Initial state fluctuations from SPS to LHC

J. Milošević
University of Belgrade and
Vinča Institute of Nuclear Sciences,
Belgrade, Serbia

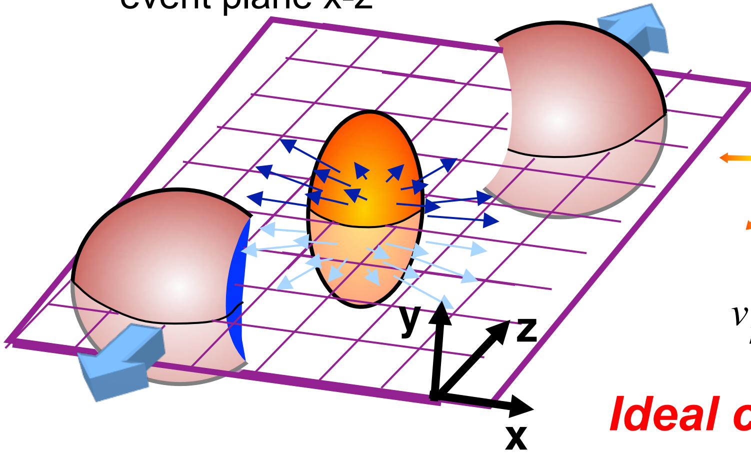
Outline

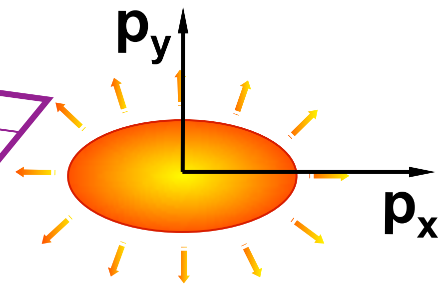
- Azimuthal anisotropy
  - conventional methods
  - Initial-state fluctuations (ISF) and higher order Fourier harmonics
- Triangular flow at SPS, RHIC and LHC energies
- Collectivity over a wide p_T range in PbPb collisions
- Collectivity in small pPb and smallest pp collision systems
  - ISF on sub-nucleonic level
- Factorization breaking – mechanism
  – p_T dependent event plane
  – η dependent event plane
- Principal Component Analysis (PCA) – method
- PCA method in flow physics – leading and sub-leading flow modes
- The PCA analysis in pPb and PbPb collisions at the LHC energy
- Conclusions

Anisotropy harmonics v_n – conventional methods

Event Plane (EP) method

event plane x-z



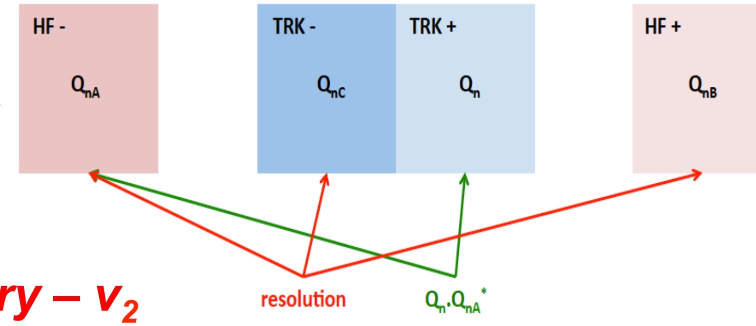


$$v_n = \langle \cos n(\phi - \Psi_n) \rangle$$

Scalar Product (SP) method

$|\Delta\eta| > 3$

$$v_n\{SP\} = \frac{\boxed{\langle Q_n \cdot Q_{nA}^* \rangle}}{\boxed{\sqrt{\frac{\langle Q_{nA} \cdot Q_{nB}^* \rangle \cdot \langle Q_{nA} \cdot Q_{nC}^* \rangle}{\langle Q_{nB} \cdot Q_{nC}^* \rangle}}}}$$



Ideal circle-like geometry – v_2

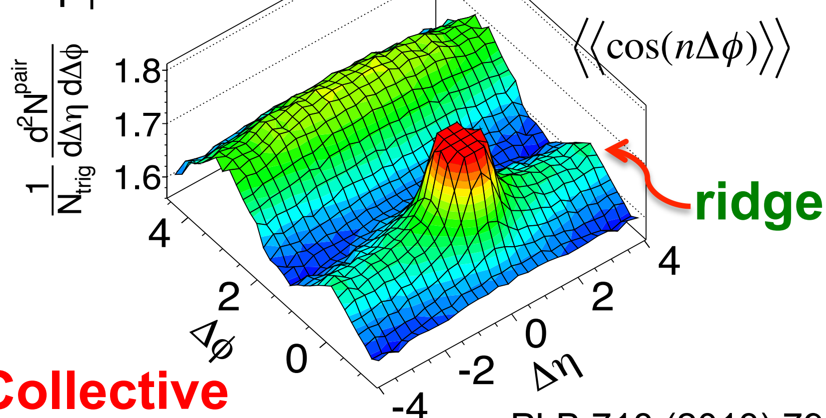
two-particle correlation method

pPb 5.02 TeV

$1<p_T<3$ GeV/c

0-3% centrality
N>110

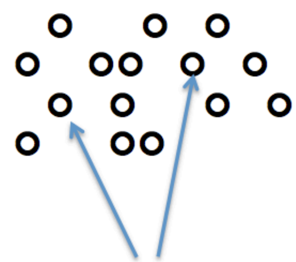
$$\langle\langle \cos(n\Delta\phi) \rangle\rangle$$



PLB 718 (2013) 795

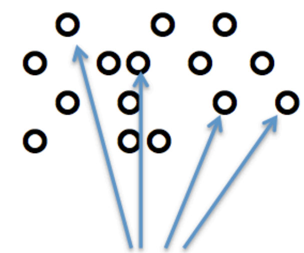
Collective behavior?

four-particle cumulant method



$$\left\langle e^{in(\varphi_1 - \varphi_2)} \right\rangle$$

Advantage wrt 2-part.corr.:
removes two- and three-particle non-flow correlation



$$\left\langle e^{in(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \right\rangle$$

- **v_n from even higher order cumulants:**
  $v_n\{6\}$, $v_n\{8\}$, ….

Lee-Yang zero method

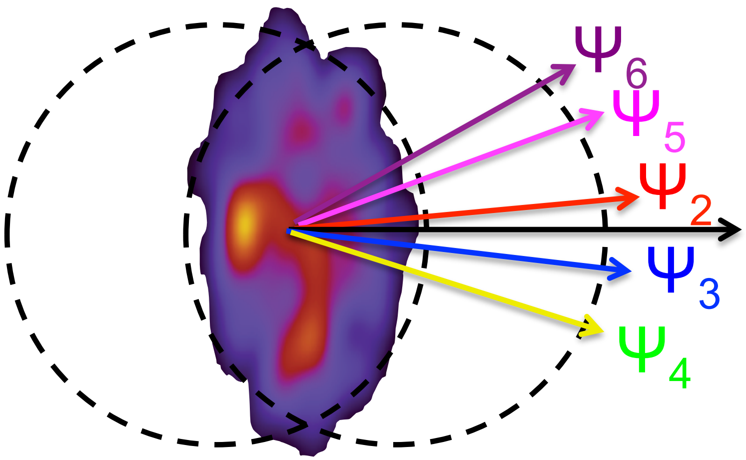
correlates all particles of interest

Role of initial state fluctuations (ISF) on anisotropy

Anisotropy harmonics with order higher than 2

geometry – v_2
ISF – v_3

Phys.Rev. C89 (2014) 044906
(arXiv:1310.8651)

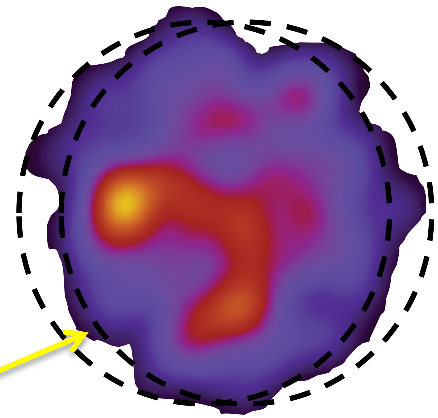


v_2, v_3, v_4, v_5 and v_6
using multiple methods

Simple, circle-like geometry does not describe the formed system precisely enough

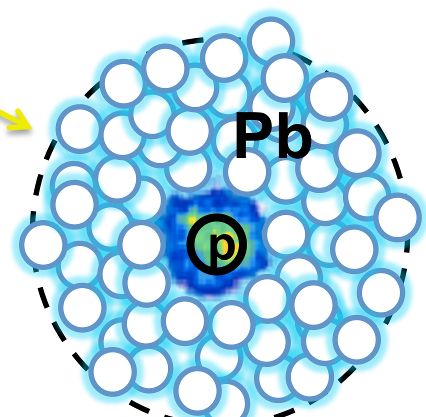
initial-state fluctuations dominates

Ultra-central collisions



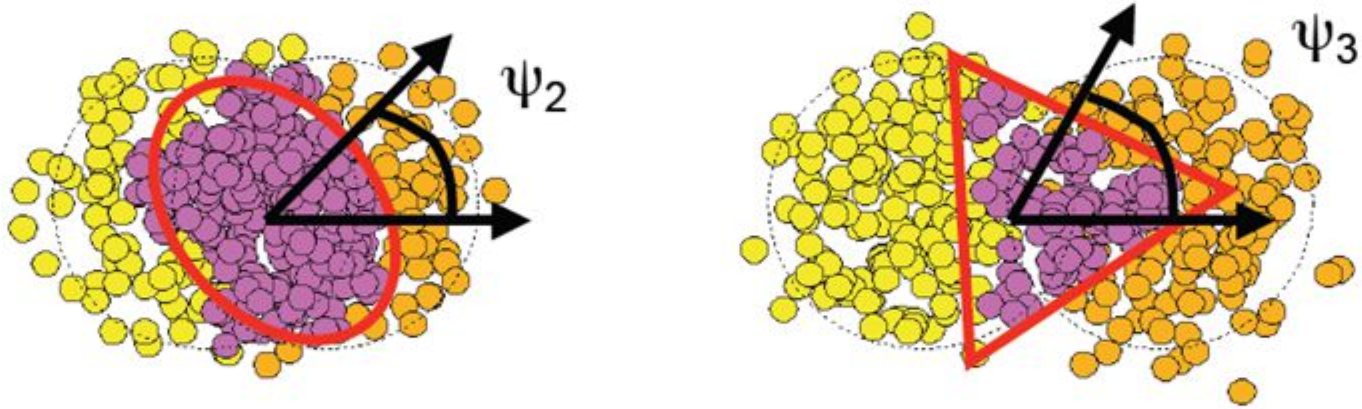
JHEP 1402 (2014) 088
(arXiv:1312.1845)

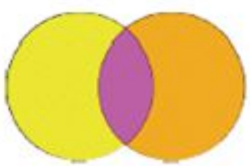
Asymmetric (pPb) high-multiplicity collisions



Phys.Lett. B724 (2013) 213
(arXiv:1305.0609)

Triangular flow – one of higher order Fourier harmonics

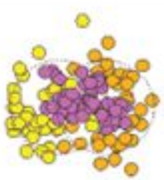




$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum 2v_n \cos\left(n(\phi - \psi_R)\right)\right)$$

$$v_2 = \left\langle \cos\left(2(\phi - \psi_R)\right)\right\rangle$$

$$v_3 = 0$$



$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum 2v_n \cos\left(n(\phi - \psi_n)\right)\right)$$

$$v_2 = \left\langle \cos\left(2(\phi - \psi_2)\right)\right\rangle$$

$$v_3 = \left\langle \cos\left(3(\phi - \psi_3)\right)\right\rangle$$

The triangular initial shape ➔ triangular hydrodynamic flow

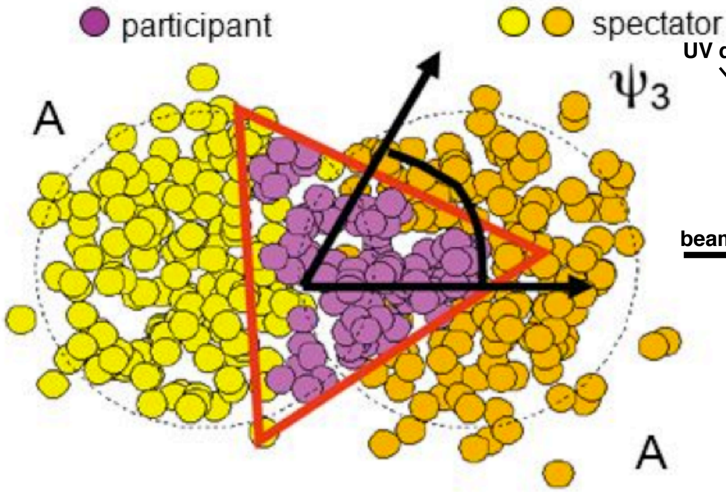
B. Alver and G. Roland
PRC 81(2010) 054905

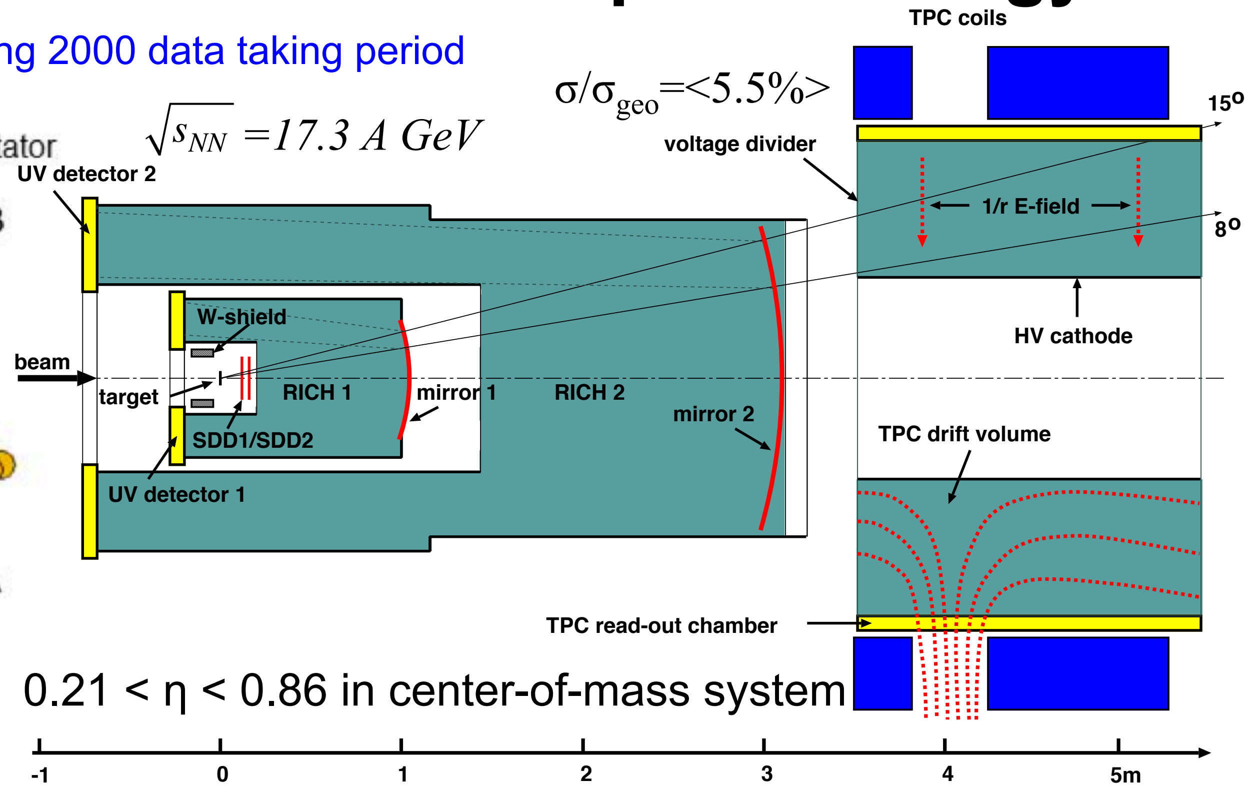
Triangular flow in PbAu at the top SPS energy

≈30M PbAu collisions collected during 2000 data taking period

$\sqrt{s_{NN}}$ =17.3 A GeV

σ/σ_{geo}=<5.5%>

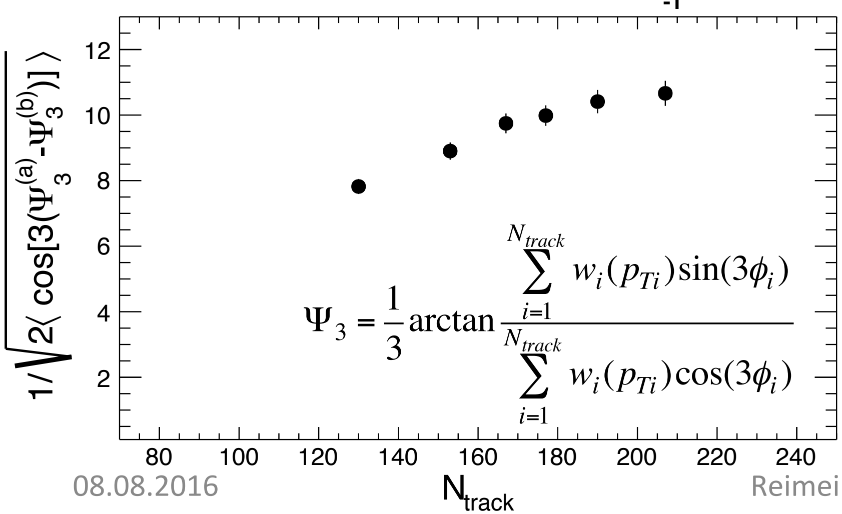




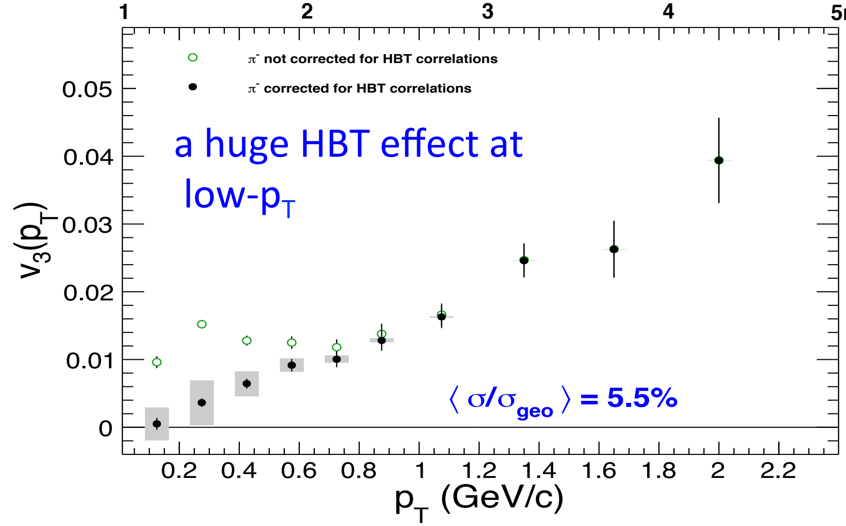
EP method is used

0.21 < η < 0.86 in center-of-mass system

accepted to **Nucl.Phys.A**



$$\Psi_3 = \frac{1}{3}\arctan\frac{\sum_{i=1}^{N_{track}} w_i(p_{Ti})\sin(3\phi_i)}{\sum_{i=1}^{N_{track}} w_i(p_{Ti})\cos(3\phi_i)}$$

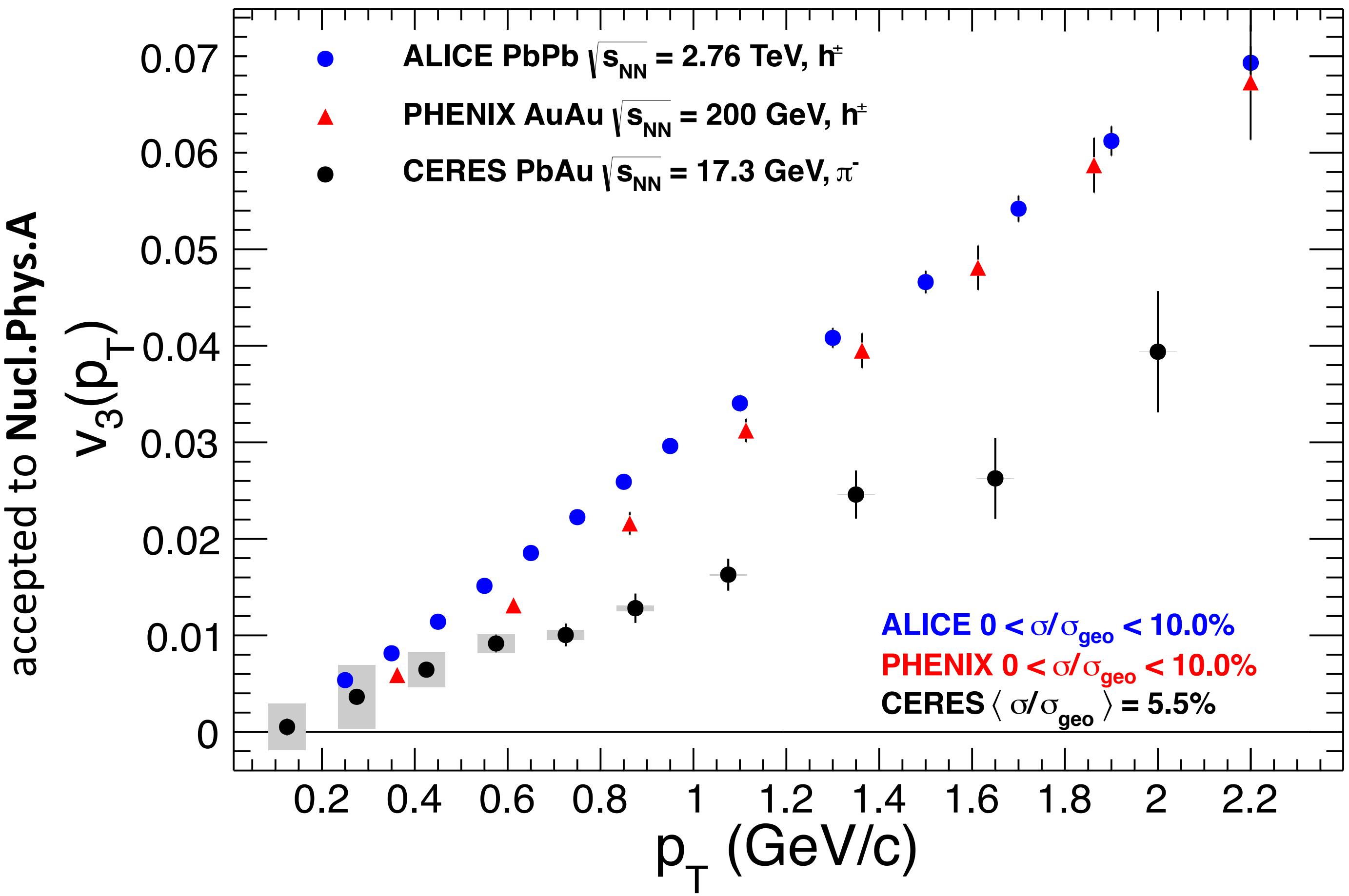


a huge HBT effect at low-p_T

$\langle \sigma/\sigma_{geo} \rangle$ = 5.5%

v_3 *vs* p_T – comparison to other experiments

- First p_T dependent measurement of the triangular flow at the top SPS energy
- Top RHIC and LHC energy gives very similar v_3 magnitudes
- The v_3 at the top SPS energy is about half of those at top RHIC and LHC
- Linear increase but with different slopes



