The radiative decays of the singly heavy baryons in chiral perturbation theory

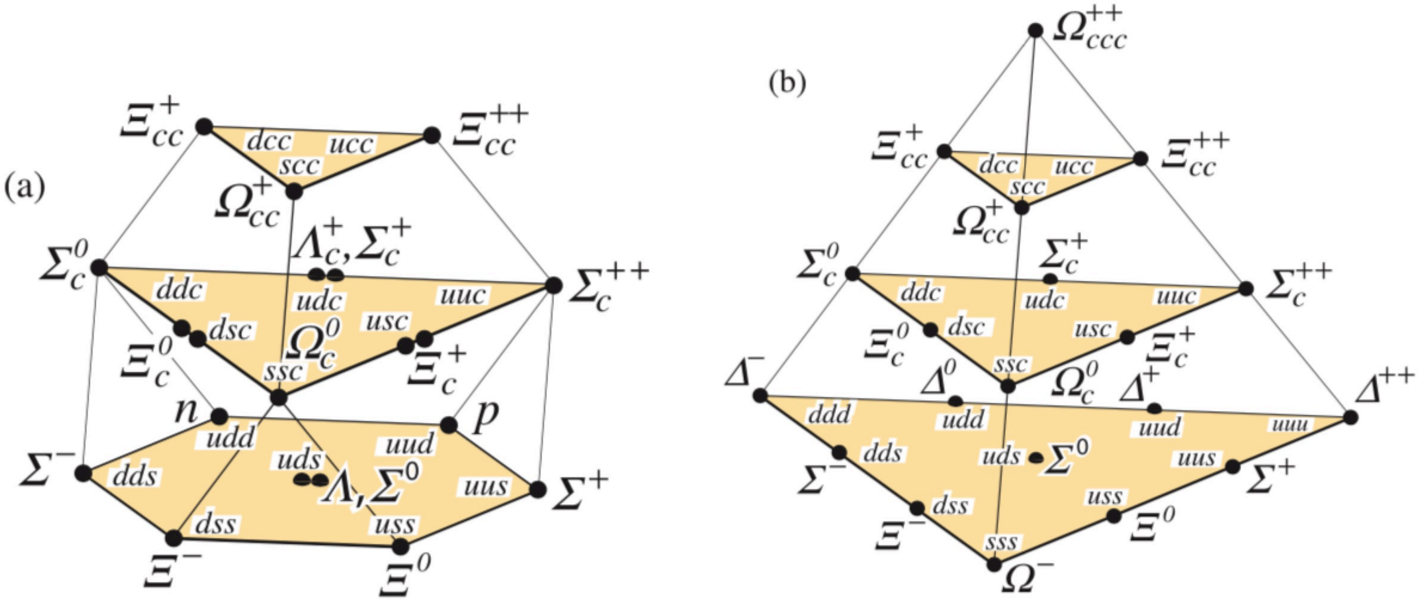
Guang-Juan Wang
Japan Atomic Energy Agency

For more details and references:
GJW, L. Meng, and S. L. Zhu, Phys.Rev. D99 (2019) no.3, 034021.

Outline

- Singly heavy baryons
- Chiral perturbation theory
- Radiative decay amplitudes of the singly heavy baryons up to next-to-next-to -leading order(NNLO).
- Lower energy constants (LECs): Heavy quark symmetry
- Numerical results
- Summary

Baryon

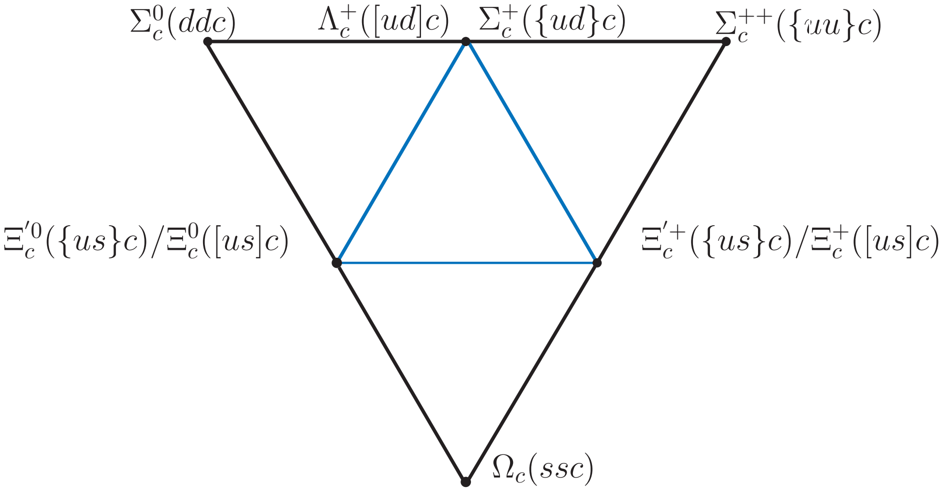


q: u,d,s
Q: c, b

- Baryon: qqq. $3 \otimes 3 \otimes 3 = 1 \oplus 8_1 \oplus 8_2 \oplus 10$.
- Singly heavy baryon: qqQ.
- Some strong decay channels are forbidden by the phase space.
- The electromagnetic form factors encode crucial information, for instance, the radiative decay width.

Singly heavy baryon

- Singly heavy baryon: qqQ. $3 \otimes 3 = \bar{3}_A \oplus 6_S$.



- diquark(qq) $\Longrightarrow \bar{3}_f \ (s_l = 0)$
  $\Longrightarrow 6_f \ (s_l = 1)$

- Spin-$\frac{1}{2}$: $\psi_{\bar{3}}, \psi_6$
  Spin-$\frac{3}{2}$: $\psi_{6^*}^{\mu}$

- $m_Q \to \infty, \psi_6$ and $\psi_{6^*}^{\mu}$ degenerate.

$$\psi_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \qquad \psi_6 = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c'^+}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c'^0}{\sqrt{2}} \\ \frac{\Xi_c'^+}{\sqrt{2}} & \frac{\Xi_c'^0}{\sqrt{2}} & \Omega_c^0 \end{pmatrix} \qquad \psi_{6^*}^{\mu} = \begin{pmatrix} \Sigma_c^{*++} & \frac{\Sigma_c^{*+}}{\sqrt{2}} & \frac{\Xi_c^{*+}}{\sqrt{2}} \\ \frac{\Sigma_c^{*+}}{\sqrt{2}} & \Sigma_c^{*0} & \frac{\Xi_c^{*0}}{\sqrt{2}} \\ \frac{\Xi_c^{*+}}{\sqrt{2}} & \frac{\Xi_c^{*0}}{\sqrt{2}} & \Omega_c^{*0} \end{pmatrix}^{\mu}$$

A: antisymmetric   S: symmetric

Chiral perturbation theory(ChPT)

- $m_q \ll 1$ GeV $\leq\ m_c, m_b, m_t$; chiral limit: $m_q\ = 0$.

- Chiral symmetry: $SU(3)_R \times SU(3)_L \times U(1)_R \times U(1)_L$

$$\mathcal{L}^0_{QCD} = \sum_{l=u,d,s} (\bar{q}_{R,l} i \not{D}\, q_{R,l} + \bar{q}_{L,l} i \not{D}\, q_{L,l}) - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a.$$

- $SU(3)_R \times SU(3)_L \to SU(3)_V$ → 8 Goldstone Bosons

- Lagrangian: chiral symmetry, CPT.. → Low energy constants ← Experiment

- expand coefficient: p/Λ_χ → power counting

$$\mathcal{L}^{(2)}_\phi = \frac{F^2_\phi}{4}\mathrm{Tr}[\nabla_\mu U \nabla^\mu U^\dagger]$$

with

$$\phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$U = \xi^2 = e^{\frac{i\phi}{F_\phi}}, \quad \nabla_\mu U = \partial_\mu U + ieA_\mu[Q, U]$$

$$Q = \mathrm{diag}(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3})$$

HBChPT

- M_B>1 GeV and $M_B \neq 0$ in chiral limit. ⟶ breaks the power counting.
- "light" component $B(p)$ and "heavy" component $H(p)$

$$p^\mu = Mv^\mu + k^\mu$$

velocity, $v^2 = 1$

$$B(p) = e^{iMv\cdot x}\frac{1+\not{v}}{2}\psi, \quad H(p) = e^{iMv\cdot x}\frac{1-\not{v}}{2}\psi$$

$$\mathcal{L} = i\bar{B}_v\underbrace{(v\cdot D)}_{\text{M=0}}B_v - \underbrace{\bar{H}_v(iv\cdot D+2M_B)}_{\text{M=}2M_B}H_v + i\bar{B}_v\not{D}_\perp H_v + i\bar{H}_v\not{D}_\perp B_v$$

$$D^\mu_\perp = D_\mu - D\cdot v v^\mu$$

- Integrate the heavy component out,

$$\begin{aligned}\mathcal{L} &= \bar{B}_v(iv\cdot D + i\not{D}_\perp\frac{1}{iv\cdot D+2M_B}\not{D}_\perp)B_v \\ &= \bar{B}_v(iv\cdot D)B_v - \mathcal{O}(\frac{1}{M_B}) + \ldots\end{aligned}$$

E. E. Jenkins et al. Phys. Lett, B255
T. R. Hemmert et al. J. Phys. 1998, G24

Power-Counting Scheme

✓ The expansion coefficients: the residue momentum of the baryons and the momentum (mass) of the pseudoscalar mesons.

✓ The chiral dimension:

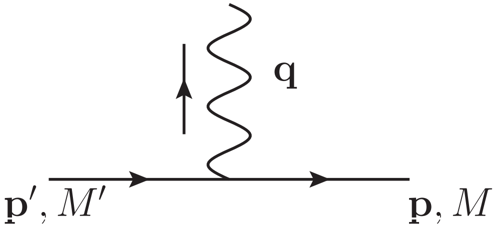
$$D = 2L + I_B + 2 + \sum_{n=1}^{\infty} 2(n-1)N_{2n}^M + \sum_{n=1}^{\infty} (n-2)N_n^B .$$

- L: the number of the loops. $\mathcal{O}(p^4)$
- I_B: the number of the internal baryon lines. $\mathcal{O}(p^{-I_B})$
- N_{2n}^M: the number of the meson interaction vertices at $\mathcal{O}(p^{2n})$
- N_n^B: the number of the baryon interaction vertices at $\mathcal{O}(p^n)$

V. Bernard et al. Int. J. Mod. Phys. 1995, E4: 193–346.

Electromagnetic multipole expansion

- spin-$\frac{1}{2}$ →spin-$\frac{1}{2}$ +γ: $\psi_6 \rightarrow \psi_{\bar{3}} + \gamma$



✓ The decay amplitude reads

$$\mathcal{M} = -ie\epsilon^{\mu}\langle\psi(p')|j_{\mu}|\psi(p)\rangle = -ie\epsilon^{\mu}\bar{\psi}(p')[(\gamma_{\mu} - \frac{\delta}{q^2}q_{\mu})F_1(q^2) + \frac{i\sigma_{\mu\nu}q^{\nu}}{M+M'}F_2(q^2)]\psi(p),$$

✓ The electromagnetic matrix element in the nonrelativistic limit

$$\begin{aligned}
\langle\psi(p')|j_{\mu}|\psi(p)\rangle &= e\bar{B}(p')[(v_{\mu} - \frac{\delta}{q^2}q_{\mu})G_E(q^2) + \frac{2[S^{\mu}, S^{\nu}]q_{\nu}}{M+M'}G_M(q^2)]B(p), \\
G_E(q^2) &= F_1(q^2) + \frac{q^2}{2(M+M')^2}F_2(q^2), \\
G_M(q^2) &= F_2(q^2) + F_1(q^2),
\end{aligned}$$

$$\Gamma = \frac{4\alpha|\mathbf{p}_{\gamma}|^3}{(M+M')^2}|G_M(0)|^2$$

In CMS:

$$\nu = (1, 0, 0, 0)$$

Pauli-Lubanski operator :

$$S^{\mu} = \frac{i}{2}\gamma^5\sigma^{\mu\nu}v_{\nu}$$

D. B. Leinweber, et al. Phys. Rev. D 43, 1659 (1991).

B. Kubis et al. Eur. Phys. J. C 18, 747 (2001).

Electromagnetic multipole expansion

- spin-$\frac{3}{2}$ →spin-$\frac{1}{2}$ γ: $\psi^{\mu}_{6^*} \to \psi_{\bar{3},6} + \gamma$

✓ The electromagnetic matrix element,

$$\langle \psi^{\rho}_{6^*} | j_\mu | \psi \rangle = e \bar{\psi}^{\rho}_{6^*}(p') \Gamma_{\rho\mu} \psi(p),$$

$$\Gamma_{\rho\mu} = G_1(q^2)(q_\rho \gamma_\mu - q\cdot\gamma g_{\rho\mu})\gamma_5 + G_2(q^2)(q_\rho p'_\mu - q\cdot p' g_{\rho\mu})\gamma_5 + G_3(q^2)(q_\rho q_\mu - q^2 g_{\rho\mu})\gamma_5$$

✓ $G_3(q^2)$ does not contribute when the photon is on-shell.

