Multiquark configurations

Su Houng Lee

YONSEI UNIVERSITY

1. Few words on multiquark states
2. Where are the compact multiquark states
3. Exotica production from heavy ion collision
4. Nuclear three-body repulsion at short distance
5. Summary

Anowledgements:

To all my former/present collaborators and students

I: Few words on “Multiquark states”

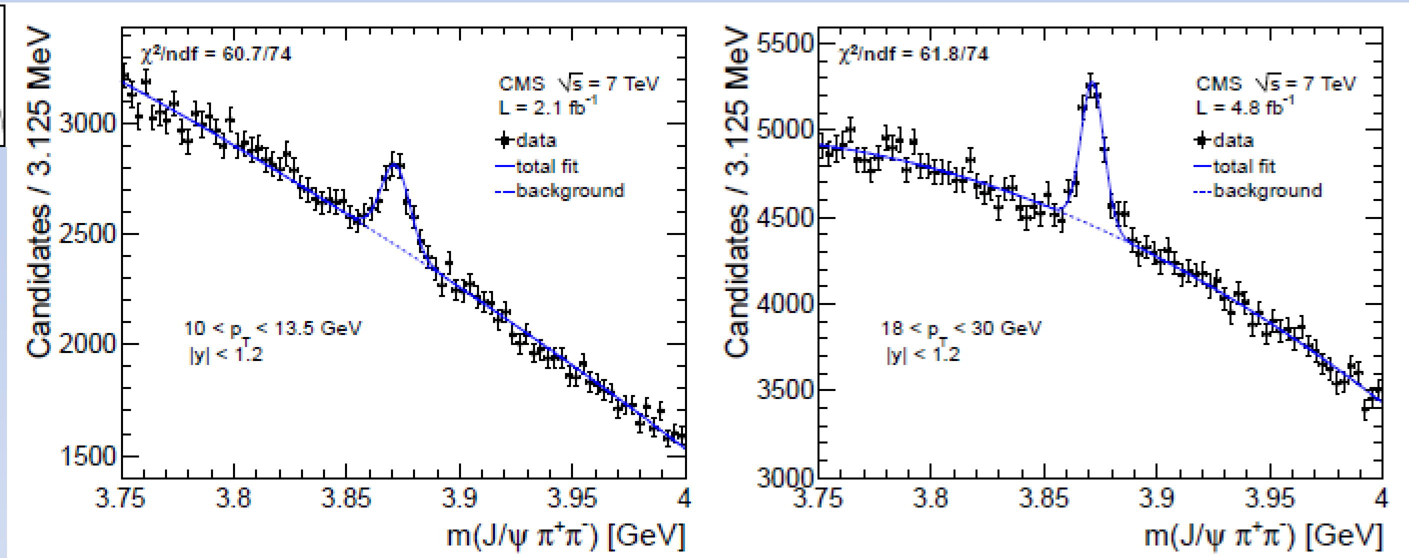
X(3872)

- 2003 -

$$B^{\pm} \rightarrow K^{\pm} \pi^{+} \pi^{-} J/\psi$$

$$M = 3872.0 \pm 0.6 \pm 0.5 \, \mathrm{MeV}$$

- 2013 -



X(3872) $I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 3871.69 \pm 0.17$ MeV

$m_{X(3872)} - m_{J/\psi} = 775 \pm 4$ MeV

$m_{X(3872)} - m_{\psi(2S)}$

Full width $\Gamma < 1.2$ MeV, CL = 90%

Z(4430)

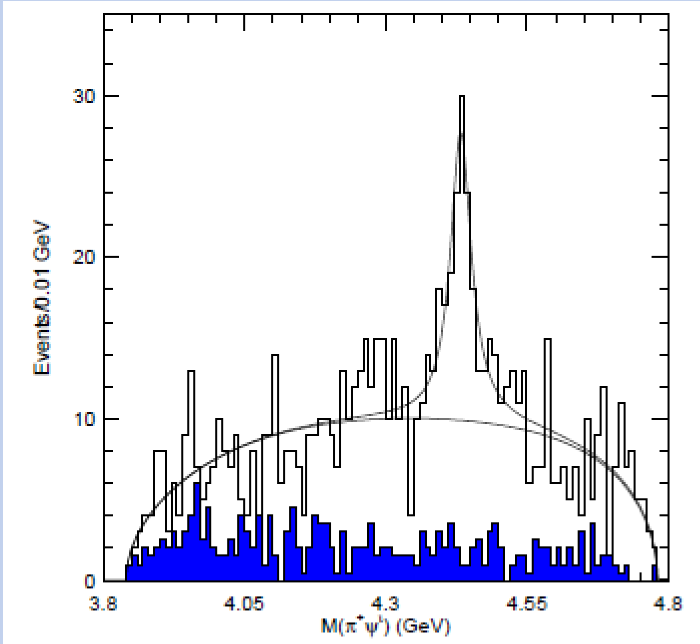
- 2007 -

BELLE

$B \to K\pi^{\pm}\psi'$

$M = 4433 \pm 4 \pm 2\,\mathrm{MeV}$

$\Gamma = 45^{+18}_{-13}(\mathrm{stat})^{+30}_{-13}(\mathrm{syst})\,\mathrm{MeV}$



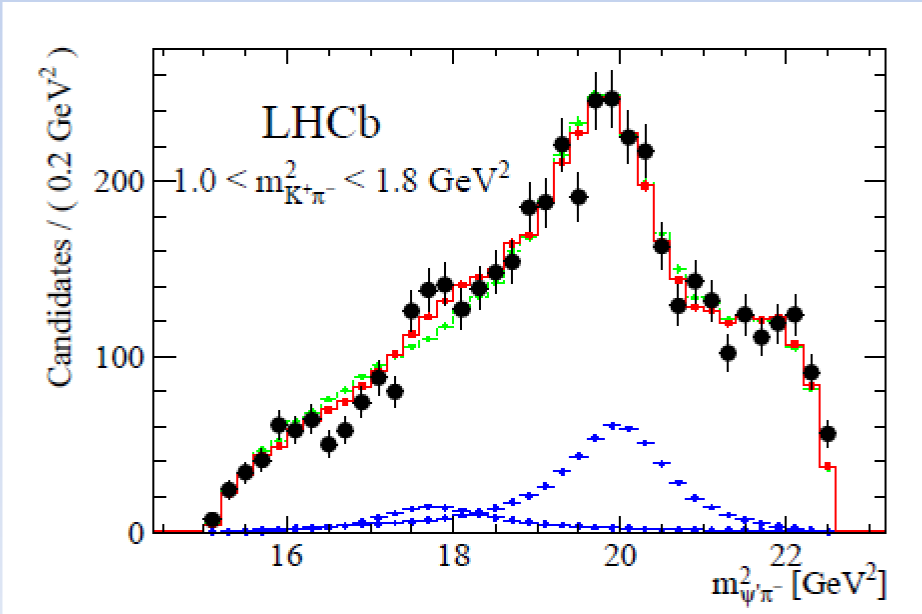
- 2014 -

LHCb

Spin parity = 1+

$\eta_G = \eta_C (-1)^I$

G=+ → will look at C=-



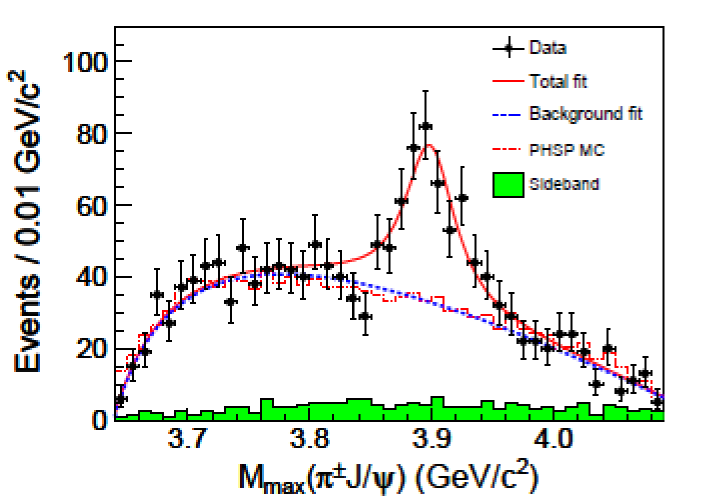
Z(3900)

- 2013 -

BESIII $e^+e^- \to \pi^+\pi^- J/\psi$

$$M = 3899.0 \pm 3.6 \pm 4.9\ \text{MeV}$$

$$\Gamma = 46 \pm 10 \pm 20\ \text{MeV}$$



Probably the same Quantum Number as Z(4430)

Hence,

$$X(3872) \to I^G\left(J^{PC}\right) = 0^+\left(1^{++}\right)$$

$$\left.\begin{array}{l} Z(3900) \to \pi^0\ J/\psi \\ Z(4430) \to \pi^0\ \psi' \end{array}\right] \to 1^+\left(1^{+-}\right)$$

Pentaquark - Pc

\- 2015 -

$$\Lambda_b^0 \rightarrow J/\psi\ p\ K^-$$

$$S = 3/2 \begin{cases} M_1 = 4380 \pm 8 \pm 29\ \mathrm{MeV} \\ \Gamma_1 = 205 \pm 18 \pm 86\ \mathrm{MeV} \end{cases}$$

$$S = 5/2 \begin{cases} M_2 = 4449.8 \pm 1.7 \pm 2.5\ \mathrm{MeV} \\ \Gamma_2 = 39 \pm 5 \pm 19\ \mathrm{MeV} \end{cases}$$

Baryon with ccu

\- 2017 -

$$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$$

$$m_{\Xi_{cc}} - m_{\Lambda_c} = 1334.94 \pm 0.72 \pm 0.27\ \mathrm{MeV}$$

$$m_{\Xi_{cc}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14(\Lambda_c^+)\ \mathrm{MeV}$$

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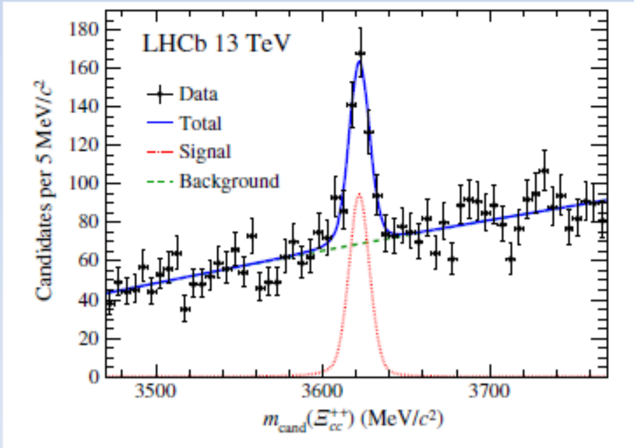
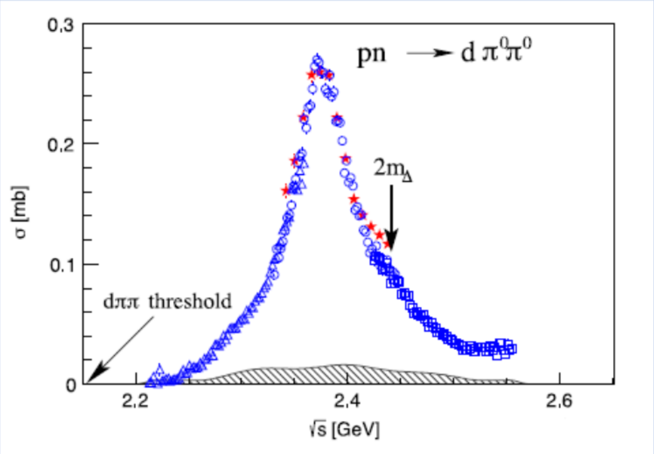