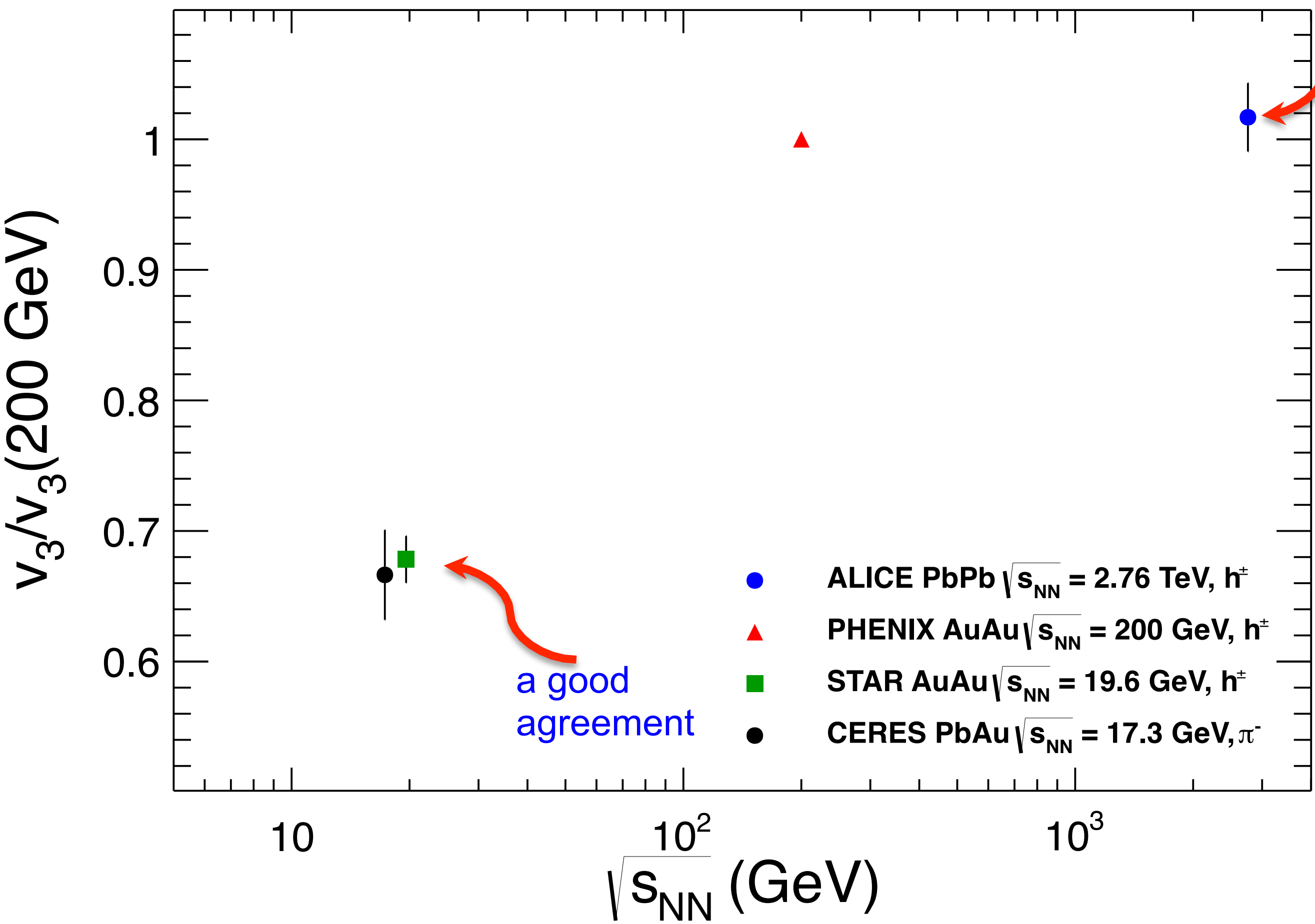
- Note limited p_T range restricted to the CERES acceptance
- ALICE uses large $|\Delta\eta|$ gaps
- Jet yield is for more than one order of magnitude smaller at SPS
- No option to include $|\Delta\eta|$ gap at CERES

PHENIX
PRL 107 (2011) 252301
ALICE
PLB 719 (2013) 18

Energy dependence

- RHIC 19.6 GeV is quite close to the top SPS energy of 17.3 GeV
- Comparison is done at very similar centralities (<σ/σ_{geo}> ≈ 5%)
- A rather good agreement with an AMPT prediction for the ratio of about 0.6 at 19.6 GeV RHIC energy

Accepted to **Nucl.Phys.A**



same v_3 at the top RHIC and LHC

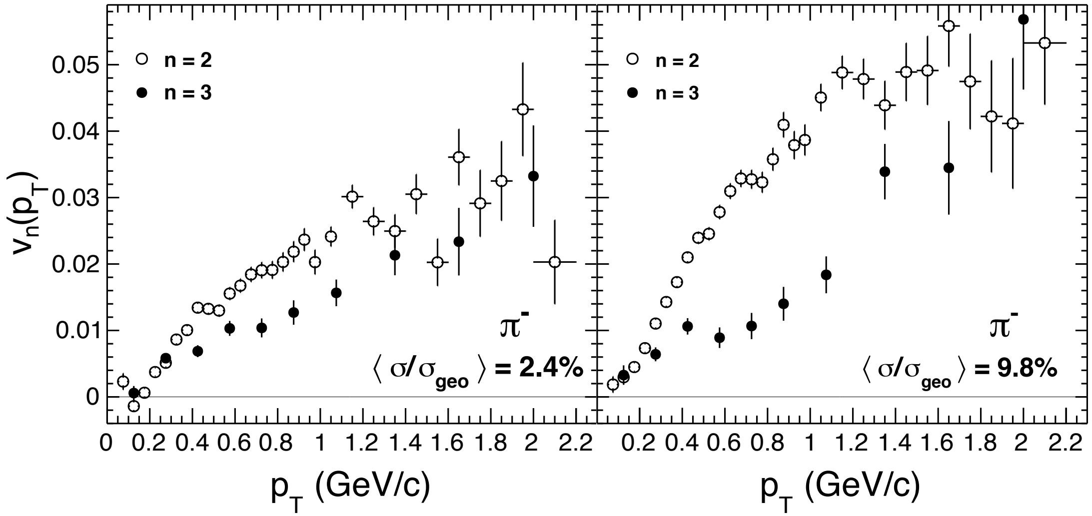
- As a referent level is taken v_3 value at the top RHIC energy
- **v_3 values integrated over $0.3 < p_T < 2.1$ GeV/c !**

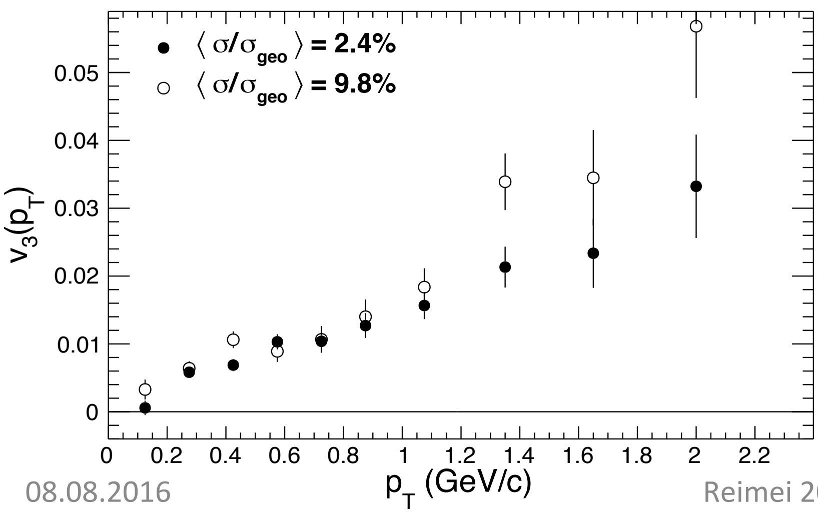
PHENIX
PRL 107 (2011) 252301
ALICE
PLB 719 (2013) 18
STAR
PRL 116 (2016) 112302

v_3 in comparison with v_2

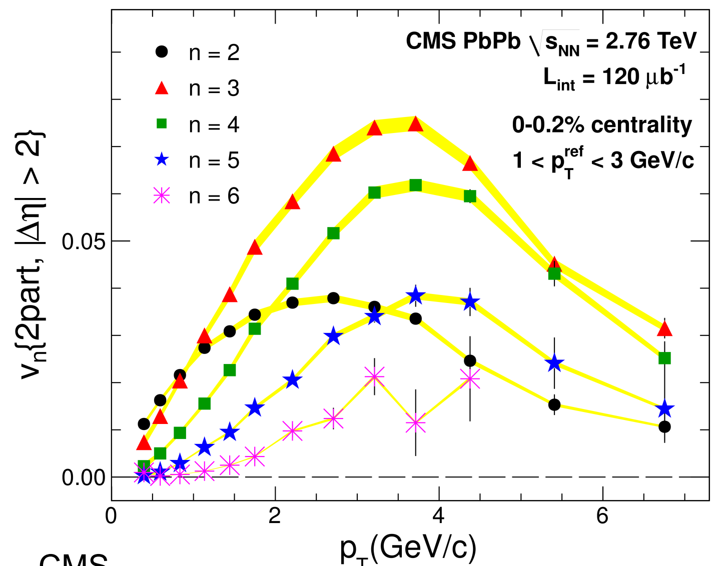
- Elliptic flow reflects the initial anisotropy and thus depends strongly on centrality
- Triangular flow comes from the ISF and weakly depends on centrality
- The different centrality behavior between v_2 and v_3
- For very central collisions ($\langle\sigma/\sigma_{geo}\rangle = 2.4\%$), v_3 becomes close to the v_2

accepted to **Nucl.Phys.A**





- Triangular flow is dominant anisotropy for ultra-central collisions at the LHC energies



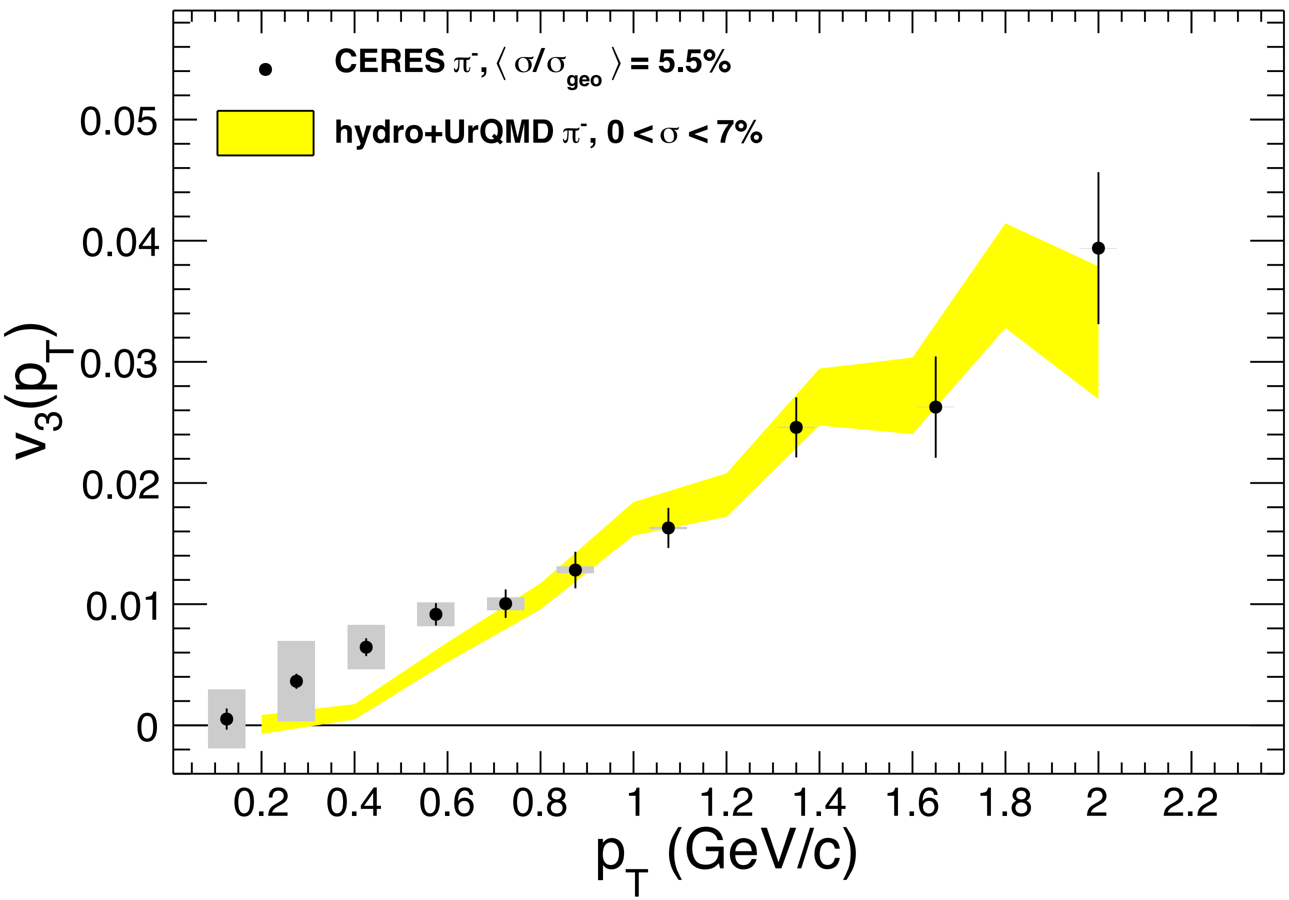
CMS

JHEP 1402 (2014) 088

Comparision with hydro+UrQMD predictions

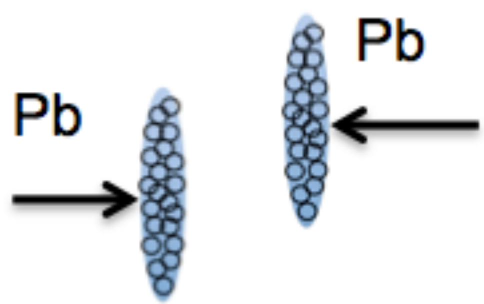
- ❖ Relativistic hydrodynamics + transport models (hybrid models)
- ✧ vHLLE viscous hydrosolver + UrQMD hadron cascade (I. Karpenko, P. Huovinen, H. Petersen and M. Bleicher **PRC 91 (2015) 064901)**
- ❖ The model predictions for hadrons within $0.2 < p_T < 2.0$ GeV/c and $-1 < \eta < 1$
- ❖ Cerentrality samples roughly correspond to the experimental ones

accepted to **Nucl.Phys.A**

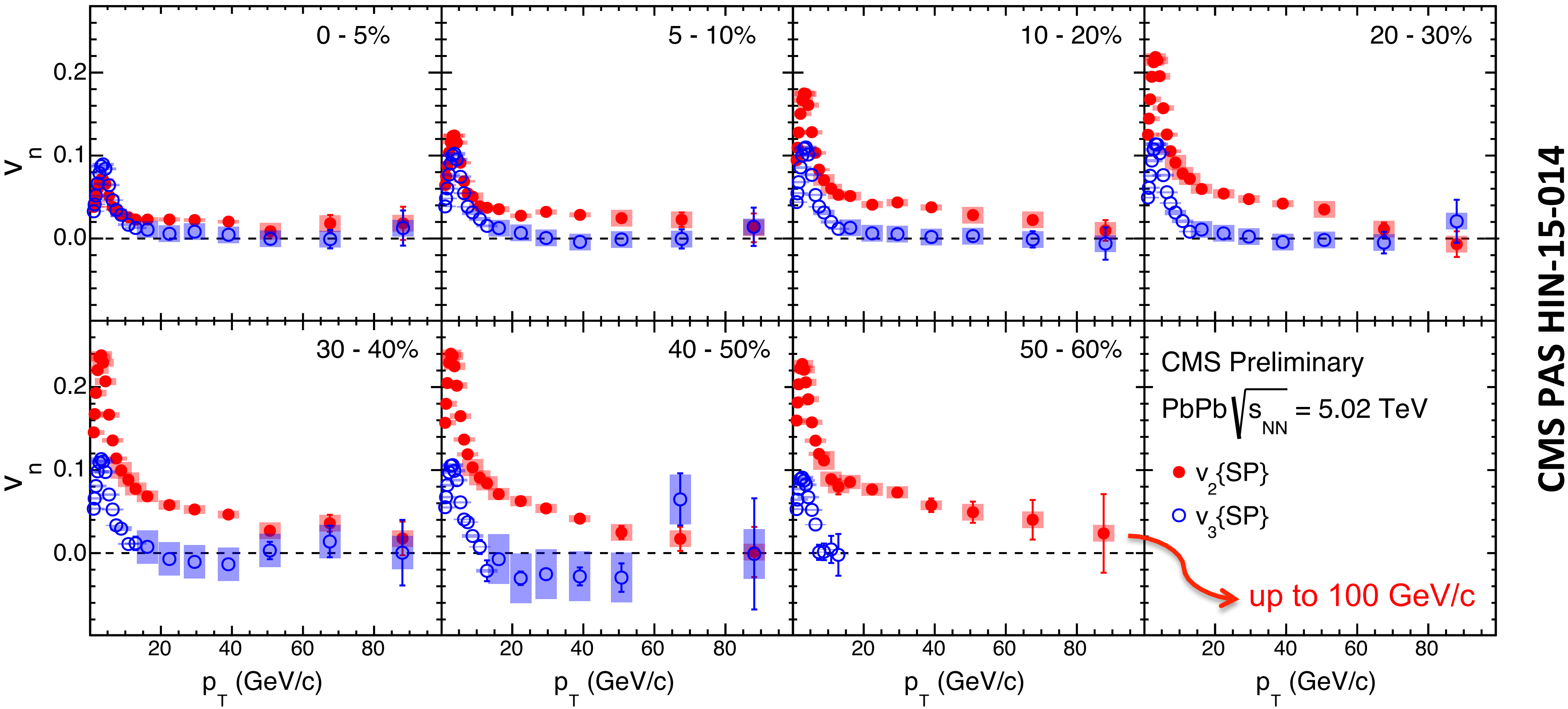


- ✧ Particlization at constant energy density 0.5 GeV fm^3
- ✧ Kinetic and chemical freeze-out are dynamical
- ✧ Model predictions in a very good agreement with the CERES results
- ✧ A small disagreement appears at low-p_T

