FEFU FAR EASTERN FEDERAL UNIVERSITY

FAIR

J-PARC and NICA Promised lands for revealing QCD phase

V. Bornyakov, D. Boyda, V. Goy, H.Iida, A. Molochkov,
A. Nakamura, M.Wakayama and V. I. Zakharov

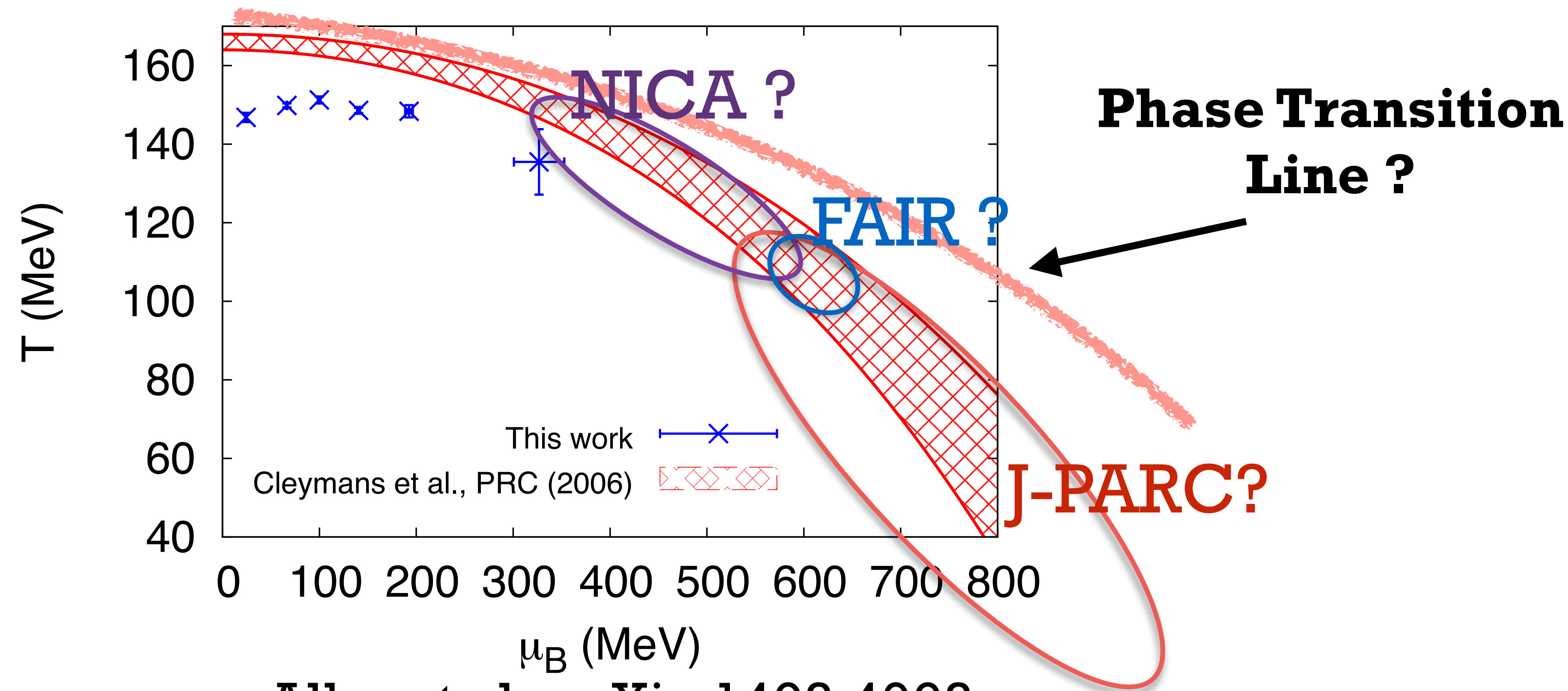
Two Messages from Vladivostok

- J-PARC and NICA can probe **Finite Baryon Density regions** in QCD phase.

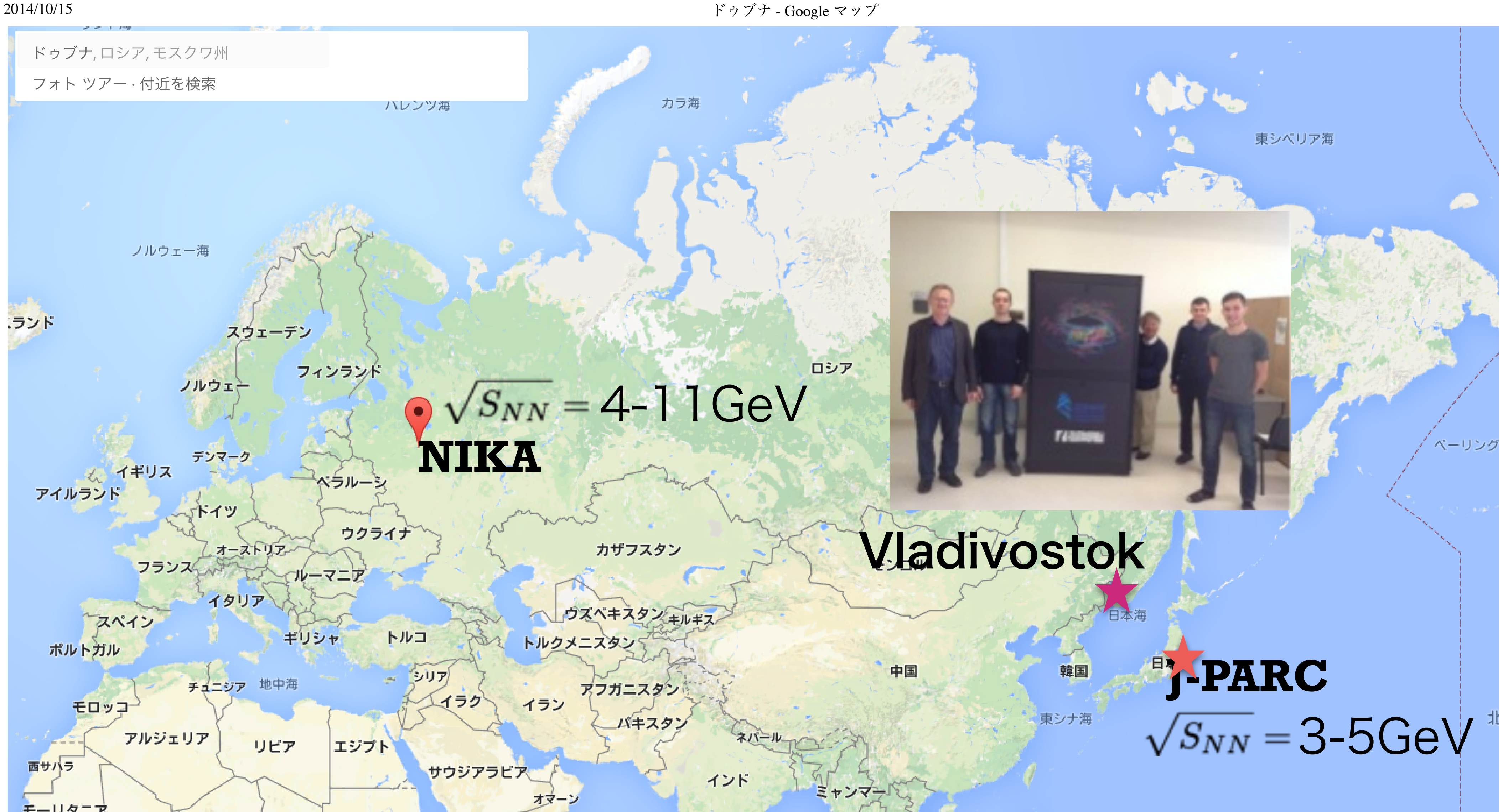
- Sign Problem of Lattice QCD simulations is essentially beaten

  →

  Study at **Finite Baryon Density regions** by First Principle calculation is possible.



Alba et al., arXiv:1403.4903



Plan of the Talk

1. Why Canonical Partition Functions, Z_N, are useful ?
2. Z_N from Experimental data
3. Z_N from Lattice QCD
4. Summary
   Experimental data + Lattice QCD

   → what we can/will learn ?

Our Tool

Canonical Approach or Fugacity Expansion

$$Z(\mu, T) = \sum_n z_n(T)\xi^n$$

Grand Canonical Partition Function — $Z(\mu, T)$

Canonical Partition Function — $z_n(T)$

μ : Chemical Potential

T : Temperature

$$\xi \equiv e^{\mu/T}$$ Fugacity

$$Z(\mu, T) = \sum_n z_n(T)\xi^n$$

$$\xi \equiv e^{\mu/T}$$

This is very useful relation because

once we have $z_n(T)$, then

we know $Z(\mu, T)$ at any density.

Z_n can be obtained from

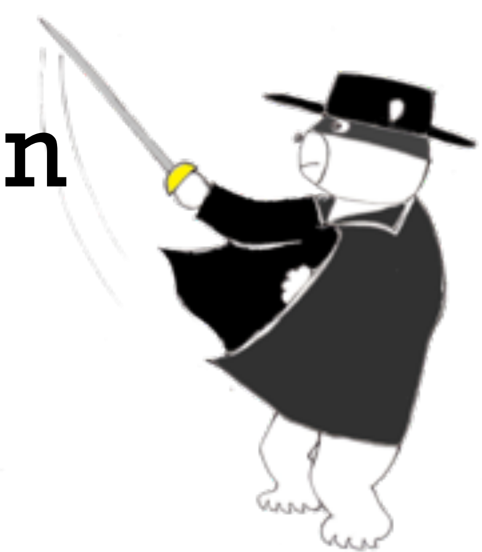
- ★ Experiment

and

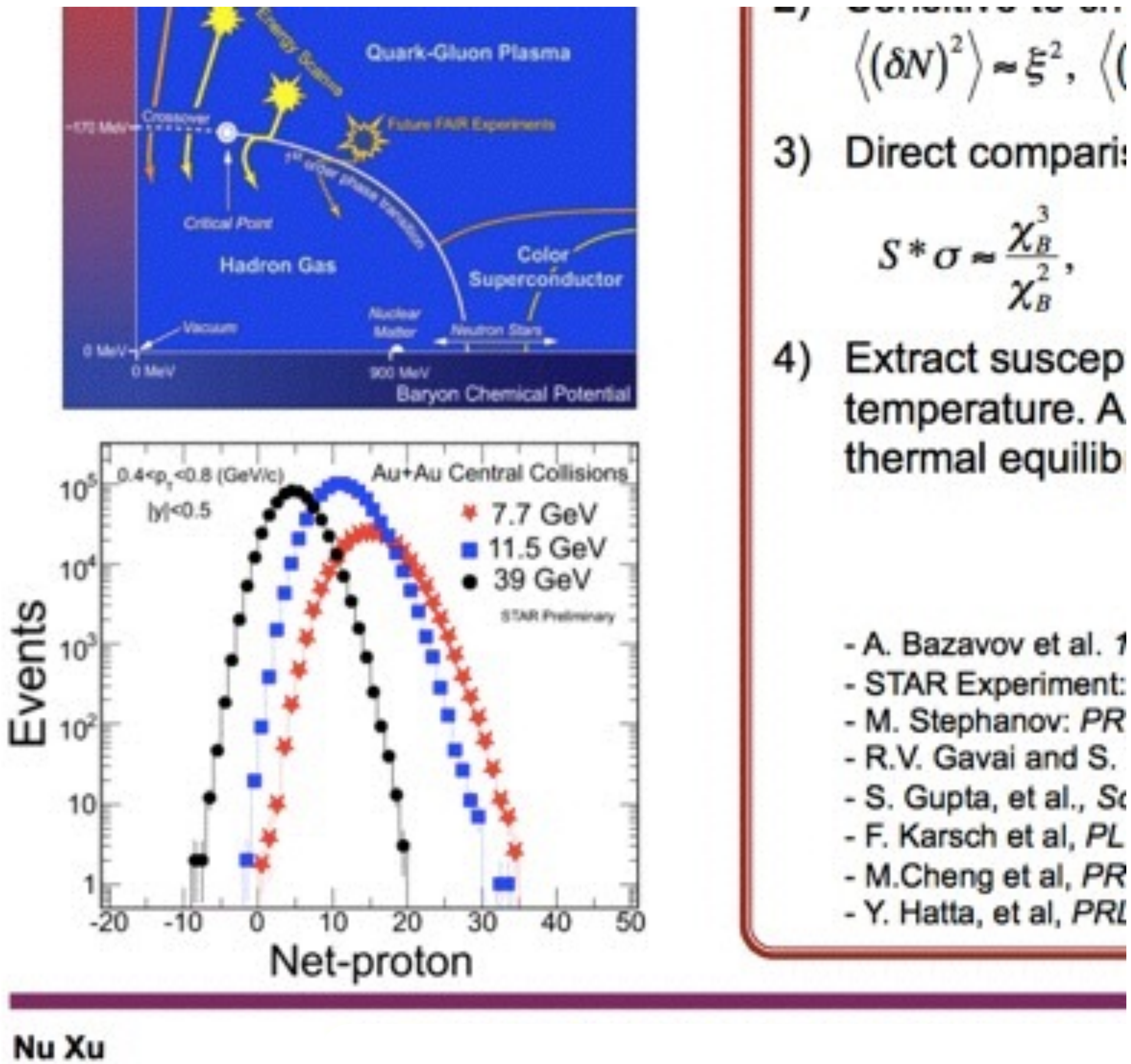
- ★ lattice QCD

From Experiments → Z_N ← From Lattice

Canonical Partition Functions



I) How to get z_n from Heavy Ion Collision data

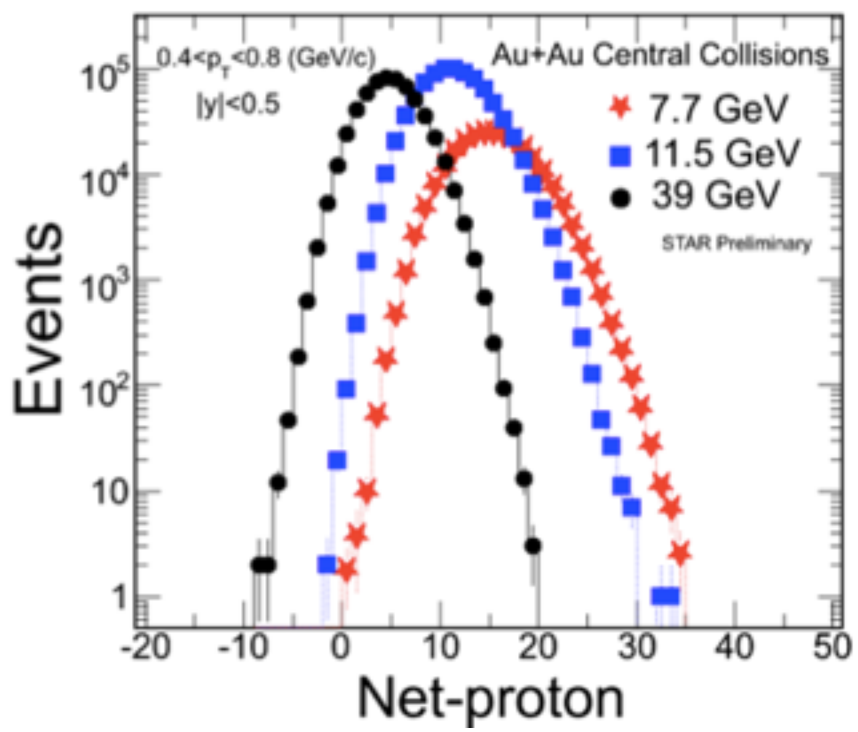


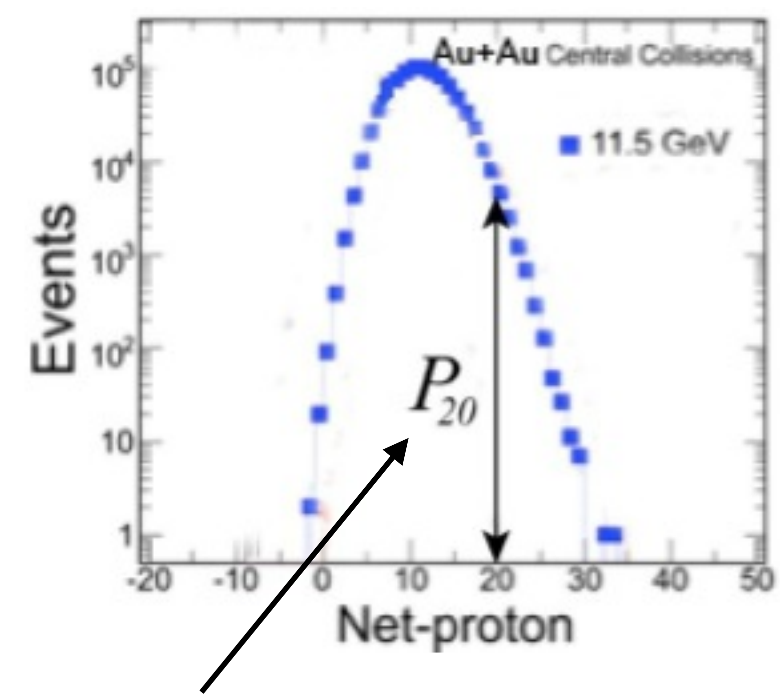
Oh,
Multiplicity
fluctuations !
It is almost Z_N