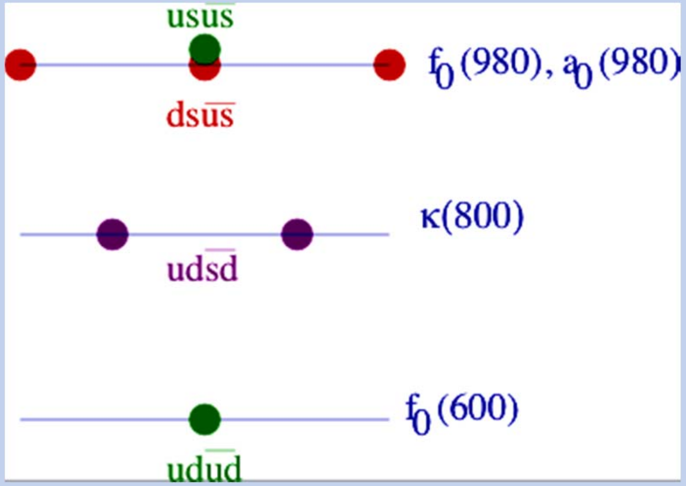
d*(2380) $I(J^P) = 0(3^+)$ $\Gamma = 70$ MeV

- WASA-at-COSY [H. Clement]-

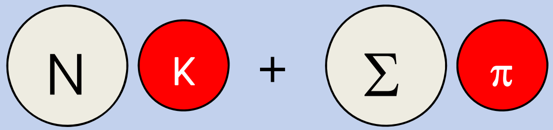


Revival of an old topic

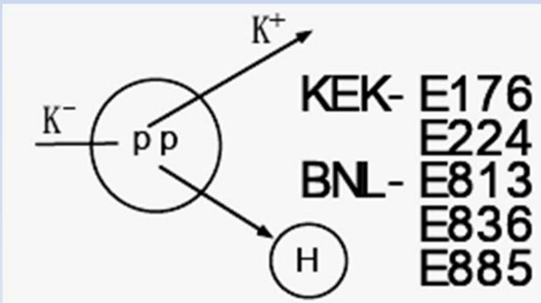
Scalar tetraquark (Jaffe 76)



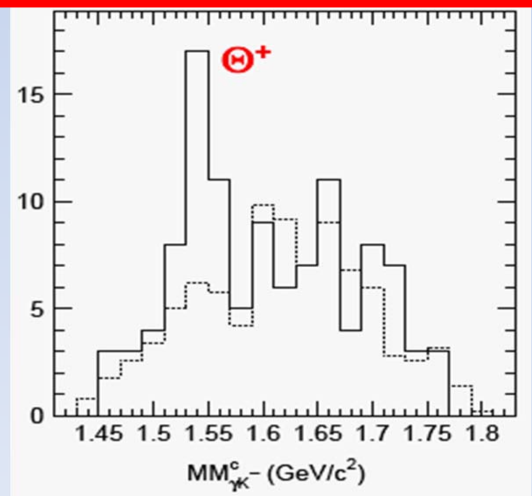
L(1405) (Weise, Oset, Jido, Sekihara..)



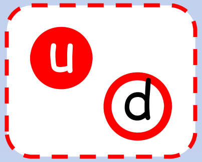
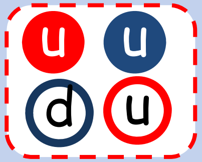
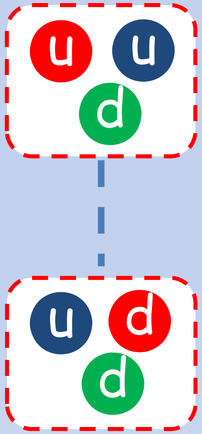
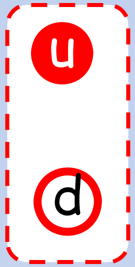
Search for H dibaryon



Search for Θ+ pentaquark



Normal meson, compact multiquark, molecules, resonances

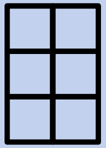
	Normal meson	Compact multiquark	Molecules	Resonance
Geometrical configuration				
Examples	Nucleon, pion, kaon	?	X(3872)	K*, rho meson

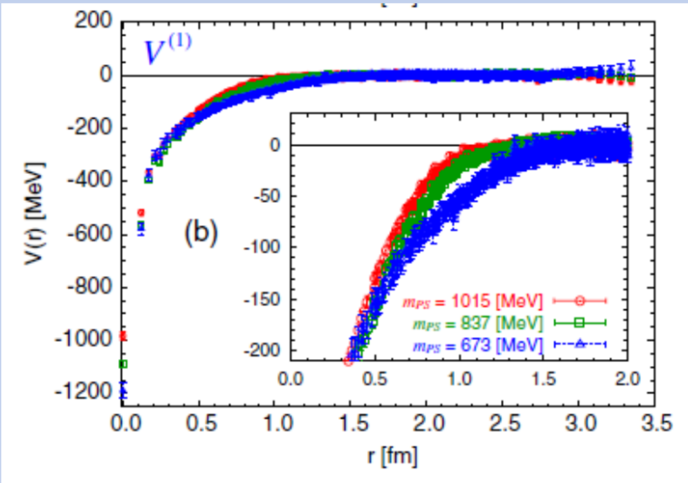
Pc, d*

II: Where are the compact “Multiquark states”

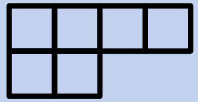
- Lattice Results : HAL QCD collaboration for H dibaryon in SU(3) symmetric limit

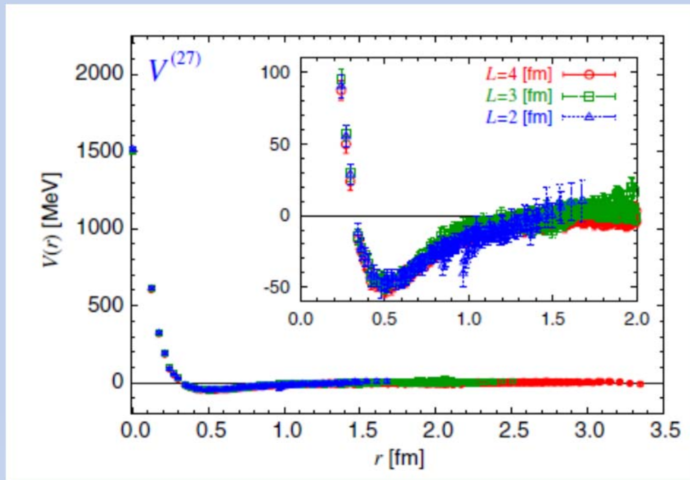
SU(3) flavor 1 state





SU(3) flavor 27 state





→ Flavor 1 channel could give compact configuration

Compact multiquark states could exists if there is a strong short range attraction

The r→0 can be understood from quark model

Quark wave function for multiquark states (W.Park, A.Park, S.Cho, SHL)

- Some Previous works have limited Fock space: diquark picture …

- Hard to picture interplay between various contribution

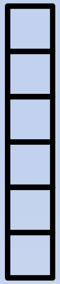
- Hard to understand SU(3) breaking effects.

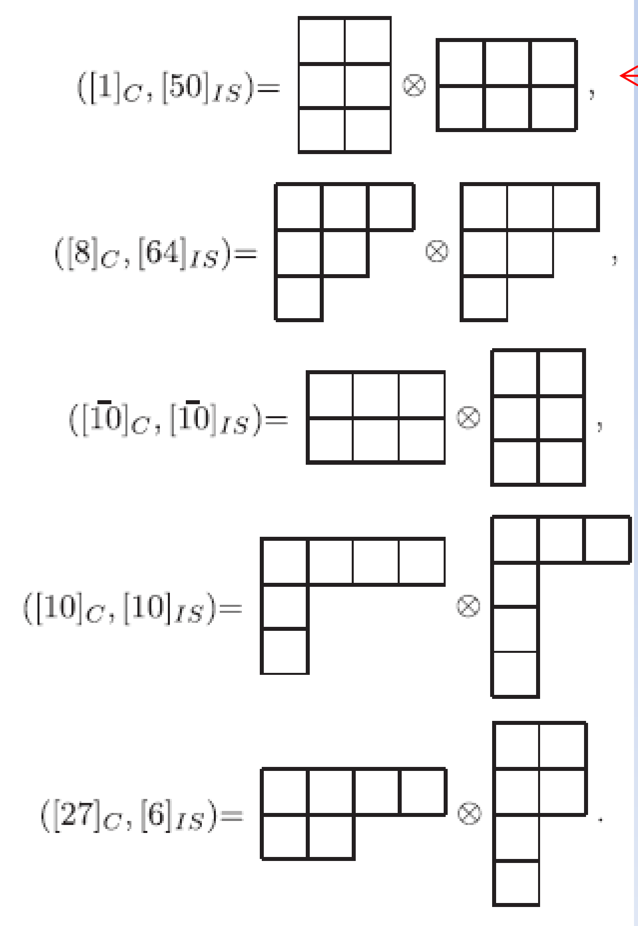
→ Work out the full (color) x (spin) x (flavor) wave function for all multiquark configurations at least for the ground state s-wave states

- Choose the spatial part to be symmetric

- Choose the Color-Isospin-Spin part to be antisymmetric : SU(12)

$$[1^6]_{CIS} = ([1]_C,[50]_{IS}) \oplus ([8]_C,[64]_{IS}) \oplus ([10]_C,[10]_{IS}) \oplus ([10]_C,[10]_{IS}) \oplus ([27]_C,[6]_{IS})$$





Physical State

- Dibaryon: 5 Independent color singlet bases

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \qquad \left| C_1 \right\rangle = \left\{ \left[(12)_6 3 \right]_8 \left[4 (56)_6 \right]_8 \right\}_1$$

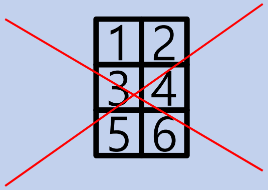
$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \qquad \left| C_2 \right\rangle = \left\{ \left[(12)_{\bar{3}} 3 \right]_8 \left[4 (56)_6 \right]_8 \right\}_1$$

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline \end{array} \qquad \left| C_3 \right\rangle = \left\{ \left[(12)_6 3 \right]_8 \left[4 (56)_{\bar{3}} \right]_8 \right\}_1$$

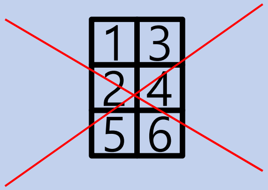
$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline \end{array} \qquad \left| C_4 \right\rangle = \left\{ \left[(12)_{\bar{3}} 3 \right]_8 \left[4 (56)_{\bar{3}} \right]_8 \right\}_1$$

$$\begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array} \qquad \left| C_5 \right\rangle = \left\{ \left[(12)_{\bar{3}} 3 \right]_1 \left[4 (56)_{\bar{3}} \right]_1 \right\}_1$$

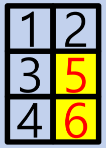
- Pentaquark: 3 Independent color singlet bases (W.Park, A. Park, S.Cho, SHL PRD95,054027)



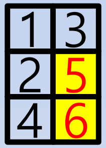
$$|C_1\rangle = \{[(12)_6 3]_8 [4(56)_6]_8\}_1$$



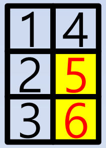
$$|C_2\rangle = \{[(12)_{\bar{3}} 3]_8 [4(56)_6]_8\}_1$$



$$|C_3\rangle = \{[(12)_6 3]_8 [4(5)_{\bar{3}}]_8\}_1$$



$$|C_4\rangle = \{[(12)_{\bar{3}} 3]_8 [4(5)_{\bar{3}}]_8\}_1$$



$$|C_5\rangle = \{[(12)_{\bar{3}} 3]_1 [4(5)_{\bar{3}}]_1\}_1$$

- Heptaquark: 11 Independent color singlet bases (W.Park, A. Park, SHL PRD96,034029)