✓ In HBChPT scheme,

$$\Gamma_{\rho\mu} = 2G_1(q^2)(q_\rho S_\mu - q\cdot S g_{\rho\mu}) + G_2(q^2)\frac{2M'}{M+M'}(q_\rho v_\mu - q\cdot v g_{\rho\mu}) q\cdot S$$

$$G_{M1}(q^2) = \frac{1}{4}\left[G_1 \frac{M_+(3M'+M) - q^2}{M'} + G_2(M_+M_- - q^2) + 2(G_3+G_2)q^2\right],$$

$$G_{E2}(q^2) = \frac{1}{4}\left[G_1 \frac{M_+M_- + q^2}{M'} + G_2(M_+M_- - q^2) + 2(G_2+G_3)q^2\right],$$

With $M_+ = M + M'$, $M_- = M' - M$

A. Faessler, et al. , PRD 74, 074010.
H. F. Jones et al., Ann. Phys. (N.Y.) 81, 1.

Electromagnetic multipole expansion

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With $M_+ = M + M'$, $M_- = M' - M$

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Electromagnetic multipole expansion

The transition magnetic moment is defined as

$$\mu = \sqrt{\frac{2}{3}} \frac{G_{M1}(0)}{2M}.$$

With the G_{M1} and G_{E2}, the helicity amplitudes are defined as,

$$A_{3/2}(q^2) = -\sqrt{\frac{\pi\alpha\omega}{2M^2}}[G_{M1}(q^2) + G_{E2}(q^2)],$$

$$A_{1/2}(q^2) = -\sqrt{\frac{\pi\alpha\omega}{6M^2}}[G_{M1}(q^2) - 3G_{E2}(q^2)],$$

with

$$\omega = \frac{M'^2 - M^2 + q^2}{2M'}.$$

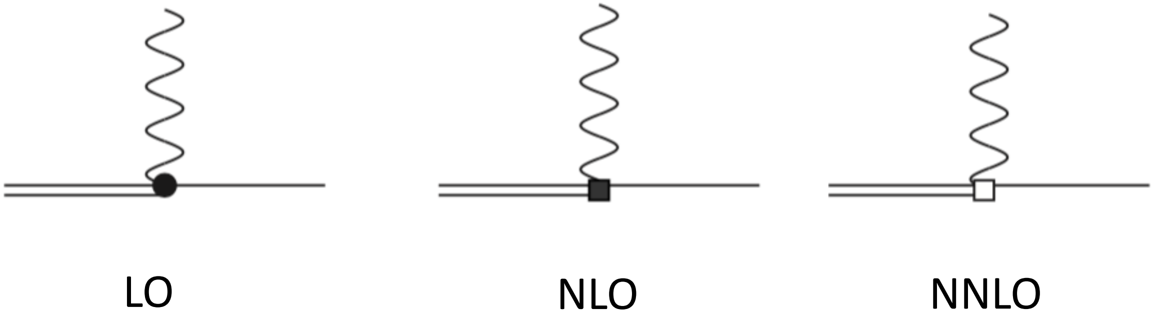
The radiative decay width reads,

A. Faessler, et al. , PRD 74, 074010.

H. F. Jones et al., Ann. Phys. (N.Y.) 81, 1.

$$\Gamma = \frac{MM'}{8\pi}\left(1 - \frac{M^2}{M'^2}\right)^2 (A^2_{3/2}(0) + A^2_{1/2}(0)).$$

Tree diagrams

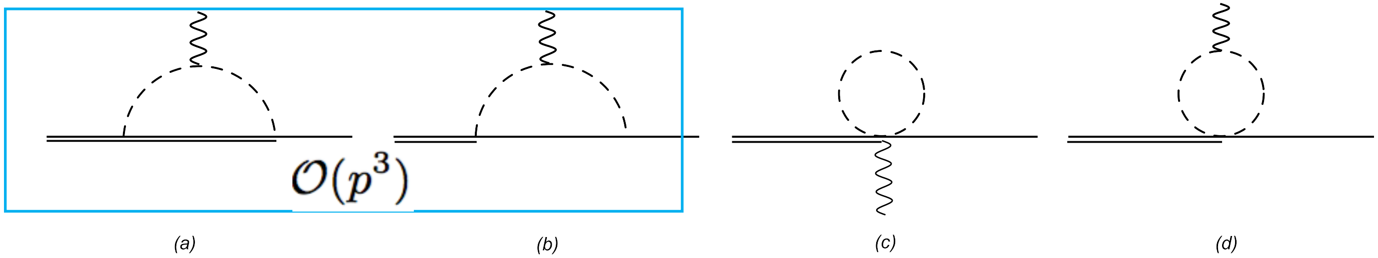


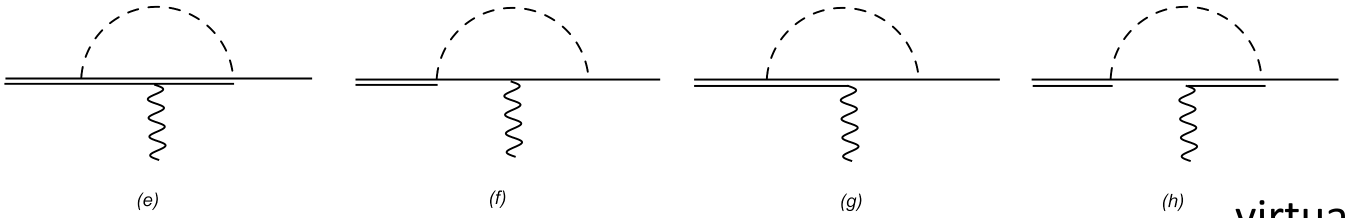
- The single line: spin-$\frac{1}{2}$ heavy baryon, the double line: spin-$\frac{3}{2}$ heavy baryon. The curve line: the photon.
- The solid circle: $\mathcal{O}(p^2)$, solid square: $\mathcal{O}(p^3)$, square: $\mathcal{O}(p^4)$
- Diagrams with chiral dimension D_χ

→ $\mathcal{O}(p^{D_\chi})$ radiative decay amplitude

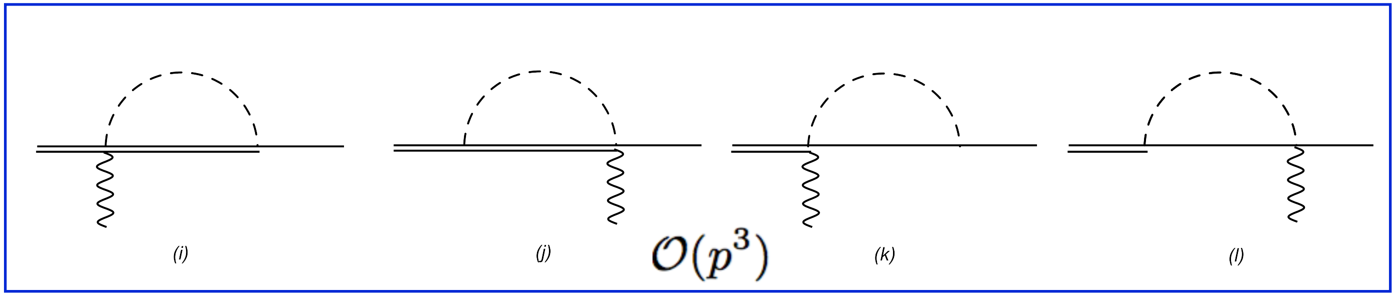
$\mathcal{O}(p^{D_\chi - 1})$ transition magnetic moment

Loop diagrams



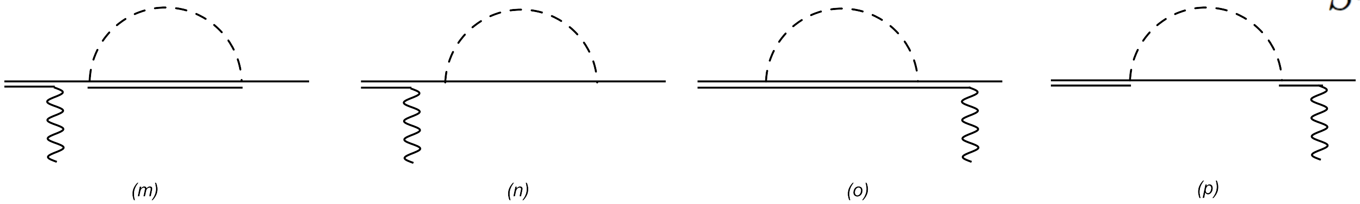


virtual line: a meson



Vanish:
$S\cdot v=0$

$$S^{\mu}=\frac{i}{2}\gamma^{5}\sigma^{\mu\nu}v_{\nu}$$



$\mathcal{O}(p)$ Lagrangian

$$\begin{aligned}\mathcal{L}^{(1)}_{B\phi} = & \frac{1}{2}\mathrm{Tr}[\bar{\psi}_{\bar{3}}(i\not{D}-M_{\bar{3}})\psi_{\bar{3}}] + \mathrm{Tr}[\bar{\psi}_6(i\not{D}-M_6)\psi_6] \\ & +\mathrm{Tr}[\bar{\psi}^{\mu}_{6^*}(-g_{\mu\nu}(i\not{D}-M_{6^*}) + i(\gamma_\mu D_\nu + \gamma_\nu D_\mu) - \gamma_\mu(i\not{D}+M_{6^*})\gamma_\nu)\psi^{\nu}_{6^*}] \\ & +g_1\mathrm{Tr}[\bar{\psi}_6 \not{u}\gamma_5\psi_6] + g_2\mathrm{Tr}[(\bar{\psi}_6\not{u}\gamma_5\psi_{\bar{3}}) + \mathrm{H.c.}] + g_3\mathrm{Tr}[(\bar{\psi}^{\mu}_{6^*}u_\mu\psi_6) + \mathrm{H.c.}] \\ & +g_4\mathrm{Tr}[(\bar{\psi}^{\mu}_{6^*}u_\mu\psi_{\bar{3}}) + \mathrm{H.c.}] + g_5\mathrm{Tr}[\bar{\psi}^{\nu}_{6^*}\not{u}\gamma_5\psi_{6^*\nu}] + g_6\mathrm{Tr}[\bar{\psi}_{\bar{3}}\not{u}\gamma_5\psi_{\bar{3}}],\end{aligned}$$

$$\begin{aligned} D_\mu\psi &= \partial_\mu\psi + \Gamma_\mu\psi + \psi\Gamma^T_\mu, \\ \Gamma_\mu &= \frac{1}{2}[\xi^\dagger,\partial_\mu\xi] + \frac{i}{2}eA_\mu(\xi^\dagger Q_B\xi + \xi Q_B\xi^\dagger), \\ u_\mu &= \frac{i}{2}\{\xi^\dagger,\partial_\mu\xi\} - \frac{1}{2}eA_\mu(\xi^\dagger Q_B\xi - \xi Q_B\xi^\dagger),\end{aligned}$$

- $Q_B = Q + \frac{1}{2}\tilde{Q}$. Q_c =diag $(1,0,0)$. Q_b =diag $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$.
- g_2 and g_4 ← $\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+)$ and $\Gamma(\Sigma_c^{*++}\to\Lambda_c^+\pi^+)$. PLB 704 547-550
- Other g_i are obtained through the quark model.

$$g_1 = 0.98, \quad g_2 = -\sqrt{\frac{3}{8}}g_1 = -0.60, \quad g_3 = \frac{\sqrt{3}}{2}g_1 = 0.85,$$

$$g_4 = -\sqrt{3}g_2 = 1.04, \quad g_5 = -\frac{3}{2}g_1 = -1.47, \quad g_6 = 0,$$

$S_l = 0$

Building Block at $\mathcal{O}(p^2)$

✓ The tensor fields:

$$f^R_{\mu\nu} = f^L_{\mu\nu} = -eQ_B(\partial_\mu A_\nu - \partial_\nu A_\mu)$$
$$f^\pm_{\mu\nu} = \xi^\dagger f^R_{\mu\nu}\xi \pm \xi^\dagger f^L_{\mu\nu}\xi,$$
$$\tilde{f}^\pm_{\mu\nu} = f^\pm_{\mu\nu} - \frac{1}{3}\mathrm{Tr}(f^\pm_{\mu\nu}).$$

- $\mathrm{Tr}(Q) = 0$ and $\mathrm{Tr}(\tilde{Q}) \neq 0$
- $\tilde{f}^+_{\mu\nu}$: the contributions from the light quarks, 8_f .
- $\mathrm{Tr}(f^+_{\mu\nu})$: the contributions from the heavy quarks, 1_f.