I will ask the data Prof. Nu Xu

Nu Xu, "QCD Structure 1" 2012 Oct. at Wuhan (武漢)

$$Z(\mu, T) = \sum_n Z_n (e^{\mu/T})^n$$





STAR@RHIC

$$P_{20} = \frac{Z_{20}(e^{\mu/T})^{20}}{Z}$$

Experimantal Data

$$\boxed{P_n} = \frac{Z_n \xi^n}{Z} \qquad \boxed{P_{-n}} = \frac{Z_{-n} \xi^{-n}}{Z} \qquad \xi \equiv e^{\mu/T}$$

$$Z_n = Z_{-n}$$

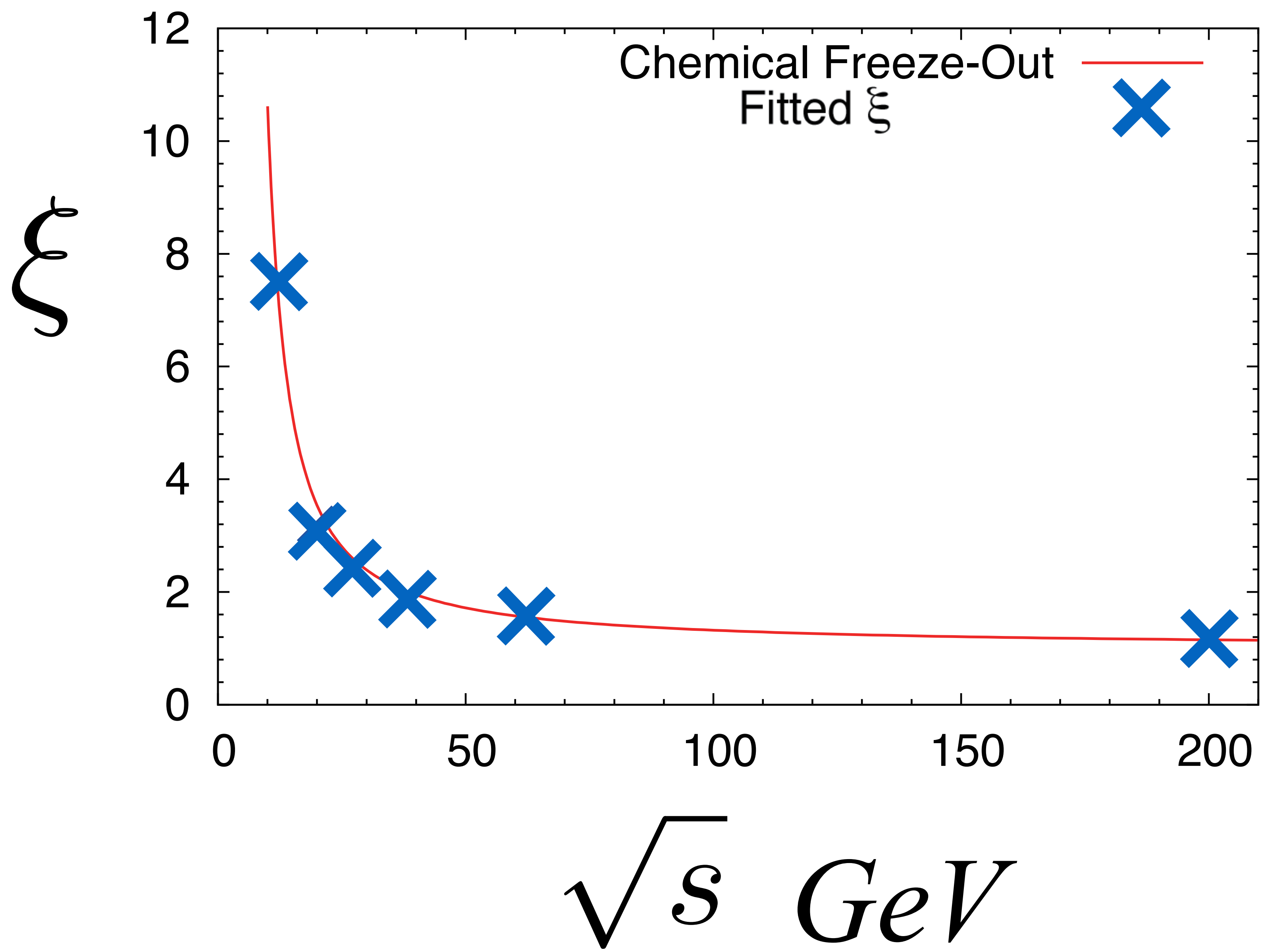
(CP-invariance, or particle anti-particle symmetry)

$$P_n / P_{-n} = \xi^{2n}$$

Now ξ is determined.

$$\frac{Z_n}{Z} = P_n / \xi^n$$

Fitted $\xi = e^{\mu/T}$ are consistent with those by Freeze-out Analysis ?



This work

Freeze-out J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

Comparison of obtained ξ

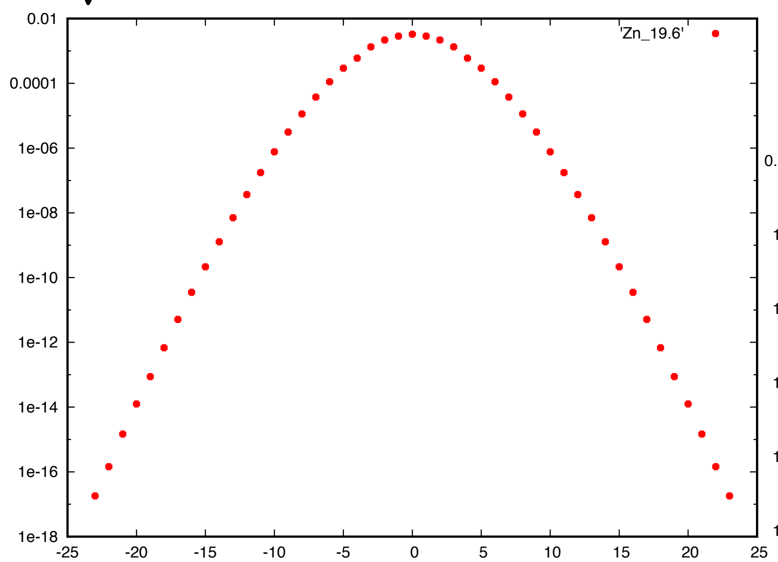
$$\xi \equiv e^{\mu/T}$$

$\sqrt{s_{NN}}$ GeV	Cleymans	Alba	Our
11.5	8.04	11.1	7.48
19.6	3.62	3.65	3.21
27	2.62	2.58	2.43
39	1.98	1.93	1.88
62.4	1.55	1.53	1.53
200	1.18	1.18	1.18

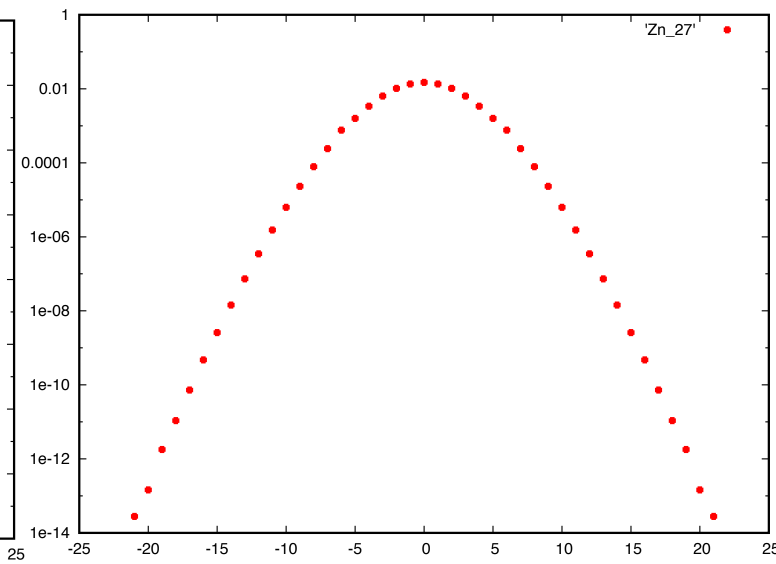
Cleymans et al., Phys. Rev. C 73, 034905 (2006).
Alba et al., Physics Letters B 738 (2014)

Z_n from RHIC data

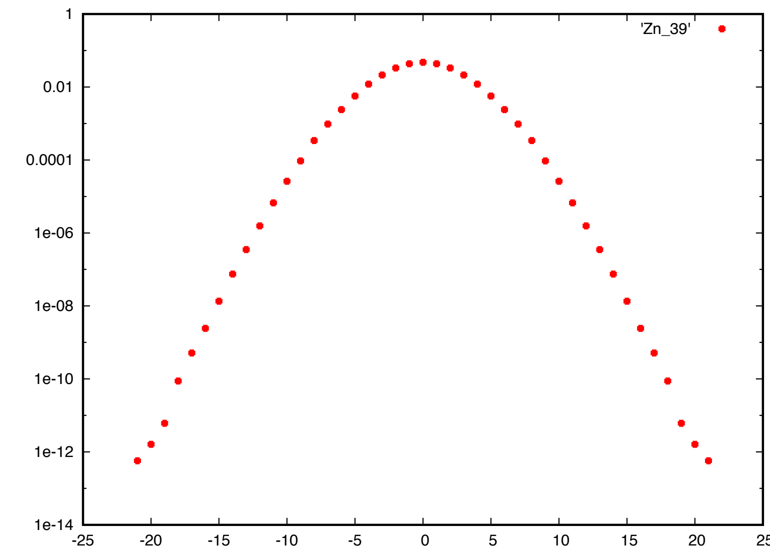
$\sqrt{s} = 19.6\mathrm{GeV}$



$\sqrt{s} = 27\mathrm{GeV}$

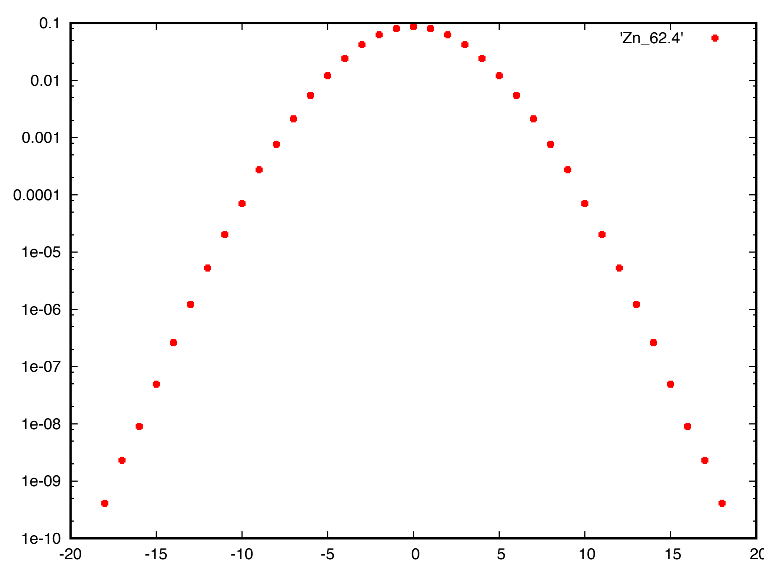


$\sqrt{s} = 39\mathrm{GeV}$

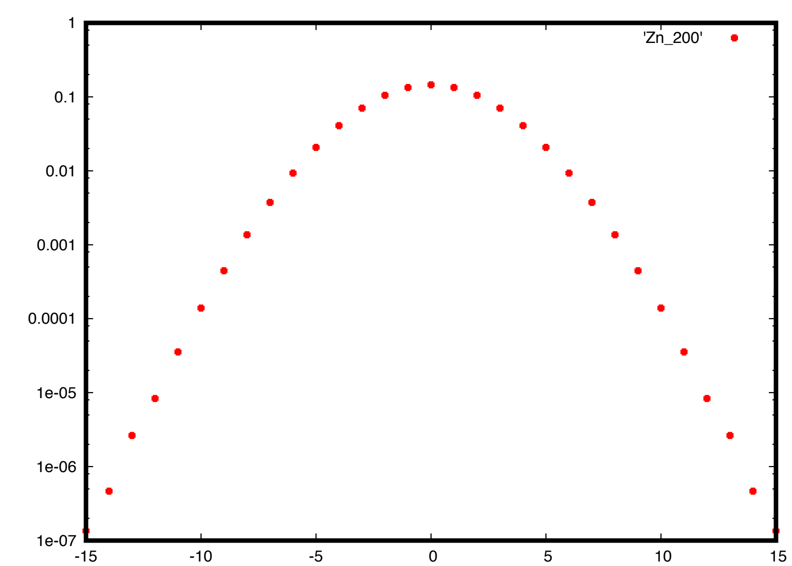


Can I see Difference?