Collectivity over a wide p_T range in PbPb

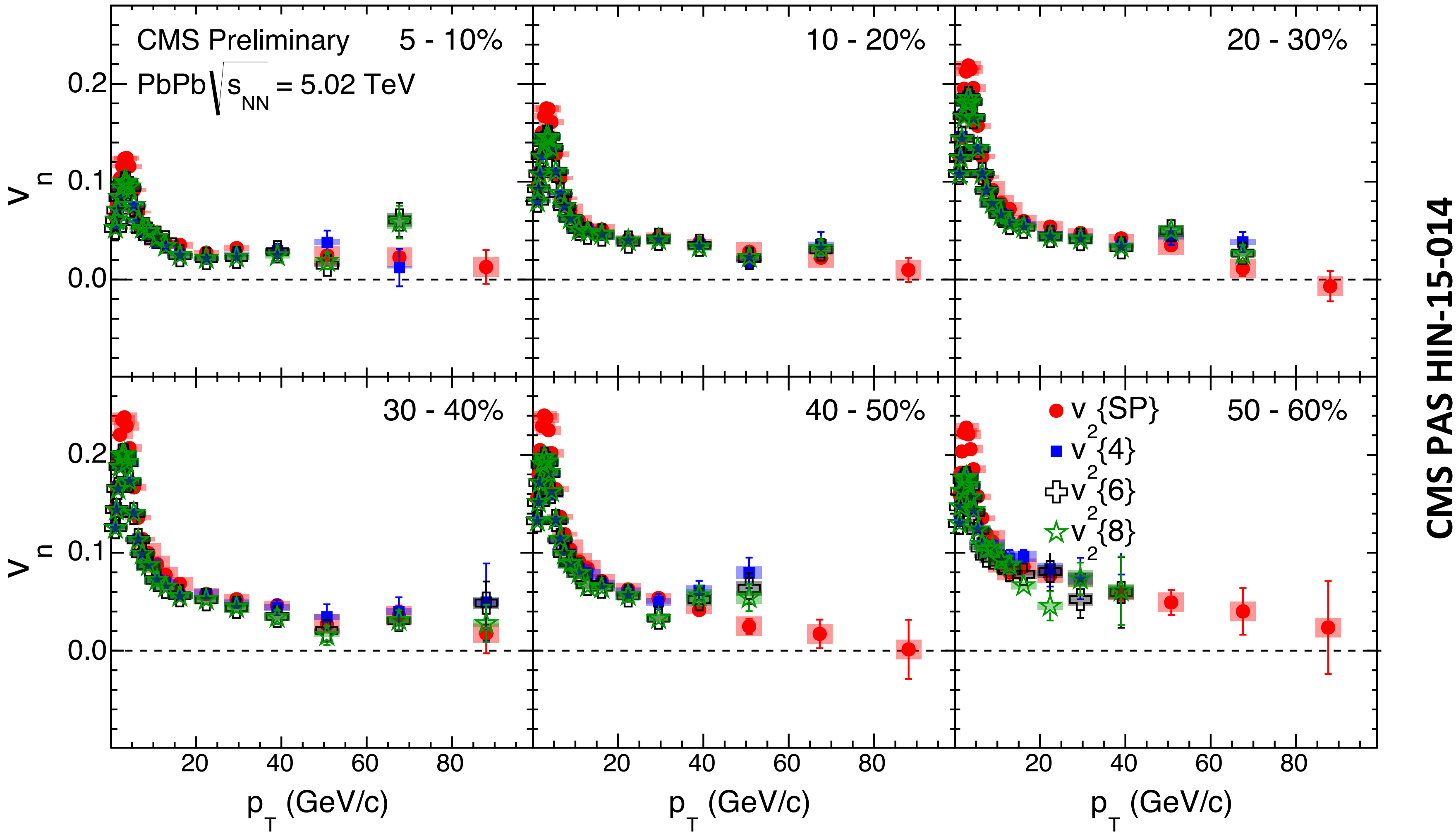


$v_n\{SP\}$ over a wide p_T range



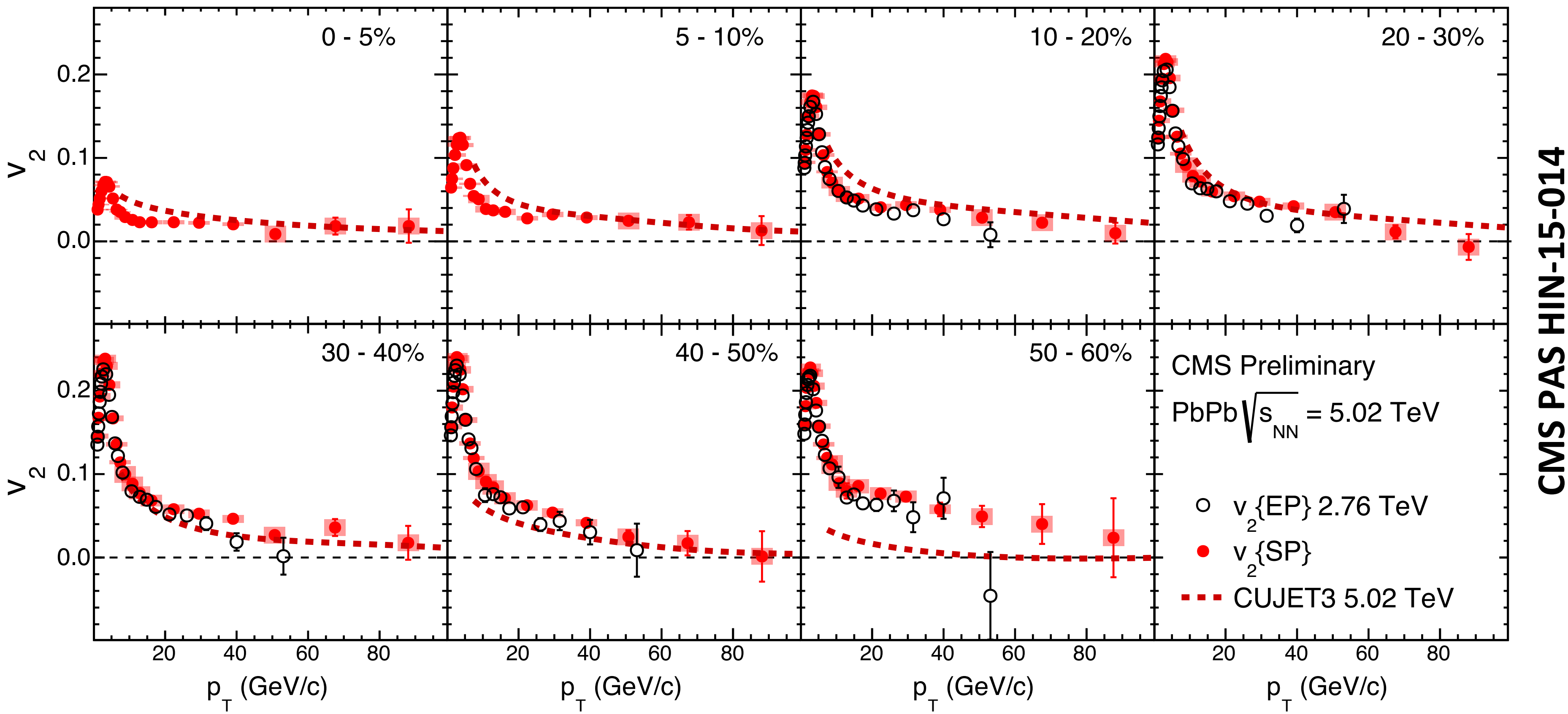
- low-p_T - hydrodynamic flow (v_2 – geometry, v_3 – ISF on nucleonic level)
- v_2 non-zero up to very high p_T
- high-p_T - may reflect the path-length dependence of parton energy loss
- v_2 is complementary to R_{AA} measurements
- v_3 mainly consistent with zero at high-p_T

Collectivity over a wide p_T range



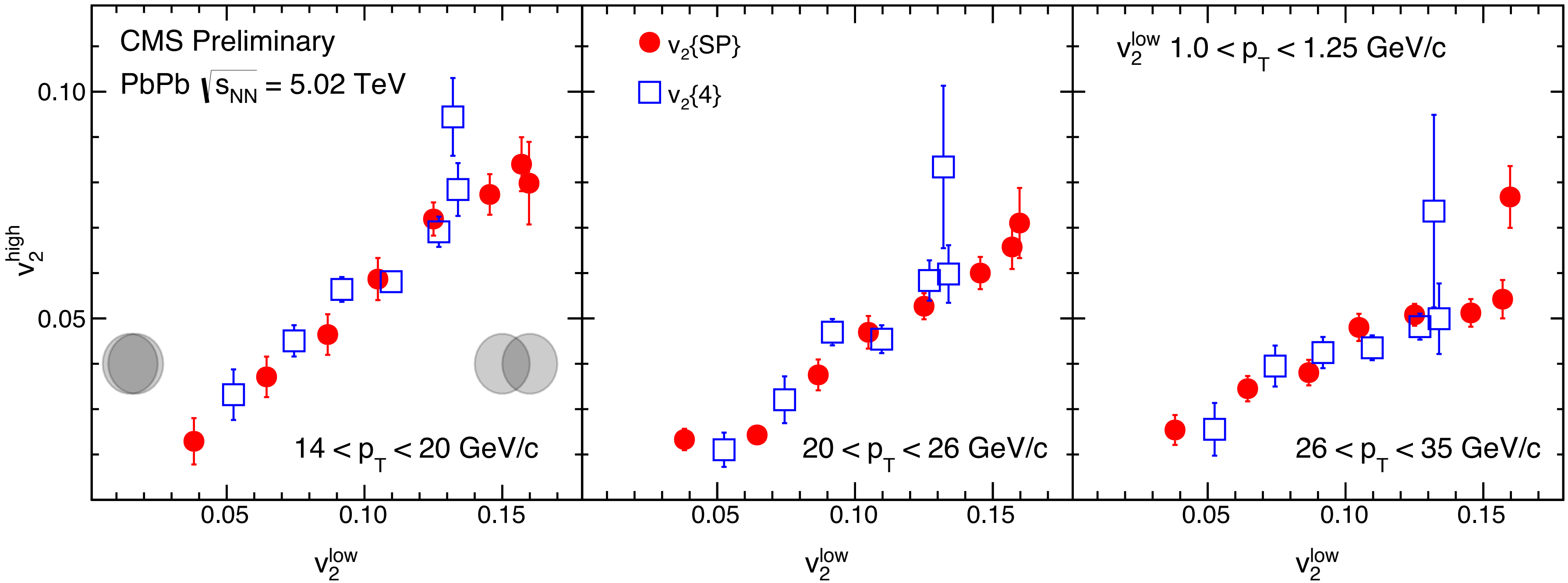
- ❖ low-p_T – ratio $v_2\{2k\}/v_2\{SP\} \approx 0.8$ and $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ ← hydrodynamics
- ❖ high-p_T – SP and multi-particle correlation tend to converge to the same value
- ❖ $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \neq 0$ ← collectivity (likely to be related to jet quenching)

Comparison with lower energy and CUJET3



- A slight increase of v_2 wrt results (EP method) from 2.76 TeV collision energy
- CUJET3 predictions roughly compatible with the data at high-p_T (over 40 GeV/c)
- At lower p_T, CUJET3 overpredicts the experimental v_2

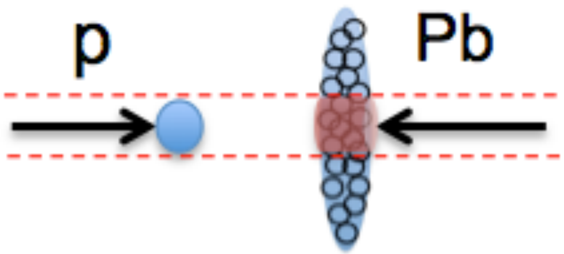
Collectivity over a wide p_T and centrality range

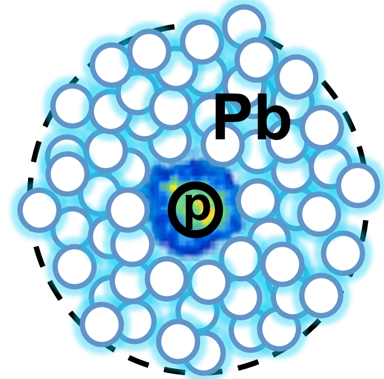


CMS PAS HIN-15-014

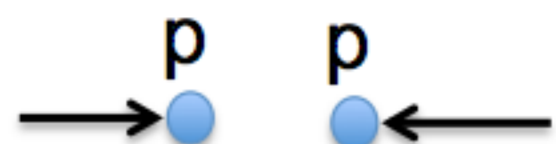
Soft and hard correlation

- Correlation between low-p_T v_2 and high-p_T v_2 over a wide centrality range
- Each point represents one centrality bin
- Strong correlation may indicate that low-p_T v_2 and high-p_T v_2 may have the same origin
- Within uncertainties, slopes between $v_2\{SP\}$ and $v_2\{2k\}$ are compatible
- Extrapolations compatible to 0 within uncertainties

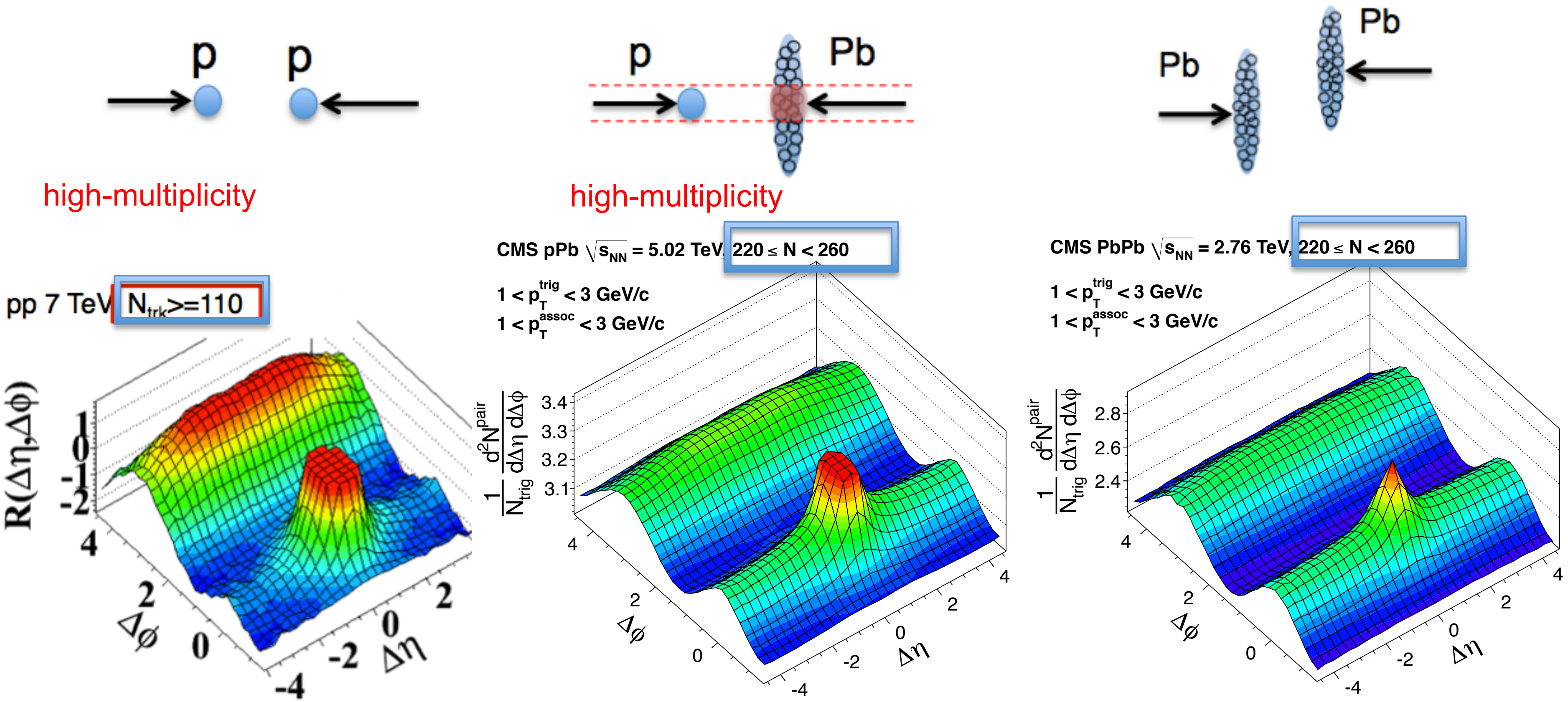




Collectivity in small pPb and smallest pp systems?

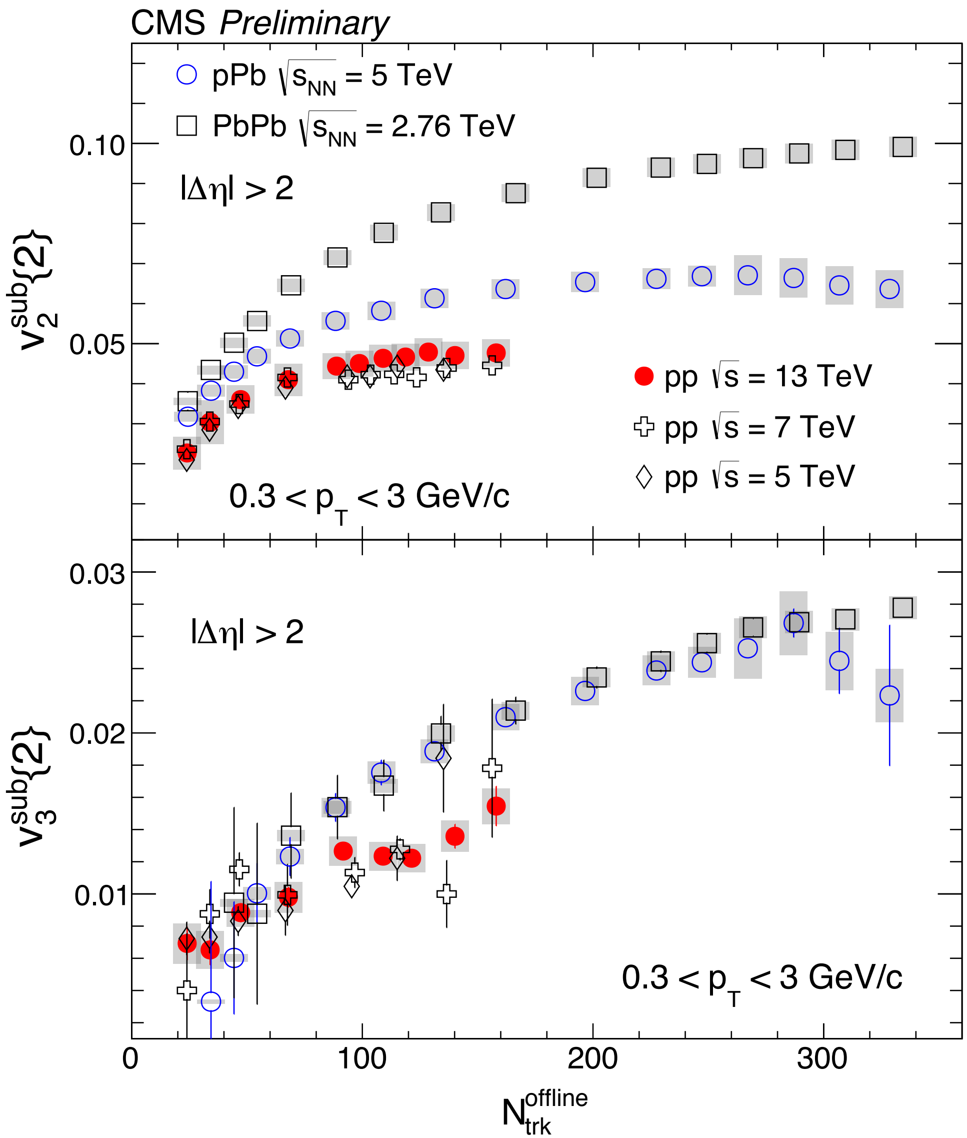
The ridge seen in all colliding systems at LHC



JHEP 09 (2010) 091 **PLB 718 (2013) 795** **PLB 724 (2013) 213**

❖ Does the ridge in *pp* and *pPb* collisions originate from hydrodynamics flow like in PbPb collisions or it is connected with color-glass condensate (CGC)

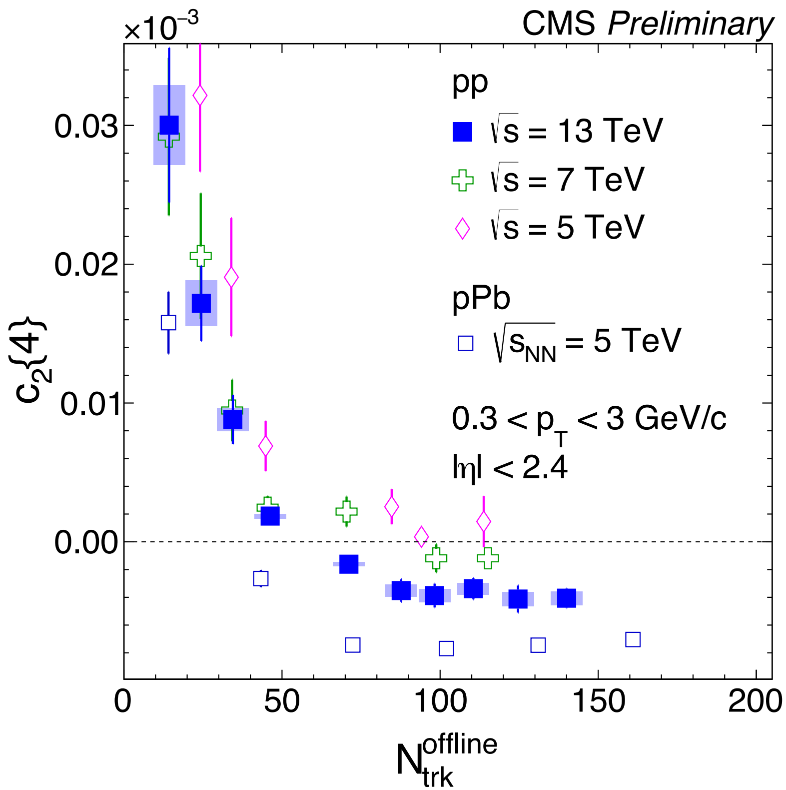
$v_2\{2\}$ and $v_3\{2\}$ in pp at different collision energies

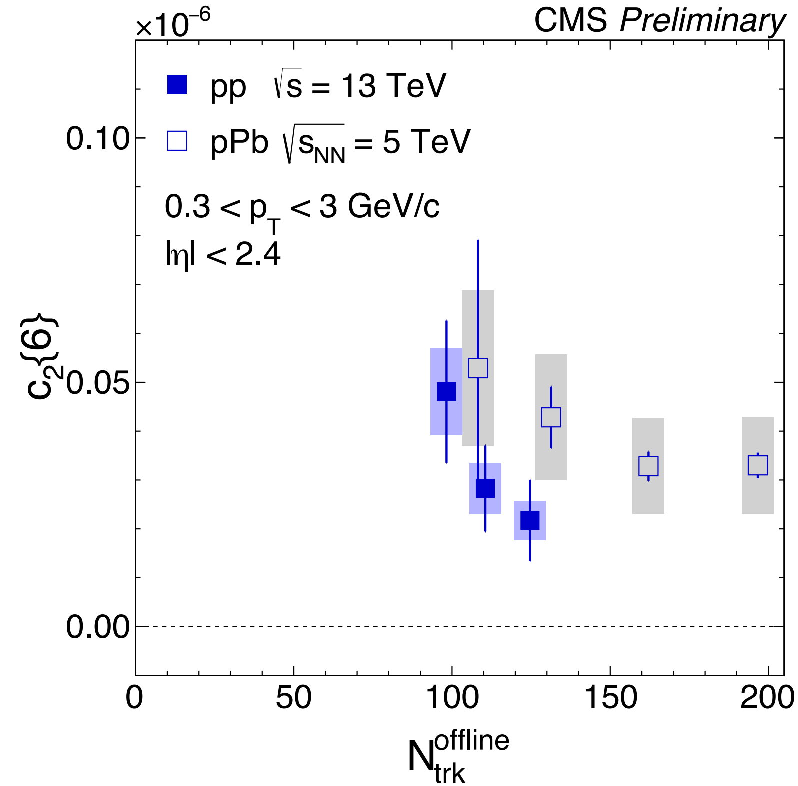


CMS PAS HIN-16-010

- There is no or a very weak energy dependence of v_2 in pp collisions
- $v_2\{2\}$ in pp collisions shows a similar pattern as the one seen in pPb collisions (gets flat at the highest multiplicities)
- The $v_2\{2\}$ magnitude is ordered: it is highest in PbPb, gets smaller in pPb and become smallest in pp collisions
- In difference of the v_2, the v_3 magnitude is comparable to those in pPb and PbPb collisions
- At low multiplicities, the systematic uncertainties are large for all the three systems
- At high multiplicities, v_3 in pp increases at a slower rate than in pPb and PbPb systems