✓ $\chi_\pm$, 8_f:

$$\chi_\pm = \xi^\dagger\chi\xi^\dagger \pm \xi\chi^\dagger\xi,$$
$$\chi = 2B_0\mathrm{diag}(m_u, m_d, m_s),$$

- At the leading order $\chi_+ = \mathrm{diag}(0,0,1)$

✓ $8 \otimes 8 = 1 \oplus 8_1 \oplus 8_1 \oplus 10 \oplus \overline{10} \oplus 27$

$\mathcal{S}_{123}$: symmetrization operator

ϵ_{ijk}: Levi-Civita symbol

1	8_1	8_2	10	$\overline{10}$	27
$\mathrm{Tr}(u_\mu u_\nu)$	$[u_\mu, u_\nu]$	$\{u_\mu, u_\nu\}$	$\mathcal{S}_{ijk}\mu^{\mu i}_a \mu^{\nu j}_b \epsilon^{abk}$	$\mathcal{S}_{abc}\mu^{\mu i}_a \mu^{\nu j}_b \epsilon_{ijc}$	$\{u^{\mu a}_b u^{\nu i}_j\}^{\{ai\}}_{\{bj\}}$

$\mathcal{O}(p^2)$ Lagrangian

$$
\begin{aligned}
\mathcal{L}^{(2)}_{B\gamma} = & -\frac{id_2}{8M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}[S^\mu, S^\nu]\tilde{f}^+_{\mu\nu}B_{\bar{3}}) - \frac{id_3}{8M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}[S^\mu, S^\nu]B_{\bar{3}})\mathrm{Tr}(f^+_{\mu\nu}) \\
& -\frac{id_5}{4M_N}\mathrm{Tr}(\bar{B}_6[S^\mu, S^\nu]\tilde{f}^+_{\mu\nu}B_6) - \frac{id_6}{4M_N}\mathrm{Tr}(\bar{B}_6[S^\mu, S^\nu]B_6)\mathrm{Tr}(f^+_{\mu\nu}) \\
& -\frac{id_8}{2M_N}\mathrm{Tr}(\bar{B}^\mu_{6*}\tilde{f}^+_{\mu\nu}B^\nu_{6*}) - \frac{id_9}{2M_N}\mathrm{Tr}(\bar{B}^\mu_{6*}B^\nu_{6*})\mathrm{Tr}(f^+_{\mu\nu}) \\
& -\frac{2if_2}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}\tilde{f}^+_{\mu\nu}[S^\mu, S^\nu]B_6) - i\frac{f_4}{2M_N}\mathrm{Tr}(\bar{B}^{*\mu}_6\tilde{f}^+_{\mu\nu}S^\nu B_{\bar{3}}) \\
& -i\frac{f_3}{2M_N}\mathrm{Tr}(\bar{B}^{*\mu}_6\tilde{f}^+_{\mu\nu}S^\nu B_6) - i\frac{\tilde{f}_3}{2M_N}\mathrm{Tr}(\bar{B}^{*\mu}_6 S^\nu B_6)\mathrm{Tr}(f^+_{\mu\nu}) + \mathrm{H.c.}.
\end{aligned}
$$

✓$3 \otimes \bar{3} = 1 \oplus 8$ and $6 \otimes \bar{6} = 1 \oplus 8 \oplus 27$.

- $d_{2,5,8}$: $8 \otimes 8 \rightarrow 1$, $d_{3,4,9}$: $1 \otimes 1 \rightarrow 1$

✓$3 \otimes 6 = 8 \oplus 10$

- only couples with $\tilde{f}^+_{\mu\nu}$
- The leading order transition $B_6/B^\mu_{6*} \rightarrow B_{\bar{3}}\gamma$ arises from the the dynamics of the light quark sector.

✓ the $\mathcal{O}(p^2)$ Lagrangians with χ_+ ,$\mathrm{Tr}(\bar{B}_{\bar{3}}\chi_+ B_6)$ $\nrightarrow$ $\mathcal{O}(p^2)$ radiative decay amplitudes.

$\mathcal{O}(p^2)$ Lagrangian

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LO Magnetic moment
NNLO radiative decay amplitude

NNLO Magnetic moment
LO radiative decay amplitude

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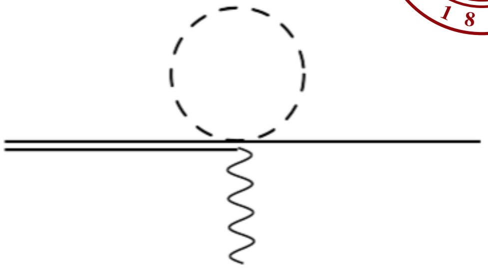
✓ the $\mathcal{O}(p^2)$ Lagrangians with χ_+ $\mathrm{Tr}(\bar{B}_{\bar{3}}\chi_+B_6)$ $\cancel{\longrightarrow}$ $\mathcal{O}(p^2)$ radiative decay amplitudes.

$\mathcal{O}(p^2)$ Lagrangian

The double π vertex: contribute through the $\mathcal{O}(p^4)$ loop diagrams.



(c)

For the magnetic moments:

$$\begin{aligned}\mathcal{L}^{(2)}_{MB} &= \frac{d_1}{2M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}[S^\mu, S^\nu][u_\mu, u_\nu]B_{\bar{3}}) + \frac{d_4}{M_N}\mathrm{Tr}(\bar{B}_6[S^\mu, S^\nu][u_\mu, u_\nu]B_6) \\ &\quad + \frac{d_7}{M_N}\mathrm{Tr}(\bar{B}^\mu_{6^*}[u_\mu, u_\nu]B^\nu_{6^*}).\end{aligned}$$

For the radiative transition:

$$\begin{aligned}\mathcal{L}^{(2)}_{B\phi\phi} &= \frac{a_1}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}[S^\mu, S^\nu][u_\mu, u_\nu]B_6) + \frac{a_2}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}ab}[S^\mu, S^\nu]u^b_{i\mu}u^a_{j\nu}B^{ij}_6) \\ &\quad + \frac{a_3}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}}S^\mu[u_\mu, u_\nu]B^\nu_{6^*}) + \frac{a_4}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}ab}S^\mu u^b_{i\mu}u^a_{j\nu}B^{\nu ij}_{6^*}) \\ &\quad + \frac{a_5}{M_N}\mathrm{Tr}(\bar{B}_6 S^\mu[u_\mu, u_\nu]B^\nu_{6^*}) + \mathrm{H.c.}.\end{aligned}$$

✓ $\mathrm{Tr}(\bar{B}_{\bar{3}}[S^\mu, S^\nu]\{u_\mu, u_\nu\}B_6)$ vanishes due to the antisymmetry of the Lorentz indices μ and ν.

$\mathcal{O}(p^3)$ Lagrangian

The Lagrangian at $\mathcal{O}(p^3)$ contributes through the $\mathcal{O}(p^3)$ diagrams.

$$
\begin{aligned}
\mathcal{L}_{B\gamma}^{(3)} = & \frac{n}{8M_N^2 M_6}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\nabla_\lambda \tilde{f}^+_{\mu\nu}\sigma^{\mu\nu}D^\lambda\psi_6) + \frac{n_1}{8M_N^2 M_{6^*}}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\nabla_\lambda \tilde{f}^+_{\mu\nu}\gamma^\lambda\gamma_5 iD^\mu\psi^\nu_{6^*}) \\
& + \frac{n_2}{8M_N^2 M_{6^*}}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\nabla_\lambda \tilde{f}^+_{\mu\nu}\gamma^\mu\gamma_5 iD^\lambda\psi^\nu_{6^*}) + \frac{m_1}{8M_N^2 M_{6^*}}\mathrm{Tr}(\bar{\psi}_6\nabla_\lambda \tilde{f}^+_{\mu\nu}\gamma^\lambda\gamma_5 iD^\mu\psi^\nu_{6^*}) \\
& + \frac{m_2}{8M_N^2 M_{6^*}}\mathrm{Tr}(\bar{\psi}_6\nabla_\lambda \tilde{f}^+_{\mu\nu}\gamma^\mu\gamma_5 iD^\lambda\psi^\nu_{6^*}) + \frac{\tilde{m}_1}{8M_N^2 M_{6^*}}\mathrm{Tr}[\bar{\psi}_6\nabla_\lambda \mathrm{Tr}(f^+_{\mu\nu})\gamma^\lambda\gamma_5 iD^\mu\psi^\nu_{6^*}] \\
& + \frac{\tilde{m}_2}{8M_N^2 M_{6^*}}\mathrm{Tr}[\bar{\psi}_6 \mathrm{Tr}(\nabla_\lambda f^+_{\mu\nu})\gamma^\mu\gamma_5 iD^\lambda\psi^\nu_{6^*}] + \mathrm{H.c.}.
\end{aligned}
$$

✓ n, n_2, m_2, and $\tilde{m}_2$ terms contribute to the G_1 → Important for M1 transition

- cancel the divergences of the $\mathcal{O}(p^3)$ loop diagrams
- be absorbed into the lower order f_{2-4} and $\tilde{f}_3$ terms

✓ The n_1, m_1 and $\tilde{m}_1$ terms contribute to G_2 → E2 transition

Building Block at $\mathcal{O}(p^4)$

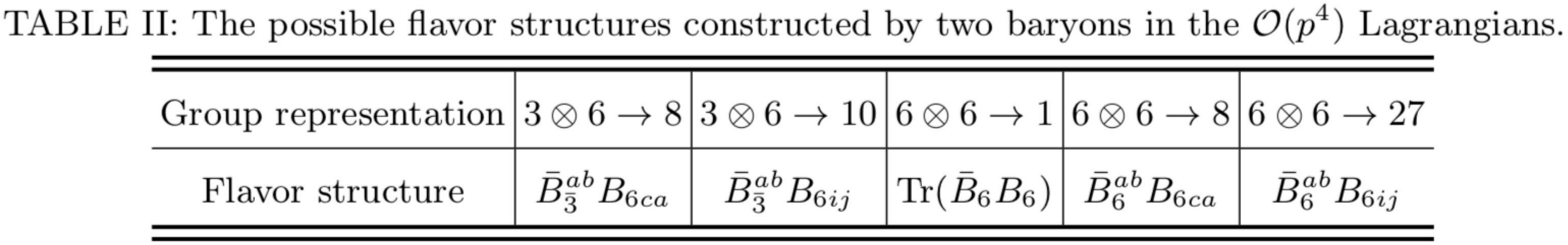


TABLE II: The possible flavor structures constructed by two baryons in the $\mathcal{O}(p^4)$ Lagrangians.