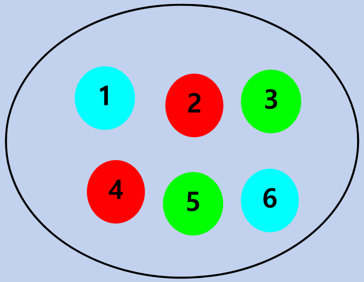
$$|C_1\rangle = \left(\begin{array}{cc} 1 & 2 \\ 3 & 4 \\ 5 & \end{array}, \begin{array}{c} \bar{6} \\ \bar{7} \end{array} \right),\ |C_2\rangle = \left(\begin{array}{cc} 1 & 3 \\ 2 & 4 \\ 5 & \end{array}, \begin{array}{c} \bar{6} \\ \bar{7} \end{array} \right),\ |C_3\rangle = \left(\begin{array}{cc} 1 & 2 \\ 3 & 5 \\ 4 & \end{array}, \begin{array}{c} \bar{6} \\ \bar{7} \end{array} \right),\ |C_4\rangle = \left(\begin{array}{cc} 1 & 3 \\ 2 & 5 \\ 4 & \end{array}, \begin{array}{c} \bar{6} \\ \bar{7} \end{array} \right),\ |C_5\rangle = \left(\begin{array}{cc} 1 & 4 \\ 2 & 5 \\ 3 & \end{array}, \begin{array}{c} \bar{6} \\ \bar{7} \end{array} \right),$$

$$|C_6\rangle = \left(\begin{array}{ccc} 1 & 2 & 3 \\ 4 & & \\ 5 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right),\ |C_7\rangle = \left(\begin{array}{ccc} 1 & 2 & 4 \\ 3 & & \\ 5 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right),\ |C_8\rangle = \left(\begin{array}{ccc} 1 & 3 & 4 \\ 2 & & \\ 5 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right),\ |C_9\rangle = \left(\begin{array}{ccc} 1 & 2 & 5 \\ 3 & & \\ 4 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right),$$

$$|C_{10}\rangle = \left(\begin{array}{ccc} 1 & 3 & 5 \\ 2 & & \\ 4 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right),\ |C_{11}\rangle = \left(\begin{array}{ccc} 1 & 4 & 5 \\ 2 & & \\ 3 & & \end{array}, \begin{array}{cc} \bar{6} & \bar{7} \end{array} \right).$$

In quark model: wave function should follow Pauli Principle

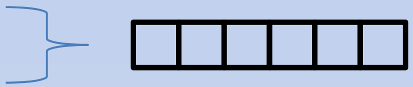
- Totally antisymmetric (color x spin x flavor) wave function (s-wave ground state)



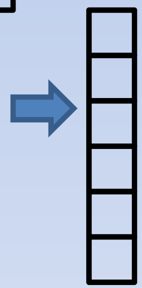
Example: ΩΩ in the Spin=3 channel is highly repulsive because

→ Flavor is totally symmetric

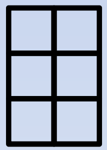
→ Spin is totally symmetric



→ Remaining part should be totally antisymmetric



→ But color singlet implies



→ Hence, assuming all quarks are in the S wave, Pauli principle forbids compact configuration.

Such forbidden configuration are highly repulsive at r→0 **(Oka et al quark cluster model)**

Constituent quark model

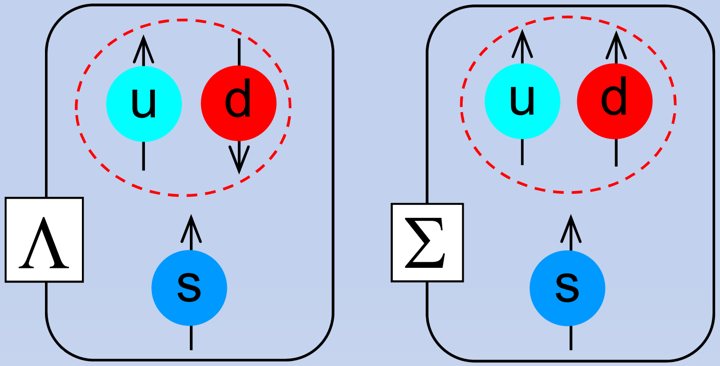
- In Constituent quark model (Can fit experimental hadron spectrum well)

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}\left(r_{ij}\right)$$

- Baryon Mass splitting in a simplified version

$$\text{Mass} = \text{Kinetic} + \text{confining..} + \sum_{i,j} \frac{C_B}{m_i m_j}\left[s_i \cdot s_j\right]$$

Example



$$\Lambda_c \ \text{Mass} = \text{Kinetic} + \text{conf.} - \frac{3}{4}\frac{C_B}{m_u m_d}$$

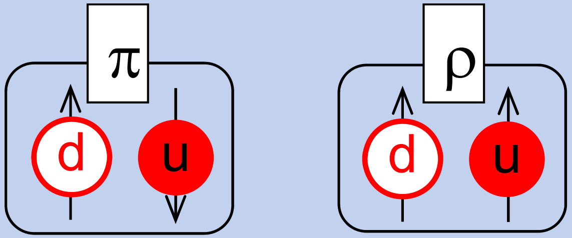
$$\Sigma_c \ \text{Mass} = \text{Kinetic} + \text{conf.} + \frac{1}{4}\frac{C_B}{m_u m_d} - \frac{C_B}{m_u m_s}$$

$$m_u = m_d = 300 \text{ MeV}, \quad m_s = 500 \text{ MeV}, \quad m_c = 1500 \text{ MeV}, \quad m_b = 4700 \text{ MeV}$$

Mass diff	$M_\Delta - M_N$	$M_\Sigma - M_\Lambda$	$M_{\Sigma c} - M_{\Lambda c}$	$M_{\Sigma b} - M_{\Lambda b}$
Formula	290 MeV	77 MeV	154 MeV	180 MeV
Experiment	290 MeV	75 MeV	170 MeV	192 MeV

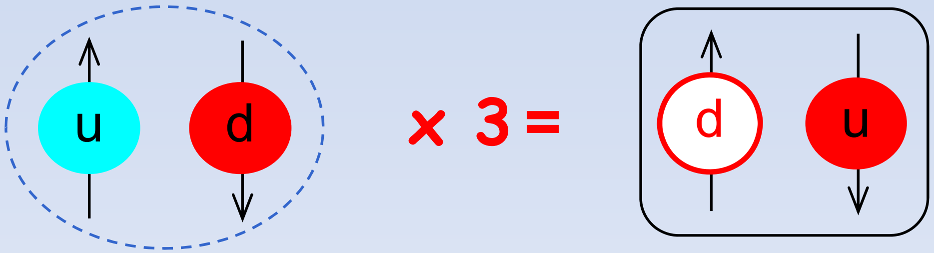
- Meson Mass splitting in a simplified version

$$\text{Mass} = \text{Kinetic} + \text{confining..} + \sum_{i,j} \frac{C_M}{m_i m_j}\left[s_i \cdot s_j\right]$$



Mass diff	$M_\rho - M_\pi$	M_{K*}-M_K	M_{D*}-M_D	M_{B*}-M_B
Formula	635 MeV	381 MeV	127 MeV	41 MeV
Experiment	635 MeV	397 MeV	137 MeV	46 MeV

Works very well with **$3\times C_B = C_M = 635\, m_u^2$**



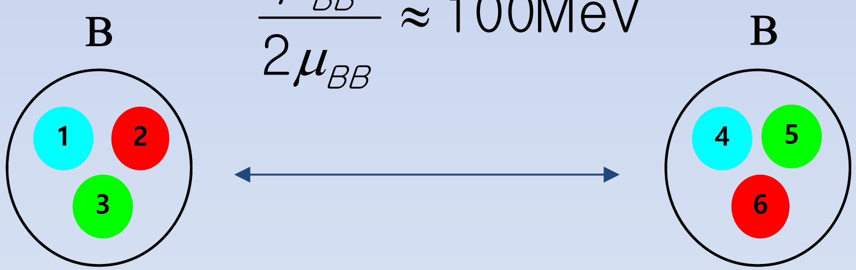
When allowed, Where are the Compact multiquark configuration?

- In Constituent quark model (Can fit experimental hadron spectrum well)

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right)V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$

1) Additional Kinetic energy compared to separated hadrons

$$\frac{p_{BB}^2}{2\mu_{BB}} \approx 100\text{MeV}$$



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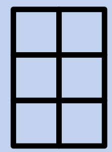


2) Color-color will not add much

$$(\lambda_1^c+\lambda_2^c+\ ....\ \lambda_n^c)^2=2\sum_{i<j}\lambda_i^c\lambda_j^c+\sum_i(\lambda_i^c)^2$$

If color singlet

→ If a color singlet configuration is possible

$$\sum_{i<j}\lambda_i^c\lambda_j^c=-\tfrac{1}{2}\times\sum_i(\lambda_i^c)^2=-\frac{2}{3}\,N_{Total}=--\frac{2}{3}\,(N_{B1}+N_{B2})$$

When allowed, Where are the Compact multiquark configuration?

- In Constituent quark model

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}\left(r_{ij}\right)$$

3) Color-spin interaction is important $K=\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s)$

for 2 body, quark-quark vs quark-antiquark

	qq				$\bar{q}q$			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(1)	S(3)	S(3)	A(1)	1	1	3	3
K	-8	-4/3	8/3	4	-16	2	16/3	-2/3

- Color spin interaction - General remarks

$$-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s)$$

1) Part of a larger group

- Color: SU(3) and 8 generators λ^c

- Spin: SU(2) and 3 generators σ^s

- SU(6) generator: $\lambda^c \times \sigma^s$ (24) + $\lambda^c \times 1$ (8) + $1 \times \sigma^s$ (3) = A (35 generators)

Therefore SU(6) Casmir of N quarks $C_6 = \sum (A_1 + \cdots A_N)^2 = 2\sum_{i<j} A_i A_j + N\,(A_1^2)$

where $(A_1^2) = \frac{35}{6}$ and $2\sum_{i<j} A_i A_j = \sum_{i<j}(\frac{1}{3}\sigma_i\sigma_j + \frac{1}{2}\lambda_i\lambda_j + \frac{1}{3}(\lambda\sigma)_i(\lambda\sigma)_j)$

→ $\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) \propto C_6 - N\frac{35}{6} - \frac{1}{2}\left(\sigma_{Total}^2 - \sum\sigma_i^2\right) - \frac{3}{4}\left(\lambda_{Total}^2 - \sum\lambda_i^2\right)$

total spin–N x(quark spin) , total color-N x(quark color)

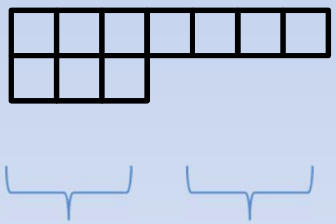
- Color spin interaction - General remarks II

$$\rightarrow \sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) \;\propto\; C_6 - N\frac{35}{6} - \frac{1}{2}\left(\sigma_{Total}^2 - \sum\sigma_i^2\right) - \frac{3}{4}\left(\lambda_{Total}^2 - \sum\lambda_i^2\right)$$

2) Color –flavor-spin wave function should be totally antisymmetric. Then

For SU(2) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

For SU(3) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$



q p

$$4C_F = \frac{4}{3}(p^2 + q^2 + 3p + 3q + qp)$$

using $p + 2q = N$ $\rightarrow 4C_F = (4I(I+1) + N(N+6)/3)$

For SU(4) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = \frac{5}{6}N\left(N - \frac{72}{5}\right) + 4C_F^{SU(4)} + \frac{4}{3}S(S+1) + 2C_c$

- Color spin interaction

For SU(3) flavor: $K= -\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$

Nucleon and Λ → $K = -8$ even in the SU(3) broken limit

- Jaffe (77) : K for H-dibaryon vs two Λ

$K = -24$ $\quad\quad$ $K = (-8) + (-8) = -16$

→ using Nucleon(K=-8) to Delta (K=+8) mass difference of 290 MeV

ΔK=-8 corresponds to about 145 MeV attraction ≫ additional Kinetic energy of 100 MeV

- Dibaryons with 6 light quarks: W.Park, A. Park, SHL, PRD92(2015)014037

For SU(2) flavor: $K = \sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

$K_{\text{dibaryon}} - (K_{\text{baryon 1}} - K_{\text{baryon 2}})$ →

Color spin interaction of 6 quark state and their decays

(I,S)	(3,0)	(2,1)	(1,2)	(1,0)	(0,3)	(0,1)
V_d	48	$\frac{80}{3}$	16	8	16	$\frac{8}{3}$
ΔV	32	$\frac{80}{3}$	16	24	0	$\frac{56}{3}$

→ The only non repulsive channel, but also "no attraction"

→ Strong indication that d*(2380) is a molecular configuration

(A. Gal, PLB769(2017)436) $s_\Delta = (1232 - B_{\Delta\Delta}/2)^2 - p_{\Delta\Delta}^2, \quad \bar{s}_\Delta = (1232 - B_{\Delta\Delta}/2)^2 - P_{\Delta\Delta}^2,$

→ No compact dibaryon in flavor SU(2)

→ Two body nuclear force is always repulsive at short distance : (Oka quark cluster model)

- H dibaryon with realistic quark masses: W.Park, A. Park, SHL, PRD93(2016)074007

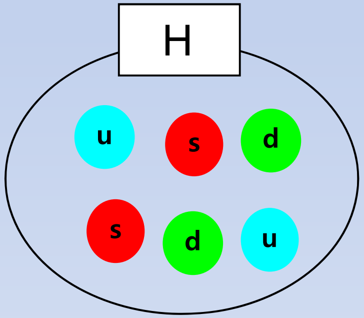
$$-\sum_{i<j}^{n} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}(r_{ij}) \quad \rightarrow K$$

$m_{u,d} = 300$ MeV, $m_s = 500$ MeV

TABLE III. The matrix element of $-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ for hyperfine potential of the dibaryon with respect to isospin and flavor.