$c_2\{4\}$ and $c_2\{6\}$ in pp at different collision energies

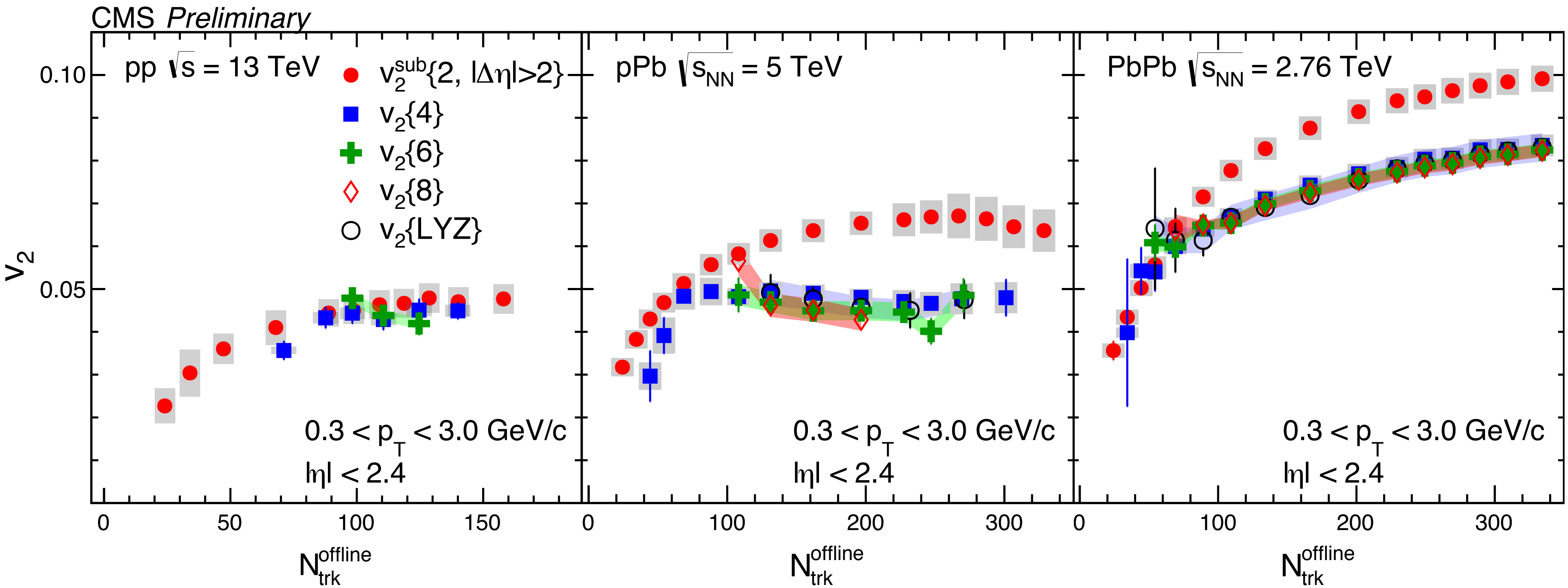




CMS PAS HIN-16-010

- Multi-particle correlations are used to reduce jet correlations from the away side and to explore collective nature of the long-range correlations in pp. $v_2\{4\}$ and $v_2\{6\}$ are extracted
- Clear negative $c_2\{4\}$ at high multiplicities in pp at 13 TeV is seen $v_n\{4\} = \sqrt[4]{-c_n\{4\}}$
- and positive $c_2\{6\}$ $v_n\{6\} = \sqrt[6]{\frac{1}{4}c_n\{6\}}$
- Statistical limitations

v_2 in pp compared to pPb and PbPb results

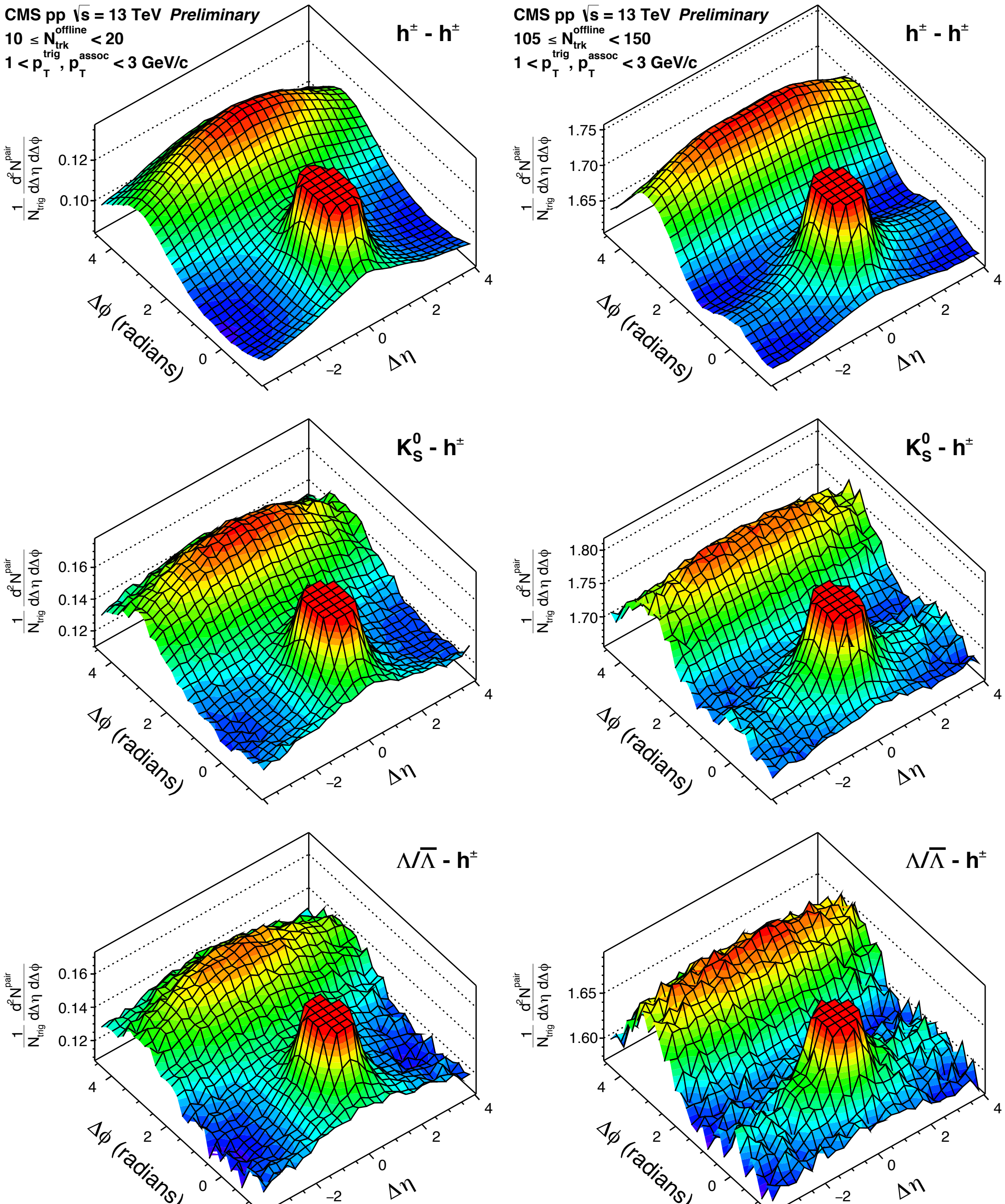


$v_2\{2\} \geq v_2\{4\} \approx v_2\{6\}$

collectivity!

CMS PAS HIN-16-010

- Elliptic flow in pp measured using 2- and multi-particle correlations – compared to pPb and PbPb results
- $v_2\{2\}/v_2\{4\}(pp) \leq v_2\{2\}/v_2\{4\}(pPb)$ ← related to initial-state (IS) fluctuations
- smaller $v_2\{2\}/v_2\{4\}$ ← less IS fluctuating sources (PRL **112** (2014) 082301)



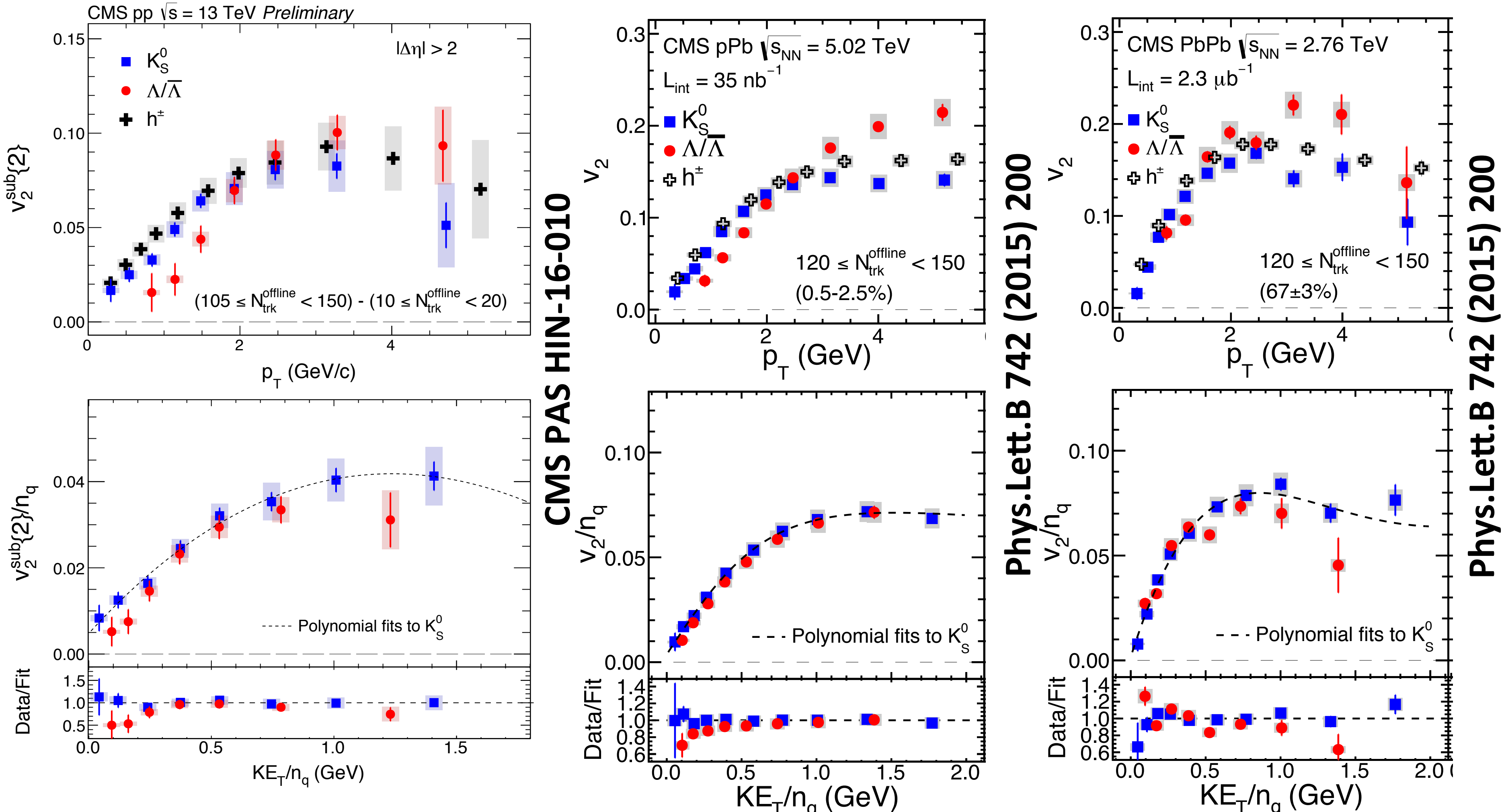
2D 2-particle corr. function in low- and high-multiplicity pp

CMS PAS HIN-16-010

- charged-charged or charged-strange ($K_S{}^0$ and $\Lambda/\bar{\Lambda}$) particles
- particles are correlated within given multiplicity bin
- The ridge, at $\Delta\phi \approx 0$ and elongated at $\Delta\eta$, is seen only in high-multiplicity pp events
- The ridge is present not only for charged, but also for strange particles
- What is the origin of the ridge in the smallest pp system?

collective behavior in pp?

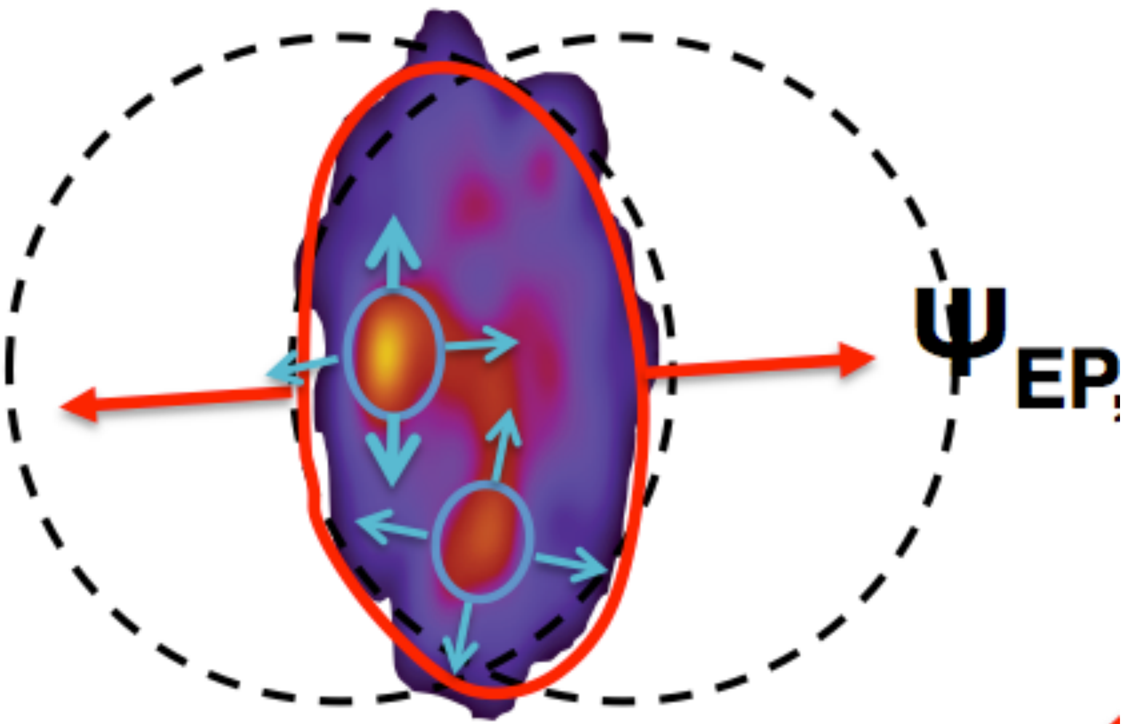
NCQ scaled v_2 in pp collisions compared to pPb and PbPb



collectivity!

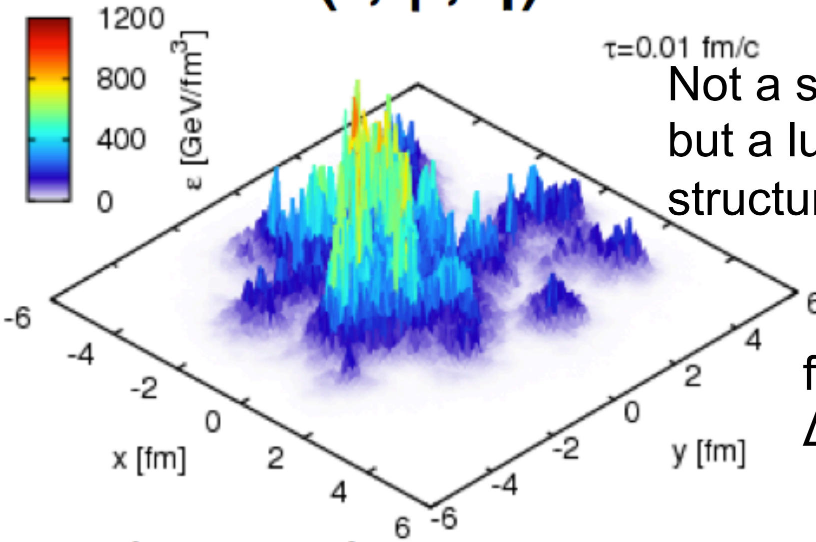
❖ Significant magnitude of the NCQ scaled v_2 in *pp,* comparable to the ones seen in pPb and PbPb collisions

Factorization breaking – p_T dependent event plane fluctuations



Initial-state inhomogeneity

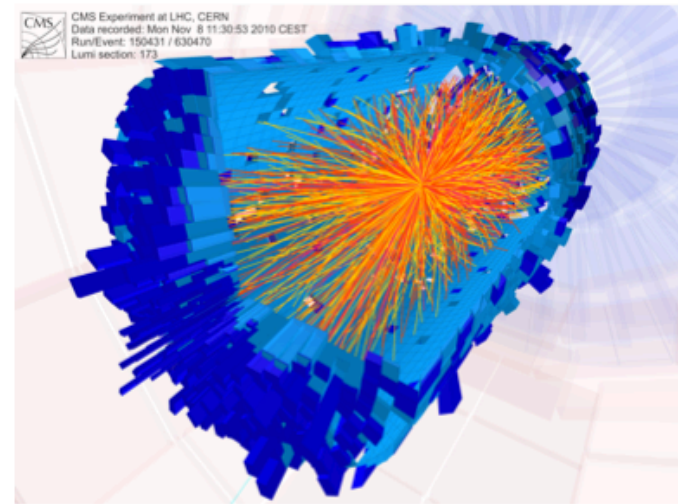
Initial state $\varepsilon(r,\varphi,\eta)$ —— EbE hydro. ——> Final state $f(p_T,\varphi,\eta)$



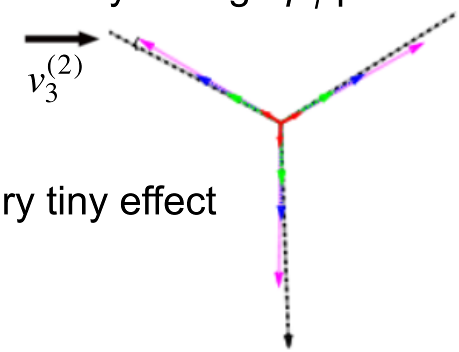
Not a smooth but a lumpy structure

fluctuations $\Delta\varepsilon(r,\phi,\eta)$

overlap zone in x-y



only for high-p_T particles



a very tiny effect

Example: sub-leading triangular flow

- ✧ The goal is to map initial-state and its fluctuations in 3D
- ✧ Local hotspots perturb the EP of a smooth medium, so $\Psi_n(p_T)$ contains information about initial-state fluctuations Phys.Rev.C **92** (2015) 034911
- ✧ Within hydrodynamics, initial-state fluctuations could appear as (sub-leading) flows

Factorization breaking

❖ How to connect $v_n(p_T)$ *and* $V_{n\Delta}(p_T)$?
❖ Usual assumption that EP angle Ψ_n does not depend on p_T leads to factorization

$$V_{n\Delta}(p_{T1}, p_{T2}) = \sqrt{V_{n\Delta}(p_{T1}, p_{T1})} \times \sqrt{V_{n\Delta}(p_{T2}, p_{T2})} = v_n(p_{T1}) \times v_n(p_{T2})$$

❖ *Gardim et al.*, PRC 87 (2013) 031901 and *Heinz et al.*, PRC 87 (2013) 034913 proposed that not only v_n depends on p_T, but also Ψ_n could depends on p_T due to event-by-event (EbE) fluctuating initial state

❖ then:

$$V_{n\Delta}(p_{T1}, p_{T2}) = \left\langle v_n(p_{T1}) v_n(p_{T2}) \cos\left[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))\right]\right\rangle$$
$$\neq \sqrt{V_{n\Delta}(p_{T1}, p_{T1})} \times \sqrt{V_{n\Delta}(p_{T2}, p_{T2})}$$

even if hydro flow is the only source of the correlation

initial state fluctuations $\rightarrow$ $\Psi_n(p_T)$ $\rightarrow$ factorization breaking

Factorization breaking

❖ new observable: $$r_n = \frac{V_{n\Delta}(p_T^{trig}, p_T^{assoc})}{\sqrt{V_{n\Delta}(p_T^{trig}, p_T^{trig})}\sqrt{V_{n\Delta}(p_T^{assoc}, p_T^{assoc})}} =$$

$$\frac{\left\langle v_n(p_T^{trig}) v_n(p_T^{assoc}) \cos\left[n(\Psi_n(p_T^{trig}) - \Psi_n(p_T^{assoc}))\right]\right\rangle}{\sqrt{v_n^2(p_T^{trig}) v_n^2(p_T^{assoc})}} = \left\{ \begin{array}{ll} 1 & \text{fact. holds} \\ <1 & \text{fact. breaks} \\ >1 & \text{non-flow} \end{array} \right.$$