Group representation	$3 \otimes 6 \to 8$	$3 \otimes 6 \to 10$	$6 \otimes 6 \to 1$	$6 \otimes 6 \to 8$	$6 \otimes 6 \to 27$
Flavor structure	$\bar{B}_{\bar{3}}^{ab} B_{6ca}$	$\bar{B}_{\bar{3}}^{ab} B_{6ij}$	$\mathrm{Tr}(\bar{B}_6 B_6)$	$\bar{B}_6^{ab} B_{6ca}$	$\bar{B}_6^{ab} B_{6ij}$

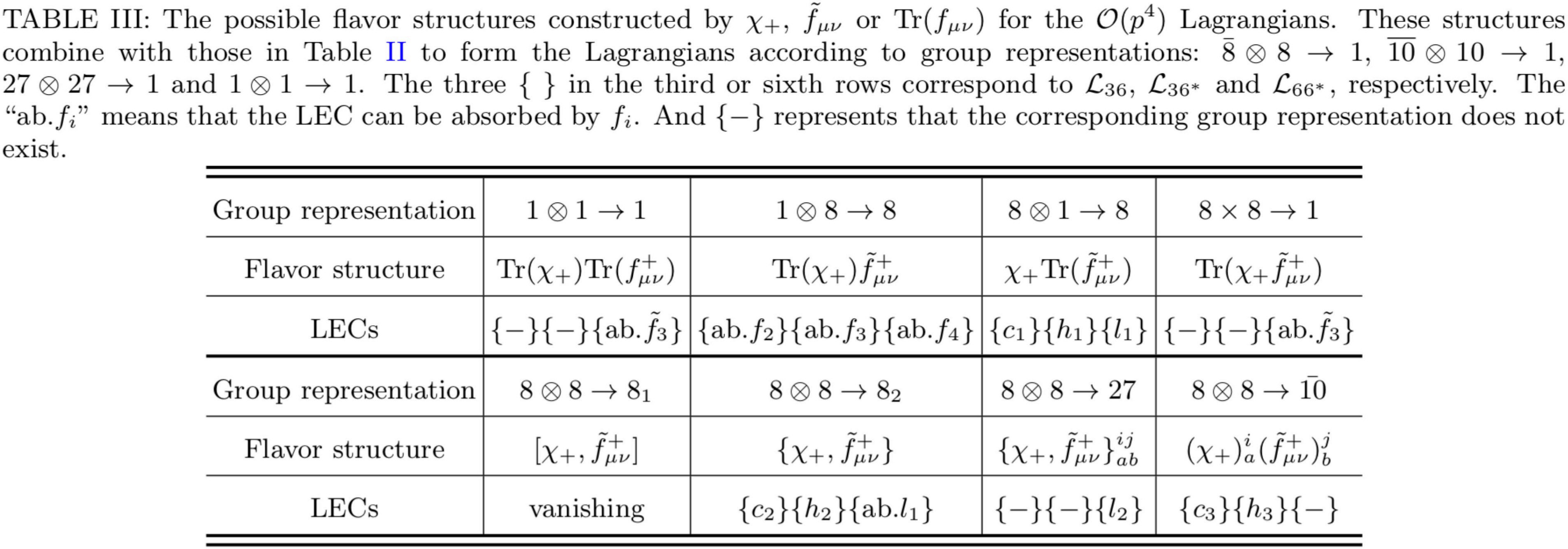


TABLE III: The possible flavor structures constructed by χ_+, $\tilde{f}_{\mu\nu}$ or $\mathrm{Tr}(f_{\mu\nu})$ for the $\mathcal{O}(p^4)$ Lagrangians. These structures combine with those in Table II to form the Lagrangians according to group representations: $\bar{8} \otimes 8 \to 1$, $\overline{10} \otimes 10 \to 1$, $27 \otimes 27 \to 1$ and $1 \otimes 1 \to 1$. The three $\{\ \}$ in the third or sixth rows correspond to $\mathcal{L}_{36}$, $\mathcal{L}_{36^*}$ and $\mathcal{L}_{66^*}$, respectively. The "ab.f_i" means that the LEC can be absorbed by f_i. And $\{-\}$ represents that the corresponding group representation does not exist.

Group representation	$1 \otimes 1 \to 1$	$1 \otimes 8 \to 8$	$8 \otimes 1 \to 8$	$8 \times 8 \to 1$
Flavor structure	$\mathrm{Tr}(\chi_+)\mathrm{Tr}(f^+_{\mu\nu})$	$\mathrm{Tr}(\chi_+)\tilde{f}^+_{\mu\nu}$	$\chi_+\mathrm{Tr}(\tilde{f}^+_{\mu\nu})$	$\mathrm{Tr}(\chi_+\tilde{f}^+_{\mu\nu})$
LECs	$\{-\}\{-\}\{\mathrm{ab}.\tilde{f}_3\}$	$\{\mathrm{ab}.f_2\}\{\mathrm{ab}.f_3\}\{\mathrm{ab}.f_4\}$	$\{c_1\}\{h_1\}\{l_1\}$	$\{-\}\{-\}\{\mathrm{ab}.\tilde{f}_3\}$
Group representation	$8 \otimes 8 \to 8_1$	$8 \otimes 8 \to 8_2$	$8 \otimes 8 \to 27$	$8 \otimes 8 \to \bar{10}$
Flavor structure	$[\chi_+, \tilde{f}^+_{\mu\nu}]$	$\{\chi_+, \tilde{f}^+_{\mu\nu}\}$	$\{\chi_+, \tilde{f}^+_{\mu\nu}\}^{ij}_{ab}$	$(\chi_+)^i_a(\tilde{f}^+_{\mu\nu})^j_b$
LECs	vanishing	$\{c_2\}\{h_2\}\{\mathrm{ab}.l_1\}$	$\{-\}\{-\}\{l_2\}$	$\{c_3\}\{h_3\}\{-\}$

$\mathcal{O}(p^4)$ Lagrangian

The Lagrangian at $\mathcal{O}(p^4)$ contributes to the $\mathcal{O}(p^4)$ tree diagram.

For magnetic moments:

$$\begin{aligned}\mathcal{L}_{B\gamma}^{(4)} &= -\frac{is_2}{4M_N}\mathrm{Tr}(\bar{B}_6[S^\mu, S^\nu]\chi_+ B_6)\mathrm{Tr}(f_{\mu\nu}^+) - \frac{is_3}{4m_N}\mathrm{Tr}(\bar{B}_6^{ab}[S^\mu, S^\nu]\{\chi_+, \tilde{f}_{\mu\nu}^+\}_{ab}^{ij} B_{6ij}^\nu) \\ &+ \frac{is_7}{4m_N}\mathrm{Tr}(\bar{B}_{6^*}^\mu \chi_+ B_{6^*}^\nu)\mathrm{Tr}(\tilde{f}_{\mu\nu}^+) + \frac{is_8}{4m_N}\mathrm{Tr}(\bar{B}_{6^*}^\mu \{\chi_+, \tilde{f}_{\mu\nu}^+\}_{ab}^{ij} B_{6^*}^\nu).\end{aligned}$$

For the radiative decay:

$$\begin{aligned}\mathcal{L}_{B\gamma}^{(4)} &= \frac{c_1}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\chi_+\sigma_{\mu\nu}\psi_6)\mathrm{Tr}(f_{\mu\nu}^+) + \frac{c_2}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\{\chi_+, \tilde{f}_{\mu\nu}^+\}\sigma_{\mu\nu}\psi_6) + \frac{c_3}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}^{ab}\chi_{a+}^i \tilde{f}_{b\mu\nu}^{j+}\sigma_{\mu\nu}\psi_{6ij}) \\ &- \frac{ih_1}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\chi_+\gamma_\mu\gamma_5\psi_{6^*}^\nu)\mathrm{Tr}(f_{\mu\nu}^+) - \frac{ih_2}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}\{\chi_+, \tilde{f}_{\mu\nu}^+\}\gamma_\mu\gamma_5\psi_{6^*}^\nu) - \frac{ih_3}{8m_N}\mathrm{Tr}(\bar{\psi}_{\bar{3}}^{ab}\chi_{a+}^i \tilde{f}_{b\mu\nu}^{j+}\gamma_\mu\gamma_5\psi_{6^*ij}^\nu) \\ &- \frac{il_1}{8m_N}\mathrm{Tr}(\bar{\psi}_6\chi_+\gamma_\mu\gamma_5\psi_{6^*}^\nu)\mathrm{Tr}(f_{\mu\nu}^+) - \frac{il_2}{8m_N}\mathrm{Tr}(\bar{\psi}_6^{ab}\{\chi_+ \tilde{f}_{\mu\nu}^+\}_{ab}^{ij}\gamma_\mu\gamma_5\psi_{6^*ij}^\nu) + \mathrm{H.c.}.\end{aligned}$$

U-spin symmetry

✓ In the U-spin transformation, $d \leftrightarrow s,\ \bar{d} \leftrightarrow \bar{s}$

$$\pi^{\pm} \leftrightarrow K^{\pm} \qquad K^0 \leftrightarrow \bar{K}^0$$

$\Lambda_c^+ \leftrightarrow \Xi_c^+,\ \Xi_c^0 \leftrightarrow \Xi_c^0,\ \Omega_c^0 \leftrightarrow \Sigma_c^0,\ \Sigma_c^+ \leftrightarrow \Xi_c'^+$ and $\Xi_c'^0 \leftrightarrow \Xi_c'^0$

✓ In the neutral decays $\Xi_c'^0 \to \Xi_c^0\gamma$ and $\Xi_c^{*0} \to \Xi_c^0\gamma$

- The contributions from s and d cancel out in the SU(3) flavor symmetry
- Their decay widths totally arise from the U-spin symmetry breaking effects up to NNLO

✓ Up to NLO

$$\boxed{G_{M1}(\Sigma_c^{*++} \to \Sigma_c^{++}\gamma) + G_{M1}(\Sigma_c^{*0} \to \Sigma_c^0\gamma) \ = \ 2G_{M1}(\Sigma_c^{*+} \to \Sigma_c^+\gamma).}$$

Holds up to NNLO

$$\begin{aligned} G_{M1}(\Sigma_c^{*++} \to \Sigma_c^{++}\gamma) + 2G_{M1}(\Xi_c^{*0} \to \Xi_c'^0\gamma) &= G_{M1}(\Sigma_c^{*0} \to \Sigma_c^0\gamma) + 2G_{M1}(\Xi_c^{*+} \to \Xi_c'^+\gamma) \\ &= G_{M1}(\Omega_c^{*0} \to \Omega_c^0\gamma) + 2G_{M1}(\Sigma_c^{*+} \to \Sigma_c^+). \end{aligned}$$

LECs: Heavy quark symmetry

- The superfield:

$$\psi^\mu = B^\mu_{6*} - \sqrt{\frac{1}{3}}(\gamma^\mu + v^\mu)\gamma_5 B_6$$

$$\bar{\psi}_\mu = \bar{B}^\mu_{6*} + \sqrt{\frac{1}{3}}\bar{B}_6\gamma_5(\gamma_\mu + v_\mu)$$

Cheng, at.al PRD 49, 5857;
Wise PRD 45, R2188;