$\sqrt{s} = 62.4\mathrm{GeV}$



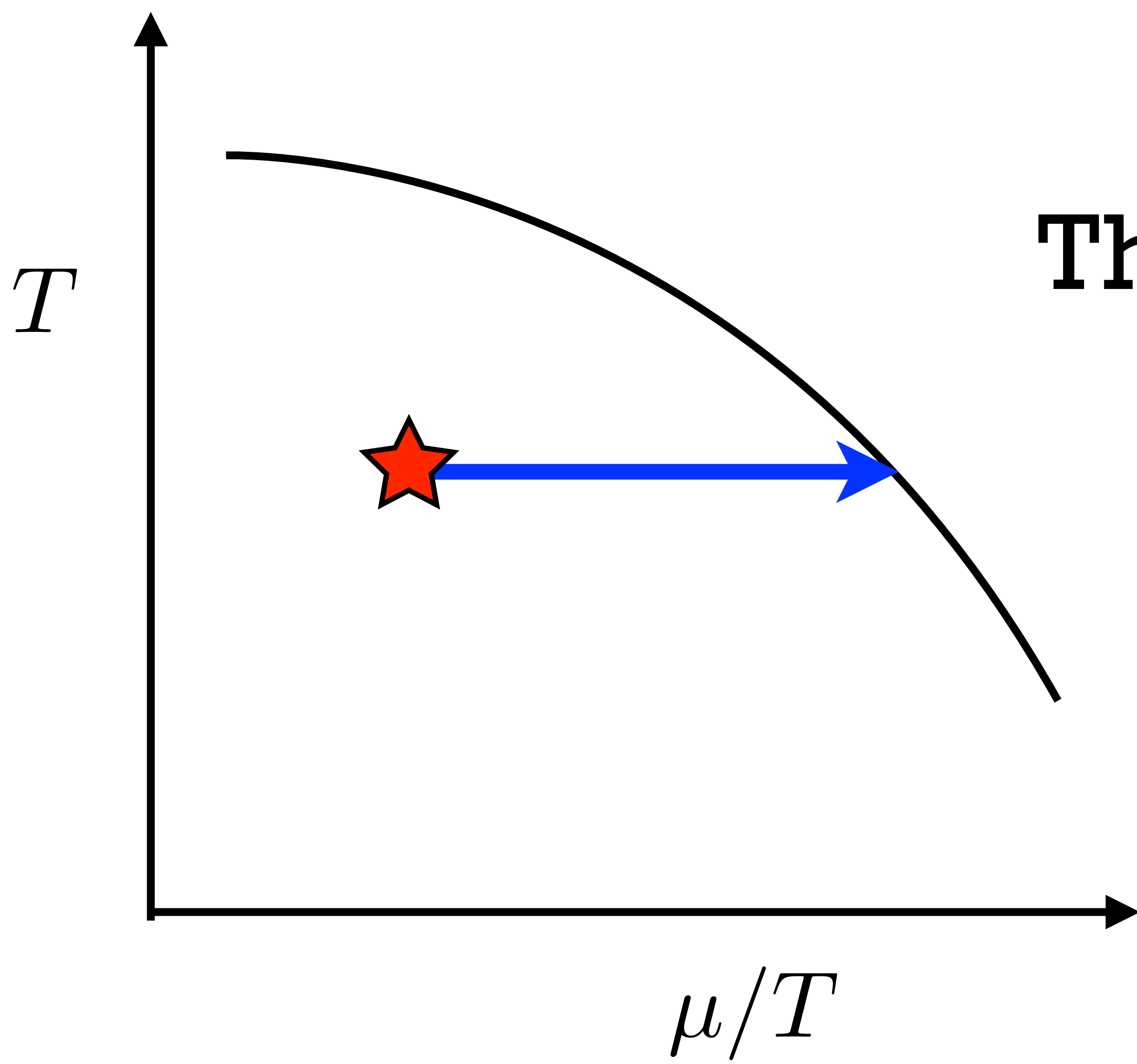
$\sqrt{s} = 200\mathrm{GeV}$



Yes, You Can !
We will see it.

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

Now we have Zn of RHIC data
(sqrt(s)= 10.5,19.6, 27, 39, 62.4, 200 GeV)

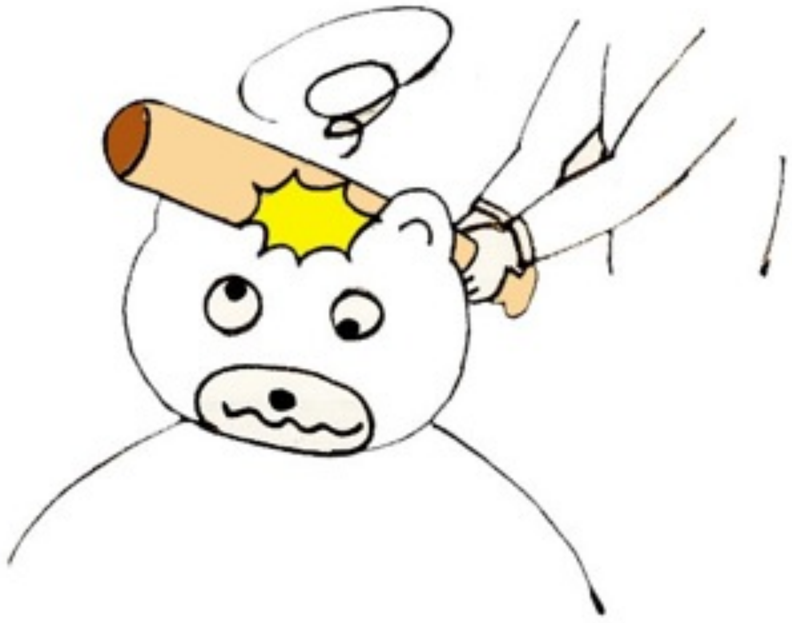


Wao ! We can calculate
at any density !
This includes all QCD Phase
information !

$(\xi \equiv e^{\mu/T})$

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Do not forget that your *n* is finite !

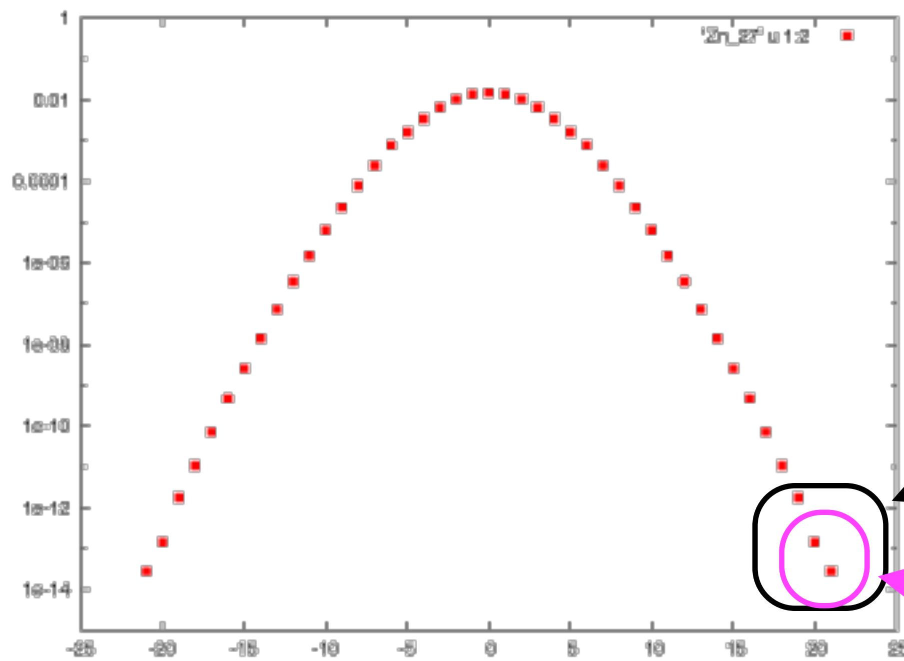


Moments λ_k

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

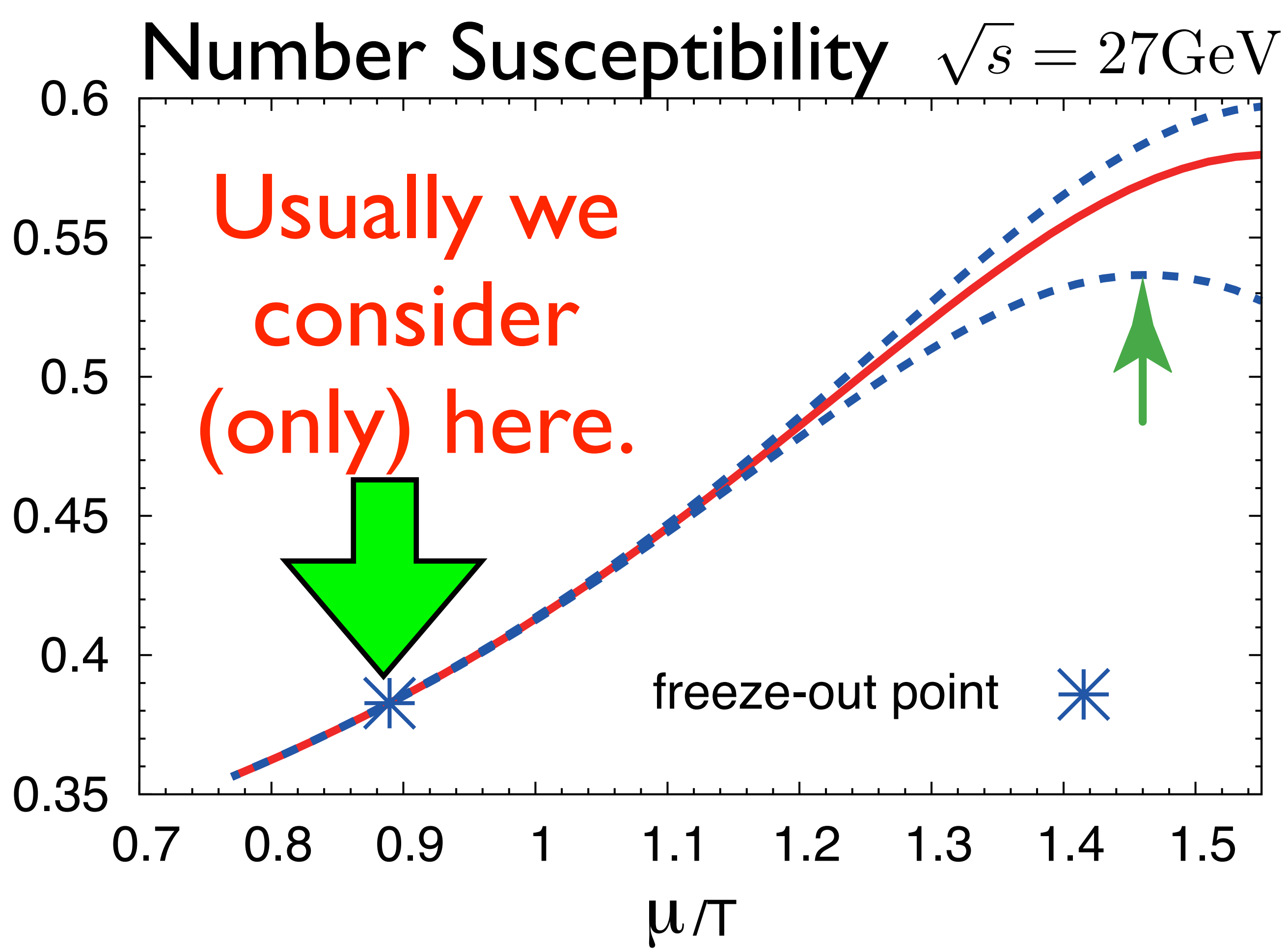
$$\lambda_k \equiv \left(T \frac{\partial}{\partial \mu} \right)^k \log Z$$

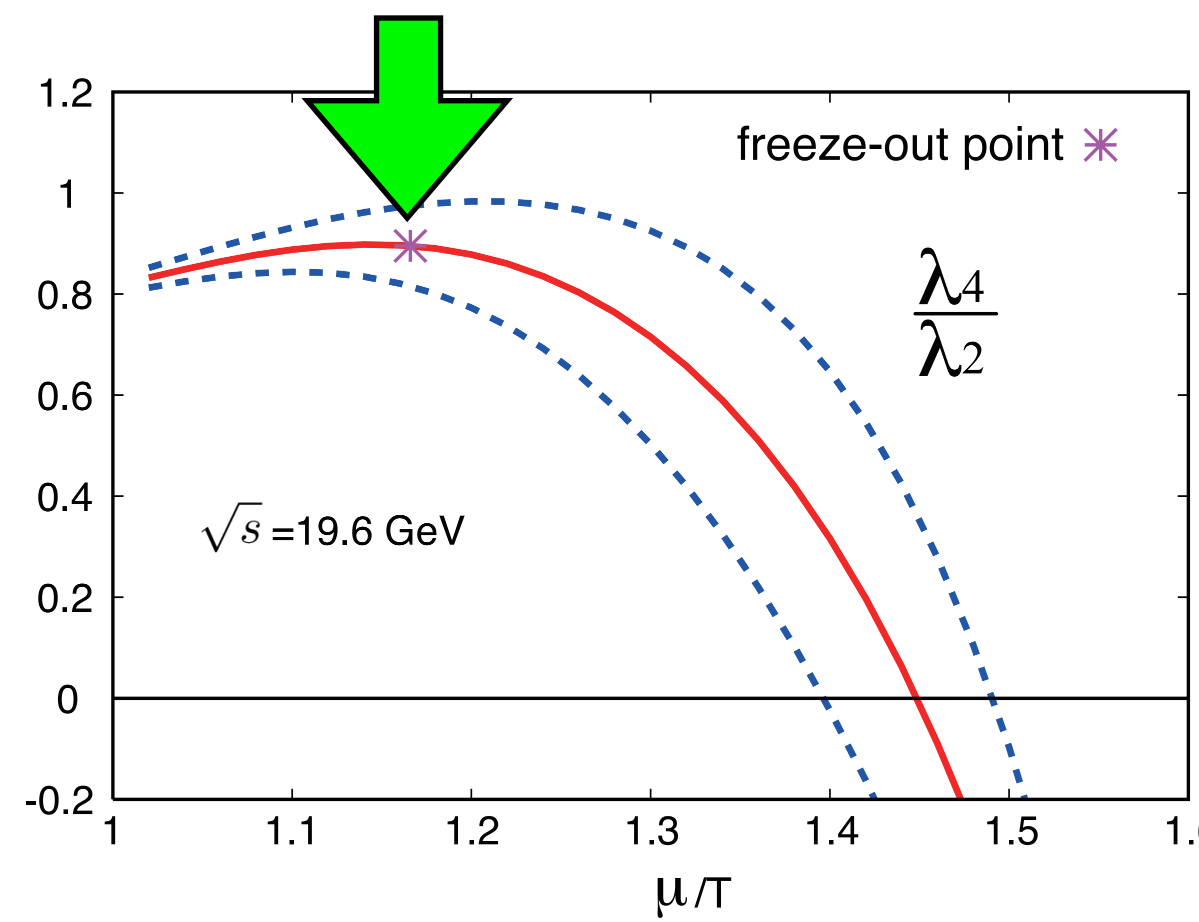
What happens ?