❖ Large effect is expected and confirmed in ultra central PbPb collisions

***CMS collaboration**: Studies of azimuthal dihadron correlations in ultra-central PbPb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV , JHEP **1402** (2014) 088*

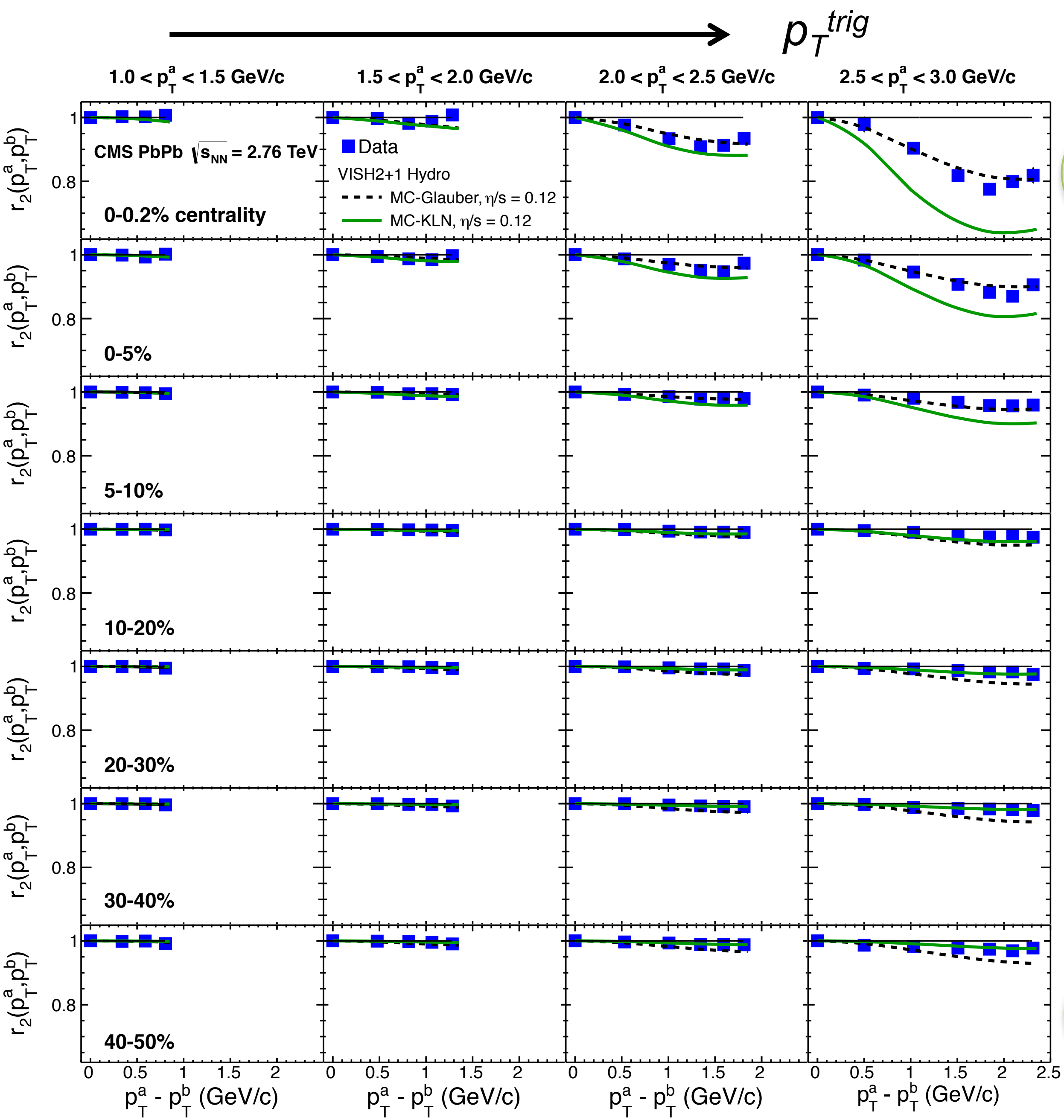
❖ As in pPb collisions initial-state fluctuations play a dominant role could we expect a similar (in size) effect?

❖ Two hydro models with different initial conditions and η/s were developed:

✧ *Heinz-Shen VISH2+1: PRC **87** (2013) 034913*

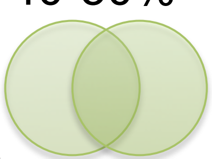
✧ *Kozlov et. al.: arXiv:1405.3976*

❖ Constraining of initial conditions and η/s by comparing to the exp. data?

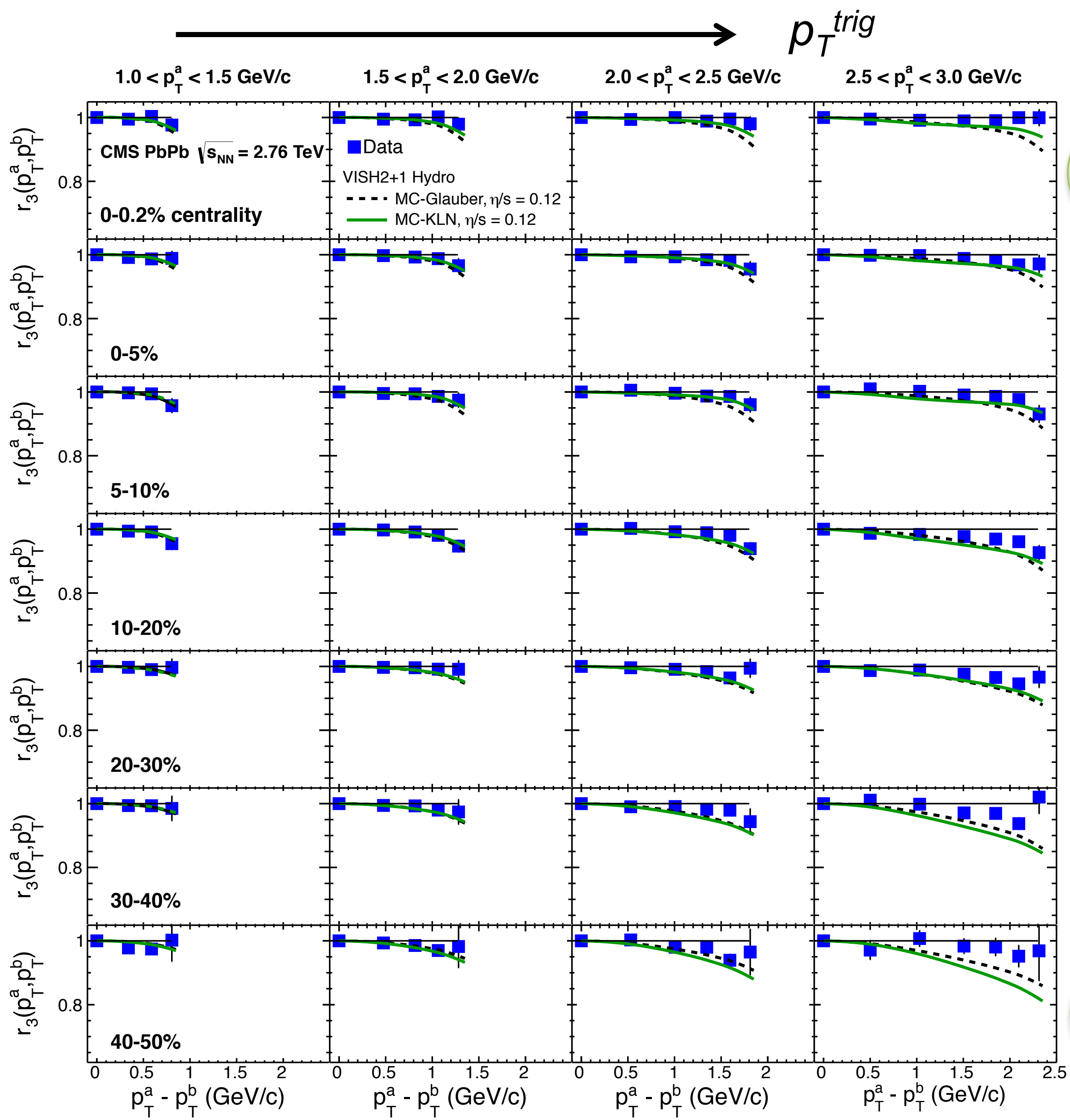


PbPb case

- The effect increases with rise of p_T^{trig} and p_T^{trig}-p_T^{assoc}
- Approaching the central collisions, the effect dramatically increases achieving value over 20%
- For semi-central collisions, the effect achieves only a size of 2−3%



arXiv: 1503.01692
PRC 92 (2015) 034911



0-0.2%

PbPb case

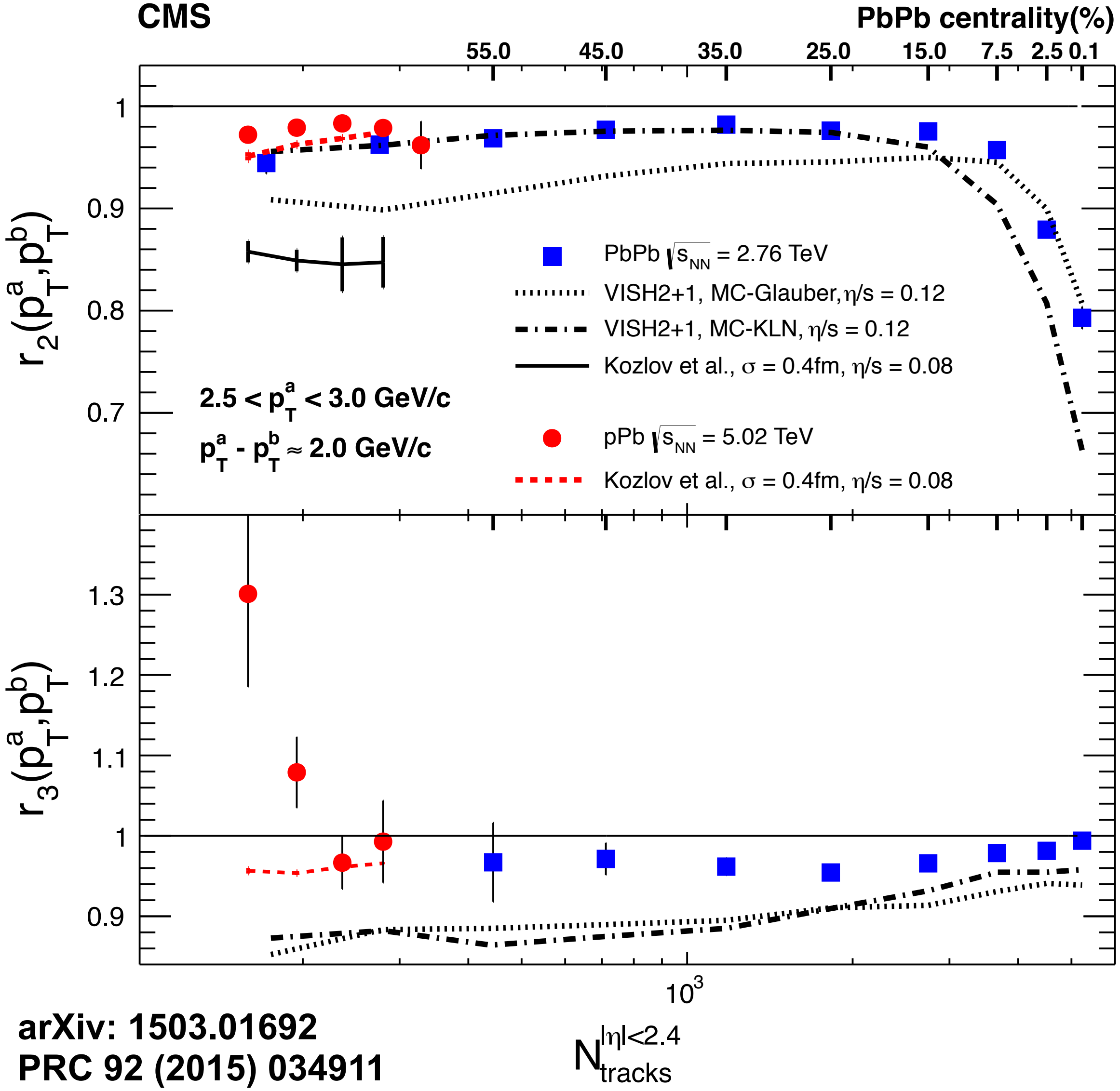
- Factorization holds better for V_3
- Breaking visible only for the highest p_T^{trig}-p_T^{assoc}
- Very weakly depends on centrality

40-50%

arXiv: 1503.01692

PRC 92 (2015) 034911

r_n multiplicity dependence at the highest Δp_T

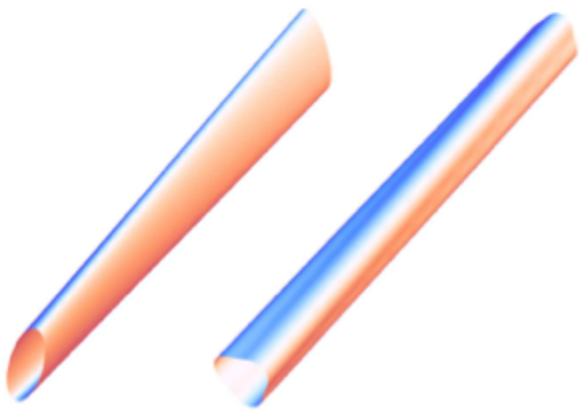


arXiv: 1503.01692
PRC 92 (2015) 034911

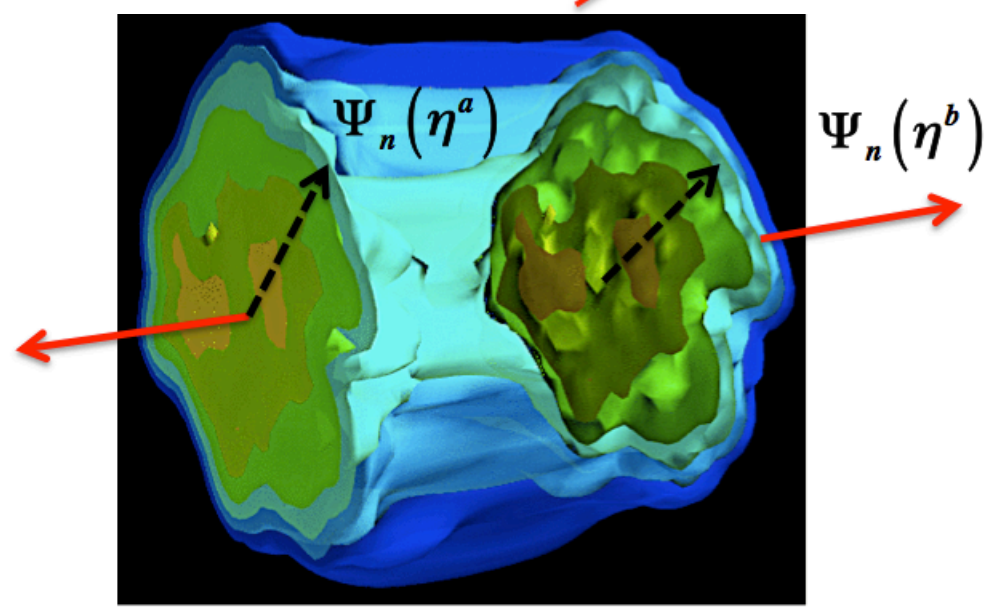
- Dramatic increase at ultra-central PbPb. For small centralities (>5%) ≈ few %
- The r_2 in pPb is a bit smaller than in PbPb
- Strong r_3 multiplicity dependence in pPb, but very weak in PbPb
- A non-flow effect in pPb for the highest p_T^{trig} in lower multiplicities
- VISH2+1 qualitatively describes CMS data
- Kozlov et al. hydro model describes pPb. Gives stronger effect for PbPb and fails for r_3 at lower multiplicity

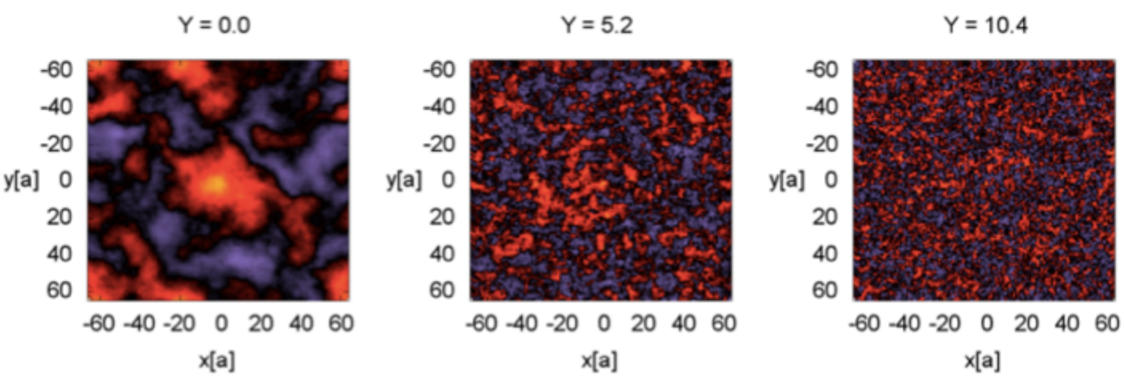
Factorization breaking – η dependence

$$f(p_T,\phi,\eta) \sim 1 + 2\sum_{n=1}^{\infty} v_n(p_T,\eta)\cos\left[n\left(\phi - \Psi_n\left(p_T,\boxed{\eta}\right)\right)\right]$$



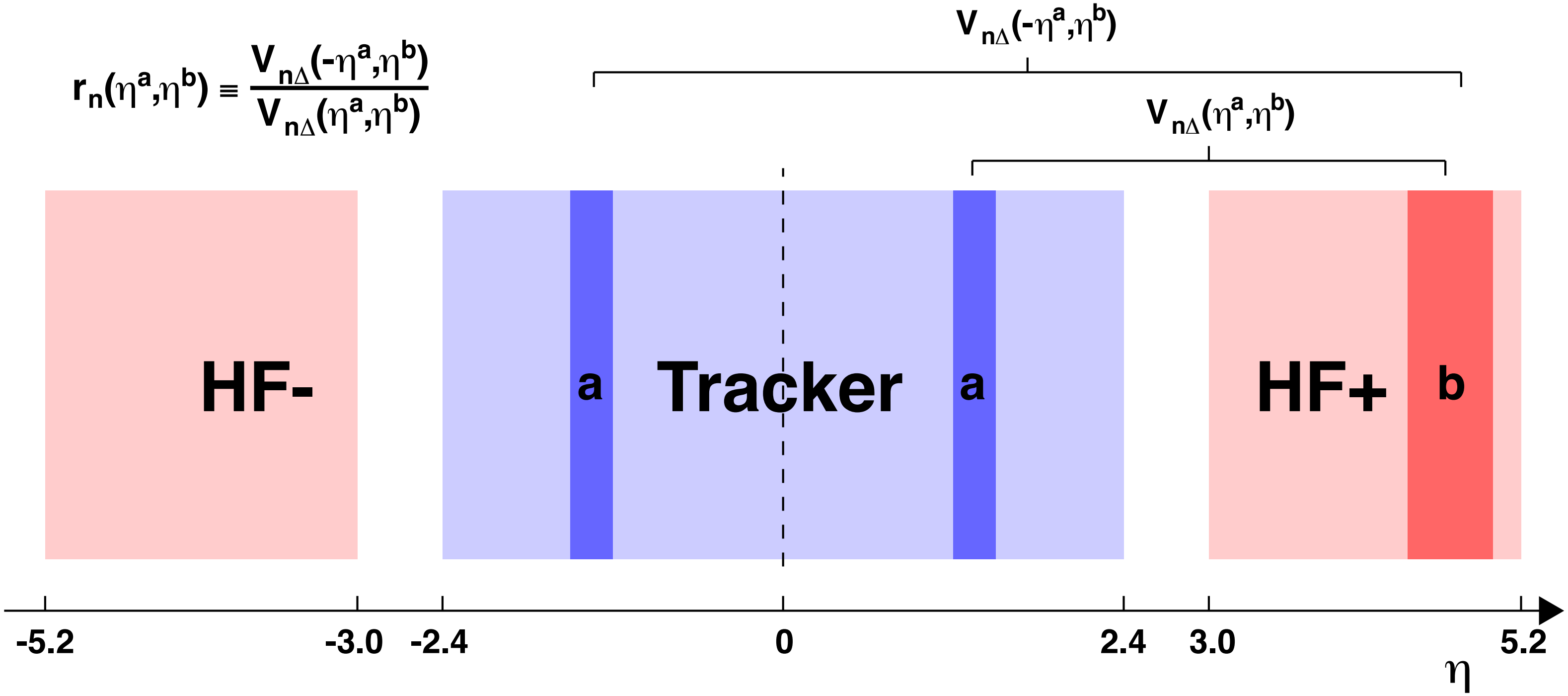
Bozek et al., arXiv: 1011.3354
Global twist





Dumitru et al., arXiv: 1108.4764

η-dependent r_n using Hadronic Forward (HF)