- The Lagrangians in the heavy quark limit:

$$\mathcal{L}^{(2)}_{HQSS} = i\frac{\kappa_1}{M_N}\mathrm{Tr}(\bar{\psi}^\mu \tilde{f}^{+}_{\mu\nu}\psi^\nu) + \frac{\kappa_2}{M_N}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\bar{\psi}^\mu \tilde{f}^{\alpha\beta}v^\nu B_{\bar{3}}),$$

$$\mathcal{L}^{(2)}_{QB} = \frac{\kappa_3}{M_N}\mathrm{Tr}(\bar{\psi}^\lambda\sigma_{\mu\nu}\psi_\lambda)\mathrm{Tr}(f^{\mu\nu+}),$$

$$d_5 = -\tfrac{8}{3}\kappa_1,\;\; d_8 = -2\kappa_1,\;\; f_3 = 4\sqrt{\tfrac{1}{3}}\kappa_1,$$

$$f_4 = 8\kappa_2,\;\; f_2 = \tfrac{1}{\sqrt{3}}\kappa_2,$$

$$d_6 = \tfrac{8}{3}\kappa_3,\;\; d_9 = -4\kappa_3,\;\; \tilde{f}_3 = -\tfrac{16}{\sqrt{3}}\kappa_3.$$

LECs: Heavy quark symmetry

Double π vertex

$$\mathcal{L}^{(2)}_{B\phi\phi} = \frac{\kappa_4}{M_N}\mathrm{Tr}(\bar{\psi}^{\mu}[u_\mu, u_\nu]\psi^{\nu}) + \frac{i\kappa_5}{M_N}\epsilon^{\sigma\mu\nu\rho}\mathrm{Tr}(\bar{B}_{\bar{3}}[u_\mu, u_\nu]v_\rho\psi_\sigma) \\ + \frac{i\kappa_6}{M_N}\mathrm{Tr}(\bar{B}_{\bar{3}ab}\epsilon^{\sigma\mu\nu\rho}u^{b}_{i\mu}u^{a}_{j\nu}v_\rho\psi^{ij}_{\sigma})$$

$$a_5 = -2\sqrt{\tfrac{1}{3}}\kappa_4,\ \ d_4 = \tfrac{2}{3}\kappa_4,\ \ d_7 = \kappa_4,$$

$$a_3 = 2\kappa_5,\ \ a_1 = 4\sqrt{\tfrac{1}{3}}\kappa_5,$$

$$a_4 = 4\kappa_6,\ \ a_2 = 2\sqrt{\tfrac{1}{3}}\kappa_6.$$

The $\mathcal{O}(p^4)$ Lagrangian reads

$$\mathcal{L}^{(4)}_{B\gamma} = \frac{i\kappa_7}{m_N}\mathrm{Tr}(\bar{\psi}^{ab}_{\mu}\{\chi_+, \tilde{f}^{+}_{\mu\nu}\}^{ij}_{ab}\psi^{\nu}_{ij}) + \frac{\kappa_8}{m_N}\mathrm{Tr}(\bar{\psi}^{\lambda}\chi_+\sigma^{\mu\nu}\psi_\lambda)\mathrm{Tr}(f^{+}_{\mu\nu}),$$

$$l_2 = 8\sqrt{\tfrac{1}{3}}\kappa_7,\ \ s_3 = -\tfrac{8}{3}\kappa_7,\quad s_8 = 4\kappa_7,$$

$$l_1 = -32\sqrt{\tfrac{1}{3}}\kappa_8,\quad s_2 = \tfrac{8}{3}\kappa_8,\quad s_7 = 8\kappa_8.$$

The LECs are reduced to κ_{1-8}, n_1, m_1 and $\tilde{m}_1$ in the heavy quark limit.

Lattice QCD

- The magnetic moments from the Lattice QCD.

$\mu^{\ddagger}_{\Xi_c^+}$	$\mu_{\Xi_c^0}$	$\mu_{\Xi_c^{\prime +}}$	$\mu_{\Xi_c^{\prime 0}}$	$\mu^{\ddagger}_{\Xi_c^{\prime +}\to\Xi_c^+\gamma}$	$\mu_{\Xi_c^{\prime 0}\to\Xi_c^0\gamma}$
$0.235(25)$	$0.192(17)$	$0.315(141)$	$-0.599(71)$	$0.729(103)$	$0.009(13)$
$\mu^{\ddagger}_{\Sigma_c^{++}}$	$\mu_{\Sigma_c^0}$	$\mu^{\ddagger}_{\Omega_c^0}$	$\mu^{\ddagger}_{\Omega_c^{*0}}$	$G_{M1}(\Omega_c^{*0}\to\Omega_c^0\gamma)$	
$1.499(202)$	$-0.875(103)$	$-0.688(31)$	$-0.730(23)$	$G^q_{M1}(0)=0.671$	$G^c_{M1}(0)=0.145$

H. Bahtiyar et al. PLB 747, 281

K.U. Can et al. PRD 92,114515

K.U. Can et al. JHEP 1405, 125

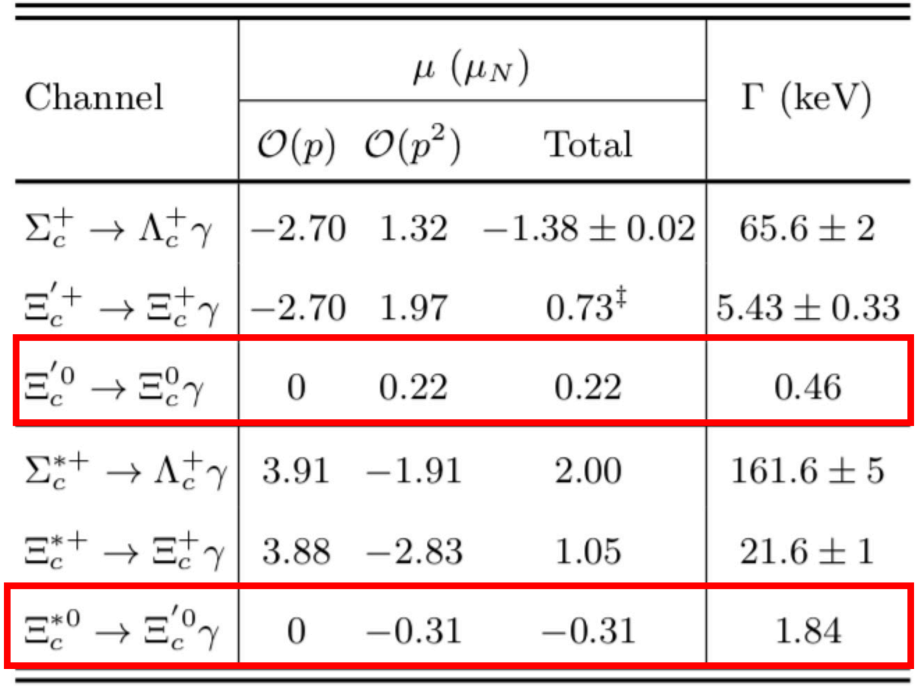
H. Bahtiyar et al. PLB 772, 121

- Many common LECs for the magnetic moments and the radiative decay amplitudes.

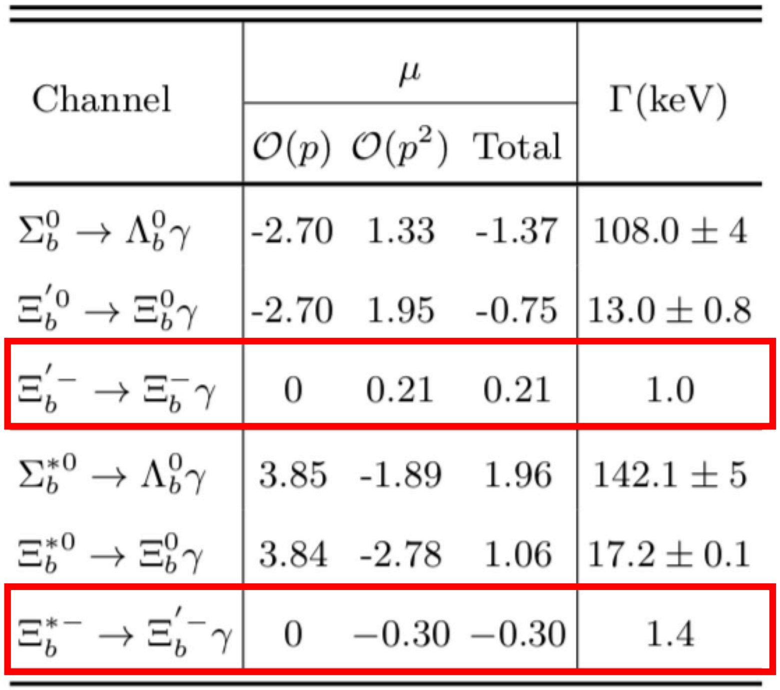
- The magnetic moments from the Lattice QCD → LECs → Γ

Numerical results

- $B_6 \to B_{\bar{3}}\gamma$: one unknown LECs κ_2 up to NLO. ← $\mu(\Xi_c'^+ \to \Xi_c^+\gamma)$
- $B_{6*}^{\mu} \to B_{\bar{3}}\gamma$: κ_2 and n_1(contribution$\propto M' - M \approx 0$).
- $\Gamma(\Xi_c'^0 \to \Xi_c^0\gamma)$ and $\Gamma(\Xi_c^{*0} \to \Xi_c^0\gamma)$: loops (a) and (b). → Independent of the input
- $\mu = \mu^q + \mu^Q, \quad \mathcal{M} = \mathcal{M}^q + \mathcal{M}^Q$ ← $\tilde{f}_{\mu\nu}^{+}$ and $\mathrm{Tr}(f_{\mu\nu}^{+})$
- $B_6 \to B_{\bar{3}}\gamma$: only light quarks contribute.



Channel	$\mu\ (\mu_N)$			Γ (keV)
	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	Total	
$\Sigma_c^+ \to \Lambda_c^+\gamma$	-2.70	1.32	-1.38 ± 0.02	65.6 ± 2
$\Xi_c'^+ \to \Xi_c^+\gamma$	-2.70	1.97	$0.73^{\ddagger}$	5.43 ± 0.33
$\Xi_c'^0 \to \Xi_c^0\gamma$	0	0.22	0.22	0.46
$\Sigma_c^{*+} \to \Lambda_c^+\gamma$	3.91	-1.91	2.00	161.6 ± 5
$\Xi_c^{*+} \to \Xi_c^+\gamma$	3.88	-2.83	1.05	21.6 ± 1
$\Xi_c^{*0} \to \Xi_c'^0\gamma$	0	-0.31	-0.31	1.84



Channel	μ			Γ(keV)
	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	Total	
$\Sigma_b^0 \to \Lambda_b^0\gamma$	-2.70	1.33	-1.37	108.0 ± 4
$\Xi_b'^0 \to \Xi_b^0\gamma$	-2.70	1.95	-0.75	13.0 ± 0.8
$\Xi_b'^- \to \Xi_b^-\gamma$	0	0.21	0.21	1.0
$\Sigma_b^{*0} \to \Lambda_b^0\gamma$	3.85	-1.89	1.96	142.1 ± 5
$\Xi_b^{*0} \to \Xi_b^0\gamma$	3.84	-2.78	1.06	17.2 ± 0.1
$\Xi_b^{*-} \to \Xi_b'^-\gamma$	0	-0.30	-0.30	1.4

Numerical results

For the transition $B_{6*}^{\mu} \to B_6\gamma$,

- E2 transition $\propto M' - M \approx 0$

- In the heavy quark limit, only the M1 transition contribute.