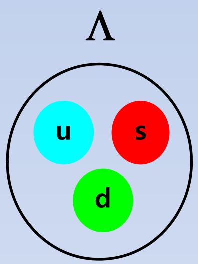
Isospin Flavor	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i<j=1\text{–}4$	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i=1\text{–}4,\ j=5,6$	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i=5,\ j=6$
$I=0,\ F^1$	$-5/6$	$-11/4$	3
$I=0,\ F^{27}$	$-13/18$	$13/12$	$11/3$
Cross terms	$1/(6\sqrt{3})$	$-1/(4\sqrt{3})$	$1/\sqrt{3}$
$I=1,\ F^{27}$	$4/9$	$1/3$	$8/3$
$I=2,\ F^{28}$	$16/5$	$16/5$	$16/5$
$I=2,\ F^{27}$	$146/45$	$-28/15$	$52/15$
Cross terms	$-2\sqrt{2}/(15\sqrt{3})$	$\sqrt{2}/(5\sqrt{3})$	$-4\sqrt{2}/(5\sqrt{3})$

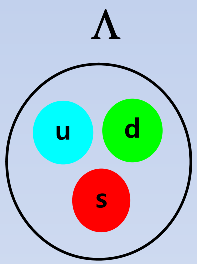
→ If the SU(3) breaking is taken into account. Color spin with constituent quark mass



$\leftarrow \dfrac{p_{BB}^2}{2\mu_{BB}} \approx 100$ MeV

VS





$-\sum_{i<j}^{n} \dfrac{K}{m_i m_j}$	**H dibaryon**	$\Lambda + \Lambda$	$\Delta\ E_{hyperfine}$	$\Delta\ E_{kinetic}$
$m_{u,d} = m_s$	$-\dfrac{24}{m_u^2}$	$-\dfrac{8}{m_u^2} - \dfrac{8}{m_u^2}$	-145 MeV	+100MeV
$m_{u,d} \approx \dfrac{3}{5} m_s$	$\left(-\dfrac{5}{m_u^2} - \dfrac{22}{m_u m_s} + \dfrac{3}{m_s^2}\right) \approx -\dfrac{17.12}{m_u^2}$	$-\dfrac{8}{m_u^2} - \dfrac{8}{m_u^2}$	-20 MeV	+ 84 MeV

Where are the compact multiquark states? - What we need

1) Need Strong Color spin interaction

$$-\sum_{i<j}^{n} \frac{(\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j)}{m_i m_j} v_{ij}^{SS}(r_{ij})$$

→ that survive in the SU(3) breaking limit

2) Need heavy quarks to suppress additional kinetic term

$$\frac{p_{BB}^2}{2\mu_{BB}} \approx \frac{(1/size)^2}{2\mu_{BB}} \qquad \mu_{BB} \approx \frac{m_{baryon1} m_{baryon2}}{m_{baryon1} + m_{baryon2}}$$

→ both baryons should have heavy quarks

- Is Pentaquark (Pc) compact ? W.park, A. Park, S.Cho, SHL, PRD95(2017) 054027

1) Color spin interaction of Pc(4380) 3/2 ⁻ state $qqq\, c\bar{c}$

- 2015 -

LHCb

$$\Lambda_b^0 \to J/\psi \; p \, K^-$$

$$S = 3/2 \quad \begin{cases} M_1 = 4380 \pm 8 \pm 29\,\text{MeV} \\ \Gamma_1 = 205 \pm 18 \pm 86\,\text{MeV} \end{cases}$$

$$S = 5/2 \quad \begin{cases} M_2 = 4449.8 \pm 1.7 \pm 2.5\,\text{MeV} \\ \Gamma_2 = 39 \pm 5 \pm 19\,\text{MeV} \end{cases}$$

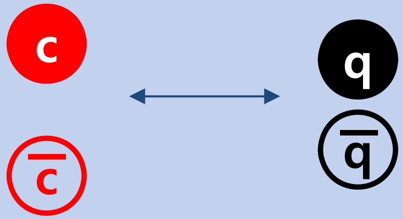
Pc(4380) can be reconstructed from $J/\psi + p$

$-\sum_{i<j}^{n} \frac{K}{m_i m_j}$ **Pc(4380)**	**P + J/ψ**	**Δ E$_{hyperfine}$**	**Δ E$_{kinetic}$**
$\left(-\frac{7.88}{m_u^2} + \frac{5.29}{m_c^2} - \frac{1.41}{m_u m_c}\right) \approx -\frac{7.95}{m_u^2}$	$\left(-\frac{8}{m_u^2} + \frac{16}{3m_c^2}\right) \approx -\frac{7.79}{m_u^2}$	-3 MeV	+ 70MeV

→ Most likely a molecular states

- Heavy Tetraquarks (Spin=1 case)

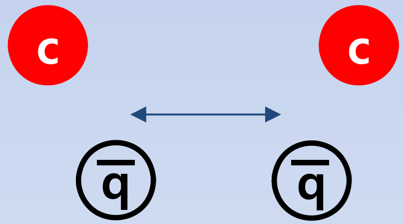
1) Heavy quark-antiquark: $c\bar{c}$



$$\mu_{BB} \approx \frac{m_{c\bar{c}} m_{q\bar{q}}}{m_{baryon1} + m_{baryon2}} \approx m_{q\bar{q}}$$

$-\sum_{i<j}^{n} \frac{K}{m_i m_j}$	**Tetraquark**	**J/ψ + π**	**Δ $E_{hyperfine}$**	**Δ $E_{kinetic}$**
$c\bar{c}$	$\left(-\frac{16}{m_u^2}+\frac{16}{3m_c^2}\right)$	$\left(-\frac{16}{m_u^2}+\frac{16}{3m_c^2}\right)$	0 MeV	+100MeV

2) Heavy quark-quark: cc



$$\mu_{MM} \approx \frac{m_{c\bar{q}} m_{c\bar{q}}}{m_{c\bar{q}} + m_{c\bar{q}}} \approx \frac{1}{2} m_{c\bar{q}}$$

$-\sum_{i<j}^{n} \frac{K}{m_i m_j}$	**Tetraquark**	**D + D***	**Δ $E_{hyperfine}$**	**Δ $E_{kinetic}$**
cc	$\left(-\frac{8}{m_u^2}+\frac{8}{3m_c^2}\right) \approx -\frac{7.47}{m_u^2}$	$\left(-\frac{8}{m_q m_c}+\frac{8}{3m_q m_c}\right)$	-97 MeV	+50MeV

- Heavy Tetraquarks

1) Previous works on Tcc

Z. Zouzou, B. Silverstre-Brac, C. Gilgnooux, J Richard (86), D. Janc, M. Rosina (04), Y. Cui, S. L. Zhu (07)

QCD sum rules: F Navarra, M. Nielsen, SHLee, PLB 649, 166 (2007)

simple diquark: SHL, S. Yasui, W.Liu, C Ko EPJ C54, 259 (2008), SHL, S. Yasui: EPJ C (09)

2) Promising final state signals

$$\mathrm{T}^{1}_{\mathrm{cc}}(\mathrm{ud}\bar{\mathrm{c}}\bar{\mathrm{c}}) \rightarrow (\overline{D}^{0} + D^{*-}) \rightarrow K^{+}\pi^{-} + K^{+}\pi^{-}\pi^{-}$$

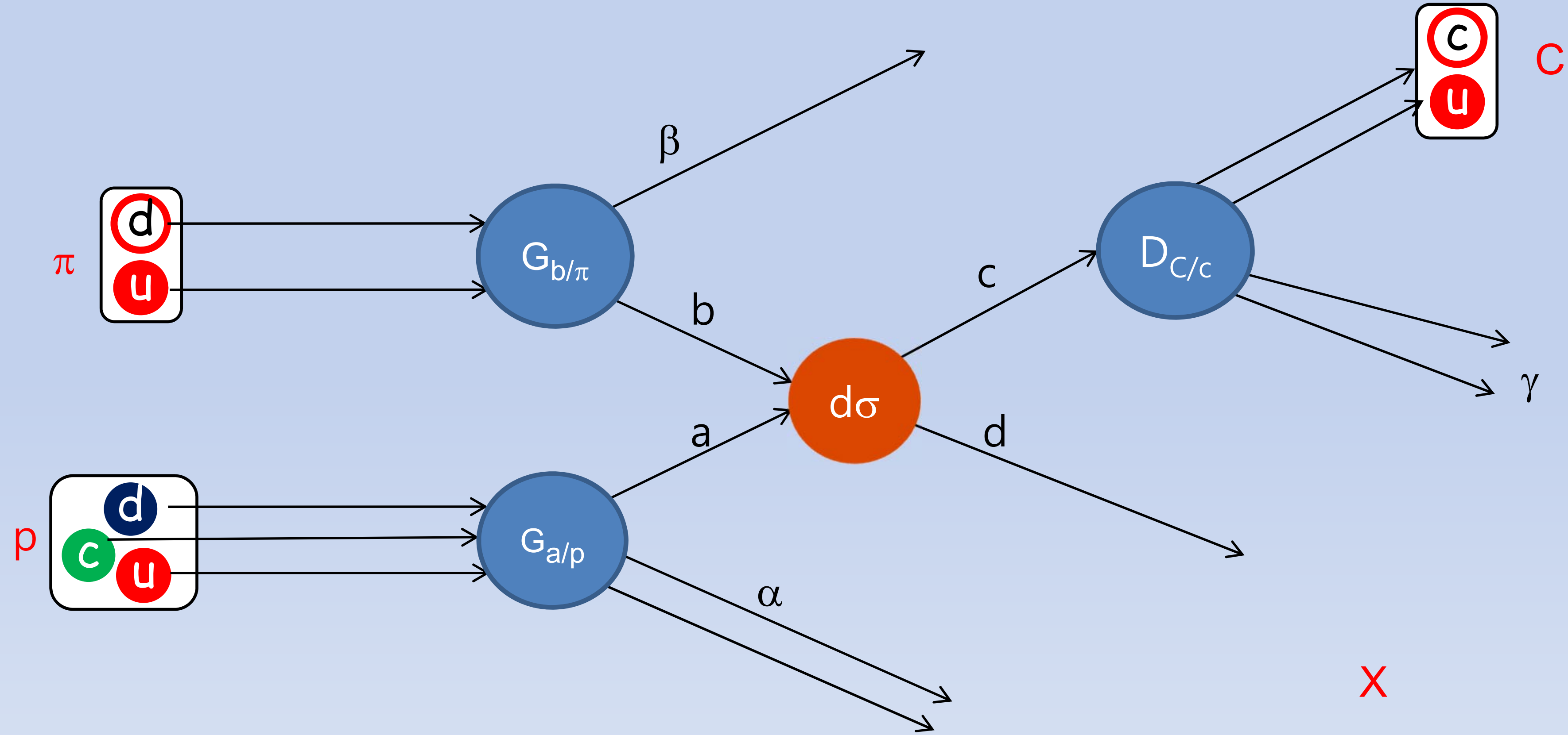
threshold	decay mode	lifetime
$M_{T_{cc}} > M_{D*} + M_D$	$D^{*-}\bar{D}^0$	hadronic decay
$2M_D + M_\pi < M_{T_{cc}} < M_{D*} + M_D$	$\bar{D}^0\bar{D}^0\pi^-$	hadronic decay
$M_{T_{cc}} < 2M_D + M_\pi$	$D^{*-}K^+\pi^-$, $D^{*-}K^+\pi^+\pi^-\pi^-$	0.41×10^{-12} sec.