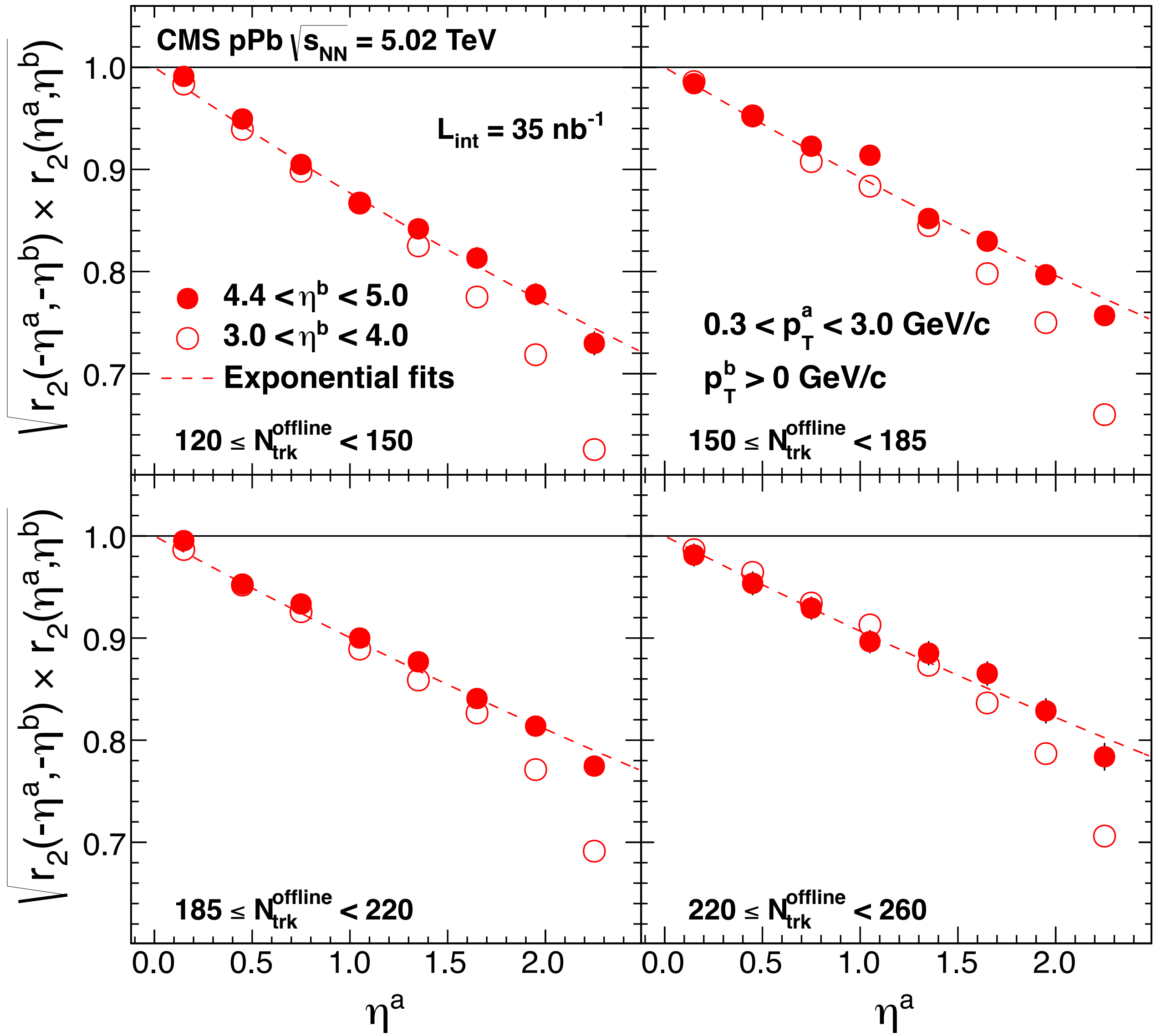
For symmetric collision:

$$r_n\left(\eta^a,\eta^b\right) \approx \frac{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}$$

For asymmetric collision:

$$\sqrt{r_n\left(\eta^a,\eta^b\right)\times r_n\left(-\eta^a,-\eta^b\right)} \approx \sqrt{\frac{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}\frac{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(-\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(-\eta^b\right)\right)\right]\right\rangle}}$$

η-dependent r_n in pPb

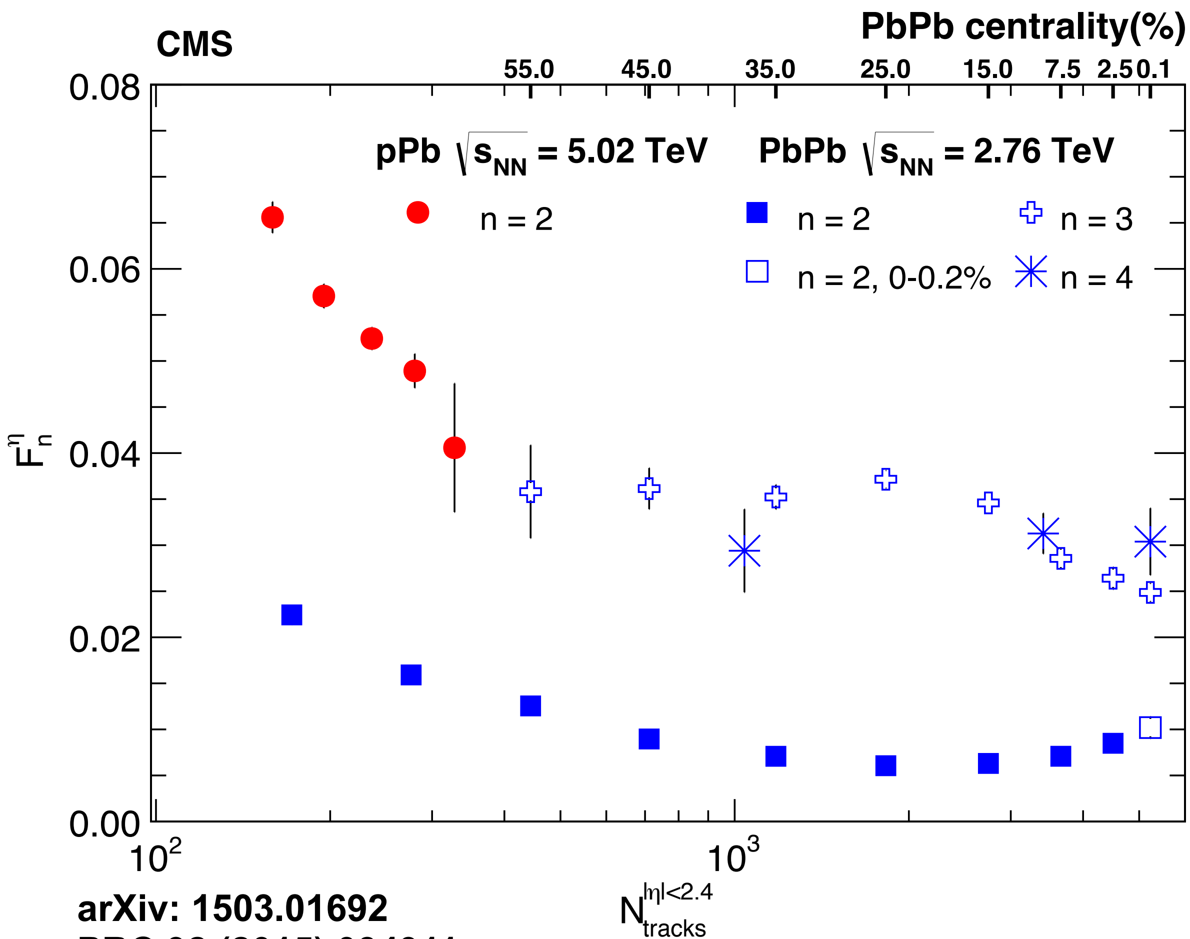


- A significant factorization breakdown in η found in pPb collisions with increase of η^a
- The effect increases approximately linearly with η^a
- Parameterization with F_n^η is purely empirical introduced just to quantify behavior of the data

$$r_n\left(\eta^a,\eta^b\right) \approx e^{-2F_n^\eta \eta^a}$$

arXiv: 1503.01692
PRC 92 (2015) 034911

η-dependent r_n vs multiplicity

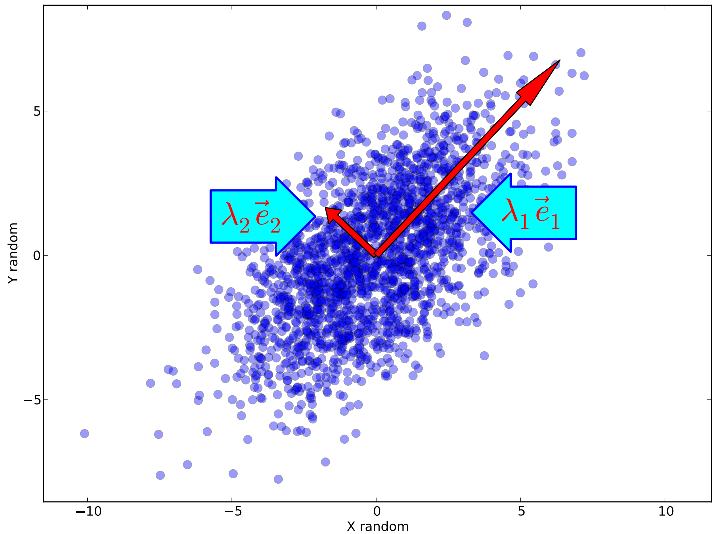


arXiv: 1503.01692
PRC 92 (2015) 034911

- The F_2^η has a minimum around midcentral PbPb and increases for peripheral and most central collisions
- At similar multiplicity, F_2^η in pPb larger than the one in PbPb
- Except for the most central PbPb, there is a very weak centrality dependence of F_3^η

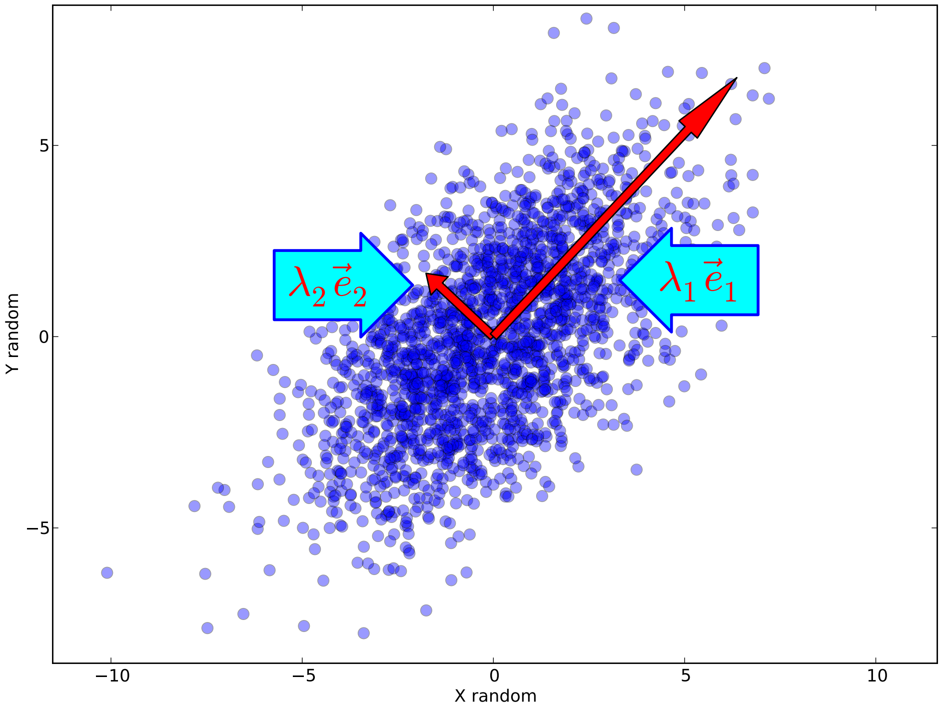
- In PbPb, higher-orders F_3^η and F_4^η, show much stronger factorization breaking than for the second order

Principal Component Analysis as a new tool to study flow



Principal Component Analysis (PCA) method

A simple 2D example



- Random data generated by 2D multivariate Gauss distribution

$$\vec{X}_n = (x_1, x_2, \ldots, x_n)$$
$$\vec{Y}_n = (y_1, y_2, \ldots, y_n)$$

- a matrix

$$\Sigma = \begin{bmatrix} \mathrm{var}(X) & \mathrm{cov}(X,Y) \\ \mathrm{cov}(X,Y) & \mathrm{var}(Y) \end{bmatrix}$$

- eigenvectors e_i and eigenvalues λ_i by diagonalization Σ

$$[e]^T \Sigma [e] = diag(\lambda_1, \lambda_2)$$

- First Principal Component: eigenvector e_1 points to maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_1} e_1$

- Second Principal Component: eigenvector e_2 points to the next maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_2} e_2$

PCA method in hydrodynamic flow - prescription

Two very recent theoretical papers: R.S.Bhalerao, J-Y. Ollitrault, S.Pal and D.Teaney, Phys.Rev.Lett. 114 (2015) 152301 and A.Mazeliauskas and D.Teaney, Phys.Rev. C91 (2015) 044902 introduced the PCA as a new method to study hydrodynamics flows

- *"The simplest correlations are pairs. The **principal component analysis** is a method which extracts all the information from pair correlations in a way which facilitates comparison between theory and experiment." J.-Y. Ollitrault*

In this analysis:

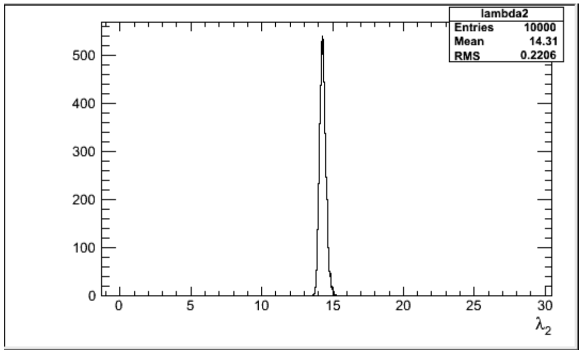
PhysRev.C **92** (2015) 034911
arXiv:1503.01692
and other CMS analyses

- **Input**: two-particle Fourier coefficients measured as $V_{n\Delta} = \langle\langle \cos(n\Delta\phi)\rangle\rangle_S - \langle\langle \cos(n\Delta\phi)\rangle\rangle_B$ where $\langle\langle \cos(n\Delta\phi)\rangle\rangle_S$ and $\langle\langle \cos(n\Delta\phi)\rangle\rangle_B$ are calculated for pairs with |Δη|>2
- 7 p_T bins (0.3<p_T<3.0 GeV/c); the eigenvalue problem of a matrix [$V_{n\Delta}(p_i,p_j)$]

$$\begin{pmatrix} e^{(1)} & e^{(2)} & . & . & . & . & e^{(7)} \end{pmatrix} \begin{bmatrix} V_{n\Delta}(p_1,p_1) & V_{n\Delta}(p_2,p_1) & V_{n\Delta}(p_3,p_1) & . & . & . & . \\ V_{n\Delta}(p_1,p_2) & V_{n\Delta}(p_2,p_2) & V_{n\Delta}(p_3,p_2) & . & . & . & . \\ V_{n\Delta}(p_1,p_3) & V_{n\Delta}(p_2,p_3) & V_{n\Delta}(p_3,p_3) & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & V_{n\Delta}(p_7,p_7) \end{bmatrix} \begin{pmatrix} e^{(1)} \\ e^{(2)} \\ . \\ . \\ . \\ . \\ e^{(7)} \end{pmatrix} = diag\begin{pmatrix} \lambda^{(1)} & \lambda^{(2)} & . & . & . & . & \lambda^{(7)} \end{pmatrix}$$

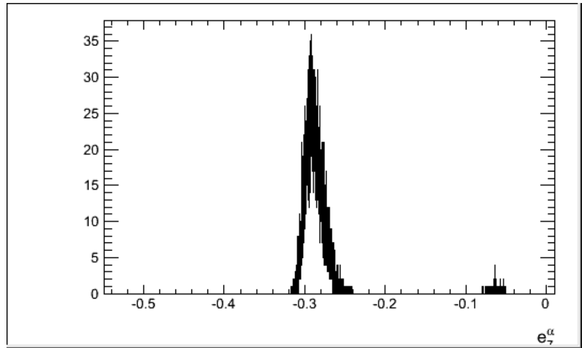
PCA method in hydrodynamic flow - prescription

λ distribution, α=2



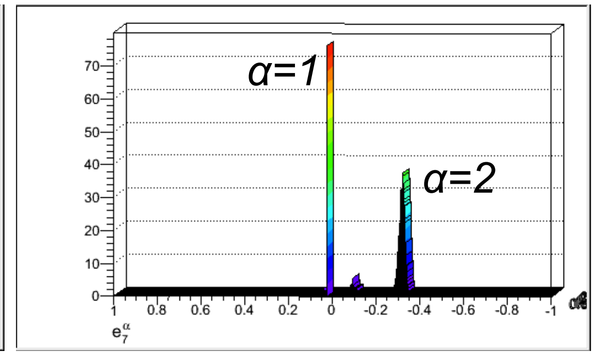
CMS Preliminary

e distribution, α=2



2.5<p_T<3.0 GeV/c

α=2 signal 200 times smaller wrt α=1

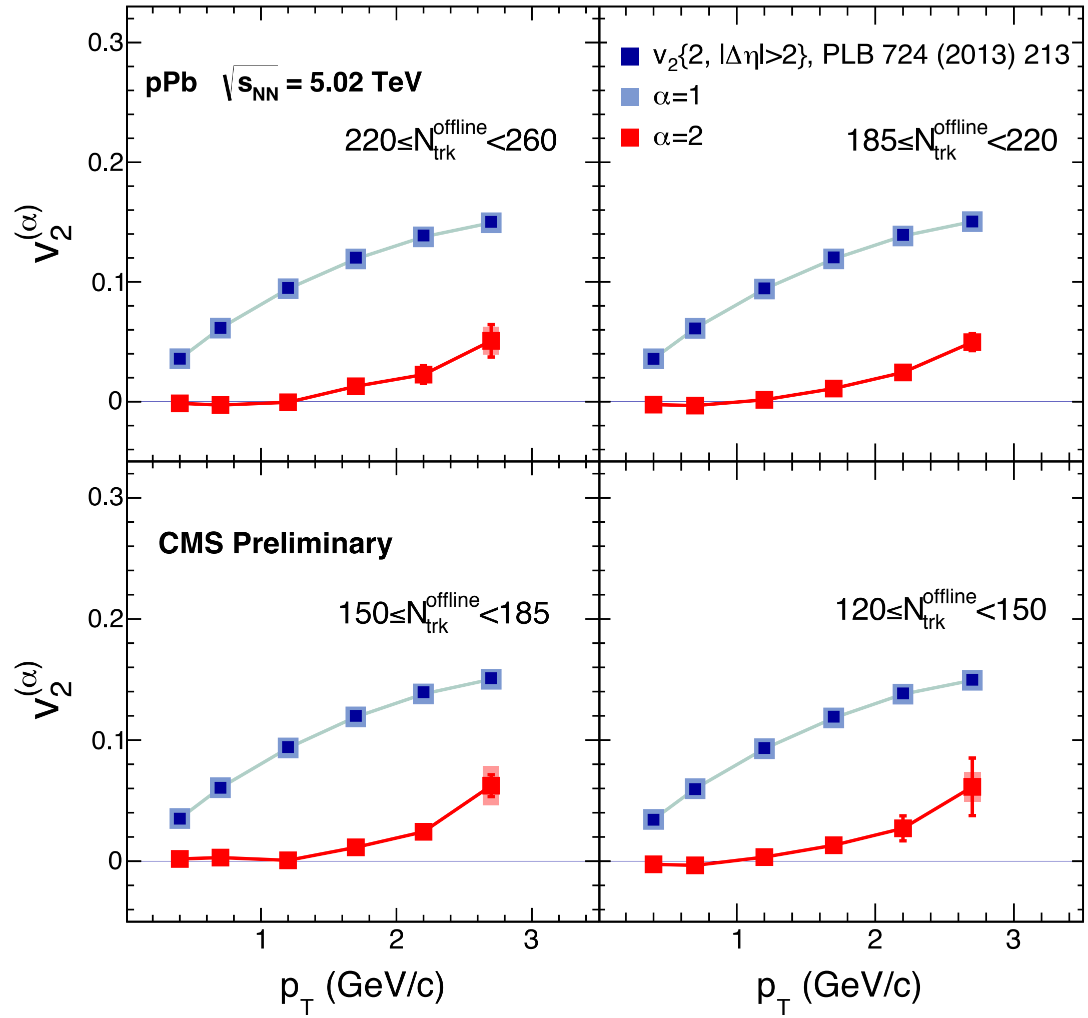


- The new introduced p_T dependent variable, **flow mode**, is defined as

$$V_n^{(\alpha)}(p_i) = \sqrt{\lambda^{(\alpha)}}\, e^{(\alpha)}(p_i)$$ where *α=1,…,7*

- corresponding single-particle flow mode $v_n^{(\alpha)}(p) = \dfrac{V_n^{(\alpha)}(p)}{\langle M(p)\rangle}$
- experimental data ➔ $V_{n\Delta}(p_i,p_j)$ ➔ it has its own statistical error $\Delta V_{n\Delta}(p_i,p_j)$
- The error propagation through $V_n^{(\alpha)}$ up to $v_n^{(\alpha)}$
- $\Delta\lambda^\alpha$ and $\Delta \mathbf{e}^\alpha$ as RMS of the distributions like ones shown above. Matrix elements $V_{n\Delta}$ were perturbed (10k times) within its $\Delta V_{n\Delta}$ → matrix $[V_{n\Delta}]$ **nonlinearly** perturbed