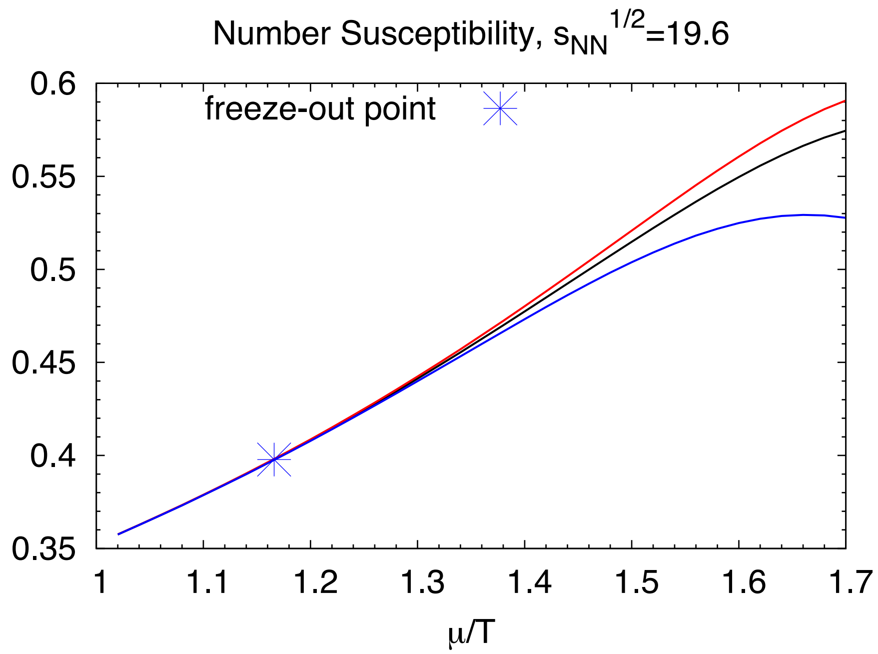
if we increase these points 15%

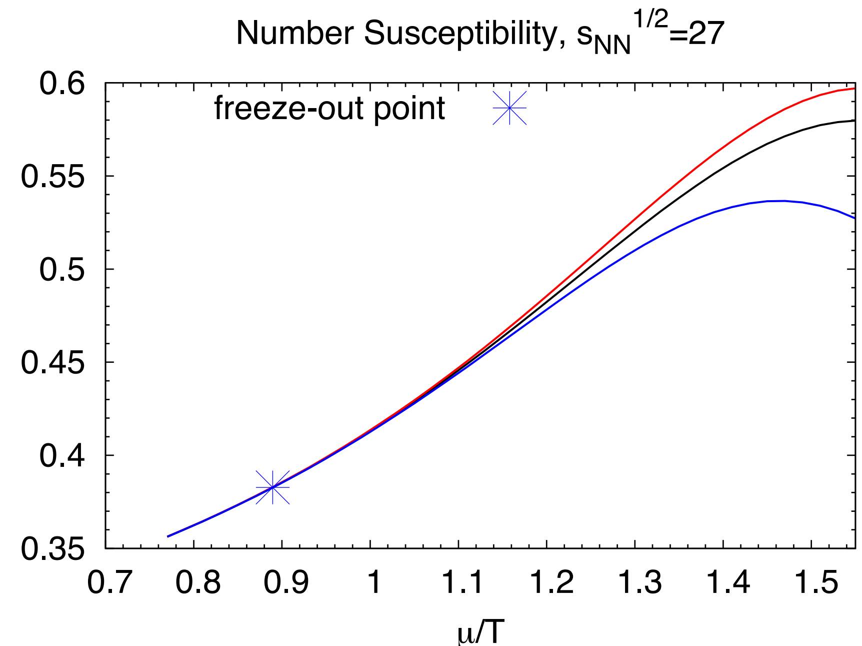
if we drop these points

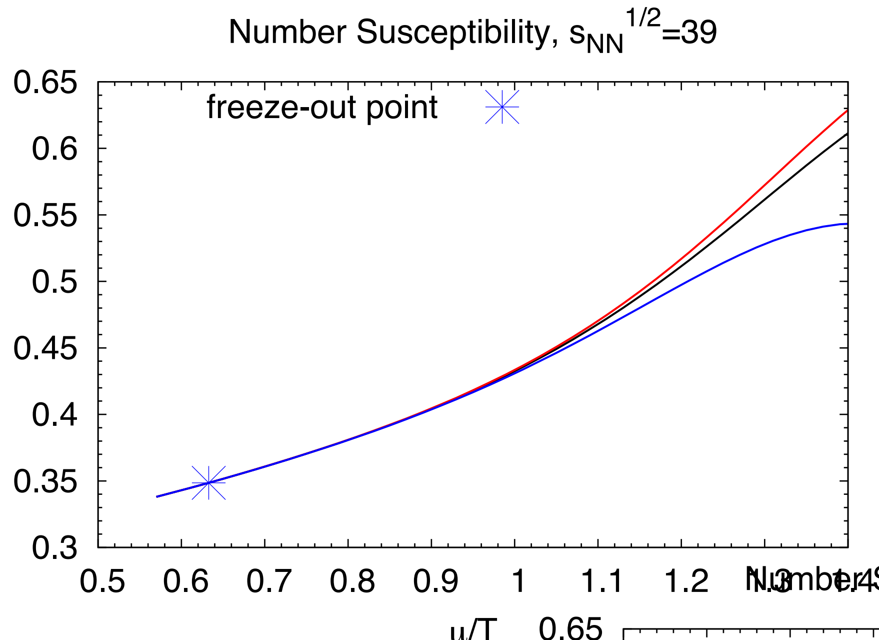


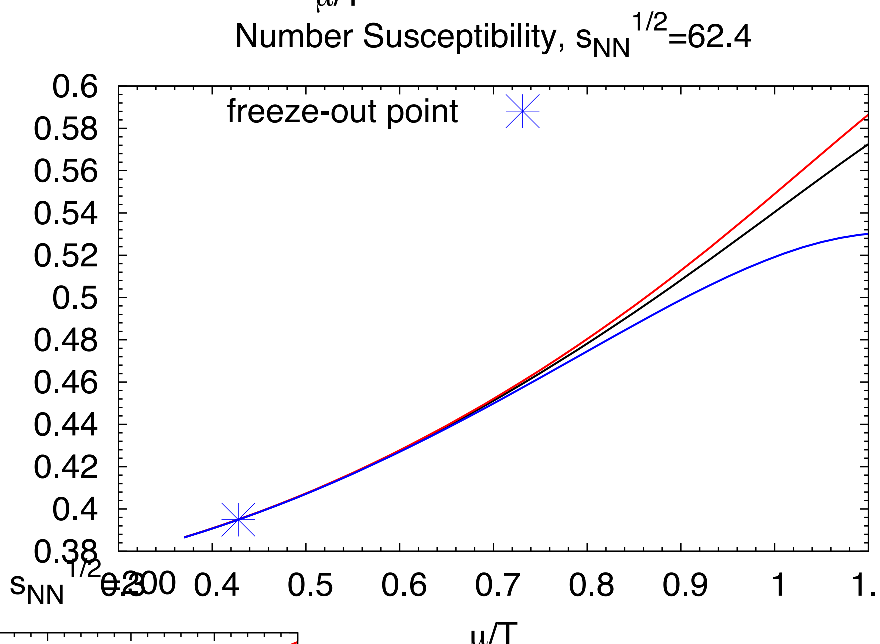


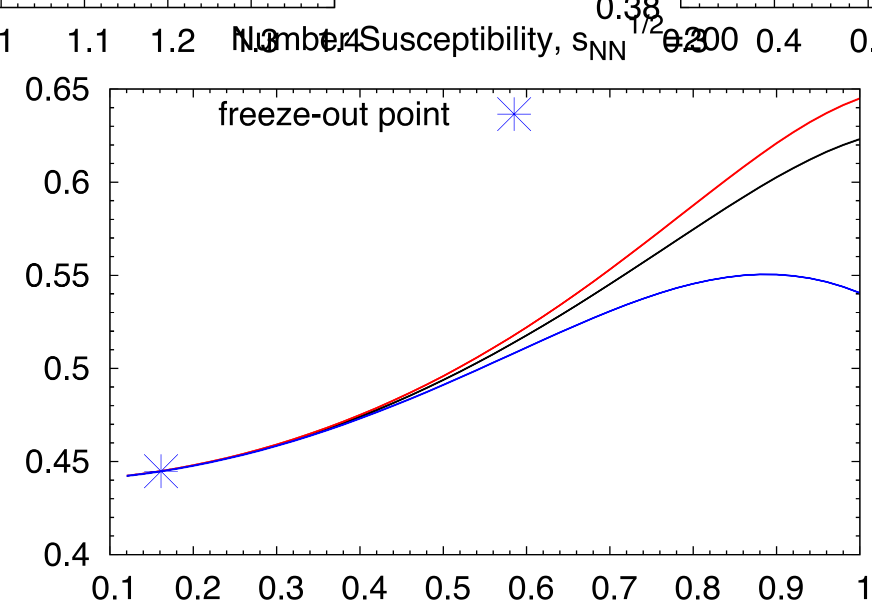
Susceptivility



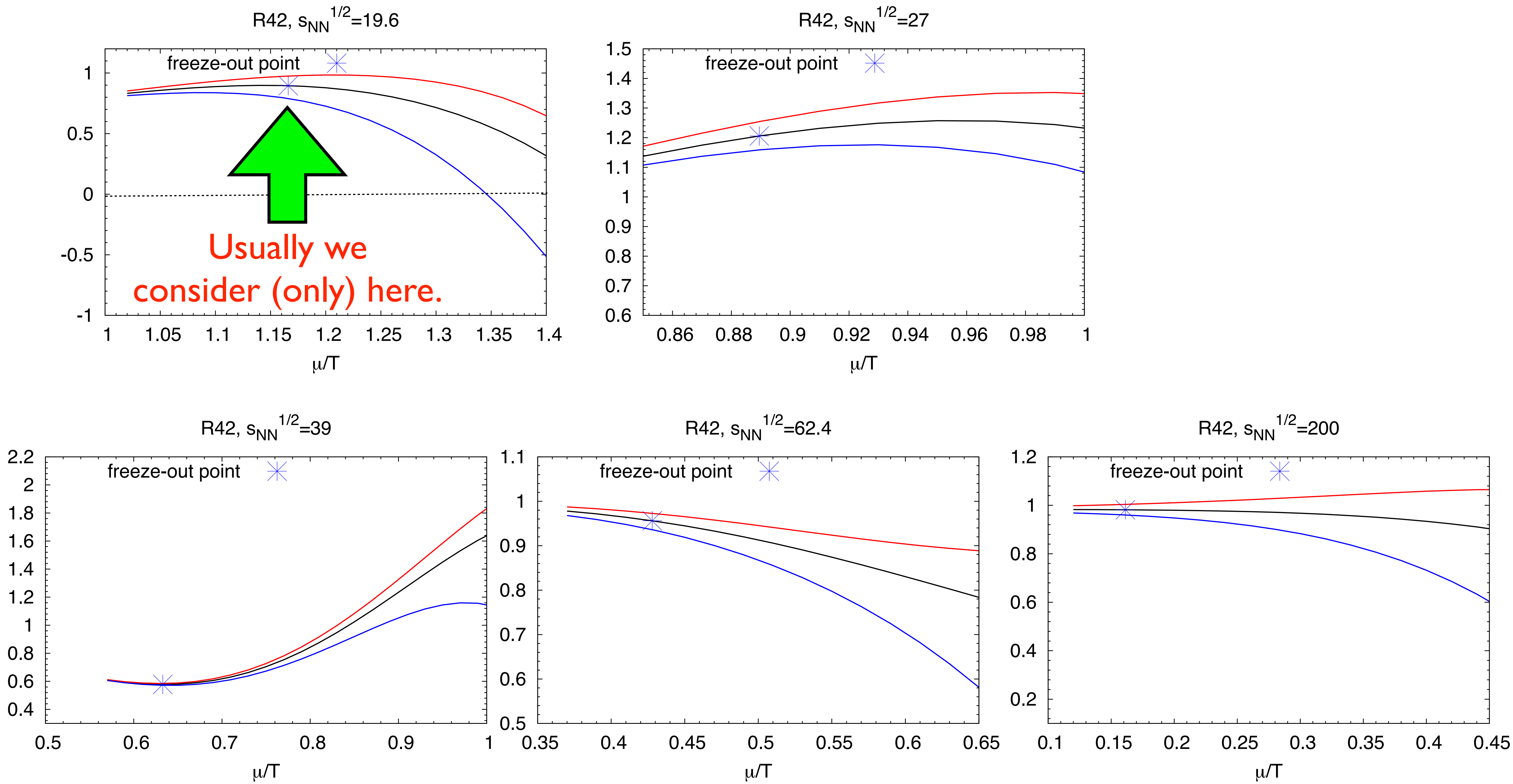








Kurtosis


R42, sNN1/2=19.6
freeze-out point
Usually we consider (only) here.
μ/T
R42, sNN1/2=27
freeze-out point
μ/T
R42, sNN1/2=39
freeze-out point
μ/T
R42, sNN1/2=62.4
freeze-out point
μ/T
R42, sNN1/2=200
freeze-out point
μ/T

We can calculate Z_n also by Lattice QCD

But Sign Problem on Lattice ?

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields}) \times \det D(\mu)\, e^{-(\text{Gluon Action})}$$

Complex if μ is real.

$$[\det D(\mu)]^* = \det D(-\mu^*)$$

For Pure Imaginary μ ➡ $\det D$ **real**

A.Hasenfratz and Toussant, 1992

$$Z_n = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

All information is in Imaginary Chemical Potential regions!

Great Idea ! But practically it did not work.

For few years,we must develop several Engineering Methds.

1) Multi-Precision Calculations
2) Integration method

Note:

$$Z(\mu, T) = \sum_n z_n(T)\xi^n$$

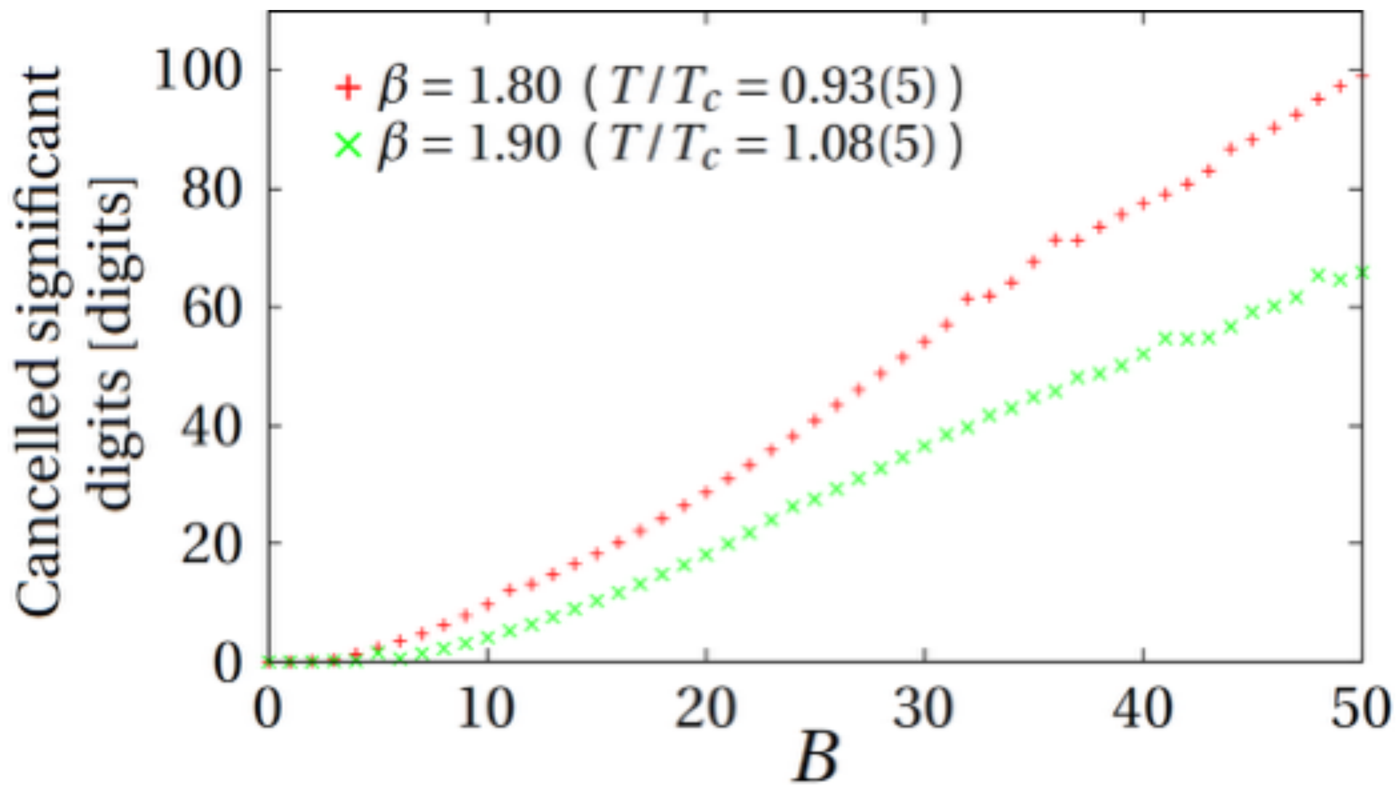
$$\xi \equiv e^{\mu/T}$$

We can extend chemical potential μ to pure imaginary.