→ Most likely a compact tetraquark states

→ Could be measured in high energy Heavy Ion Collision (ExHIC coll)

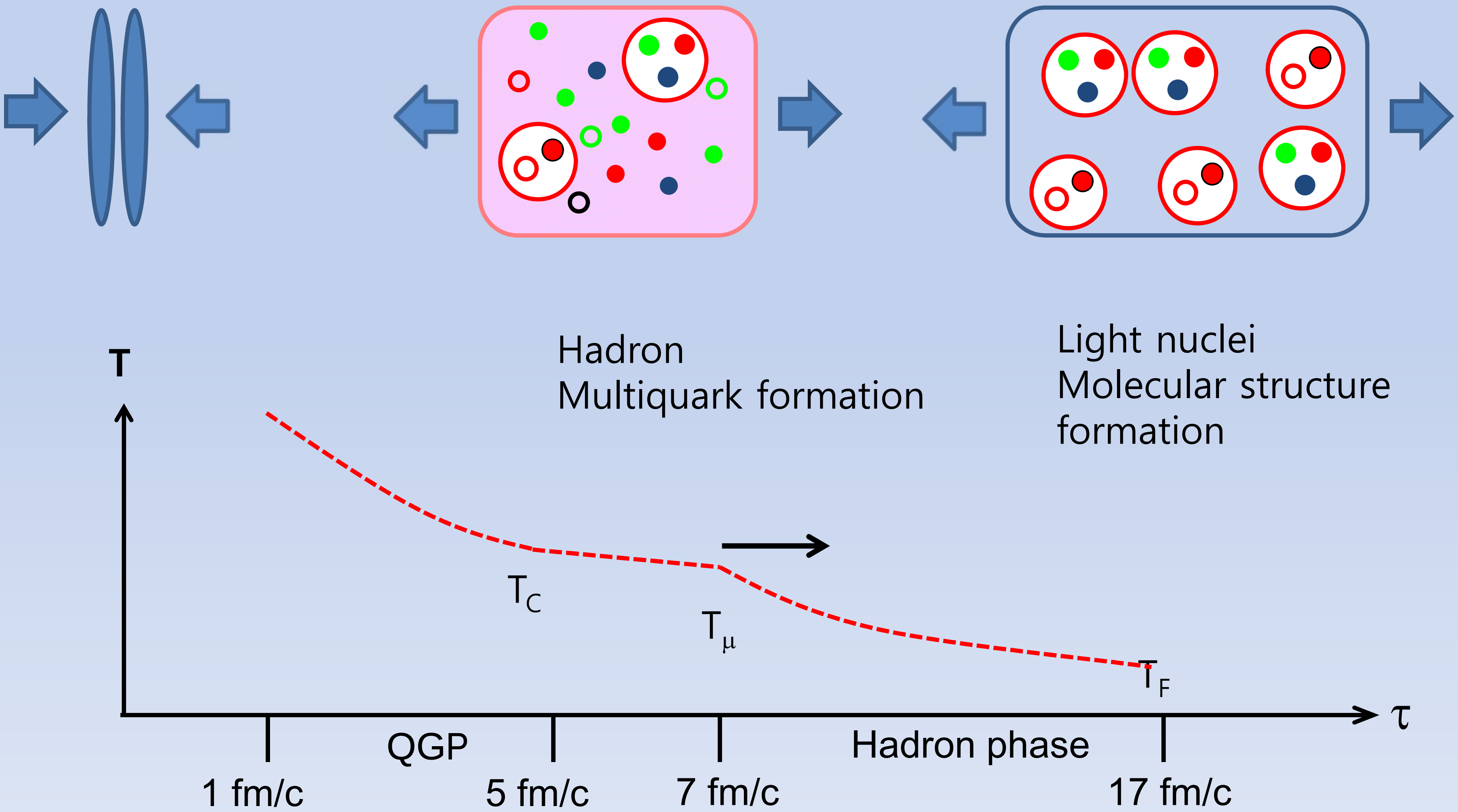
III: Exotica production from Heavy Ion Collision

Hadron production in (p+π→C+X) collision



$$d\sigma\Big|_{p+\pi\to C+X} = \int G_{b/\pi}(x_b) G_{a/p}(x_a) \times \int d\sigma\Big|_{a+b\to c+d} \times D_{C/c}(x_c)$$

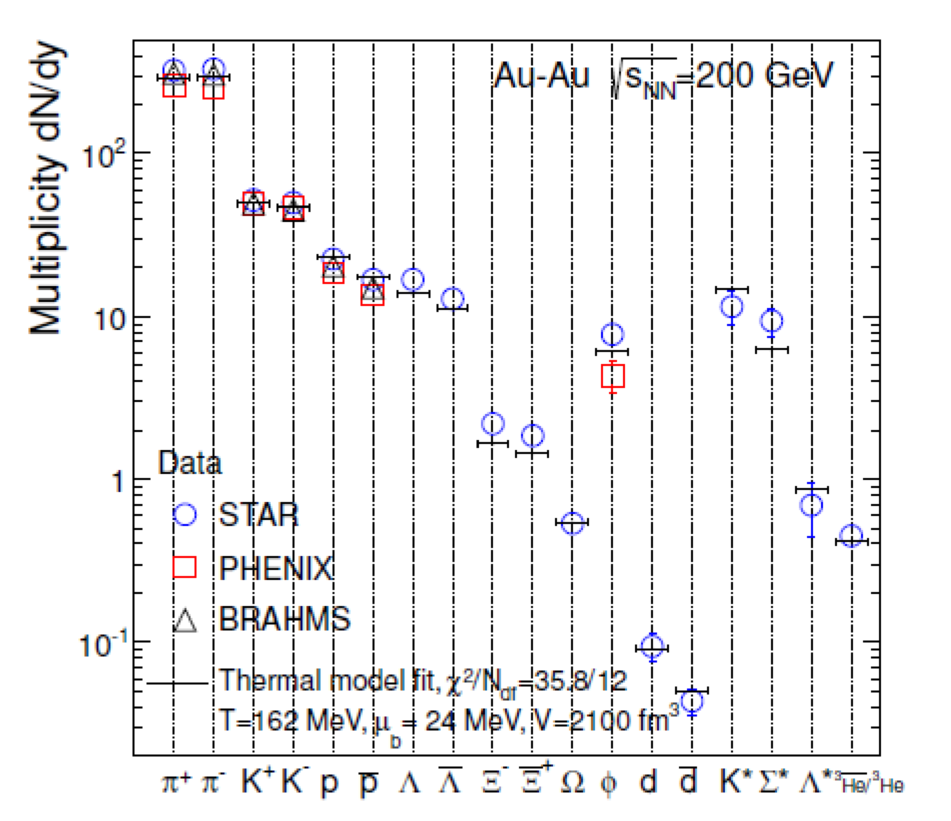
Particle production in heavy ion collision

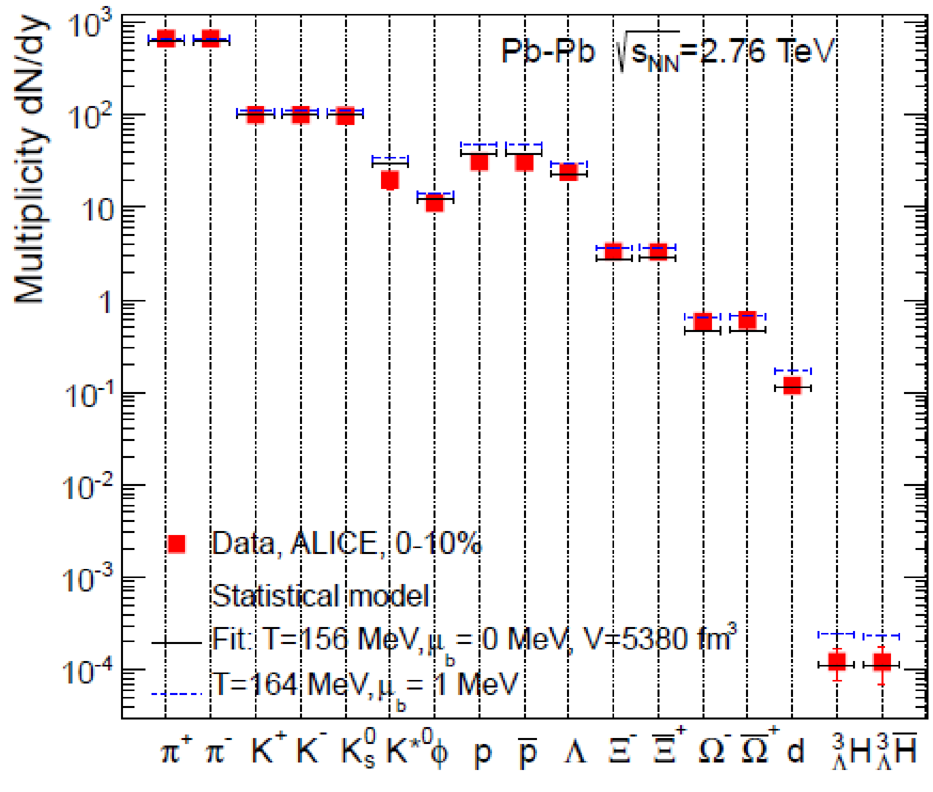


Production of hadrons near Tc

RHIC – Statistical model (PBM ..)

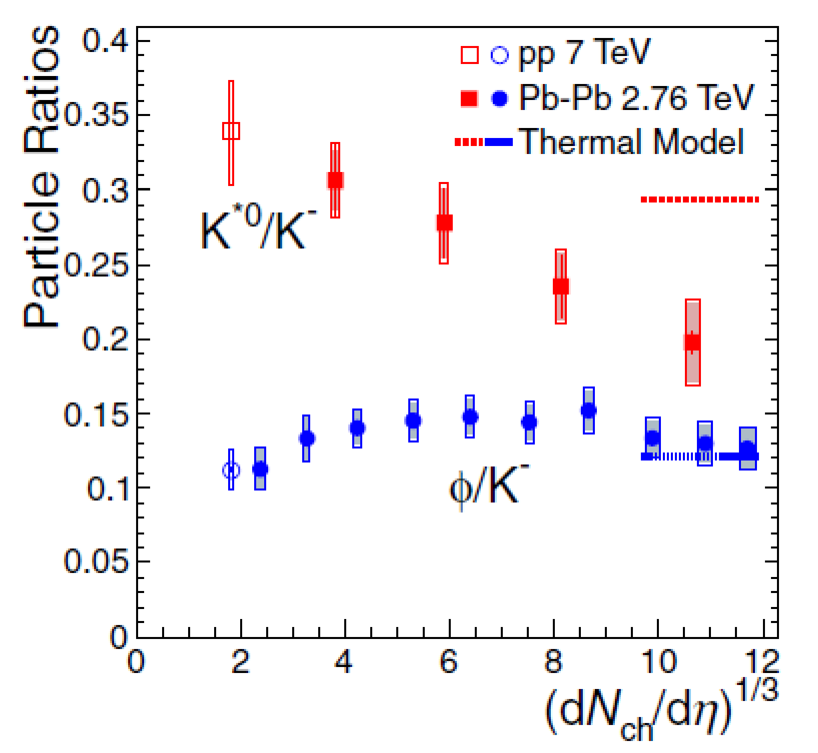
ALICE – Statistical model





Production of resonances

ALICE (2015 prc)