- The unknown LECs up to NNLO : $\xrightarrow{\text{HQS}}$ $\kappa_1, \kappa_3, \kappa_4, \kappa_7$ and κ_8

The heavy quark contribution

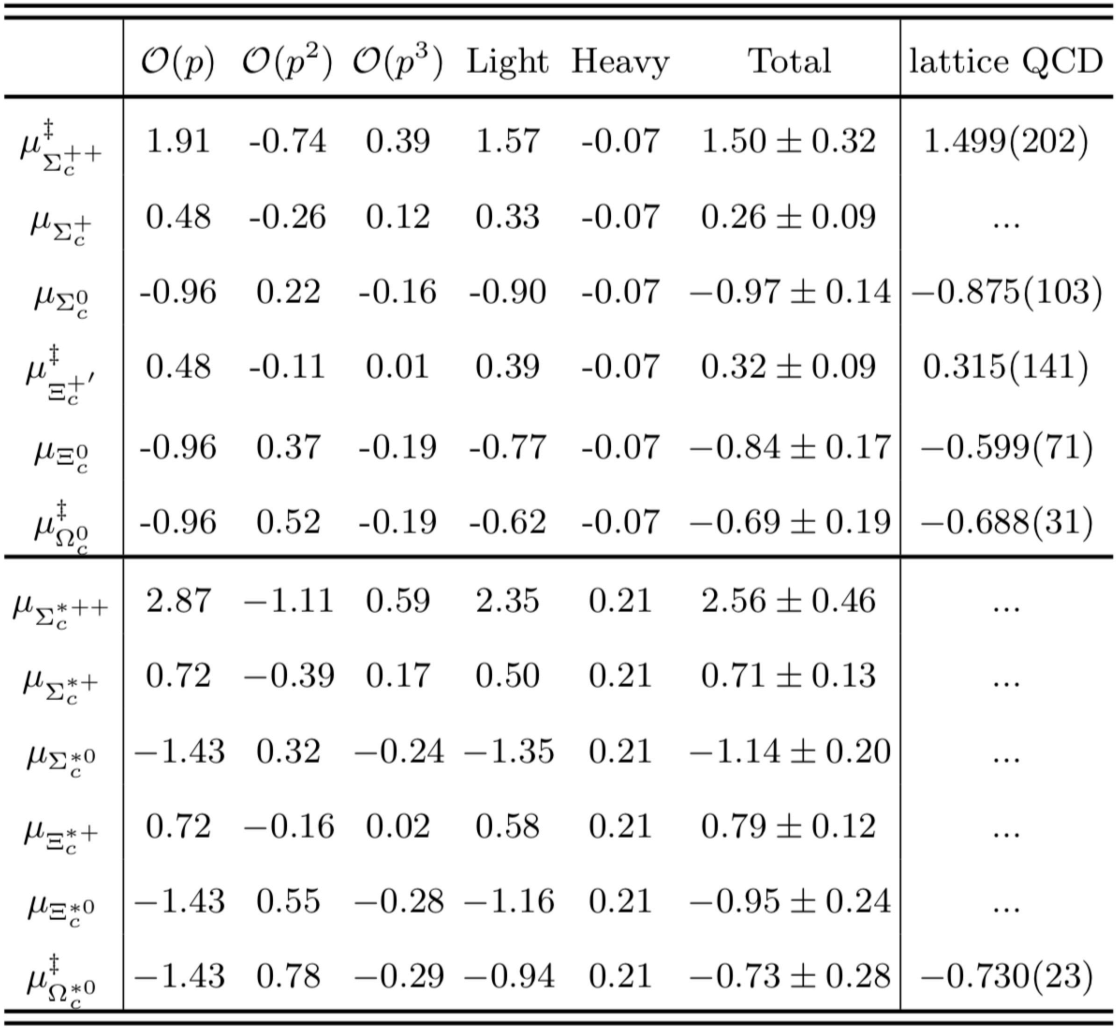
- The errors:

✓ lattice QCD simulations. $\longrightarrow$ errors on κ_i

✓ the error of the order 10% for heavy quark symmetry $\longrightarrow$ errors on d_i

Numerical results

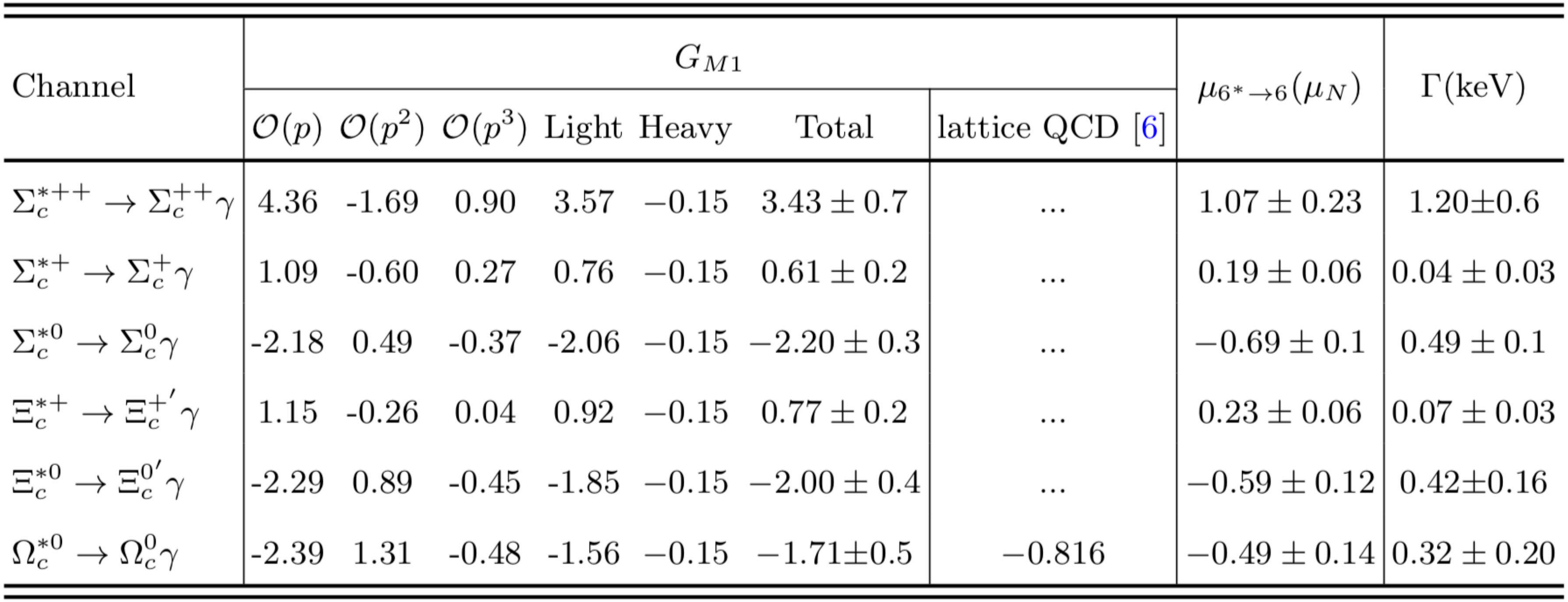
The convergence works well.



	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Light	Heavy	Total	lattice QCD
$\mu^{\ddagger}_{\Sigma_c^{++}}$	1.91	-0.74	0.39	1.57	-0.07	1.50 ± 0.32	1.499(202)
$\mu_{\Sigma_c^{+}}$	0.48	-0.26	0.12	0.33	-0.07	0.26 ± 0.09	...
$\mu_{\Sigma_c^{0}}$	-0.96	0.22	-0.16	-0.90	-0.07	-0.97 ± 0.14	−0.875(103)
$\mu^{\ddagger}_{\Xi_c^{+\prime}}$	0.48	-0.11	0.01	0.39	-0.07	0.32 ± 0.09	0.315(141)
$\mu_{\Xi_c^{0}}$	-0.96	0.37	-0.19	-0.77	-0.07	-0.84 ± 0.17	−0.599(71)
$\mu^{\ddagger}_{\Omega_c^{0}}$	-0.96	0.52	-0.19	-0.62	-0.07	-0.69 ± 0.19	−0.688(31)
$\mu_{\Sigma_c^{*++}}$	2.87	−1.11	0.59	2.35	0.21	2.56 ± 0.46	...
$\mu_{\Sigma_c^{*+}}$	0.72	−0.39	0.17	0.50	0.21	0.71 ± 0.13	...
$\mu_{\Sigma_c^{*0}}$	−1.43	0.32	−0.24	−1.35	0.21	-1.14 ± 0.20	...
$\mu_{\Xi_c^{*+}}$	0.72	−0.16	0.02	0.58	0.21	0.79 ± 0.12	...
$\mu_{\Xi_c^{*0}}$	−1.43	0.55	−0.28	−1.16	0.21	-0.95 ± 0.24	...
$\mu^{\ddagger}_{\Omega_c^{*0}}$	−1.43	0.78	−0.29	−0.94	0.21	-0.73 ± 0.28	−0.730(23)

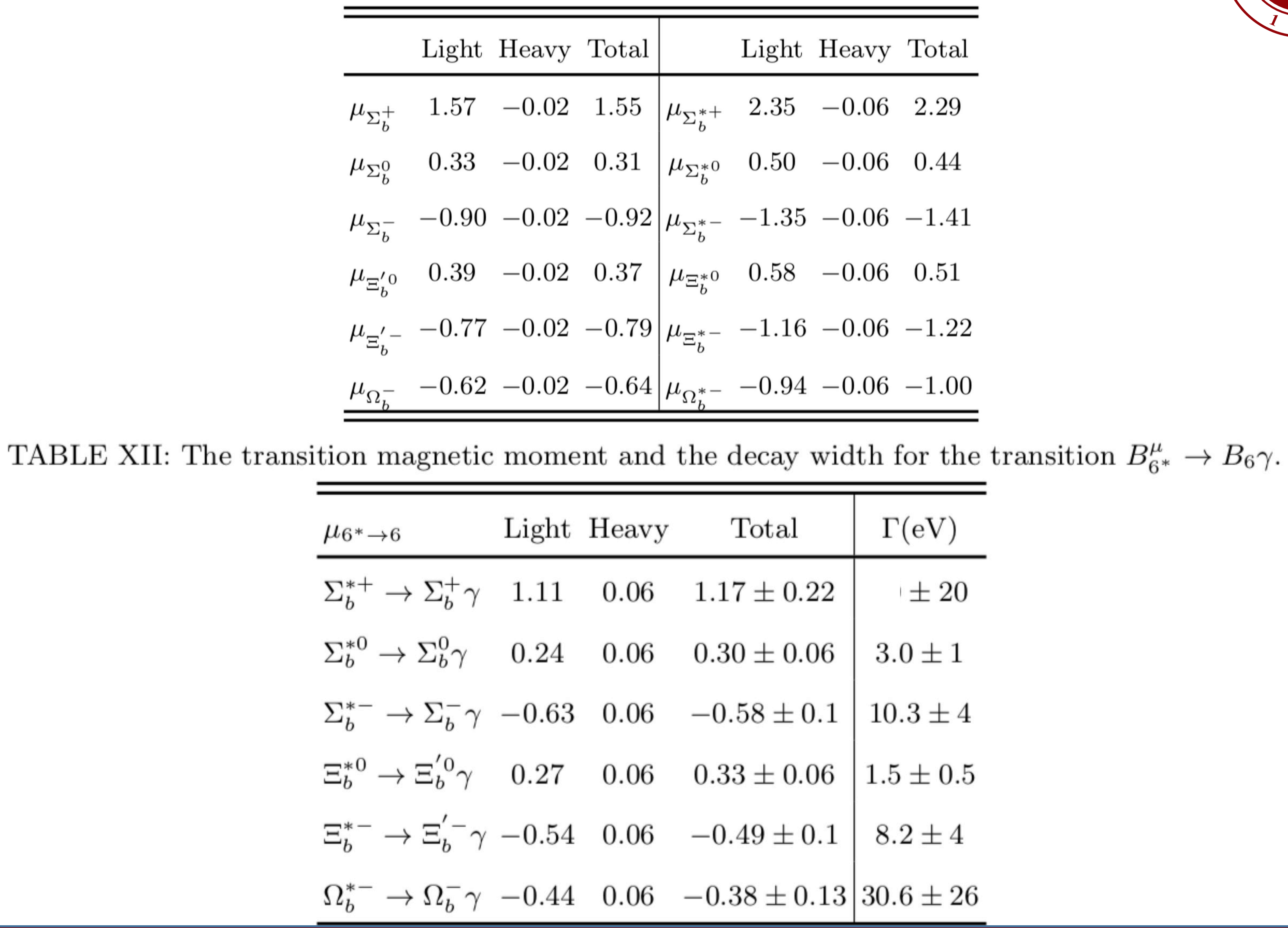
Numerical results

The convergence works well.



