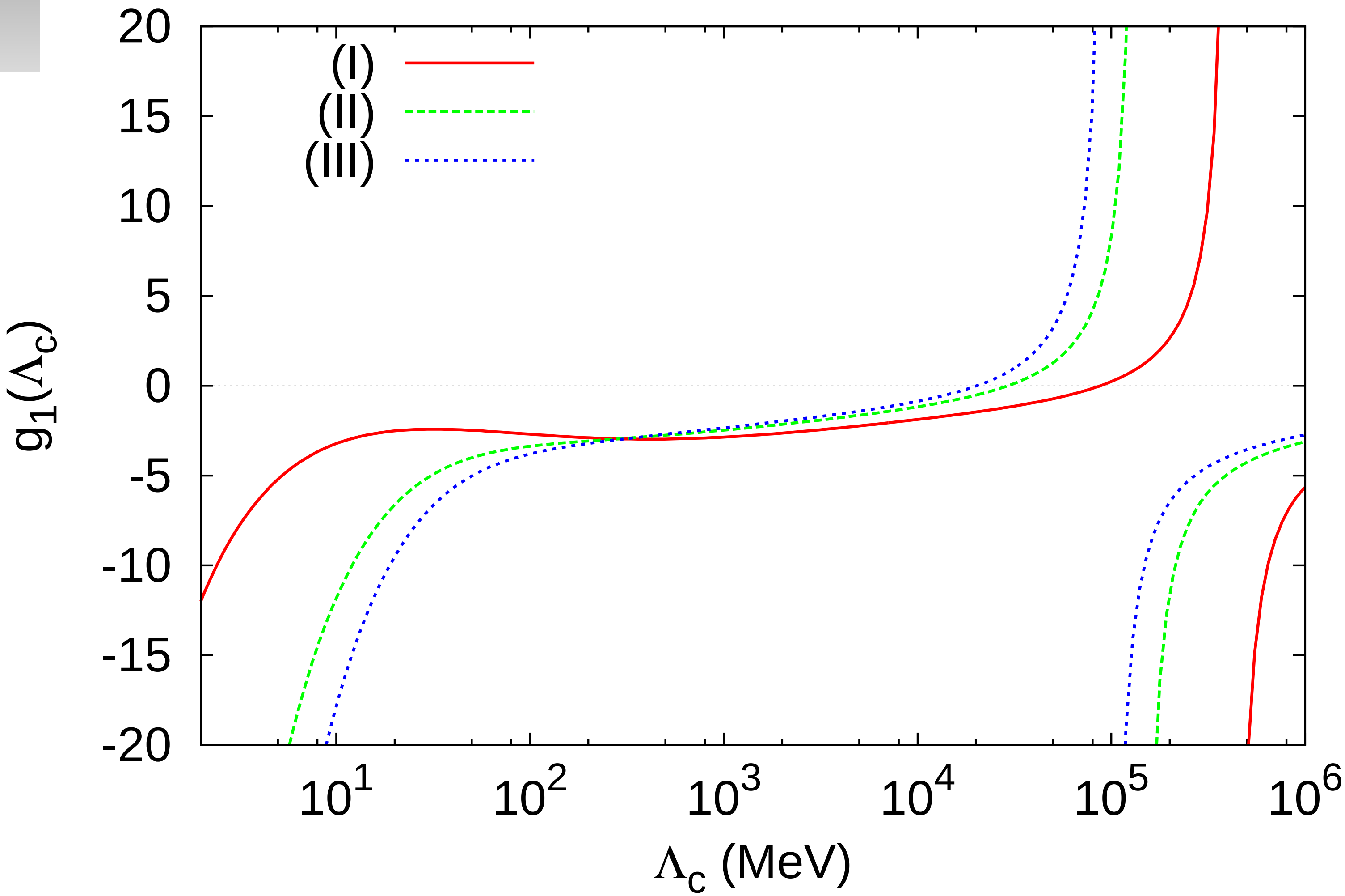
$$I_3(s) = \frac{2\pi}{s}b^{s/2}\frac{\sin[s\cot^{-1}\left(\sqrt{4b-1}\right)]}{\cos\left(\frac{\pi}{2}s\right)},$$

and $a = \frac{2\mu_{\Lambda d}}{m_d}$ and $b = \frac{m_\Lambda}{2\mu_{\Lambda d}}$.