- Reconstruction

$$K^* \to K + \pi, \quad \Gamma > 50\,\mathrm{MeV}$$

$$\phi \to K + K, \quad \Gamma > 5\,\mathrm{MeV}$$

$$\Lambda(1529) \to \bar{K} + N, \quad \Gamma > 15\,\mathrm{MeV}$$

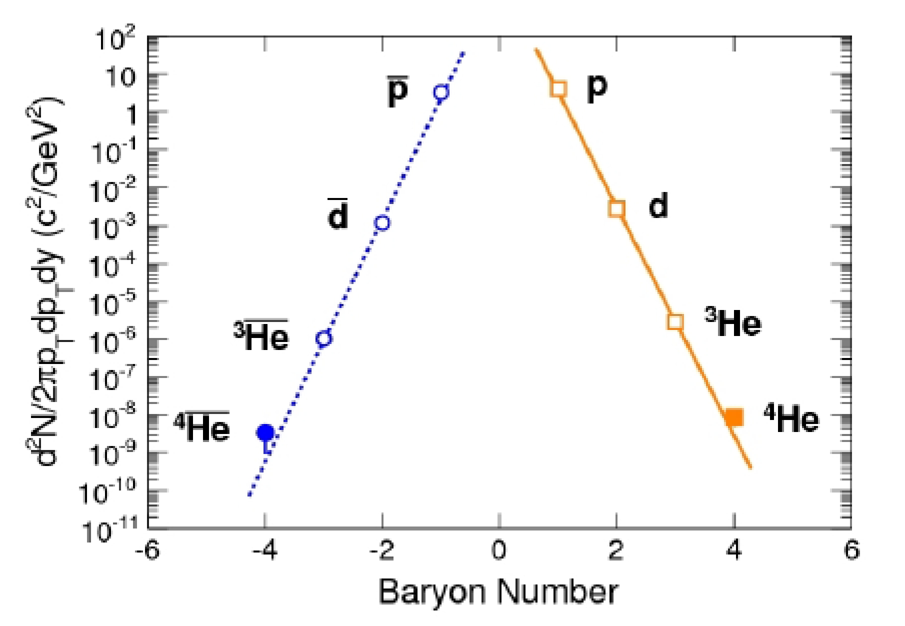
STAR collaboration (PRL 2006) find

$$\frac{\Lambda(1529)_{Au+Au}}{\Lambda(1529)_{Stat}} \approx 0.4$$

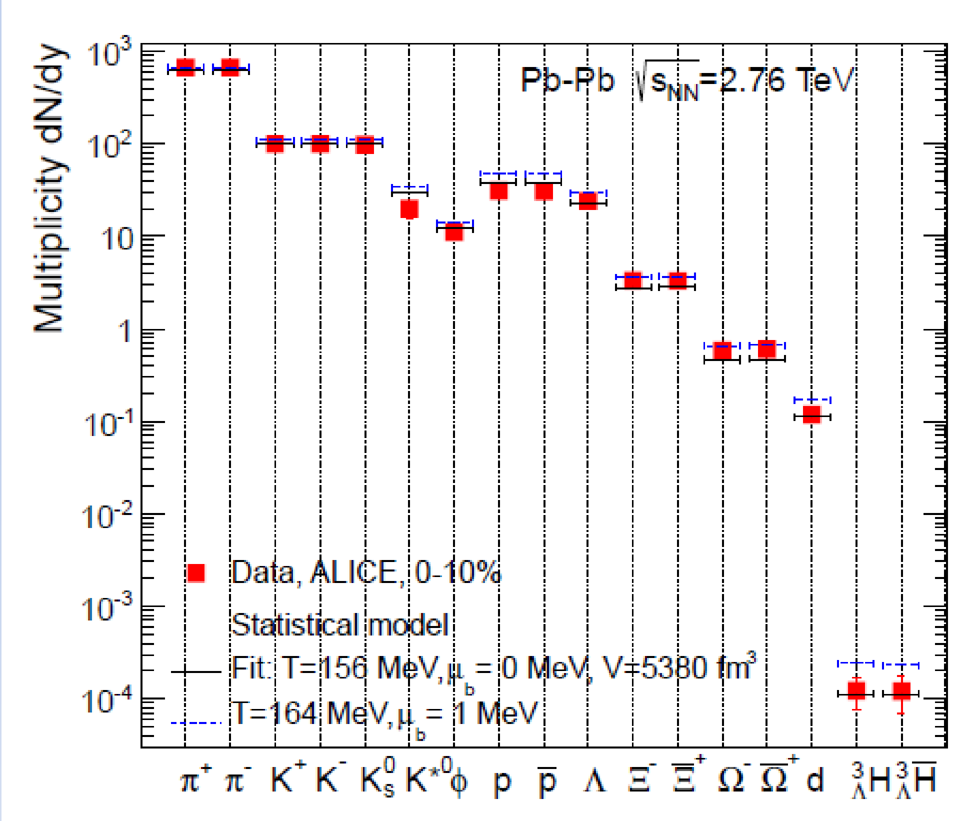
Production of light nuclear

RHIC/STAR (Yugang Ma)

ALICE – Statistical model

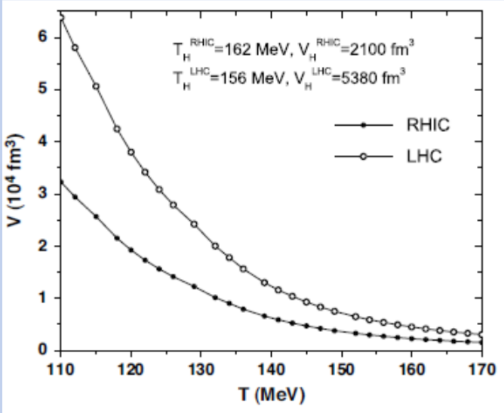


S/N i conserved (Siemens, Kapusta 79)



Details of coalescence model calculation (ExHIC PPNP 2017)

- Model central rapidity, central collision using Lattice EOS



- Heavy quark production (T Song)

Table 3.2
Estimates of heavy quark pairs dN/dy at midrapidity in 0%–10% central collision at RHIC and LHC.

	RHIC	LHC @2.76 TeV	LHC @5.02 TeV
Without shadowing			
$N_c = N_{\bar{c}}$	4.5	17	23
$N_b = N_{\bar{b}}$	0.034	0.68	1.2
With shadowing			
$N_c = N_{\bar{c}}$	4.1	11	14
$N_b = N_{\bar{b}}$	0.031	0.44	0.71

- Coalescence Parameters:

fit production of normal hadrons from statistical model

$$N_h^{\rm coal} = g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{\int d^3y_i d^3k_i f_i(k_i) f^W(y_i, k_i)}{\int d^3y_i d^3k_i f_i(k_i)}$$

$$f_s^W(y_i, k_i) = 8 \exp\left(-\frac{y_i^2}{\sigma_i^2} - k_i^2\sigma_i^2\right)$$

$$\sigma_i = 1/\sqrt{\mu_i \omega}$$

$m_{u,d} = 300$ MeV

$m_s = 500$ MeV

$m_c = 1500$ MeV

$m_b = 4700$ MeV

	RHIC Sc. 1	RHIC Sc. 2	LHC (2.76 TeV) Sc. 1	LHC (2.76 TeV) Sc. 2	LHC (5.02 TeV) Sc. 1	LHC (5.02 TeV) Sc. 2	RHIC Refs [14,15]	LHC (5 TeV) Refs [14,15]
T_H (MeV)	162		156				175	
V_H (fm^3)	2100		5380				1908	5152
μ_B (MeV)	24		0				20	0
μ_s (MeV)	10		0				10	0
γ_c	22		39		50		6.40	15.8
γ_b	4.0×10^7		8.6×10^8		1.4×10^9		2.2×10^6	3.3×10^7
T_C (MeV)	162	166	156	166	156	166	175	
V_C (fm^3)	2100	1791	5380	3533	5380	3533	1000	2700
ω (MeV)	590	608	564	609	564	609	550	
ω_s (MeV)	431	462	426	502	426	502	519	
ω_c (MeV)	222	244	219	278	220	279	385	
ω_b (MeV)	183	202	181	232	182	234	338	
$N_u = N_d$	320	302	700	593	700	593	245	662
$N_s = N_{\bar{s}}$	183	176	386	347	386	347	150	405
$N_c = N_{\bar{c}}$	4.1		11		14		3	20
$N_b = N_{\bar{b}}$	0.03		0.44		0.71		0.02	0.8
T_F (MeV)	119		115				125	
V_F (fm^3)	20355		50646				11322	30569
N_K	67.5		134				142[a]	363[a]
$N_{\bar{K}}$	59.6		134				127[a]	363[a]
N_N	20		32				62[a]	150[a]
N_Δ	18		28				–	–
N_Λ	3.8		6.5				–	–
N_Ξ	2.6		4.4				4.7	13
N_Ω	0.37		0.62				0.81	2.3
$N_D = N_{\bar{D}}$	1.5		4.0		5.2		1.0	6.9
$N_{D^*} = N_{\bar{D}^*}$	2.0		5.4		6.9		1.5	10
$N_{D_1} = N_{\bar{D}_1}$	0.20		0.49		0.63		0.19	1.3
$N_B = N_{\bar{B}}$	8.1×10^{-3}		0.12		0.20		5.3×10^{-3}	0.21
$N_{B^*} = N_{\bar{B}^*}$	1.9×10^{-2}		0.27		0.45		1.2×10^{-2}	0.49
N_{Λ_c}	0.17		0.36		0.46		–	–
N_{Σ_c}	0.2		0.41		0.52		–	–
$N_{\Sigma_c^*}$	0.28		0.56		0.71		–	–
N_{Ξ_c}	0.11		0.25		0.32		0.10	0.65

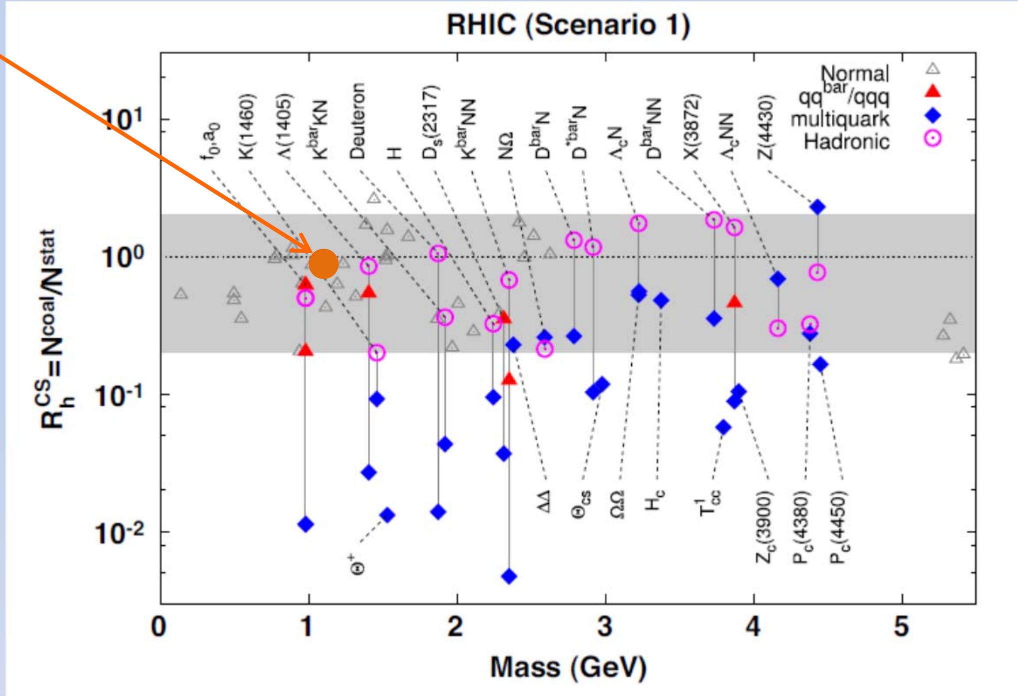
[a] Values contain feed down contributions.