Channel	G_{M1}							$\mu_{6^*\to 6}(\mu_N)$	$\Gamma(\mathrm{keV})$
	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Light	Heavy	Total	lattice QCD [6]		
$\Sigma_c^{*++}\to\Sigma_c^{++}\gamma$	4.36	-1.69	0.90	3.57	-0.15	3.43 ± 0.7	...	1.07 ± 0.23	1.20 ± 0.6
$\Sigma_c^{*+}\to\Sigma_c^{+}\gamma$	1.09	-0.60	0.27	0.76	-0.15	0.61 ± 0.2	...	0.19 ± 0.06	0.04 ± 0.03
$\Sigma_c^{*0}\to\Sigma_c^{0}\gamma$	-2.18	0.49	-0.37	-2.06	-0.15	-2.20 ± 0.3	...	-0.69 ± 0.1	0.49 ± 0.1
$\Xi_c^{*+}\to\Xi_c^{+\prime}\gamma$	1.15	-0.26	0.04	0.92	-0.15	0.77 ± 0.2	...	0.23 ± 0.06	0.07 ± 0.03
$\Xi_c^{*0}\to\Xi_c^{0\prime}\gamma$	-2.29	0.89	-0.45	-1.85	-0.15	-2.00 ± 0.4	...	-0.59 ± 0.12	0.42 ± 0.16
$\Omega_c^{*0}\to\Omega_c^{0}\gamma$	-2.39	1.31	-0.48	-1.56	-0.15	-1.71 ± 0.5	-0.816	-0.49 ± 0.14	0.32 ± 0.20

Numerical results



	Light	Heavy	Total		Light	Heavy	Total
$\mu_{\Sigma_b^+}$	1.57	-0.02	1.55	$\mu_{\Sigma_b^{*+}}$	2.35	-0.06	2.29
$\mu_{\Sigma_b^0}$	0.33	-0.02	0.31	$\mu_{\Sigma_b^{*0}}$	0.50	-0.06	0.44
$\mu_{\Sigma_b^-}$	-0.90	-0.02	-0.92	$\mu_{\Sigma_b^{*-}}$	-1.35	-0.06	-1.41
$\mu_{\Xi_b^{\prime 0}}$	0.39	-0.02	0.37	$\mu_{\Xi_b^{*0}}$	0.58	-0.06	0.51
$\mu_{\Xi_b^{\prime -}}$	-0.77	-0.02	-0.79	$\mu_{\Xi_b^{*-}}$	-1.16	-0.06	-1.22
$\mu_{\Omega_b^-}$	-0.62	-0.02	-0.64	$\mu_{\Omega_b^{*-}}$	-0.94	-0.06	-1.00

TABLE XII: The transition magnetic moment and the decay width for the transition $B_{6*}^{\mu} \to B_6\gamma$.

$\mu_{6*\to 6}$	Light	Heavy	Total	Γ(eV)
$\Sigma_b^{*+} \to \Sigma_b^+\gamma$	1.11	0.06	1.17 ± 0.22	± 20
$\Sigma_b^{*0} \to \Sigma_b^0\gamma$	0.24	0.06	0.30 ± 0.06	3.0 ± 1
$\Sigma_b^{*-} \to \Sigma_b^-\gamma$	-0.63	0.06	-0.58 ± 0.1	10.3 ± 4
$\Xi_b^{*0} \to \Xi_b^{\prime 0}\gamma$	0.27	0.06	0.33 ± 0.06	1.5 ± 0.5
$\Xi_b^{*-} \to \Xi_b^{\prime -}\gamma$	-0.54	0.06	-0.49 ± 0.1	8.2 ± 4
$\Omega_b^{*-} \to \Omega_b^-\gamma$	-0.44	0.06	-0.38 ± 0.13	30.6 ± 26

Numerical results

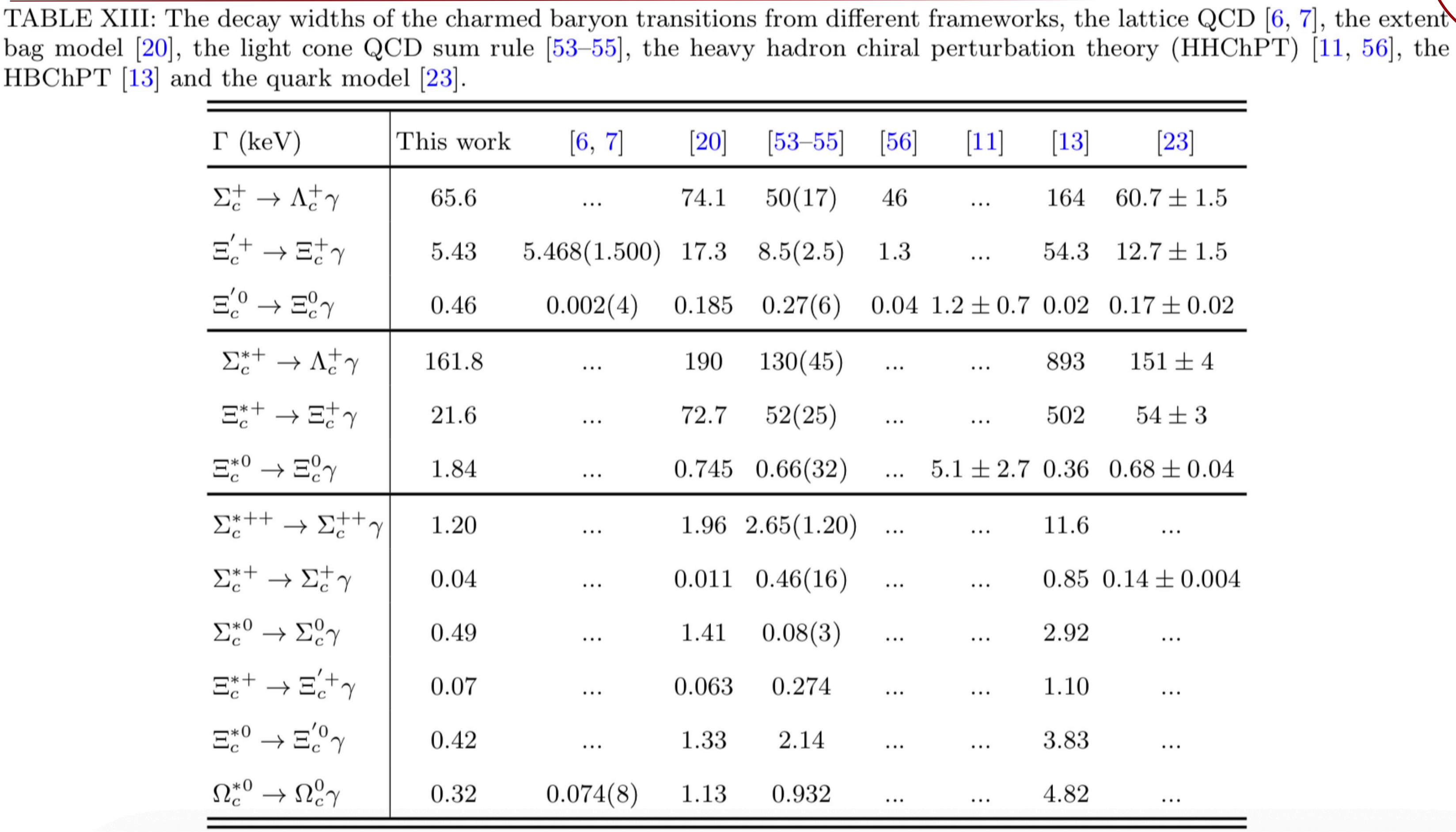


TABLE XIII: The decay widths of the charmed baryon transitions from different frameworks, the lattice QCD [6, 7], the extent bag model [20], the light cone QCD sum rule [53–55], the heavy hadron chiral perturbation theory (HHChPT) [11, 56], the HBChPT [13] and the quark model [23].

Γ (keV)	This work	[6, 7]	[20]	[53–55]	[56]	[11]	[13]	[23]
$\Sigma_c^+ \to \Lambda_c^+\gamma$	65.6	...	74.1	50(17)	46	...	164	60.7 ± 1.5
$\Xi_c'^+ \to \Xi_c^+\gamma$	5.43	5.468(1.500)	17.3	8.5(2.5)	1.3	...	54.3	12.7 ± 1.5
$\Xi_c'^0 \to \Xi_c^0\gamma$	0.46	0.002(4)	0.185	0.27(6)	0.04	1.2 ± 0.7	0.02	0.17 ± 0.02
$\Sigma_c^{*+} \to \Lambda_c^+\gamma$	161.8	...	190	130(45)	...	...	893	151 ± 4
$\Xi_c^{*+} \to \Xi_c^+\gamma$	21.6	...	72.7	52(25)	...	...	502	54 ± 3
$\Xi_c^{*0} \to \Xi_c^0\gamma$	1.84	...	0.745	0.66(32)	...	5.1 ± 2.7	0.36	0.68 ± 0.04
$\Sigma_c^{*++} \to \Sigma_c^{++}\gamma$	1.20	...	1.96	2.65(1.20)	...	...	11.6	...
$\Sigma_c^{*+} \to \Sigma_c^+\gamma$	0.04	...	0.011	0.46(16)	...	...	0.85	0.14 ± 0.004
$\Sigma_c^{*0} \to \Sigma_c^0\gamma$	0.49	...	1.41	0.08(3)	...	...	2.92	...
$\Xi_c^{*+} \to \Xi_c'^+\gamma$	0.07	...	0.063	0.274	...	...	1.10	...
$\Xi_c^{*0} \to \Xi_c'^0\gamma$	0.42	...	1.33	2.14	...	...	3.83	...
$\Omega_c^{*0} \to \Omega_c^0\gamma$	0.32	0.074(8)	1.13	0.932	...	...	4.82	...

[6,7] PLB 747, 281, PLB 772, 121;
[20] arXiv:1803.01809;
[53,54,55] PRD 79, 056005, EPJC 75, 14,PRD 93, 056007;
[11,56] PRD 79, 056005, PRD49, 5857;

[13] PRD 92, 054017 (2015);
[23] PRD 60, 094002;