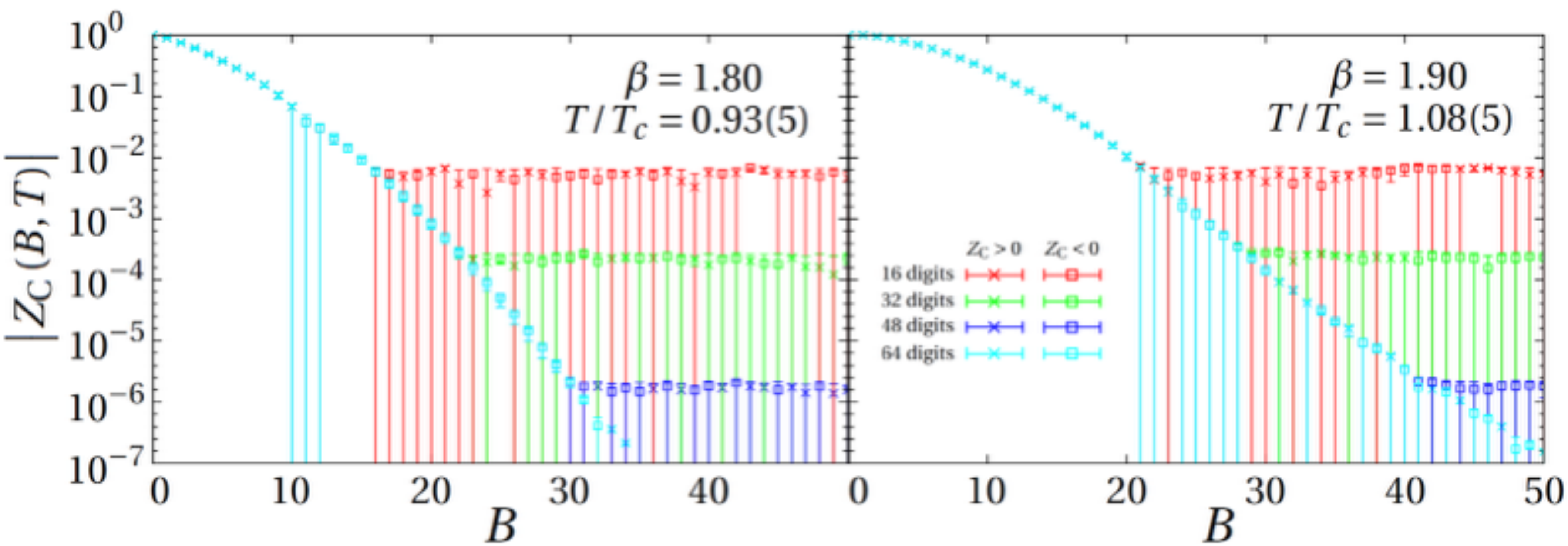
We can consider μ even of Complex number.

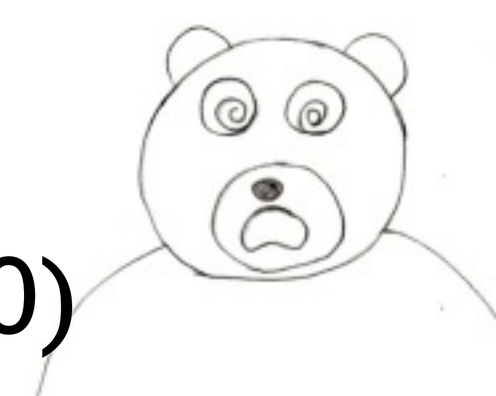
Lee-Yang zeros (If I have time,,,)

Big Cancellation in FFT !



S.Oka, arXiv:1511.04711





θ integration → Multi-Precision (50 - 100)

Integration Method

n_B *in imaginary* μ → z_n

$$n_B = \frac{1}{3V} T \frac{\partial}{\partial \mu} \log Z_G$$

$$= \frac{N_f}{3N_s^3 N_t} \int \mathcal{D}U e^{-S_G} \mathrm{Tr}\Delta^{-1} \frac{\partial \Delta}{\partial \mu} \det \Delta$$

(For pure imaginary μ, n_B is also imaginary)

Then, for fixed *T*

$$Z(\theta \equiv \frac{\mu}{T}) = \exp(V \int_0^\theta n_B d\theta')$$

$$Z_k = \frac{3}{2\pi} \int_{-\pi/3}^{+\pi/3} d\theta \exp\left(i\,k\theta + \int_0^\theta n_B d\theta' \right)$$

Advantage of Integration Method over the Original Hasenfratz-Toussant

- In Hasenfratz-Toussant

$$Z_n = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

not so easy to calculate Grand Canonical Partition function, Z_GC.
We used Hopping Parameter Expansion, i.e.,
we can not go to low quark masses.

- The phase of Z_n obtained in this way fluctuates.
(Hidden sign problem shown later)

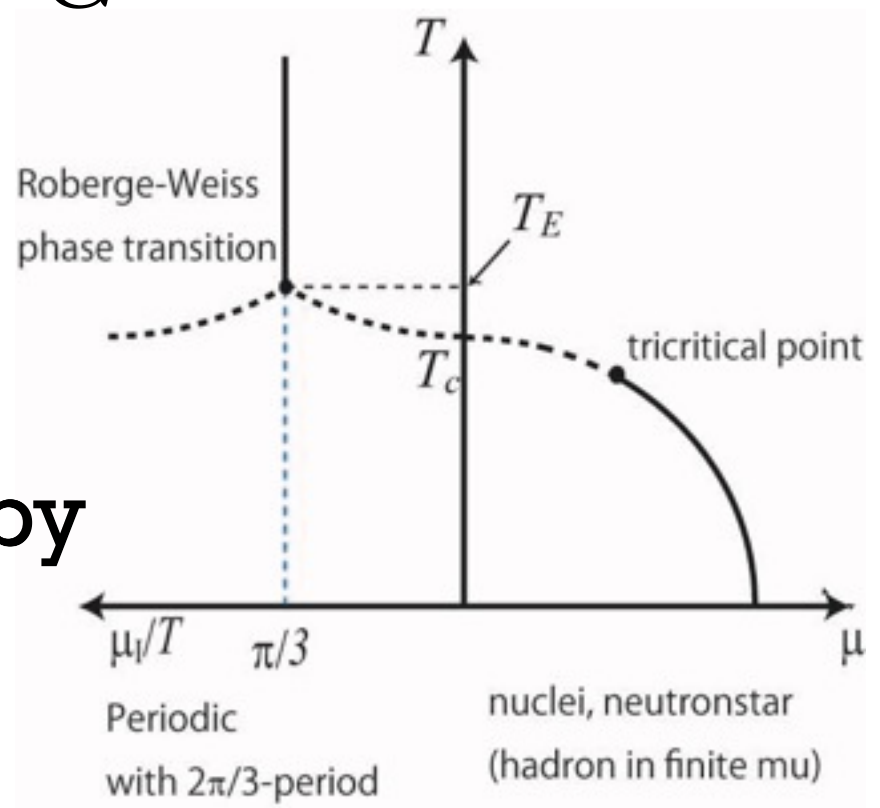
- In the integration method, we calculate the number density in whole pure imaginary chemical potential, which is under control. (although we must fight against large error bars in the confinement regions.)

We map Information in Pure Imaginary Chemical Potential to Real ones.

See also, D'Elia, Gagliardi and Sanfilippo, Phys. Rev. D 95, 094503 (2017)

- We measure the number density at many pure imaginary chemical potential $n_B(\mu_I)$.
- We construct Grand Partition Function Z_G, by integrating $n_B(\mu_I)$
- By Fourier transformation, we get z_n
- Then we can calculate Real μ regions by