- Hadron coalescence for molecules at kinetic freezeout point

$$\omega = \frac{3}{2\mu_R \langle r^2 \rangle} \qquad \text{or} \quad B \approx \frac{\eta^2}{2\mu_R a_0^2}, \quad \langle r^2 \rangle \approx \frac{a_0^2}{2}$$

Particle	m (MeV)	g	I	J^P	$2q/3q/6q$	$4q/5q/8q$	Mol.	$\omega_{\text{Mol.}}$ (MeV)	Decay mode
Mesons									
$f_0(980)$	980	1	0	0^+	$q\bar{q}, s\bar{s}(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\pi\pi$ (Strong decay)
$a_0(980)$	980	3	1	0^+	$q\bar{q}(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\eta\pi$ (Strong decay)
$K(1460)$	1460	2	1/2	0^-	$q\bar{s}$	$q\bar{q}q\bar{s}$	$\bar{K}KK$	69.0(R)	$K\pi\pi$ (Strong decay)
$D_s(2317)$	2317	1	0	0^+	$c\bar{s}(L=1)$	$q\bar{q}c\bar{s}$	DK	273(B)	$D_s\pi$ (Strong decay)
T_{cc}^{1}[a]	3797	3	0	1^+	—	$qq\bar{c}\bar{c}$	$\bar{D}\bar{D}^*$	476(B)	$K^+\pi^- + K^+\pi^- + \pi^-$
$X(3872)$	3872	3	0	$1^+, 2^-$[c]	$c\bar{c}(L=2)$	$q\bar{q}c\bar{c}$	$\bar{D}D^*$	3.6(B)	$J/\psi\pi\pi$ (Strong decay)
$Z^+(4430)$[b]	4430	3	1	0^-[c]	—	$q\bar{q}c\bar{c}(L=1)$	$D_1\bar{D}^*$	13.5(B)	$J/\psi\pi$ (Strong decay)
T_{cb}^{0}[a]	7123	1	0	0^+	—	$qq\bar{c}\bar{b}$	$\bar{D}B$	128(B)	$K^+\pi^- + K^+\pi^-$
Baryons									
$\Lambda(1405)$	1405	2	0	$1/2^-$	$qqs(L=1)$	$qqqs\bar{q}$	$\bar{K}N$	20.5(R)–174(B)	$\pi\Sigma$ (Strong decay)
$\Theta^+(1530)$[b]	1530	2	0	$1/2^+$[c]	—	$qqqq\bar{s}(L=1)$	—	—	KN (Strong decay)
$\bar{K}KN$[a]	1920	4	1/2	$1/2^+$	—	$qqqs\bar{s}(L=1)$	$\bar{K}KN$	42(R)	$K\pi\Sigma, \pi\eta N$ (Strong decay)
$\bar{D}N$[a]	2790	2	0	$1/2^-$	—	$qqqq\bar{c}$	$\bar{D}N$	6.48(R)	$K^+\pi^-\pi^- + p$
$\bar{D}^*N$[a]	2919	4	0	$3/2^-$	—	$qqqq\bar{c}(L=2)$	$\bar{D}^*N$	6.48(R)	$\bar{D} + N$ (Strong decay)
Θ_{cs}[a]	2980	4	1/2	$1/2^+$	—	$qqqs\bar{c}(L=1)$	—	—	$\Lambda + K^+\pi^-$
BN[a]	6200	2	0	$1/2^-$	—	$qqqq\bar{b}$	BN	25.4(R)	$K^+\pi^-\pi^- + \pi^+ + p$
B^*N[a]	6226	4	0	$3/2^-$	—	$qqqq\bar{b}(L=2)$	B^*N	25.4(R)	$B + N$ (Strong decay)
Dibaryons									
H[a]	2245	1	0	0^+	$qqqqss$	—	ΞN	73.2(B)	$\Lambda\Lambda$ (Strong decay)
$\bar{K}NN$[b]	2352	2	1/2	0^-[c]	$qqqqqs(L=1)$	$qqqqqq\,s\bar{q}$	$\bar{K}NN$	20.5(T)–174(T)	ΛN (Strong decay)
$\Omega\Omega$[a]	3228	1	0	0^+	$ssssss$	—	$\Omega\Omega$	98.8(R)	$\Lambda K^- + \Lambda K^-$
H_c^{++}[a]	3377	3	1	0^+	$qqqqsc$	—	$\Xi_c N$	187(B)	$\Lambda K^-\pi^+\pi^+ + p$
$\bar{D}NN$[a]	3734	2	1/2	0^-	—	$qqqqqq\,q\bar{c}$	$\bar{D}NN$	6.48(T)	$K^+\pi^- + d, K^+\pi^-\pi^- + p + p$
BNN[a]	7147	2	1/2	0^-	—	$qqqqqq\,q\bar{b}$	BNN	25.4(T)	$K^+\pi^- + d, K^+\pi^- + p + p$

Fachini [STAR]



Progress in Particle and Nuclear Physics 95 (2017) 279–322

Summary I

1] Compact multiquark states can be understood from color spin flavor wave function:

- A strong attractive short range interaction is needed in the SU(3) broken limit
- Heavy quarks are needed to reduce extra Kinetic energy

→ Tcc could be strongly bound

$$T^{1}_{cc}(ud\bar{c}\bar{c}) \rightarrow (\overline{D}^{0} + D^{*-}) \rightarrow K^{+}\pi^{-} + K^{+}\pi^{-}\pi^{-}$$

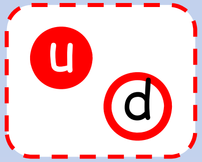
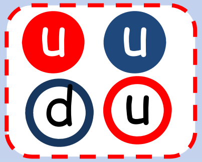
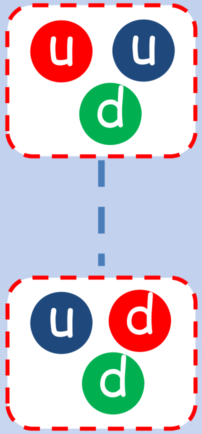
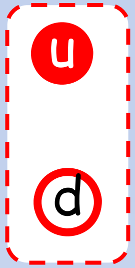
threshold	decay mode	lifetime
$M_{T_{cc}} > M_{D^*} + M_D$	$D^{*-}\bar{D}^0$	hadronic decay
$2M_D + M_\pi < M_{T_{cc}} < M_{D^*} + M_D$	$\bar{D}^0\bar{D}^0\pi^-$	hadronic decay
$M_{T_{cc}} < 2M_D + M_\pi$	$D^{*-}K^+\pi^-$, $D^{*-}K^+\pi^+\pi^-\pi^-$	0.41×10^{-12} sec.

→ Vertex detector: weakly decaying exotics : FAIR 10^4 D^0 /month,
LHC 10^5 D^0/month

→ T_{cc} production T_{cc}/D > 0.34×10^{-4} RHIC
> 0.8×10^{-4} LHC

Summary II

2] Measurements from Heavy Ion can discriminate the structures

	Normal meson	**Compact multiquark**	**Molecules**	**Resonance**
Geometrical configuration				
Examples	Nucleon, ..	Tcc , ...	Pc, d*, ..	K*, ρ meson
Production rate	= Statistical Model	<< Statistical Model	= Statistical Model	< Statistical Model

IV: Tribaryons

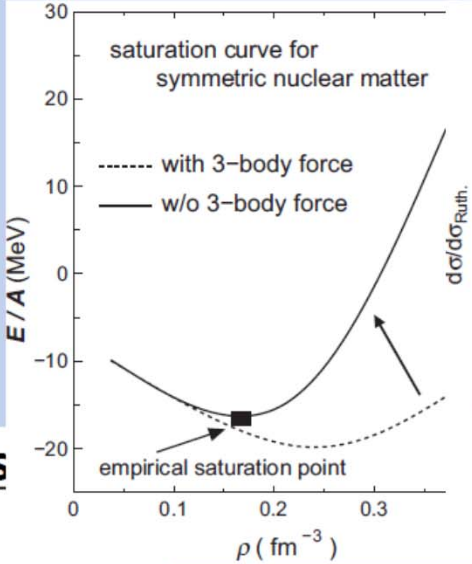
and

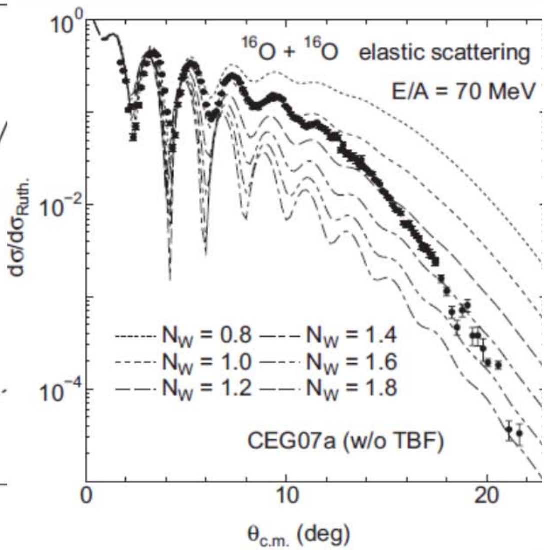
Short distance repulsive three body nuclear force

- Three body nuclear force is repulsive at short distance

1) Review by Sakuragi

(PTEP 2016,06A106)



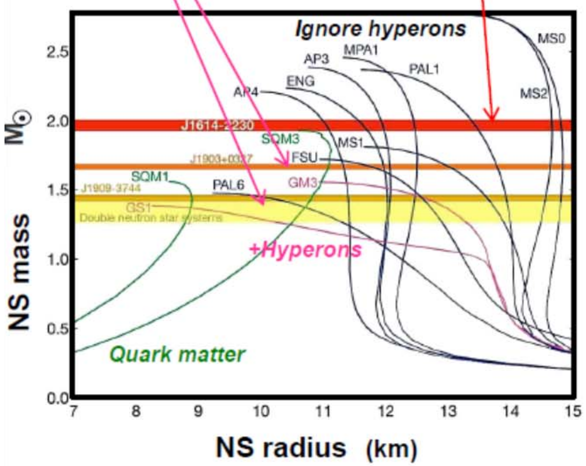


2) Hyperon Puzzle : slide by Tamura

“Hyperon puzzle” in neutron stars

- Hyperons (Λ at least) should appear at ρ ~ 2-3 ρ_0
  EOS’s with hyperons or kaons too soft => cannot support $M > 1.5\ M_{sun}$
- Heavy NS’s (~2.0 M_{sun}) were observed.

PSR J1614-2230 (2010) 1.97±0.04 M_{sun}
PSR J0348-0432 (2013) 2.01±0.04 M_{sun}



=> Unknown repulsion at high ρ

- Strong repulsion in three-body force including hyperons, NNN, YNN, YYN, YYY ?
  Chiral EFT is successful in NNN force. Extension to include hyperons requires **high quality YN scattering data.**
- Phase transition to quark matter ? (quark star or hybrid star)

We need to know YN, YY, $K^{bar}N$ interactions both <u>in free space</u> and <u>in nuclear medium</u>

- Color spin interaction - General remarks
  Tribaryon configuration (Aaron Park)

$$-\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s)$$

1) SU(2): Three nucleons

For SU(2) flavor: $-\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

For nucleon *K= -8*, *But* But Tribaryon (N=9) K>>0

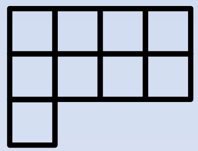
2) SU(3): Including hyperons

Flavor	$-\sum_{i<j}\lambda_i\lambda_j\sigma_i\cdot\sigma_j$				
	$S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$	$S=\frac{7}{2}$	$S=\frac{9}{2}$
1		-4	$\frac{8}{3}$		24
8	4	8	$\frac{44}{3}$	24	
10		20			
$\overline{\mathbf{10}}$		20			
27	24	28	$\frac{104}{3}$		
35	40				
$\overline{\mathbf{35}}$	40				
64		56			
V	-24	-24	-8	8	24

For SU(3) flavor: $-\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$

With one strangeness: least repulsive state is S=3/2, Flavor antidecuplet

$$K = 20 + \frac{20}{3}\left(1 - \frac{mu}{ms}\right) + 24 \gg 0$$



All tribaryon channel is very repulsive

→ Three Baryon force should be repulsive with or without strangeness

- Effects of intrinsic quark three-body force $f^{abc}F_1^a F_2^b F_3^c$ $d^{abc}F_1^a F_2^b F_3^c$