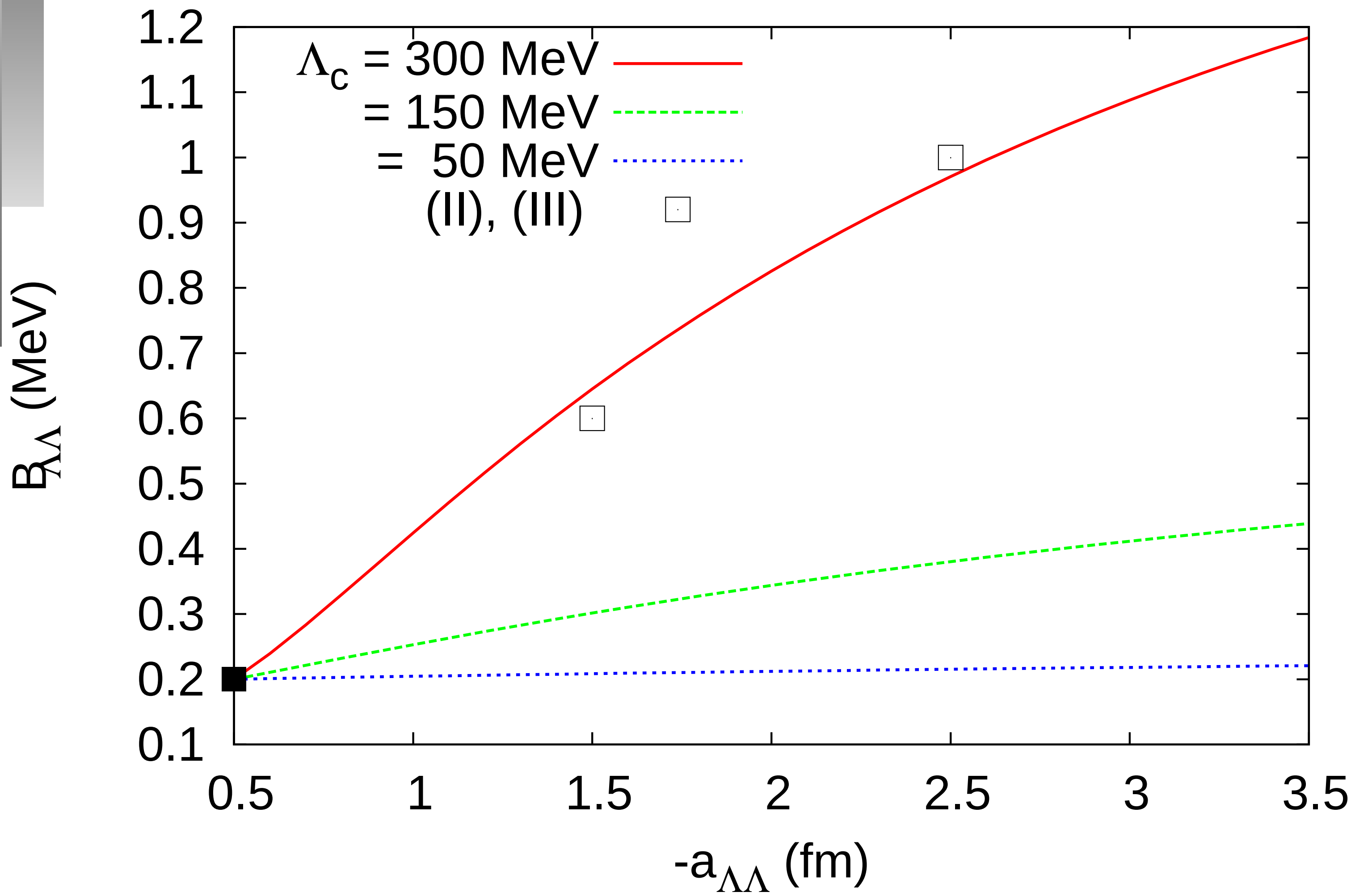
Numerical results: $S = 1$ ***channel***

- With $g_1(\Lambda_c)$,

$(B_{\Lambda\Lambda}, a_{\Lambda\Lambda}) =$ (I) (0.2 MeV, -0.5 fm), (II) (0.6, -1.5), (III) (1.0, -2.5).