Results – elliptic flows in pPb collisions

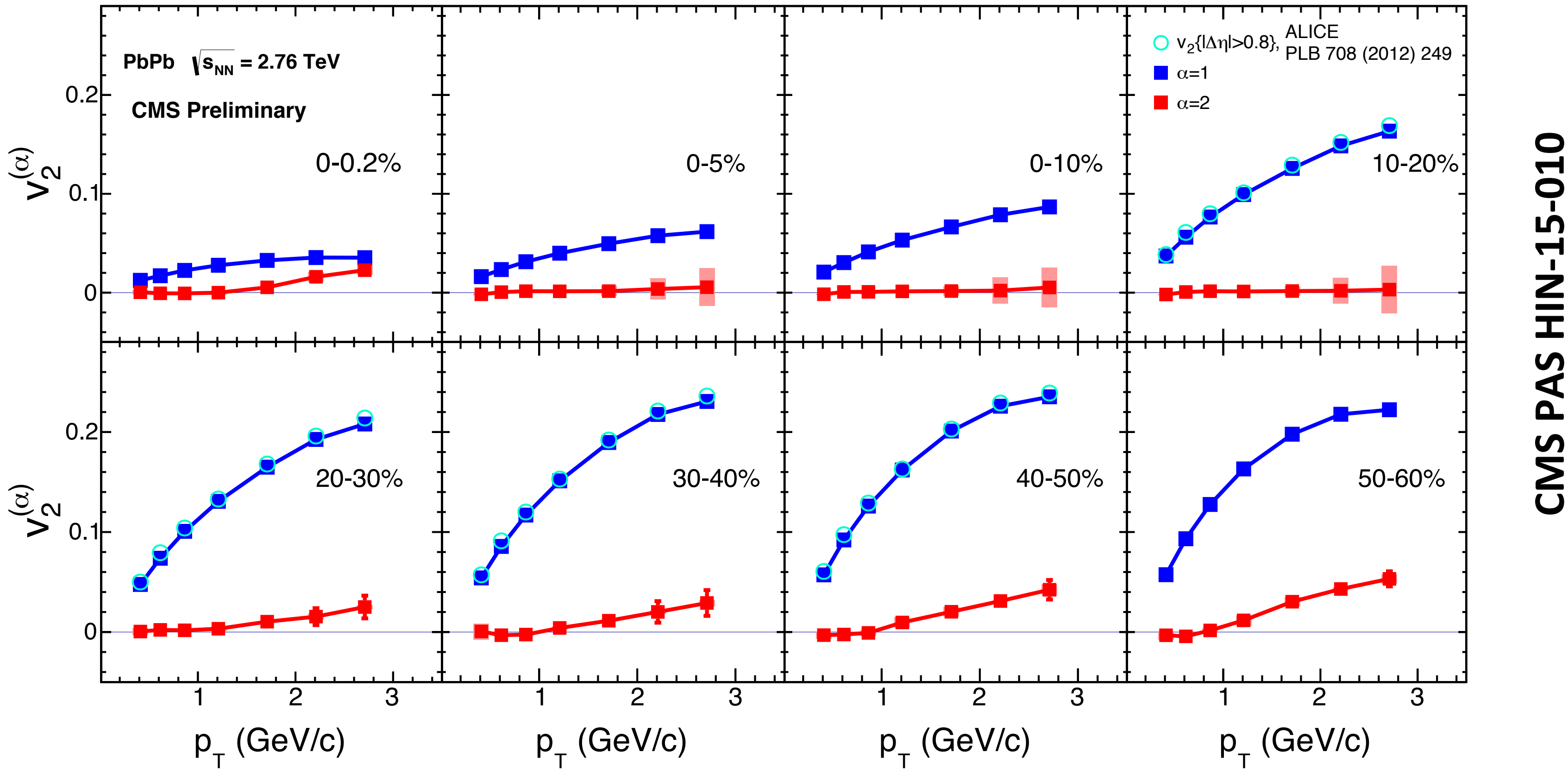


- The leading flow mode, *α=1*, practically identical to the v_2 measured using two-particle correlations
- The sub-leading flow mode, *α=2*, is essentially equal to zero at small p_T and increases up to 4-5% going to the high-p_T

CMS PAS HIN-15-010

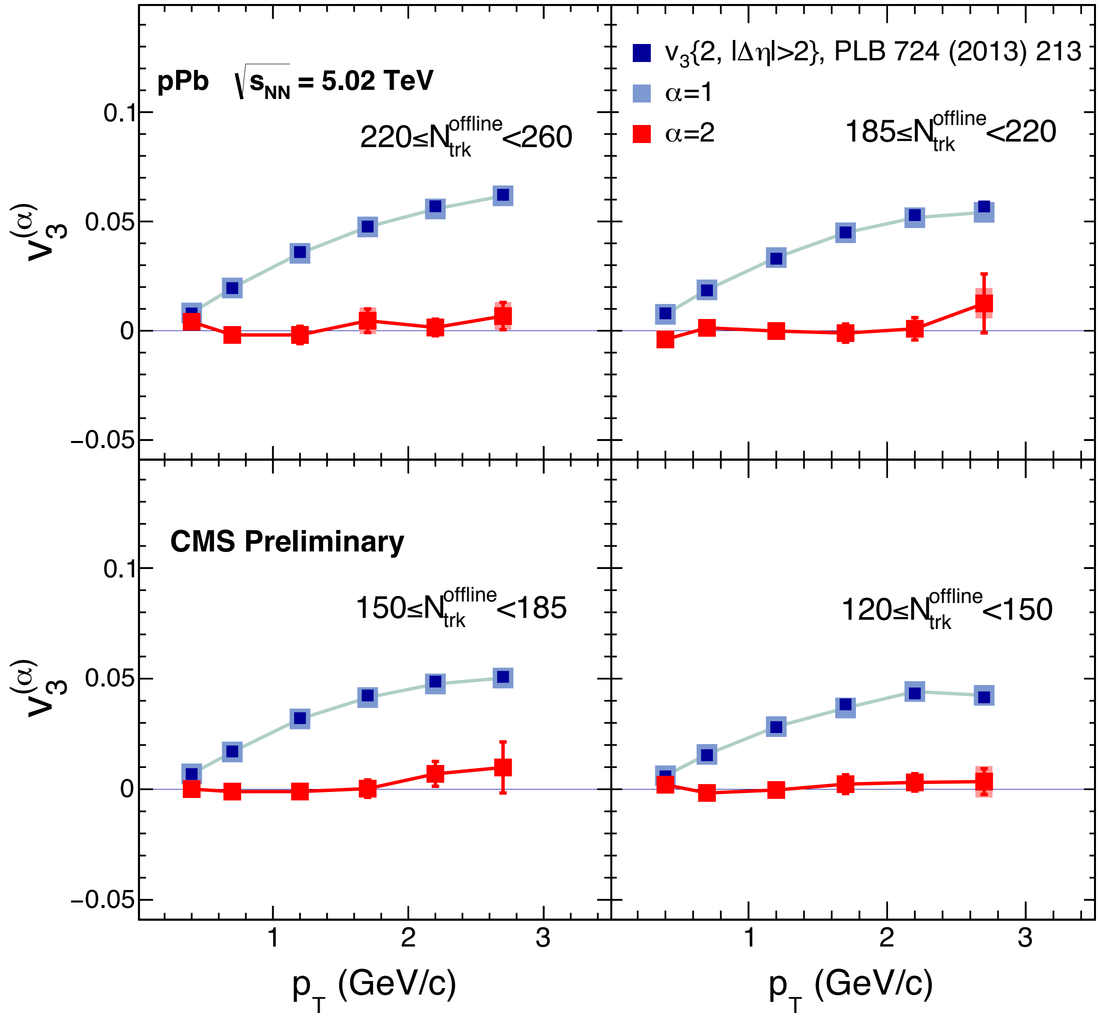
- The first experimental measurement of the elliptic sub-leading flow
- Systematical uncertainties small or comparable to statistical ones only at high-p_T

Results – elliptic flows in PbPb collisions



- The leading flow mode, *α=1,* essentially equal to the v_2 measured by ALICE using two-particle correlations
- The sub-leading flow mode, *α=2,* is positive at UCC and for collisions with centralities above 20%
- In the region 0-20% centrality comparable with zero
- Similar behavior wrt the r_2 results (10.1103/PhysRevC.92.034911, *arXiv: 1503.01692)*

Results – triangular flows in pPb collisions

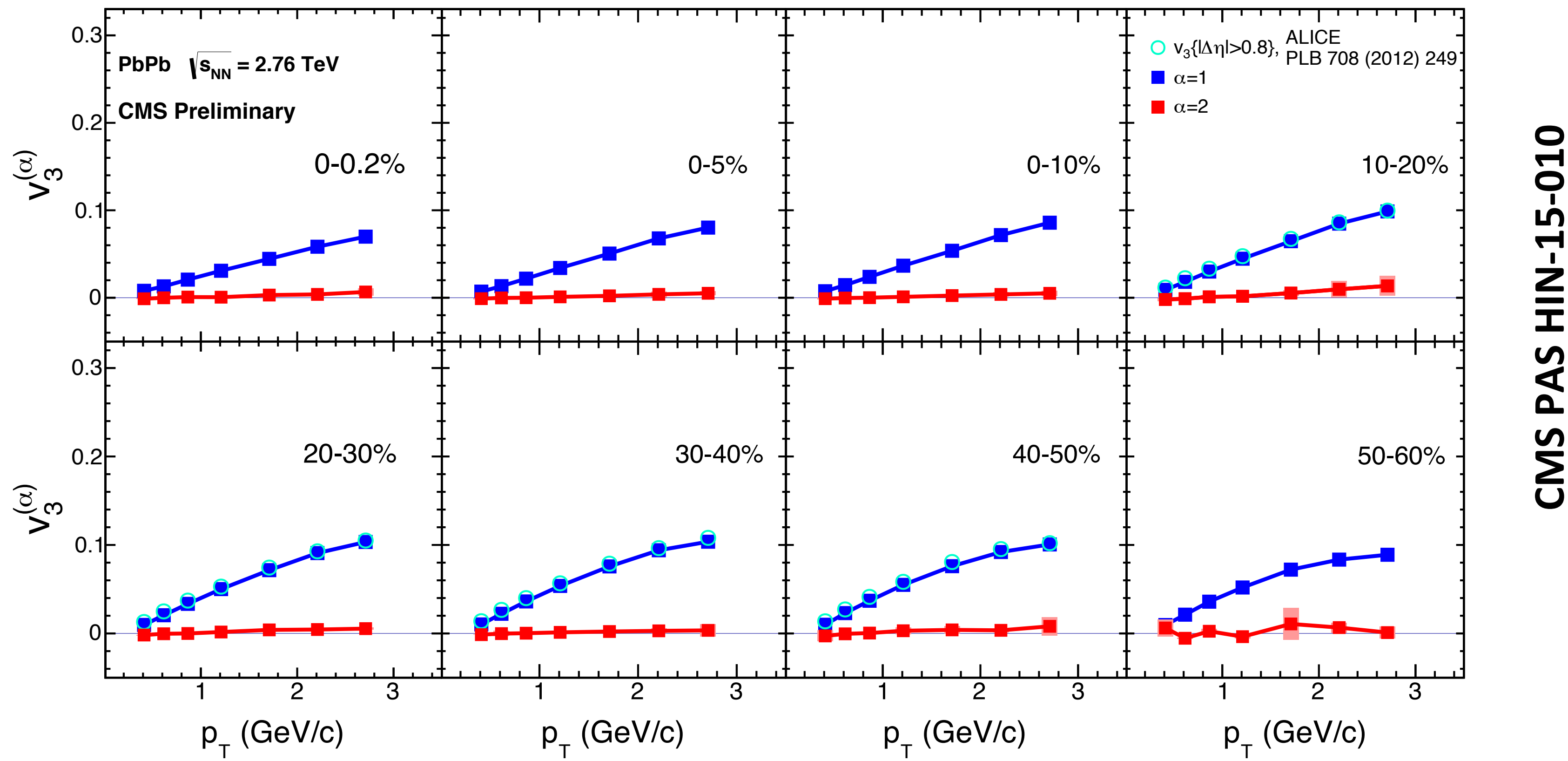


CMS PAS HIN-15-010

- The leading triangular flow mode, *α=1,* nearly identical to the v_3 measured using two-particle correlations
- The sub-leading flow mode, *α=2,* is comparable with zero within the given uncertainties.

- The first experimental measurement of the triangular sub-leading flow

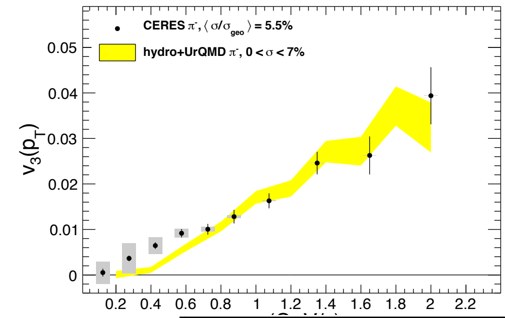
Results – triangular flows in PbPb collisions

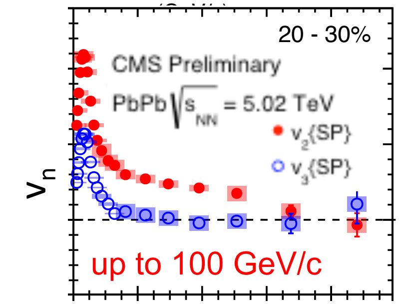


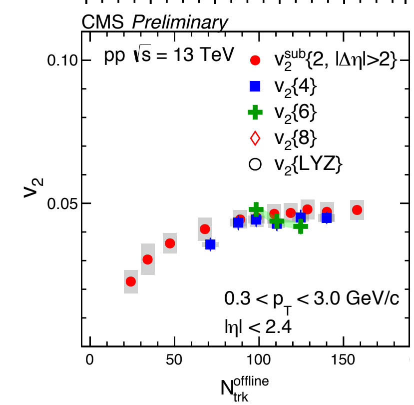
- ❖ Again, the leading flow mode, *α=1,* essentially equal to the v_3 measured by ALICE using two-particle correlations
- ❖ The sub-leading flow mode, *α=2,* is, within the uncertainties, equal to zero
- ❖ Results have a similar centrality dependence to that observed for r_3 (Phys. Rev C **92** (2015) 034911, *arXiv: 1503.01692)*

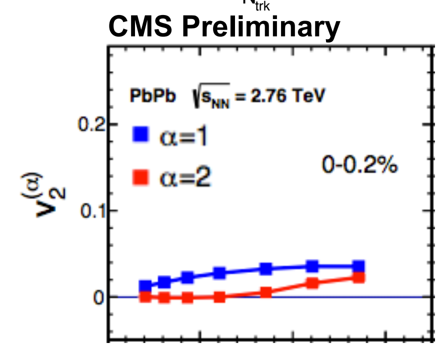
Conclusions

- The first $v_3(p_T)$ measurement at the top SPS energy with CERES using the event plane method
- The v_2 and v_3 measured up to 100 GeV/c in PbPb at 5 TeV
- The v_2 and v_3 in small pPb and smallest pp system formed in collisions at the LHC energies
- A strong factorization breaking effect for n=2 appears approaching UCC PbPb collisions
- The sub-leading flow modes are for the first time experimentally measured in both pPb and PbPb collisions at the LHC energies
- The sub-leading elliptic flow modes is in a qualitative agreement with the r_2 factorization-breaking results
- The sub-leading triangular flow modes in both collision system is small if not zero showing that the triangular flow factorizes much better than the elliptic flow
- These results could help in better understanding of the initial-state fluctuations



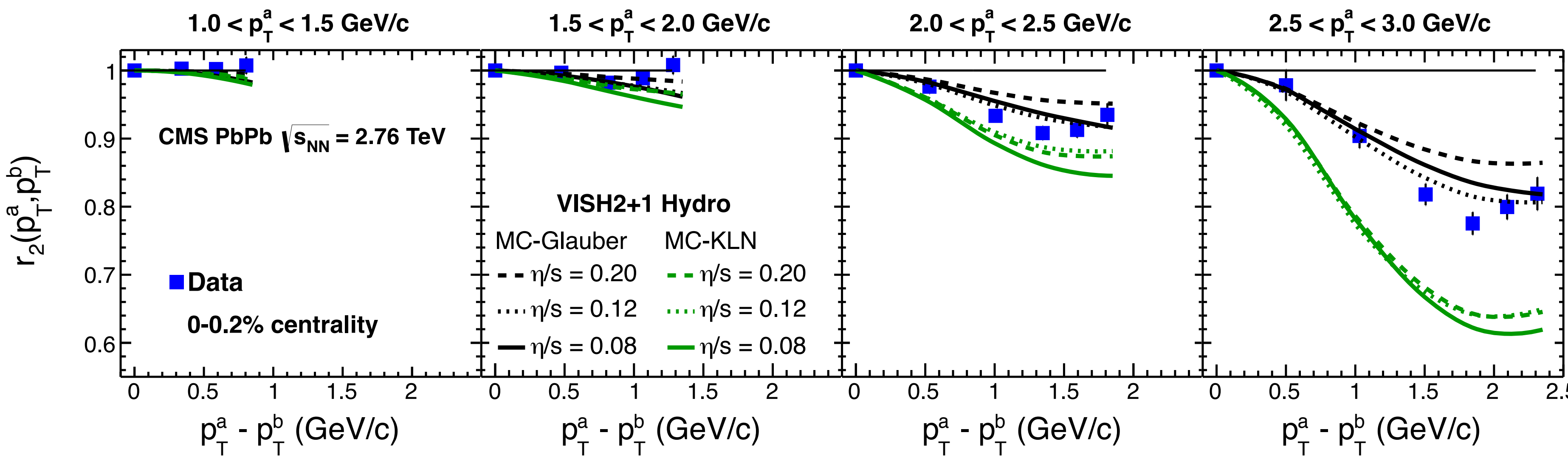






Backup slides

r_2 in ultra-central PbPb collisions and VISH2+1

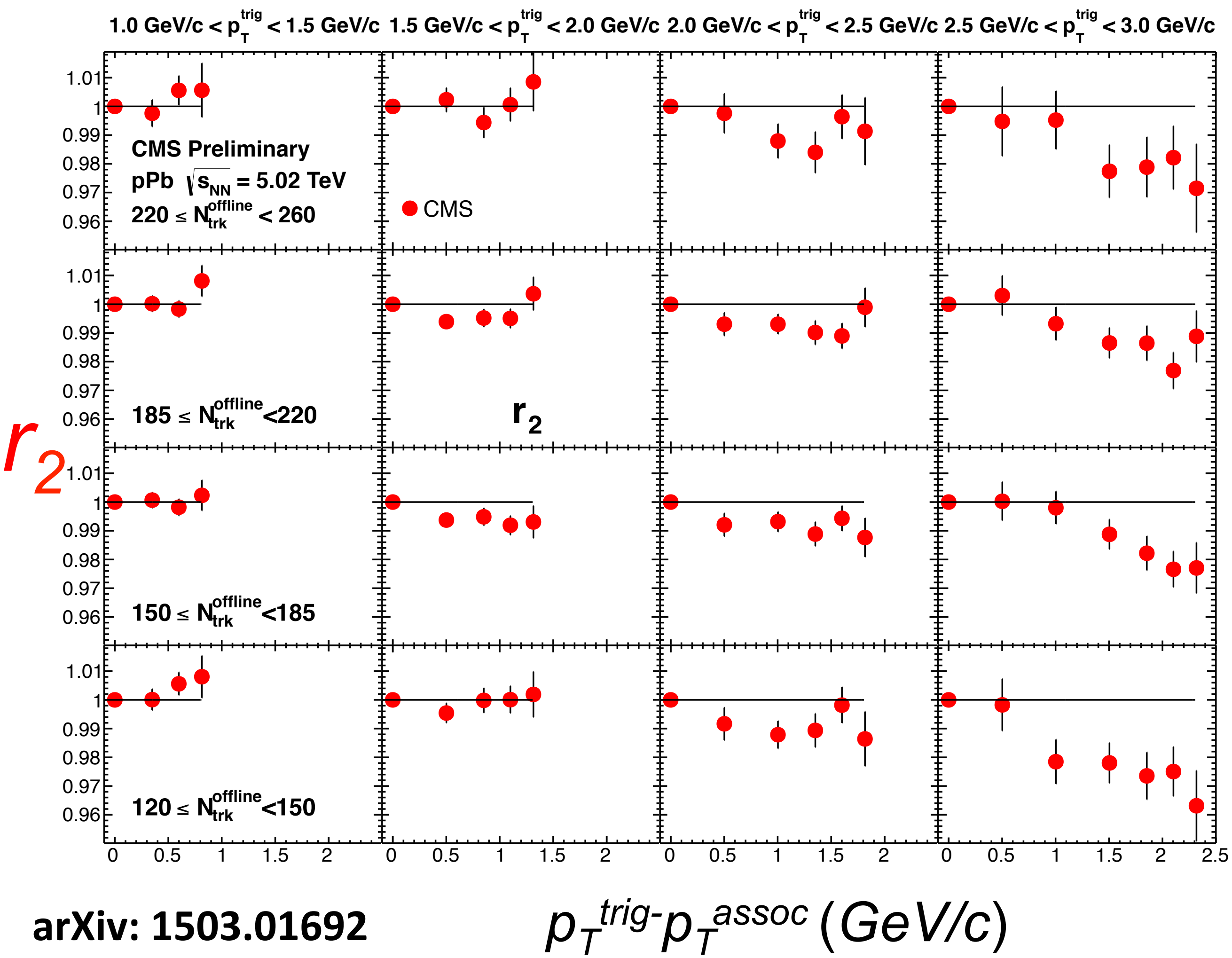


arXiv: 1503.01692, PRC 92 (2015) 034911 **VISH2+1: PRC 87 (2013) 034913**

- The effect increases with rise of p_T^{trig} and p_T^{trig}-p_T^{assoc}
- The biggest effect seen in ultra-central collisions while for semi-central collisions, the effect achieves only a size of 2−3%
- The VISH2+1 model qualitatively gives a good description of CMS data for both MC-Glauber and MC-KLN initial conditions
- Large insensitivity to η/s ➔ an independent constraint to the initial-state

r_2 from high-multiplicity pPb collisions

p_T^{trig}



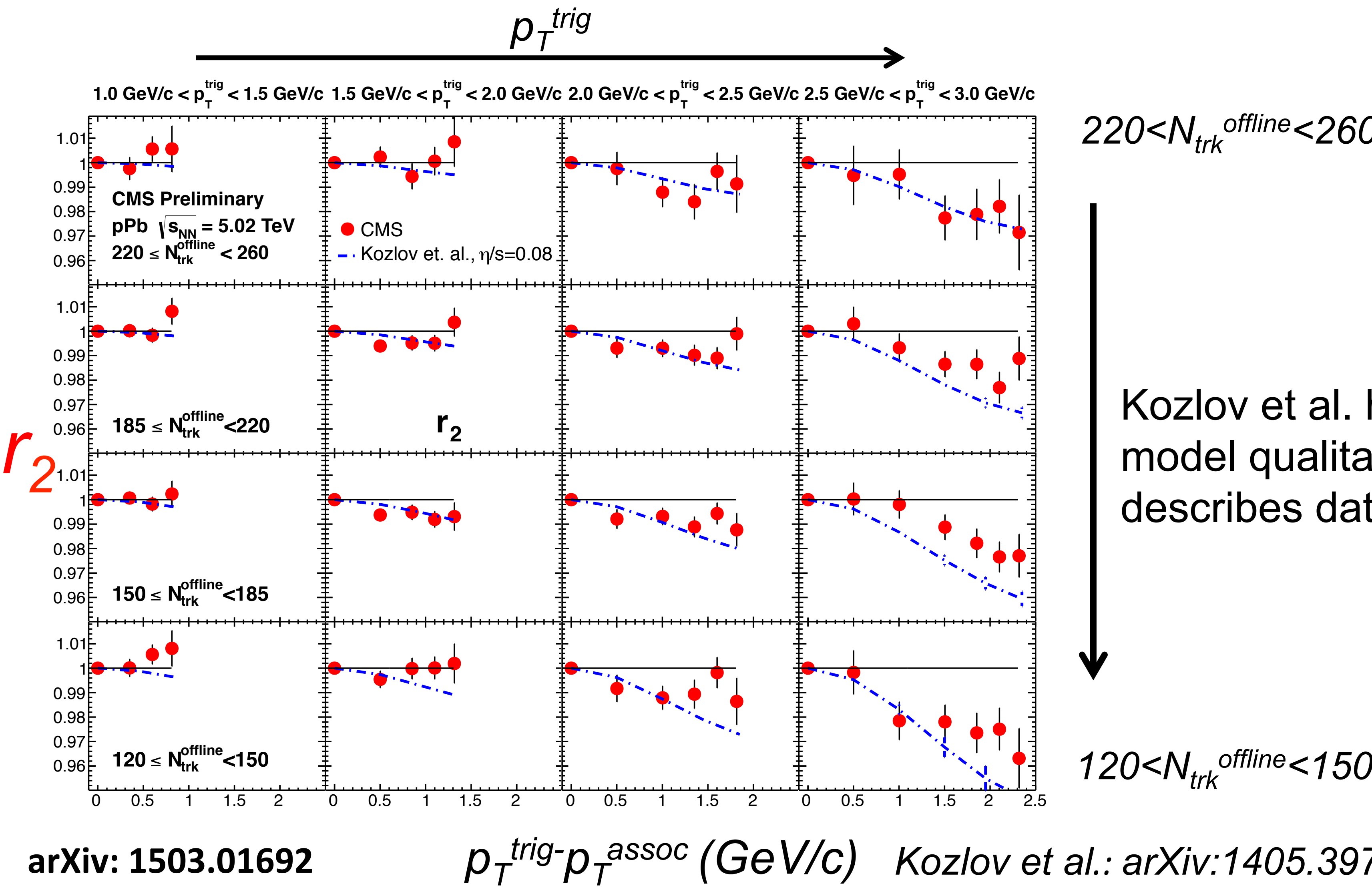
$220<N_{trk}^{offline}<260$

- The effect increases with p_T^{trig} and p_T^{trig}-p_T^{assoc}
- Maximum around 2-3%
- Nearly no dependence on multiplicity