Numerical results

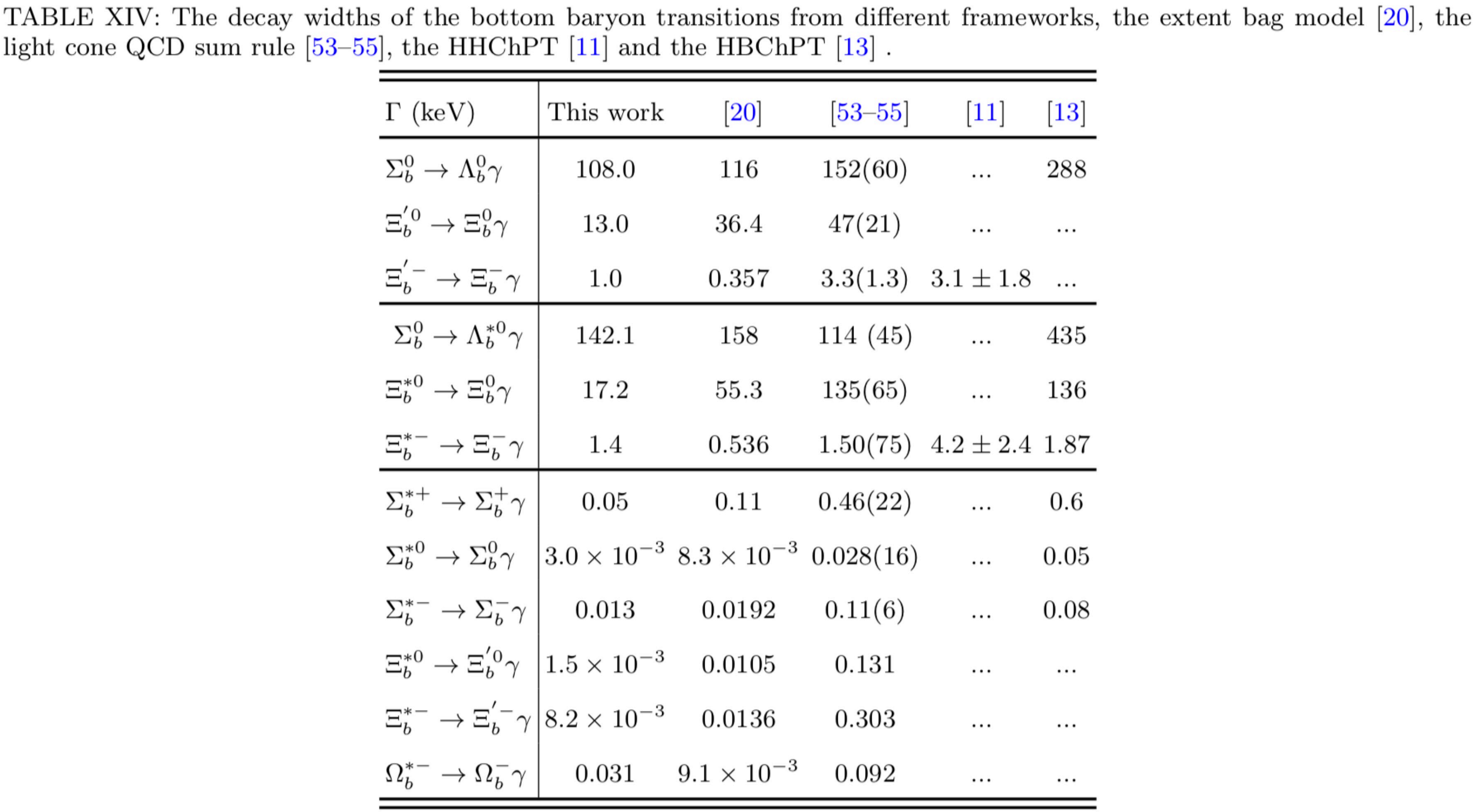


TABLE XIV: The decay widths of the bottom baryon transitions from different frameworks, the extent bag model [20], the light cone QCD sum rule [53–55], the HHChPT [11] and the HBChPT [13] .

Γ (keV)	This work	[20]	[53–55]	[11]	[13]
$\Sigma_b^0 \to \Lambda_b^0\gamma$	108.0	116	152(60)	...	288
$\Xi_b^{\prime 0} \to \Xi_b^0\gamma$	13.0	36.4	47(21)	...	...
$\Xi_b^{\prime -} \to \Xi_b^-\gamma$	1.0	0.357	3.3(1.3)	3.1 ± 1.8	...
$\Sigma_b^0 \to \Lambda_b^{*0}\gamma$	142.1	158	114 (45)	...	435
$\Xi_b^{*0} \to \Xi_b^0\gamma$	17.2	55.3	135(65)	...	136
$\Xi_b^{*-} \to \Xi_b^-\gamma$	1.4	0.536	1.50(75)	4.2 ± 2.4	1.87
$\Sigma_b^{*+} \to \Sigma_b^+\gamma$	0.05	0.11	0.46(22)	...	0.6
$\Sigma_b^{*0} \to \Sigma_b^0\gamma$	3.0×10^{-3}	8.3×10^{-3}	0.028(16)	...	0.05
$\Sigma_b^{*-} \to \Sigma_b^-\gamma$	0.013	0.0192	0.11(6)	...	0.08
$\Xi_b^{*0} \to \Xi_b^{\prime 0}\gamma$	1.5×10^{-3}	0.0105	0.131	...	...
$\Xi_b^{*-} \to \Xi_b^{\prime -}\gamma$	8.2×10^{-3}	0.0136	0.303	...	...
$\Omega_b^{*-} \to \Omega_b^-\gamma$	0.031	9.1×10^{-3}	0.092	...	...

[20] arXiv:1803.01809; [53,54,55] PRD 79, 056005, EPJC 75, 14,PRD 93, 056007;
[11] PRD 79, 056005, PRD49, 5857; [13] PRD 92, 054017 (2015);

Summary

- The analytical expressions of the radiative decay amplitudes up to NNLO.
  - ✓ $B_6 \to B_{\bar{3}}\gamma,\ B^{\mu}_{6*} \to B_{\bar{3}}\gamma$ and $B^{\mu}_{6*} \to B_6\gamma$
- The numerical results up to NLO for
  - ✓ $B_6 \to B_{\bar{3}}\gamma$ and $B^{\mu}_{6*} \to B_{\bar{3}}\gamma$
- The numerical results up to NNLO for $B^{\mu}_{6*} \to B_6\gamma$
  - ✓ The magnetic moments
  - ✓ The radiative decay width
- The electromagnetic properties of the other systems
  - ✓ The heavy meson
  - ✓ The exotic states
- The chiral extrapolations.

Thanks for your attention!

Backup slides

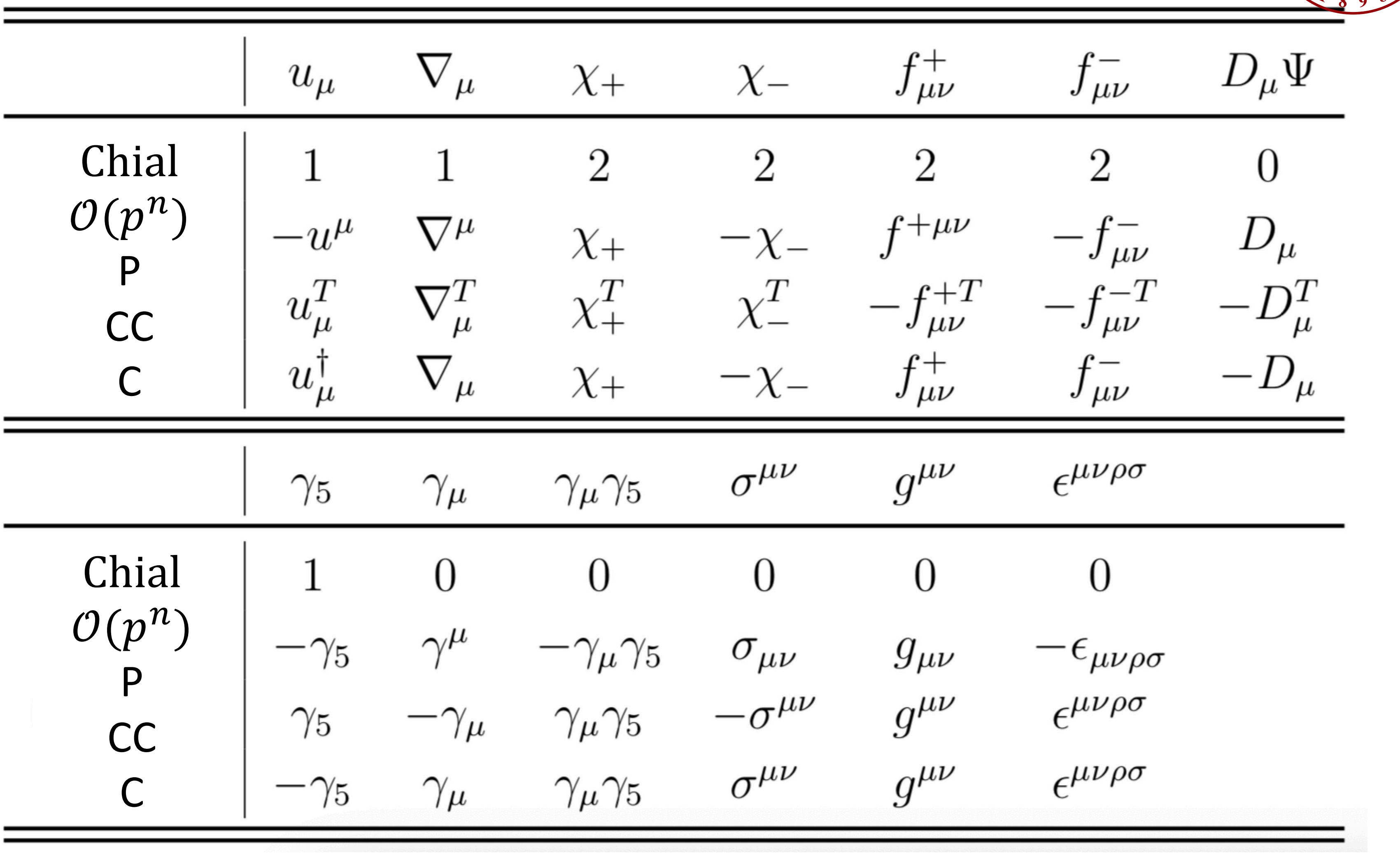
Building Block

	U	$D_\mu U$	χ	$D_\mu \chi$
Chial	$V_R U V_L^\dagger$	$V_R D_\mu U V_L^\dagger$	$V_R \chi V_L^\dagger$	$V_R D_\mu \chi V_L^\dagger$
$\mathcal{O}(p^n)$	0	1	2	3
P	$U^\dagger$	$(D^\mu U)^\dagger$	$\chi^\dagger$	$(D^\mu \chi)^\dagger$
CC	U^T	$(D_\mu U)^T$	χ^T	$(D_\mu \chi)^T$
C	$U^\dagger$	$(D_\mu U)^\dagger$	$\chi^\dagger$	$(D_\mu \chi)^\dagger$

	r_μ	l_μ	$f^R_{\mu\nu}$	$f^L_{\mu\nu}$
Chial	$V_R r_\mu V_R^\dagger + i V_R \partial_\mu V_R^\dagger$	$V_L l_\mu V_L^\dagger + i V_L \partial_\mu V_L^\dagger$	$V_R f^R_{\mu\nu} V_R^\dagger$	$V_L f^L_{\mu\nu} V_L^\dagger$
$\mathcal{O}(p^n)$	1	1	2	2
P	l^μ	r^μ	$f_L^{\mu\nu}$	$f_R^{\mu\nu}$
CC	$-l_\mu^T$	$-r_\mu^T$	$-(f^L_{\mu\nu})^T$	$-f^{RT}_{\mu\nu}$
C	r_μ	l_μ	$f^R_{\mu\nu}$	$f^L_{\mu\nu}$

Parity (P), charge conjugation (CC), conjugation (C)

Building Block



	u_μ	∇_μ	χ_+	χ_-	$f^+_{\mu\nu}$	$f^-_{\mu\nu}$	$D_\mu\Psi$
Chial $\mathcal{O}(p^n)$	1	1	2	2	2	2	0
P	$-u^\mu$	∇^μ	χ_+	$-\chi_-$	$f^{+\mu\nu}$	$-f^-_{\mu\nu}$	D_μ
CC	u^T_μ	∇^T_μ	χ^T_+	χ^T_-	$-f^{+T}_{\mu\nu}$	$-f^{-T}_{\mu\nu}$	$-D^T_\mu$
C	$u^\dagger_\mu$	∇_μ	χ_+	$-\chi_-$	$f^+_{\mu\nu}$	$f^-_{\mu\nu}$	$-D_\mu$

	γ_5	γ_μ	$\gamma_\mu\gamma_5$	$\sigma^{\mu\nu}$	$g^{\mu\nu}$	$\epsilon^{\mu\nu\rho\sigma}$
Chial $\mathcal{O}(p^n)$	1	0	0	0	0	0
P	$-\gamma_5$	γ^μ	$-\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$g_{\mu\nu}$	$-\epsilon_{\mu\nu\rho\sigma}$
CC	γ_5	$-\gamma_\mu$	$\gamma_\mu\gamma_5$	$-\sigma^{\mu\nu}$	$g^{\mu\nu}$	$\epsilon^{\mu\nu\rho\sigma}$
C	$-\gamma_5$	γ_μ	$\gamma_\mu\gamma_5$	$\sigma^{\mu\nu}$	$g^{\mu\nu}$	$\epsilon^{\mu\nu\rho\sigma}$

Parity (P), charge conjugation (CC), conjugation (C)