Numerical results: $S = 1$ ***channel***

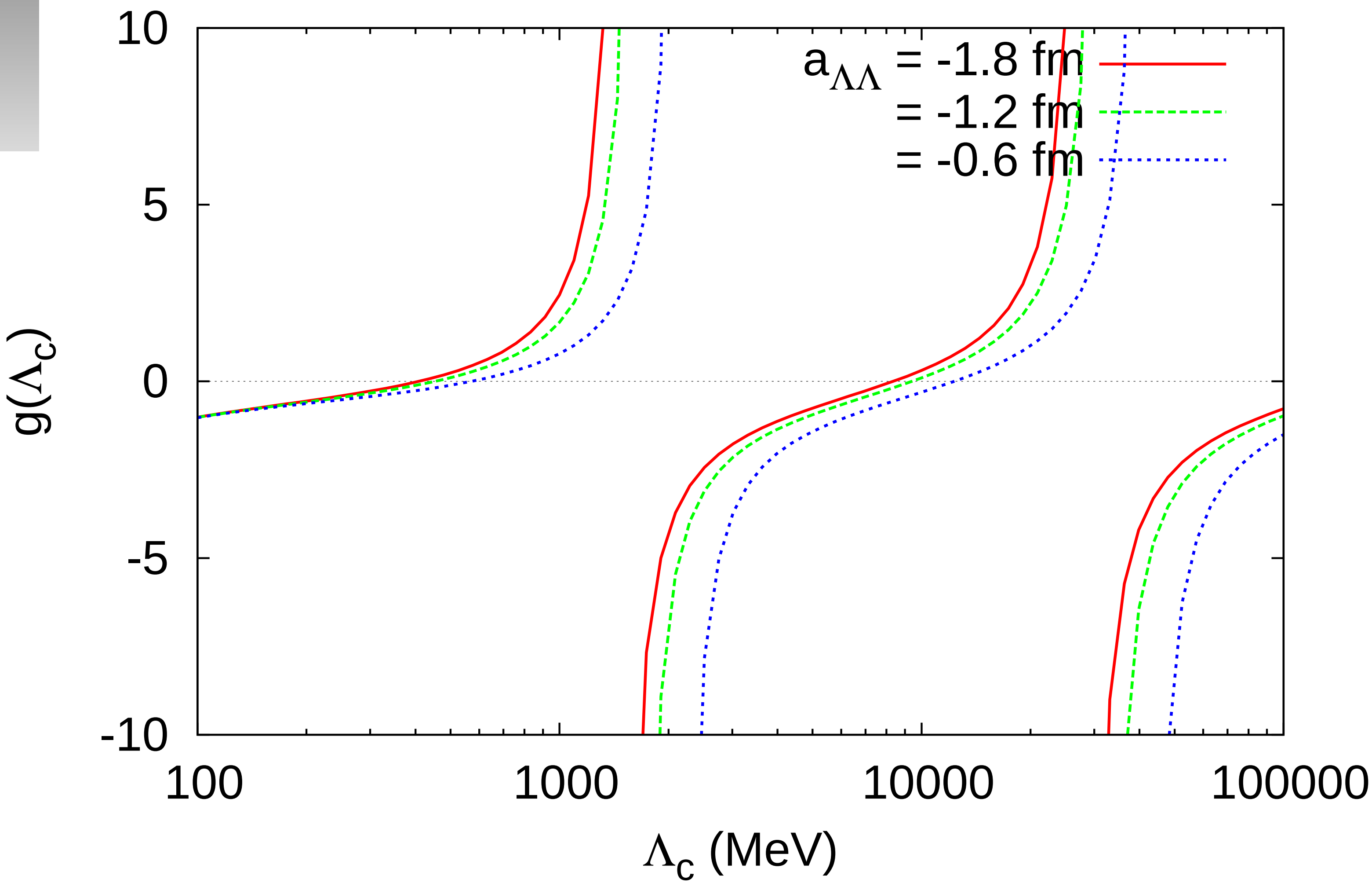


${}^{6}_{\Lambda\Lambda}$He at LO

- $\Lambda\Lambda\alpha$ ($S=0$)
- ${}^{5}_{\Lambda}$He, $B_\Lambda \simeq 3$ MeV
- First excited energy of α, $B_1 \simeq 20$ MeV
- The limit cycle appears, three parameters, $a_{\Lambda\Lambda}$, $\gamma_{\Lambda\alpha}$, $g(\Lambda_c)$, at LO, and the three-body interaction is fixed by using the Nagara event, $B_{\Lambda\Lambda} \simeq 6.93$ MeV