$$Z(\xi, T) = \sum_n z_n(T)\xi^n$$

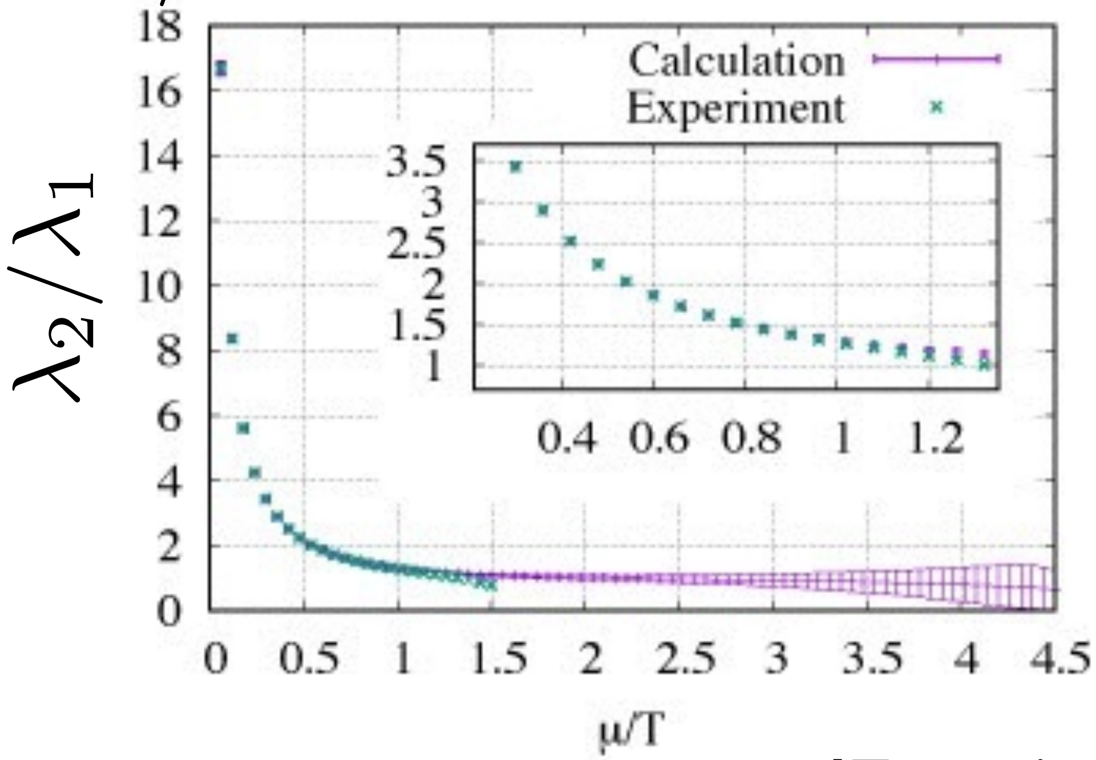


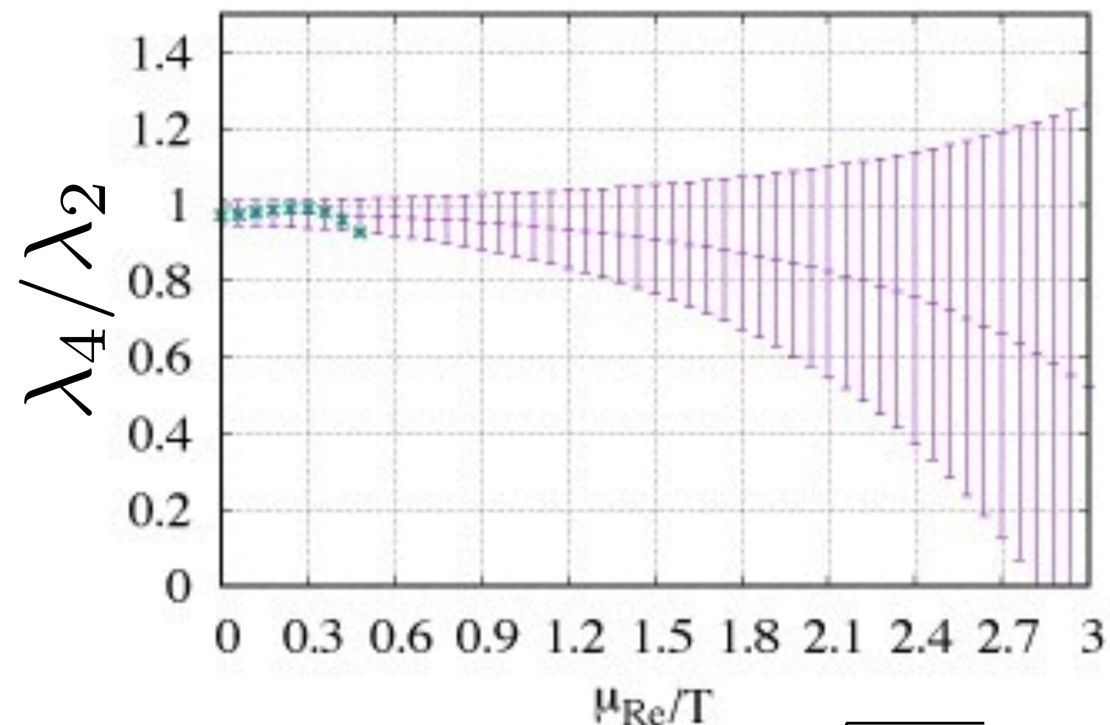
$\xi \equiv e^{\mu/T}$ Fugacity

Moments $\lambda_k = (T\frac{\partial}{\partial\mu})^k \log Z$

D.Boyda

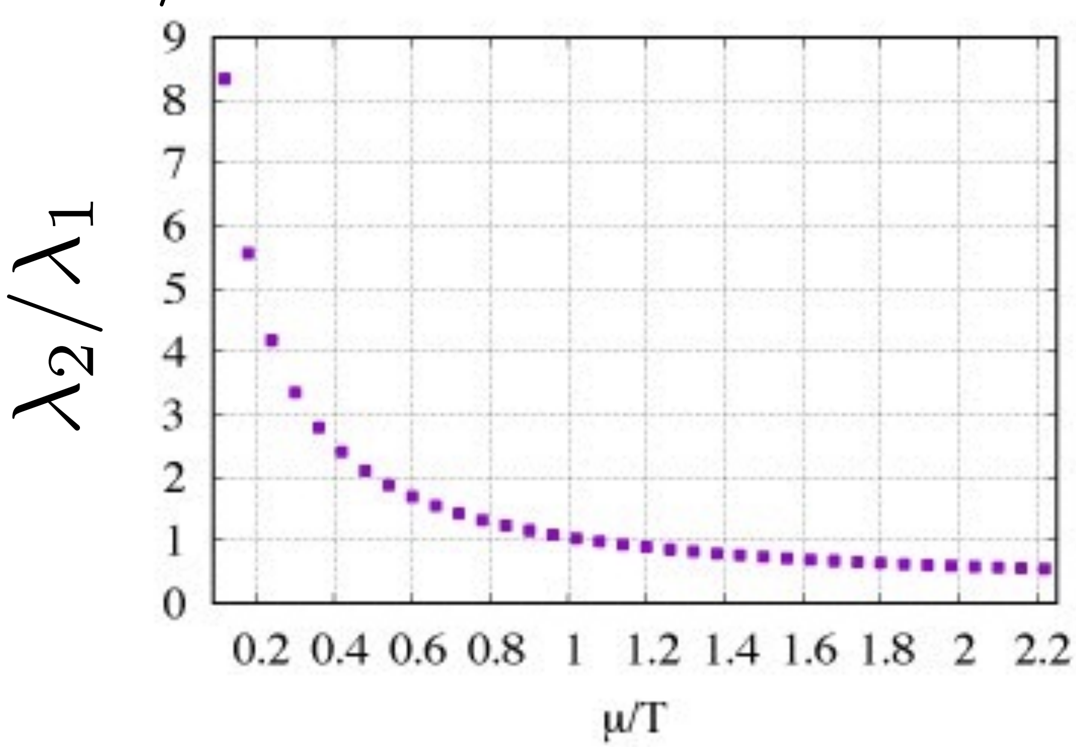
$T/T_c = 0.93$

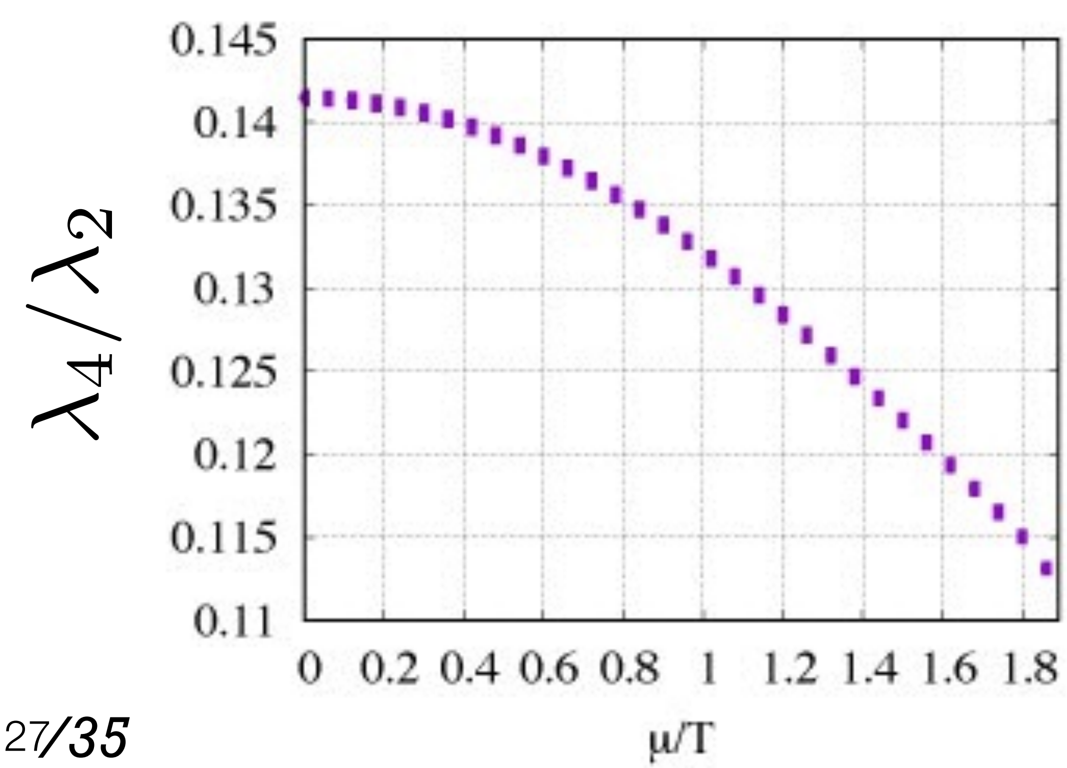




× `Experiment' constructed from RHIC Star $\sqrt{s_{NN}} = 39\,(\mathrm{GeV})$

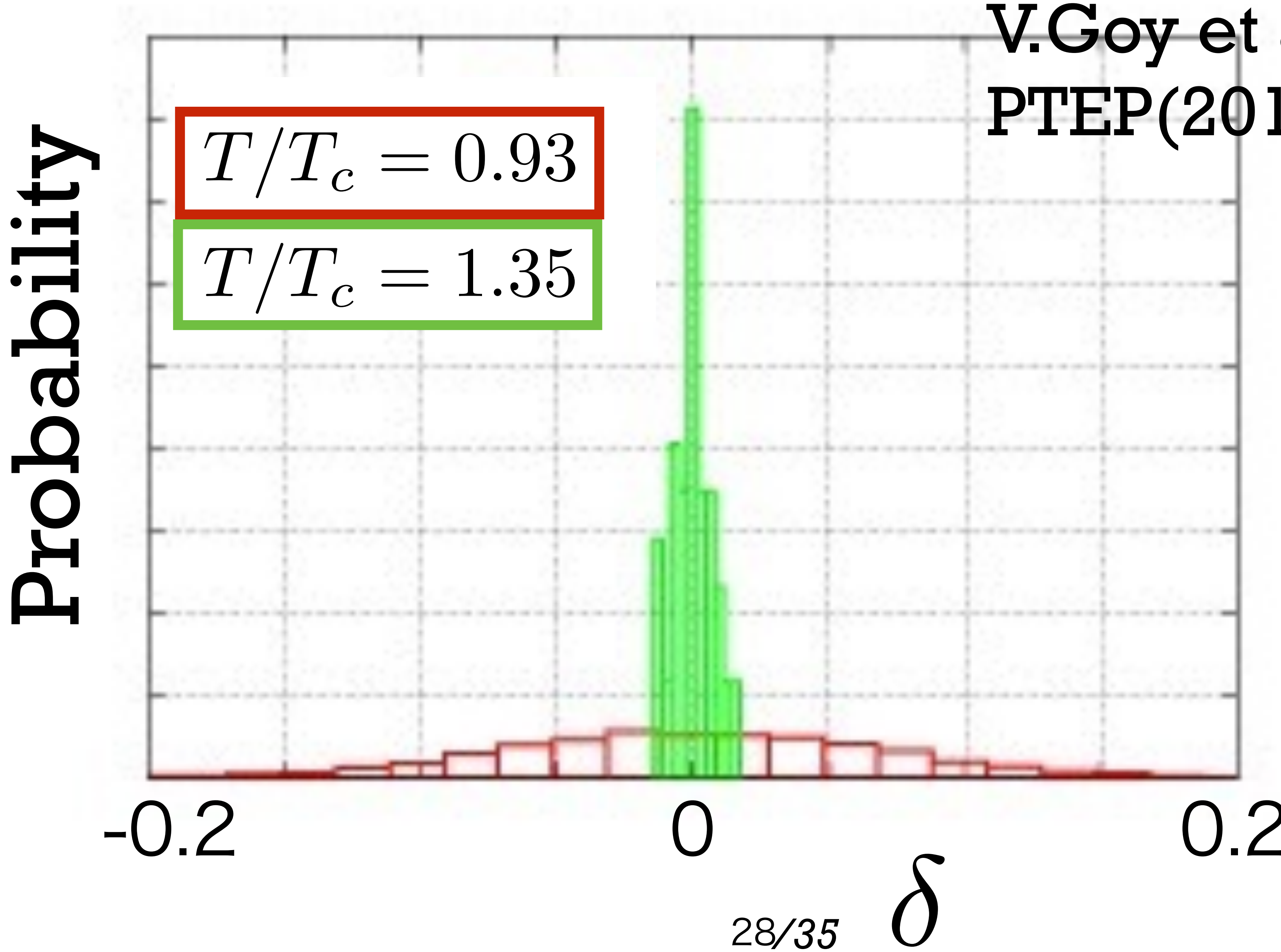
$T/T_c = 1.35$





Hidden Sign Problem ?

z_n by Hasenfratz-Toussant have phase on each configuration !



V.Goy et al., PTEP(2017) 031D01

$z_n \simeq |z_n| e^{in\delta}$

$Z_n = \langle z_n \rangle$

are real positive.

Experimental data and Lattice QCD

We introduce a Model for Lattice QCD

Number density in Imaginary μ

$$n_B/T^3 = \sum_{k=1}^{k_{max}} f_{3k} \sin(k\theta)$$

Confinement phase

$T < T_c$

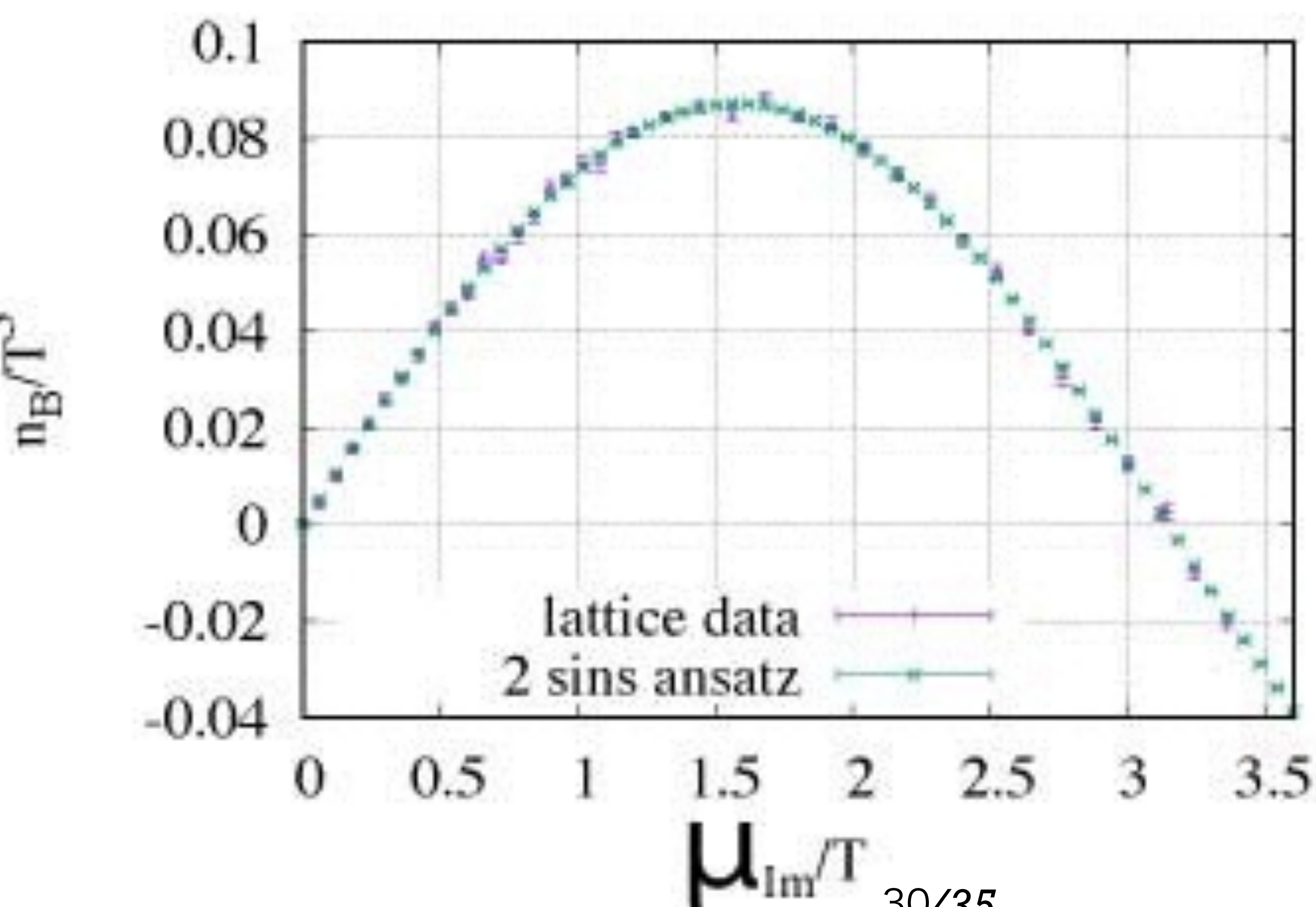
$$n_B/T^3 = \sum_{k=1}^{k_{max}} a_{2k-1} \theta^{2k-1}$$

DeConfinement phase

$T > T_c$

$$\theta \equiv \frac{\mu}{T}$$

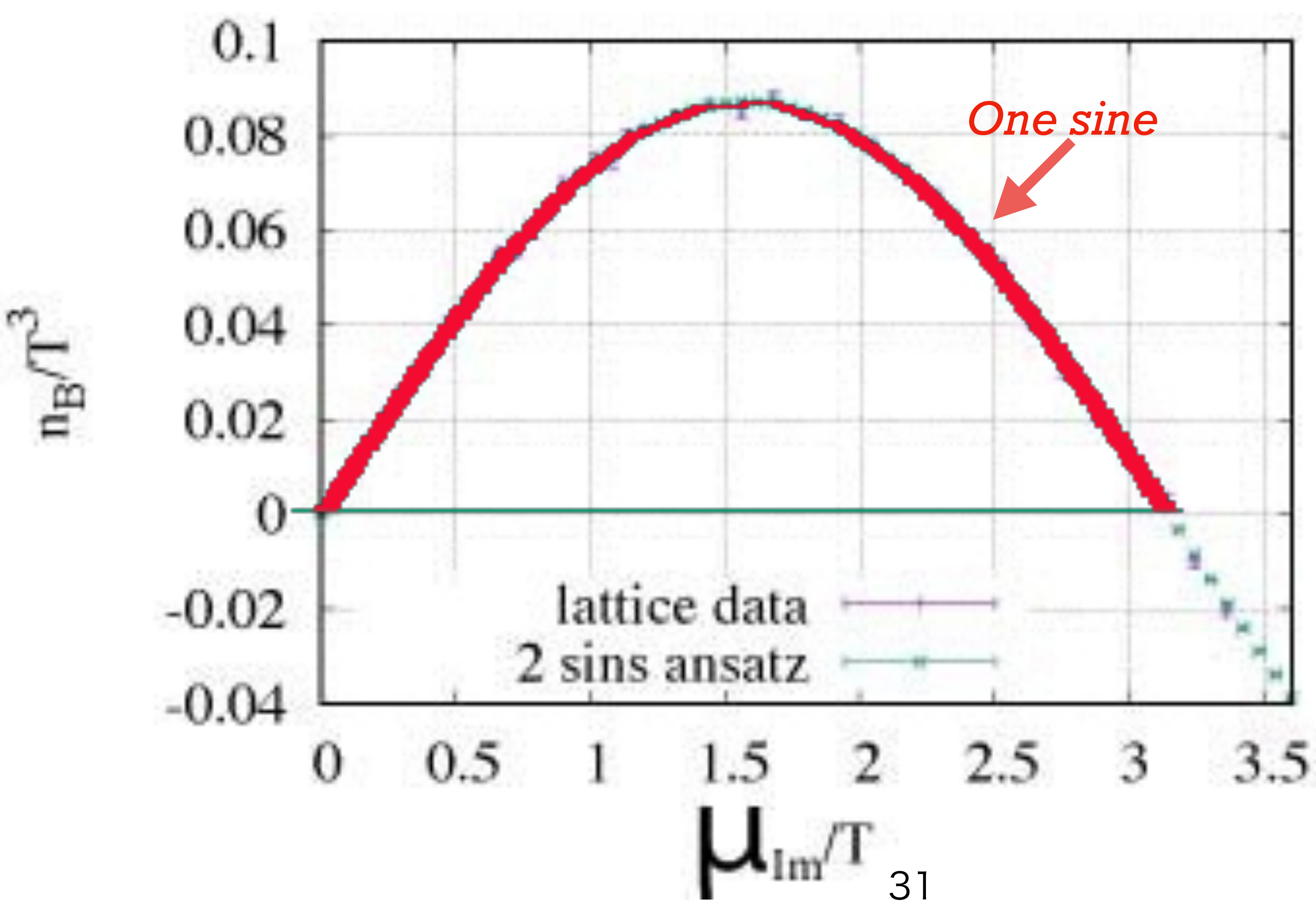
A Remark of Function Form of $n_B(\mu_I)$



$n_B(\mu_I)$ is well approx-imated by sine function at $T<Tc$.