$120<N_{trk}^{offline}<150$

arXiv: 1503.01692
PRC 92 (2015) 034911

pPb r_2: comparison to Kozlov et. al hydro model

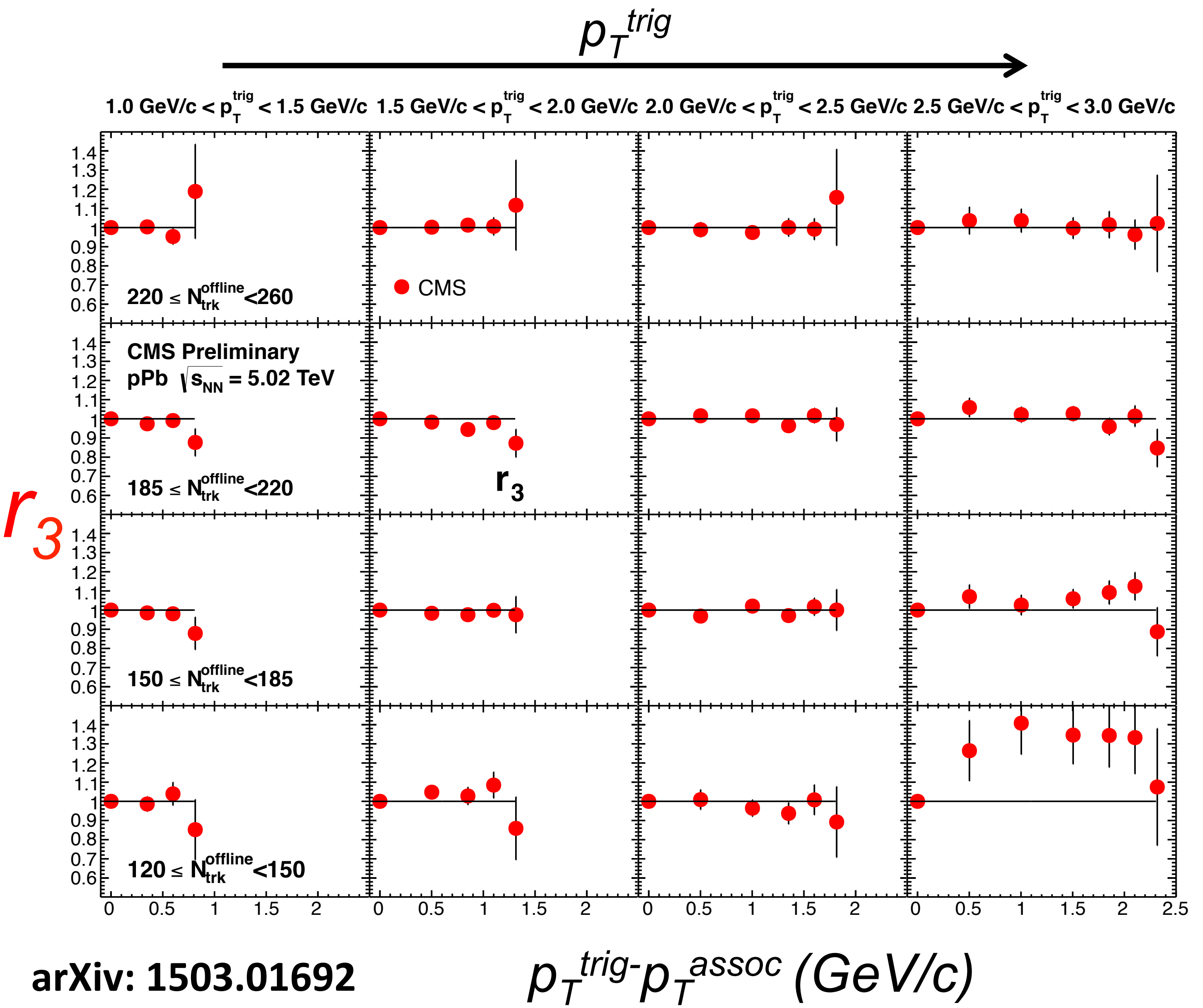


Kozlov et al. hydro model qualitatively describes data

arXiv: 1503.01692
PRC 92 (2015) 034911

Kozlov et al.: arXiv:1405.3976

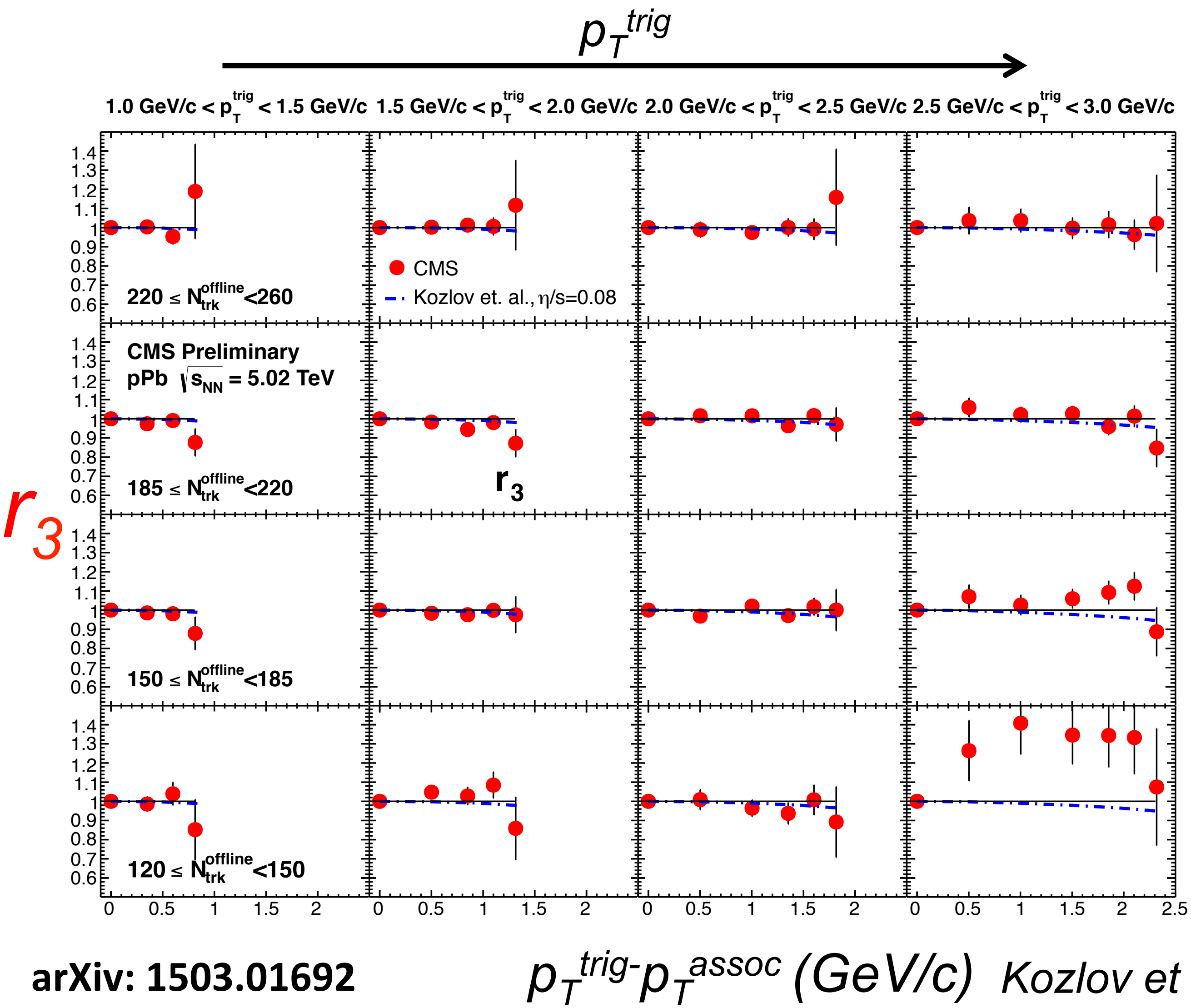
r_3 from high-multiplicity pPb collisions



- V_3 factorize better than V_2
- A direct indication of non-flow effect seen in r_3 for the highest p_T^{trig} in lower multiplicity bins

arXiv: 1503.01692
PRC 92 (2015) 034911

pPb r_3: comparison to Kozlov et. al hydro model

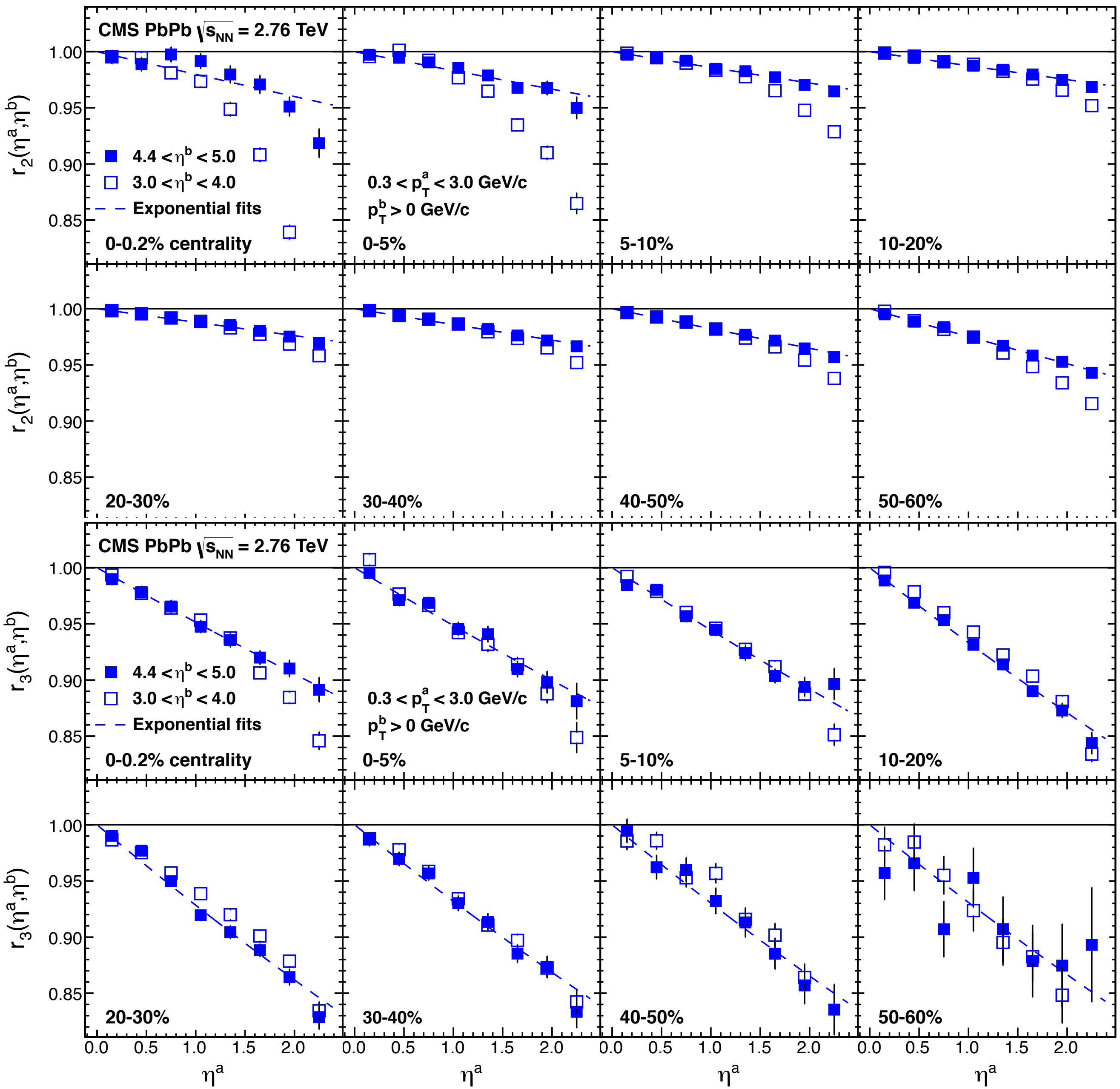


- Kozlov et al. hydro model qualitatively describes data except in lower multiplicity bins for the highest p_T^{trig}

arXiv: 1503.01692
PRC 92 (2015) 034911

Kozlov et al.: arXiv:1405.3976

η-dependent r_n in PbPb



- ❖ The r_2 factorization
- breaking effect increases with increase of η^a
- ❖ Except for the most central collisions, the increase is approximately linear

arXiv: 1503.01692
PRC 92 (2015) 034911

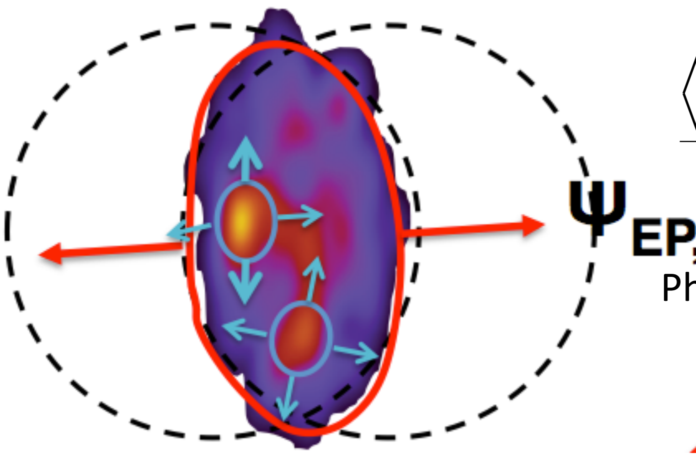
- ❖ The effect of factorization breaking is much stronger for higher-order harmonic r_3 – opposite to the p_T dependence
- ❖ Almost linear increase of the effect size
- ❖ Parameterization:

$$r_n\left(\eta^a,\eta^b\right) \approx e^{-2F_n^\eta \eta^a}$$

Factorization breaking - connection to the PCA

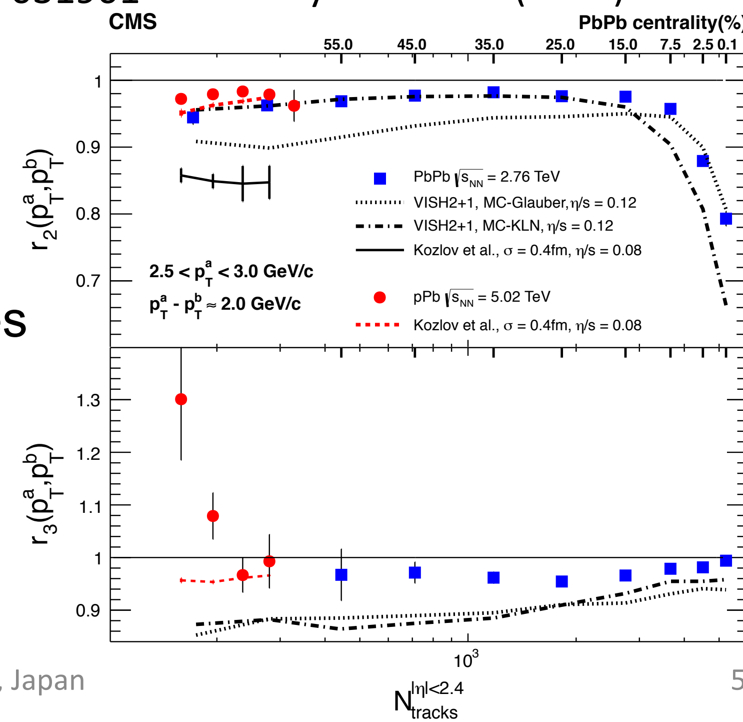
- Initial-state fluctuations → the EP (Ψ_n) depends on p_T *and on* η → factorization is broken. New observable introduced:

$$r_n = \frac{V_{n\Delta}(p_{T1},p_{T2})}{\sqrt{V_{n\Delta}(p_{T1},p_{T1})}\sqrt{V_{n\Delta}(p_{T2},p_{T2})}} = \frac{\left\langle v_n(p_{T1})v_n(p_{T2})\cos\left[n(\Psi_n(p_{T1})-\Psi_n(p_{T2}))\right]\right\rangle}{\sqrt{v_n^2(p_{T1})v_n^2(p_{T2})}} = \begin{cases} 1 & \text{holds} \\ <1 & \text{brakes} \\ >1 & \text{non-flow} \end{cases}$$



Phys.Rev. C87 (2013) 031901

Phys.Rev. C87 (2013) 034913



Phys. Rev. C **92** (2015) 034911 *(arXiv:1503.01692)*

- If there is only one principal component for each harmonic *n* → $V_{n\Delta}(p_i,p_j)$ factorizes
- r_n i.e. $V_{n\Delta}(p_i,p_j)$ results are partially integrated, while mutually orthogonal eigenmodes contain all information

Factorization breaking - connection to the PCA

❖ The given harmonic order *n* has also higher (α>2) eigenmodes ordered from largest to smallest, while in r_n they are not clearly distinguished

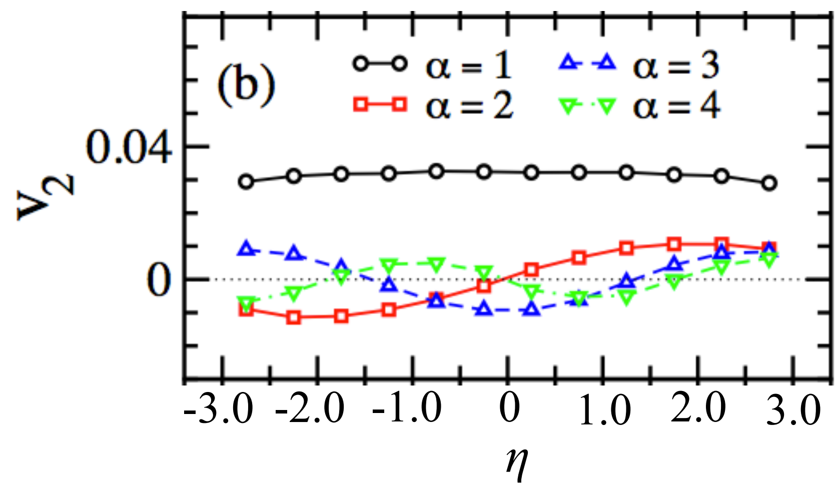
❖ The PCA can approximately reconstruct two-particle $V_{n\Delta}(p_i,p_j)$ coefficients

$$V_{n\Delta}(p_i,p_j) \approx \sum_{\alpha=1}^{k\leq N_b} V_n^{(\alpha)*}(p_i) V_n^{(\alpha)*}(p_j) \qquad \text{where } N_b=7$$

which can be used to calculate the factorization breaking ratio r_n

❖ So, the PCA is a good tool for analysis in hydrodynamics with fluctuations in the initial state

❖ Note that the PCA uses the whole p_T range simultaneously to extract the information on both leading and sub-leading flow modes



Phys.Rev.Lett. 114 (2015) 152301