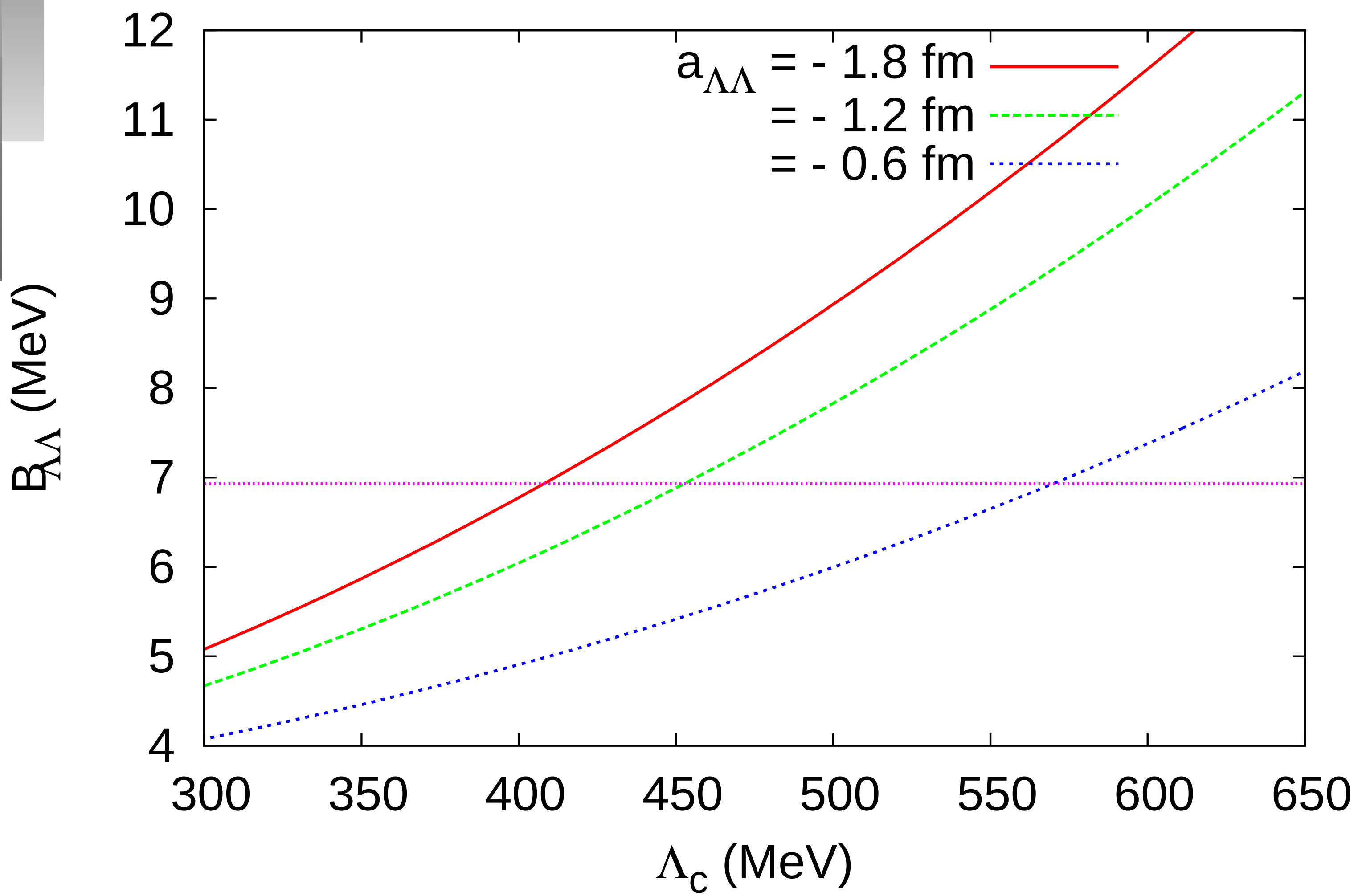
Numerical results:

- With $g(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}$ =6.93MeV)



$$\Lambda_n = \Lambda_0 \exp(n\pi/s_0)\,, \quad s_0 \simeq 1.05\,, \quad \exp(\pi/s_0) \simeq 19.9\,.$$