Takahashi et al. Phy. Rev. D 91 (1) (2015) 014501.
Bornyakov et al., Phys.Rev. D95, 094506 (2017)

A Remark of Function Form of $n_B(\mu_I)$



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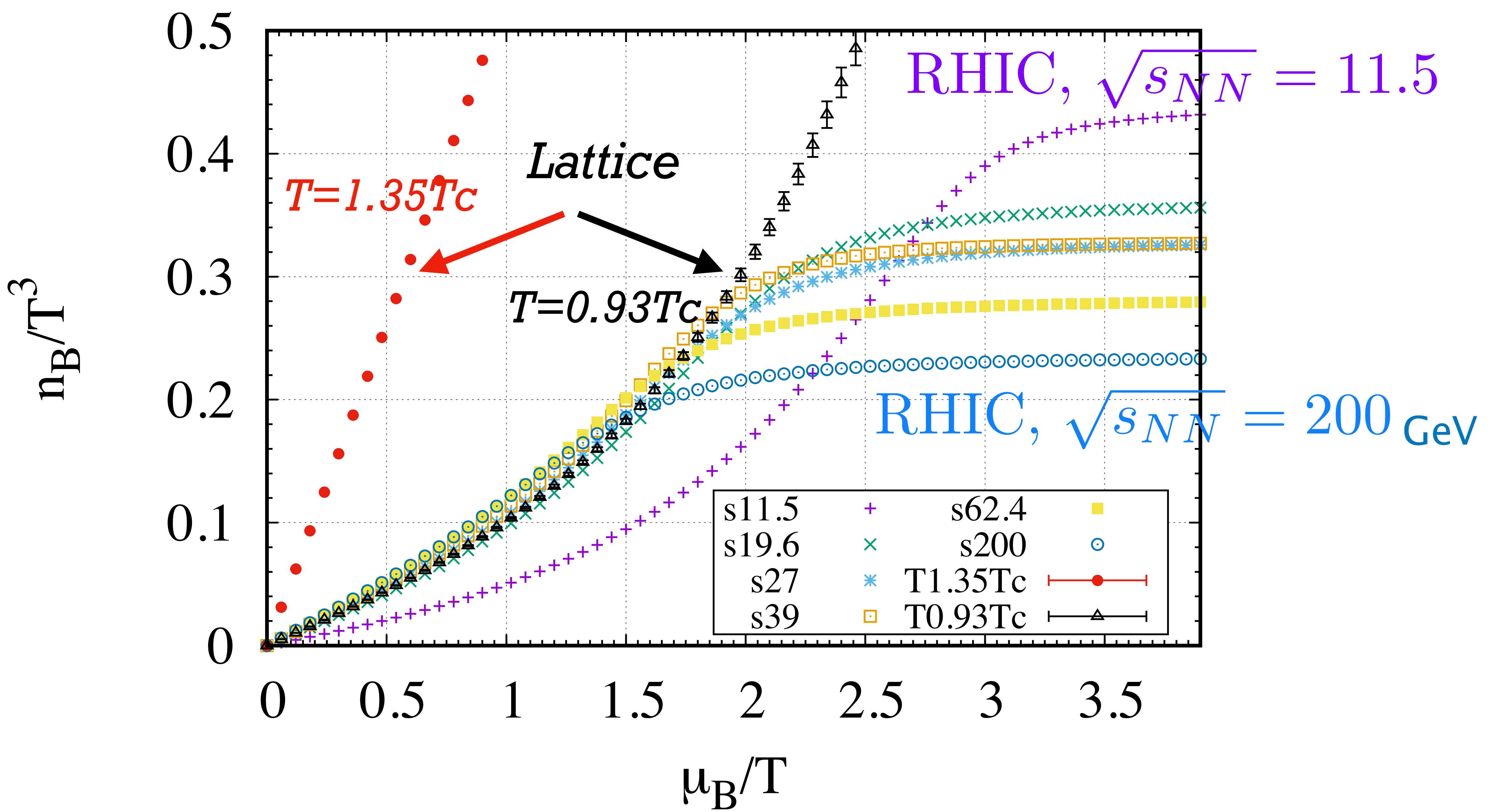
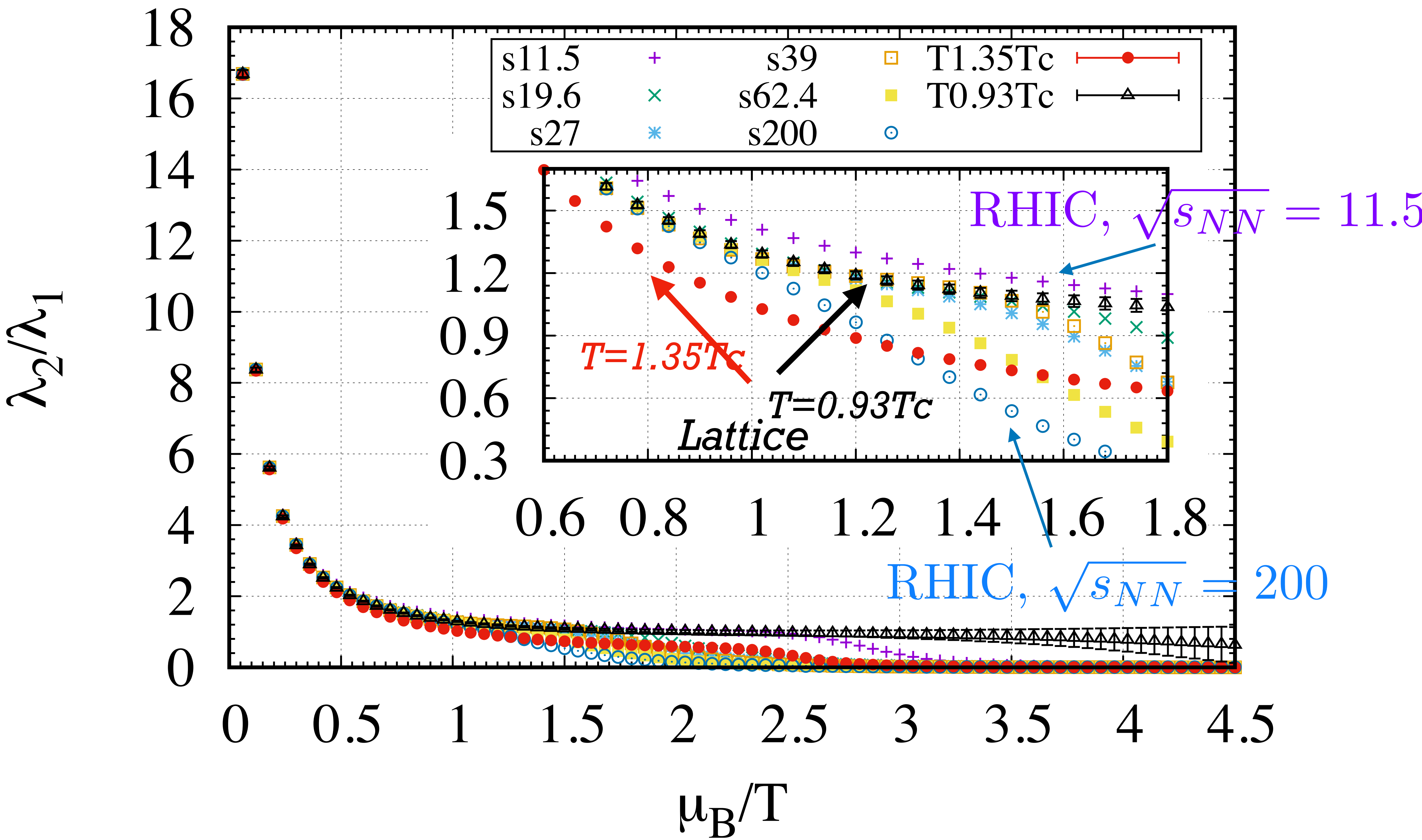
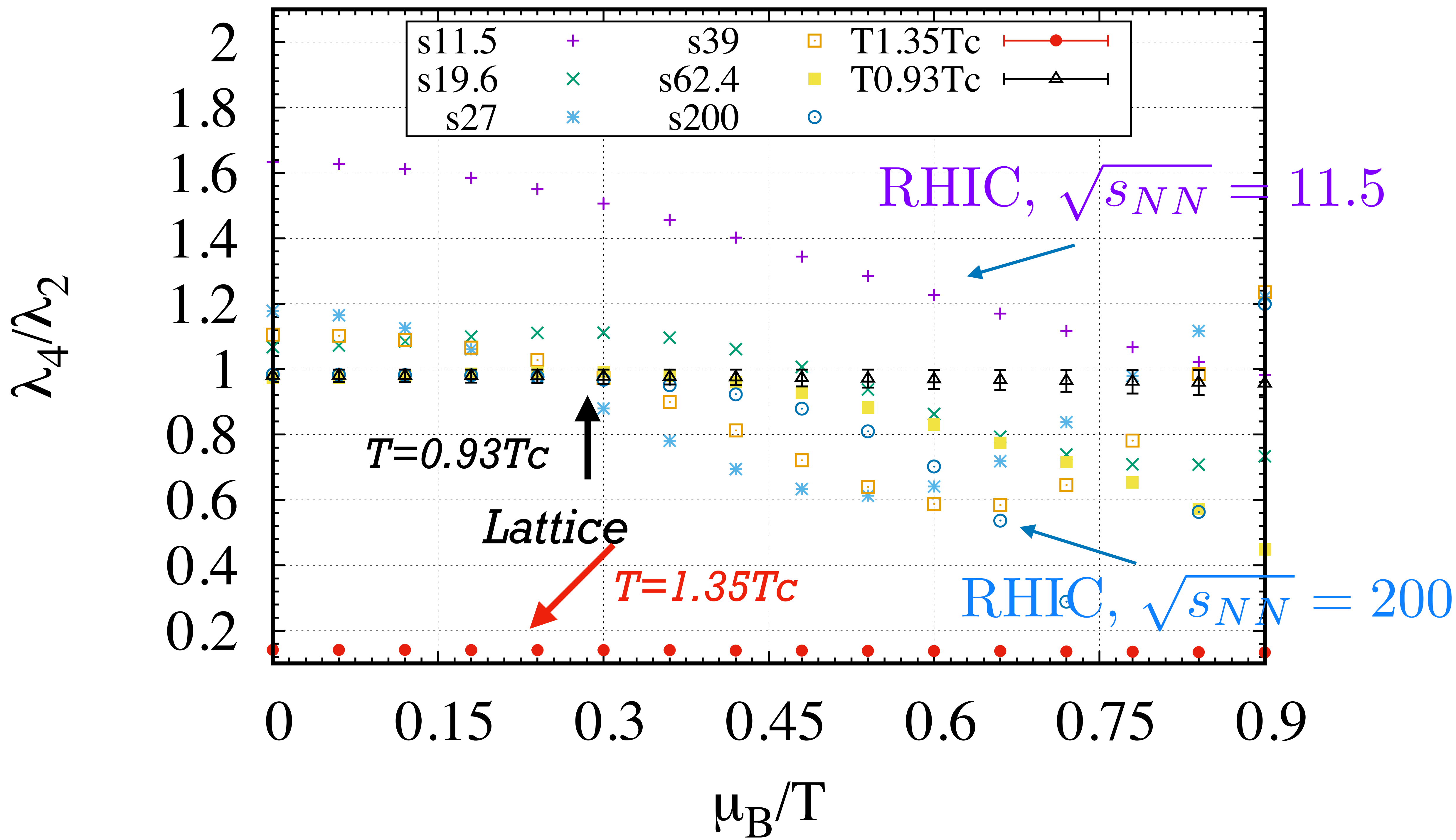
Number Density




s11.5
s19.6
s27
s39
s62.4
s200
T1.35Tc
T0.93Tc
RHIC, $\sqrt{s_{NN}} = 11.5$
RHIC, $\sqrt{s_{NN}} = 200$
T=1.35Tc
T=0.93Tc
Lattice
λ_2/λ_1
μ_B/T


s11.5
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s27
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T1.35Tc
T0.93Tc
RHIC, $\sqrt{s_{NN}} = 11.5$
RHIC, $\sqrt{s_{NN}} = 200$
T=0.93Tc
Lattice
T=1.35Tc
λ_4/λ_2
μ_B/T
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.15
0.3
0.45
0.6
0.75
0.9

Summary and Whats next

- J-PARC energy regions are very promising for finite density QCD Study. To get z_n for large n, high intensity is essential.
- Using the canonical approach, we may see the phase transition line.
- We report the baryon case, but the charge and the strangeness fluctuations are also interesting.
- For solving the sign problem, the canonical approach with the integration method seems to the best.
- Quark mass in the present lattice QCD calculation is very heavy, and we must go to more realistic quark masses.
- We plan to (should) prepare 2+1 (u, d, s) lattice QCD code.

Backup Slides

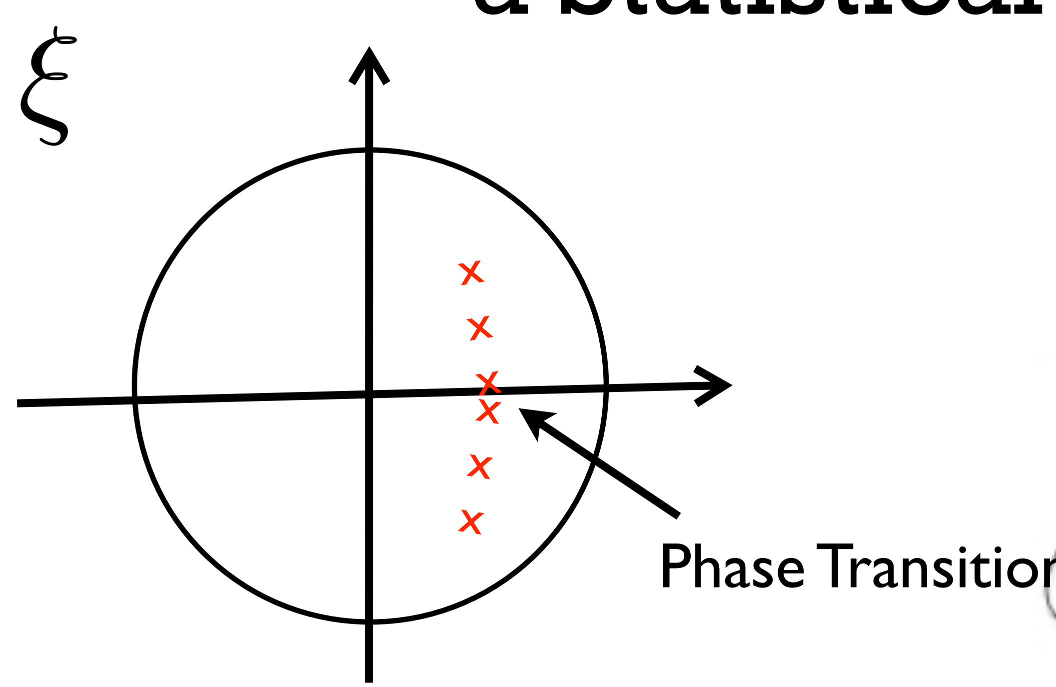
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

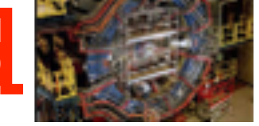
Lee-Yang Zeros (1952)

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System

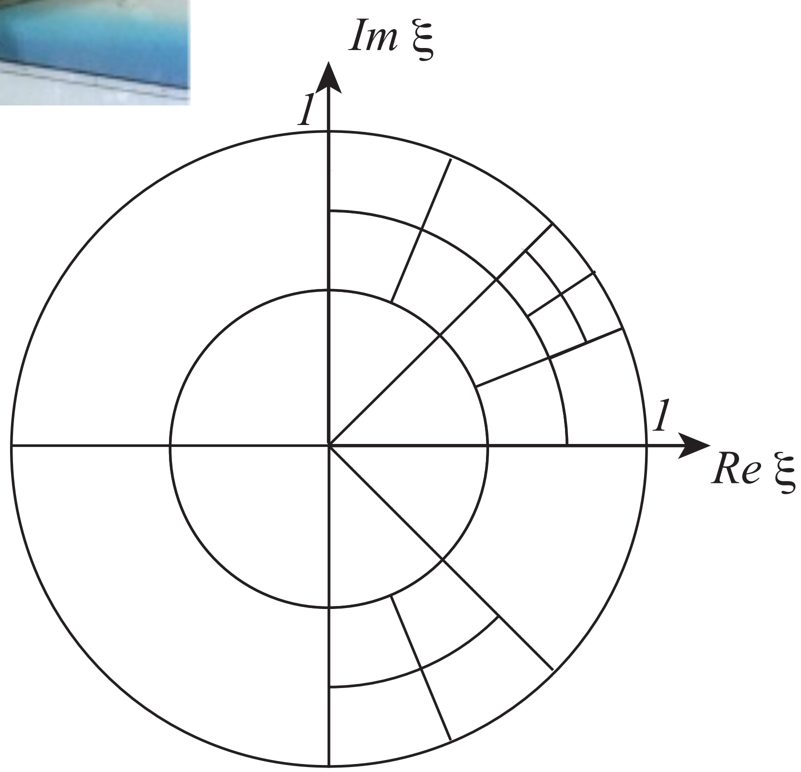


 **and** 

cut Baum-Kuchen (cBK) Algorithm

$$f(\xi) = \prod_k (\xi - \alpha_k)$$

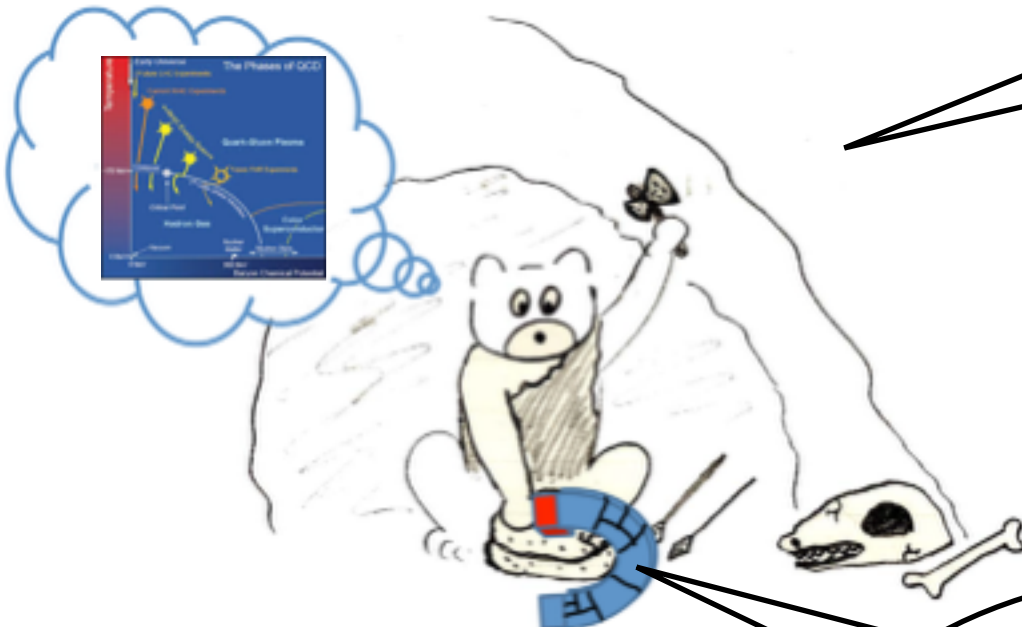
$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$



$$\frac{1}{2\pi i}\oint_C \frac{f'}{f} d\xi = \left(\begin{array}{c}\text{Number of}\\ \text{Zeros in}\\ \text{Contour } C\end{array}\right)$$

50 - 100 number of significant digits

A Coutour is cut into four pieces if there are zeros inside.



Is this my Original ?

I donot think so.

Let us wait until someone claims.

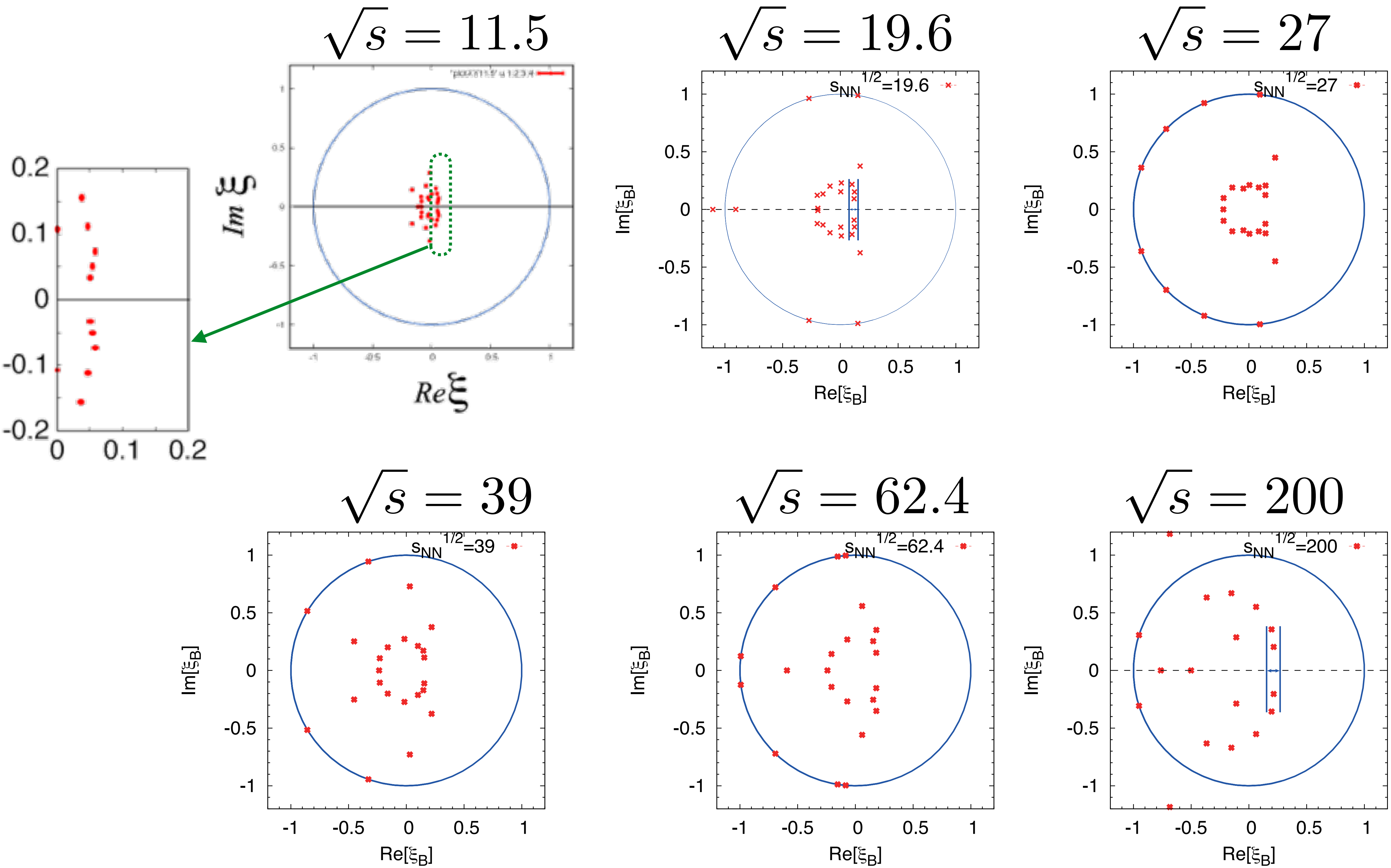
It’s ME

Riemann

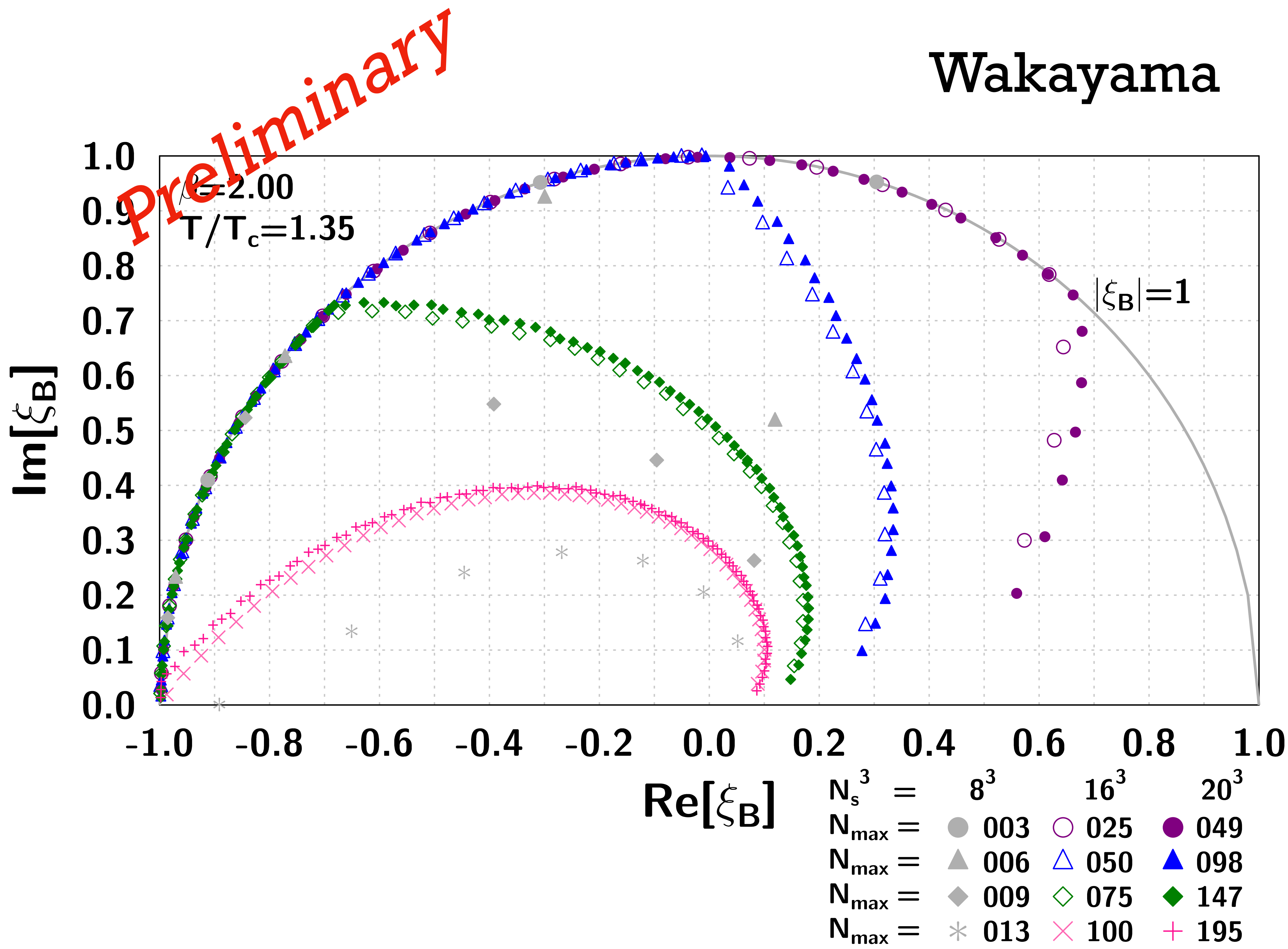
MPI parallel version will be ready soon, which is available upon request.

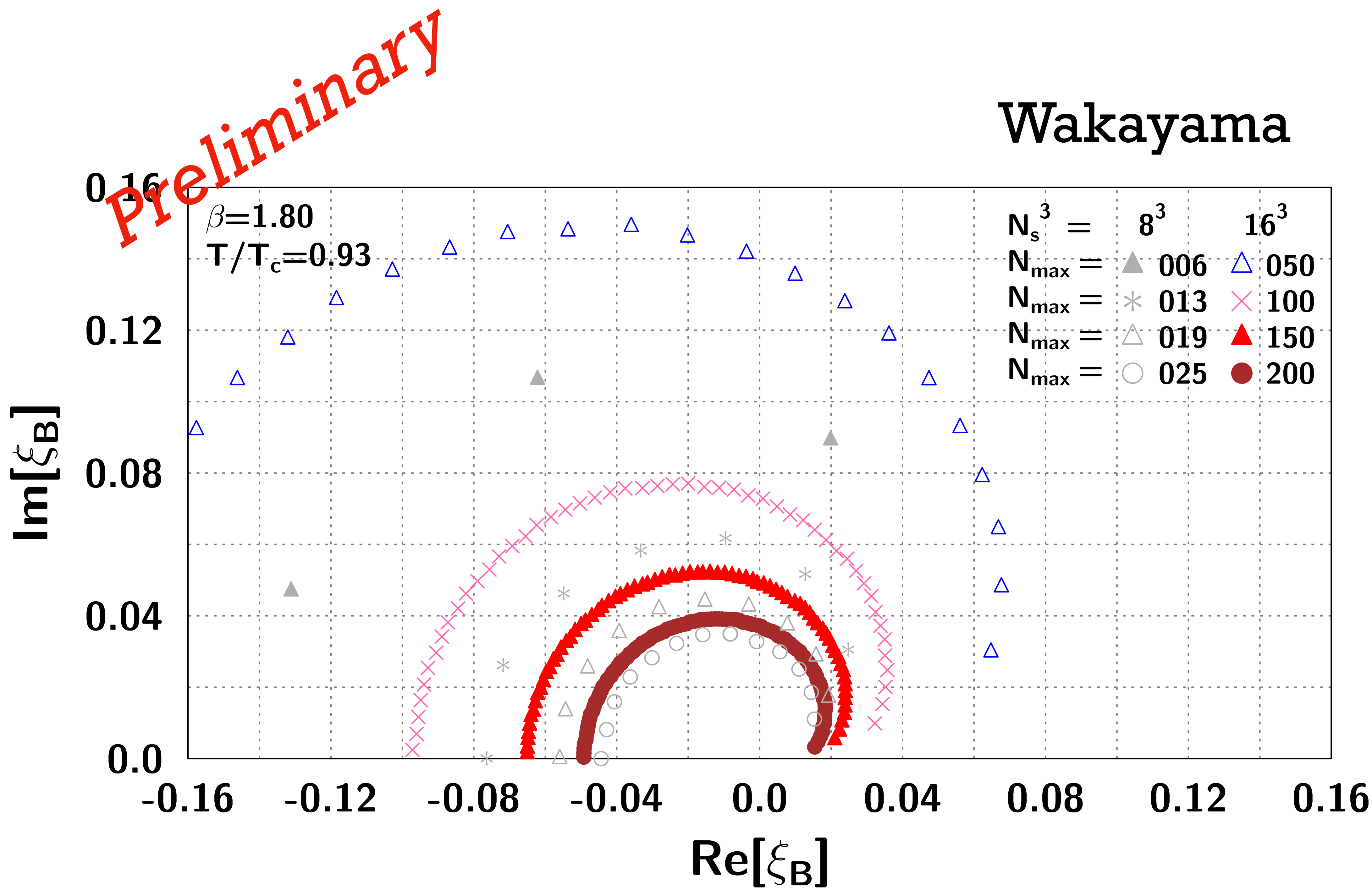
Lee-Yang Zeros Experimental Data (RHIC)

Lee-Yang Zeros: RHIC Experiments



Wakayama


Preliminary
β=2.00
T/Tc=1.35
|ξB|=1
Im[ξB]
Re[ξB]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
Ns^3 = 8^3 16^3 20^3
Nmax = 003 025 049
Nmax = 006 050 098
Nmax = 009 075 147
Nmax = 013 100 195



Preliminary
Wakayama
β=1.80
T/T$_c$=0.93
N_s^3 = 8^3 16^3
N_{max} = 006 050
N_{max} = 013 100
N_{max} = 019 150
N_{max} = 025 200
Im[ξ_B]
Re[ξ_B]
0.16
0.12
0.08
0.04
0.0
-0.16
-0.12
-0.08
-0.04
0.0
0.04
0.08
0.12
0.16