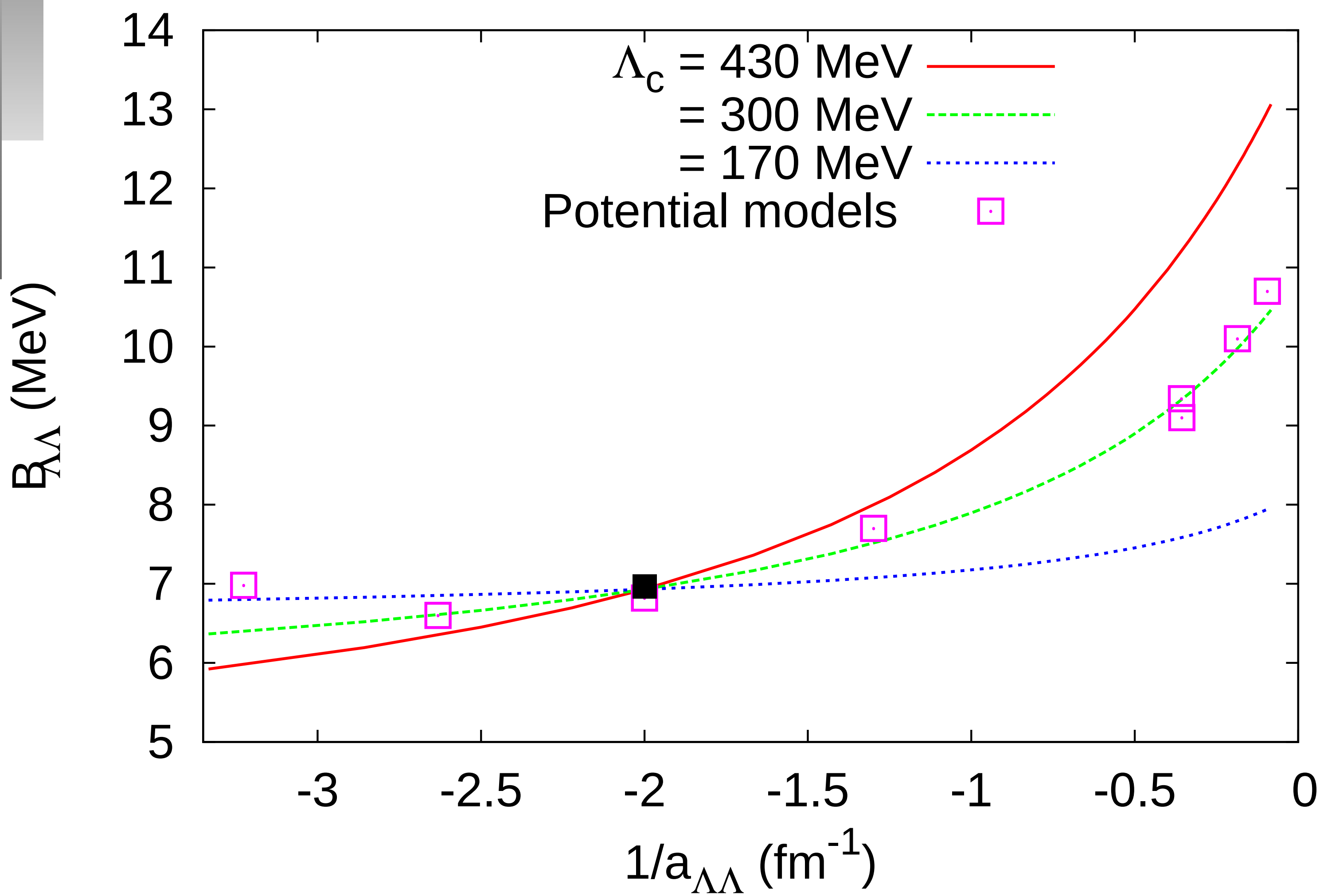
- Without $g(\Lambda_c)$



- $r_c = \Lambda_c^{-1} \simeq 0.35$ to 0.5 fm

Numerical results:

- With $g(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}=$6.93MeV, $a_{\Lambda\Lambda}=-0.5$fm)



[Filikhin and Gal, NPA707,491(2002)]

Summary

- Halo/cluster EFTs at LO for the light hypernuclei are constructed.
- Those three-body systems described by means of EFTs at LO exhibit a limit cycle in the asymptotic limit which implies the formation of bound states.
- For more conclusive results, we need to have the exp. data and include higher order corrections.
- We have applied the present approach to the study of $nn\Lambda$ system [SIA, Raha, Oh, PRC92(2015)024325].