1) Basic notation

$$
\begin{aligned}
[F^\alpha, F^\beta] &= i f^{\alpha\beta\gamma} F^\gamma \\
\{F^\alpha, F^\beta\} &= \frac{1}{3}\delta^{\alpha\beta} + d^{\alpha\beta\gamma} F^\gamma .
\end{aligned}
$$

$$
\begin{aligned}
[\bar{F}^\alpha, \bar{F}^\beta] &= i f^{\alpha\beta\gamma} \bar{F}^\gamma \\
\{\bar{F}^\alpha, \bar{F}^\beta\} &= \frac{1}{3}\delta^{\alpha\beta} - d^{\alpha\beta\gamma} \bar{F}^\gamma .
\end{aligned}
$$

$$F^\alpha = \frac{\lambda^\alpha}{2},$$

$$\bar{F} = -F^* .$$

$$
\begin{aligned}
\bar{F}^{2,5,7} &= F^{2,5,7}, \\
\bar{F}^{rest} &= -F^{rest},
\end{aligned}
$$

$$
\begin{aligned}
f^{\alpha\beta\gamma} f^{\alpha\beta\rho} &= 3\delta^{\gamma\rho} \\
d^{\alpha\beta\gamma} d^{\alpha\beta\rho} &= \frac{5}{3}\delta^{\gamma\rho}
\end{aligned}
$$

$$C_1 = (F^\alpha)^2 = -\frac{2i}{3} f^{\alpha\beta\gamma} F^\alpha F^\beta F^\gamma = -\frac{2i}{3} f^{\alpha\beta\gamma} \bar{F}^\alpha \bar{F}^\beta \bar{F}^\gamma$$

$$
\begin{aligned}
C_2 &= d^{\alpha\beta\gamma} F^\alpha F^\beta F^\gamma = C_1(2C_1 - \frac{11}{6}) \\
\bar{C}_2 &= d^{\alpha\beta\gamma} \bar{F}^\alpha \bar{F}^\beta \bar{F}^\gamma = -C_1(2C_1 - \frac{11}{6})
\end{aligned}
$$

- f –type intrinsic quark three-body force $f^{abc}F_1^a F_2^b F_3^c$

1) Consider N_1 quarks and N_2 antiquarks so that $N= N_1+N_2$.

we want to calculate $T_1 = \sum_{i\neq j\neq k} f^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma$, where the subscripts denote "j" quark

Start from $F^\alpha = \sum_i^N F_i^\alpha$

$$C_1 = (F^\alpha)^2 = -\frac{2i}{3} f^{\alpha\beta\gamma} F^\alpha F^\beta F^\gamma = -\frac{2i}{3} f^{\alpha\beta\gamma} \bar{F}^\alpha \bar{F}^\beta \bar{F}^\gamma$$

$$\begin{aligned} \frac{3i}{2} C_1(N) &= \sum_{ijk} f^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma \\ &= \left(\sum_{i\neq j\neq k} + \sum_{two-are-equal} + \sum_{i=j=k} \right) f^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma \end{aligned}$$

$= T_1 + T_{1\text{-}2} + T_{1\text{-}3}$

- f –type intrinsic quark three-body force $f^{abc}F_1^a F_2^b F_3^c$

$$\frac{3i}{2}C_1(N) = \sum_{ijk} f^{\alpha\beta\gamma}F_i^\alpha F_j^\beta F_k^\gamma$$
$$= \left(\sum_{i\neq j\neq k} + \sum_{two-are-equal} + \sum_{i=j=k}\right) f^{\alpha\beta\gamma}F_i^\alpha F_j^\beta F_k^\gamma$$

$= T_1 + T_{1\text{-}2} + T_{1\text{-}3}$

$T_{1\text{-}2} =$

$$= \sum \left(f^{\alpha\beta\gamma}F_i^\alpha F_i^\beta F_k^\gamma + f^{\alpha\beta\gamma}F_i^\alpha F_k^\beta F_i^\gamma + f^{\alpha\beta\gamma}F_k^\alpha F_i^\beta F_i^\gamma\right)$$
$$= \sum \left(f^{\alpha\beta\gamma}F_i^\alpha F_i^\beta F_k^\gamma + f^{\alpha\beta\gamma}F_i^\alpha F_k^\beta F_i^\gamma + f^{\alpha\beta\gamma}F_i^\beta F_k^\alpha F_i^\gamma\right)$$
$$= \sum f^{\alpha\beta\gamma}\left(F_i^\alpha F_i^\beta F_k^\gamma\right)$$
$$= \sum f^{\alpha\beta\gamma}\left(\frac{1}{2}[F_i^\alpha F_i^\beta] + \frac{1}{2}\{F_i^\alpha F_i^\beta\}\right)F_k^\gamma$$
$$= \sum f^{\alpha\beta\gamma}\frac{i}{2}f^{\alpha\beta\rho}F_i^\rho F_k^\gamma$$
$$= \sum_{i\neq k}\frac{i3}{2}F_i^\gamma F_k^\gamma$$
$$= \frac{i3}{2}\left(F^2 - \sum F_i^2\right)$$
$$= \frac{i3}{2}\left(C_1(N) - NC_1(q)\right)$$

$T_{1\text{-}3} =$

$$= \sum_{i=j=k} f^{\alpha\beta\gamma}F_i^\alpha F_j^\beta F_k^\gamma$$
$$= N\frac{3i}{2}C_1(q)$$

Therefore $T_1 = \sum_{i\neq j\neq k} f^{\alpha\beta\gamma}F_i^\alpha F_j^\beta F_k^\gamma$ $= 0$

- d –type intrinsic quark three-body force $d^{abc}F_1^a F_2^b F_3^c$

1) Consider N_1 quarks and N_2 antiquarks so that $N = N_1 + N_2$.

we want to calculate $T_2 = \sum_{i\neq j\neq k} d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma$, where the subscripts denote “j” quark

Start from $F^\alpha = \sum_i^N F_i^\alpha$

$$C_2 = d^{\alpha\beta\gamma} F^\alpha F^\beta F^\gamma = C_1(2C_1 - \frac{11}{6})$$
$$\bar{C}_2 = d^{\alpha\beta\gamma} \bar{F}^\alpha \bar{F}^\beta \bar{F}^\gamma = -C_1(2C_1 - \frac{11}{6})$$

$$C_2(N) = \sum_{ijk} d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma = \left(\sum_{i\neq j\neq k} + \sum_{two-are-equal} + \sum_{i=j=k}\right) d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma$$

$= T_2 + T_{2-2} + T_{3-3}$

$$T_2 = \sum_{i\neq j\neq k} d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma$$

- Effects of intrinsic quark three-body force $d^{abc} F_1^a F_2^b F_3^c$

$$
\begin{aligned}
C_2(N) &= \sum_{ijk} d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma \\
&= \left(\sum_{i \neq j \neq k} + \sum_{two-are-equal} + \sum_{i=j=k} \right) d^{\alpha\beta\gamma} F_i^\alpha F_j^\beta F_k^\gamma
\end{aligned}
$$

$= T_2 + T_{2\text{-}2} + T_{3\text{-}3}$

$T_{2\text{-}2}$ should be divided according to quark and antiquarks

(a) All quarks are quarks. The we can divide it into three terms depending on where the different quark appears

$$
\begin{aligned}
2-1nd &= \sum \left(d^{\alpha\beta\gamma} F_i^\alpha F_i^\beta F_k^\gamma + d^{\alpha\beta\gamma} F_i^\alpha F_k^\beta F_i^\gamma + d^{\alpha\beta\gamma} F_k^\alpha F_i^\beta F_i^\gamma \right) && (37) \\
&= 3 \sum d^{\alpha\beta\gamma} \left(F_i^\alpha F_i^\beta F_k^\gamma \right) && (38) \\
&= 3 \sum d^{\alpha\beta\gamma} \left(\frac{1}{2} [F_i^\alpha F_i^\beta] + \frac{1}{2} \{F_i^\alpha F_i^\beta\} \right) F_k^\gamma && (39) \\
&= 3 \sum d^{\alpha\beta\gamma} \left(\frac{1}{6} \delta^{ii} \delta^{\alpha\beta} + \frac{1}{2} d^{\alpha\beta\rho} F_i^\rho \right) F_k^\gamma && (40) \\
&= 3 \sum \left(\frac{1}{6} \delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma + \frac{1}{2} d^{\alpha\beta\gamma} d^{\alpha\beta\rho} F_i^\rho F_k^\gamma \right) && (41) \\
&= 3 \sum \left(\frac{1}{6} \delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma + \frac{1}{2} \frac{5}{3} \delta^{\gamma\rho} F_i^\rho F_k^\gamma \right) && (42) \\
&= \sum \left(\frac{3}{6} \delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma + \frac{5}{2} F_i^\gamma F_k^\gamma \right) && (43) \\
&= \sum \left(\frac{3}{6} \delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma \right) + \frac{5}{2} \left(F^2 - \sum F_i^2 \right) && (44) \\
&= \sum^{N_1} \left(\frac{3}{6} \delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma \right) + \frac{5}{2} \left(C_1(N_1) - N_1 C_1(q) \right) && (45)
\end{aligned}
$$

(b) All quarks are antiquarks.

$$\begin{aligned}
2-2nd &= 3\sum\left(\frac{1}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma - \frac{1}{2}d^{\alpha\beta\gamma}d^{\alpha\beta\rho}F_i^\rho F_k^\gamma\right)\\
&= 3\sum\left(\frac{1}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma - \frac{1}{2}\frac{5}{3}\delta^{\gamma\rho}F_i^\rho F_k^\gamma\right)\\
&= \sum\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma - \frac{5}{2}\bar{F}_i^\gamma \bar{F}_k^\gamma\right)\\
&= \sum\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}\bar{F}_k^\gamma\right) - \frac{5}{2}\left(\bar{F}^2 - \sum \bar{F}_i^2\right) = \sum^{N_2}\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma\right) - \frac{5}{2}\Big(C_1(N_2) - N_2C_1(q)\Big)
\end{aligned}$$

(c) Two quarks are quarks.

$$\begin{aligned}
2-3nd &= 3\sum\left(\frac{1}{6}\delta^{ii}d^{\alpha\alpha\gamma}\bar{F}_k^\gamma + \frac{1}{2}d^{\alpha\beta\gamma}d^{\alpha\beta\rho}F_i^\rho \bar{F}_k^\gamma\right)\\
&= 3\sum\left(\frac{1}{6}\delta^{ii}d^{\alpha\alpha\gamma}\bar{F}_k^\gamma + \frac{1}{2}\frac{5}{3}\delta^{\gamma\rho}F_i^\rho \bar{F}_k^\gamma\right)\\
&= \sum\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}\bar{F}_k^\gamma + \frac{5}{2}F_i^\gamma \bar{F}_k^\gamma\right)
\end{aligned}$$

(d) Two quarks are anti-quarks.

$$\begin{aligned}
2-4nd &= 3\sum\left(\frac{1}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma - \frac{1}{2}d^{\alpha\beta\gamma}d^{\alpha\beta\rho}\bar{F}_i^\rho F_k^\gamma\right)\\
&= \sum\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma - \frac{5}{2}\bar{F}_i^\gamma F_k^\gamma\right)
\end{aligned}$$

Adding all $T_{2\text{-}2}$ =

$$\begin{aligned}
2nd &= \sum^{all}\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma\right) + \frac{5}{2}\Big(C_1(N_1) - N_1C_1(q)\Big)\\
&\quad -\frac{5}{2}\Big(C_1(N_2) - N_2C_1(q)\Big) + \sum\left(\frac{5}{2}F_i^\gamma \bar{F}_k^\gamma - \frac{5}{2}\bar{F}_i^\gamma F_k^\gamma\right)\\
&= \sum^{all}\left(\frac{3}{6}\delta^{ii}d^{\alpha\alpha\gamma}F_k^\gamma\right) + \frac{5}{2}\Big(C_1(N_1) - C_1(N_2)\Big) - \frac{5}{2}C_1(q)\Big(N_1 - N_2\Big)
\end{aligned}$$

$T_{3\text{-}3}$ =

$$= \sum_{i=j=k} d^{\alpha\beta\gamma}\Big(F_i^\alpha F_j^\beta F_k^\gamma + \bar{F}_i^\alpha \bar{F}_j^\beta \bar{F}_k^\gamma\Big)$$
$$= (N_1 - N_2)C_1(q)\Big(2C_1(q) - \frac{11}{6}\Big)$$

Summing all the contributions, we find

$$T_2 = C_1(N)\Big(2C_1(N) - \frac{11}{6}\Big) - \frac{5}{2}\Big(C_1(N_1) - C_1(N_2)\Big) - (N_1 - N_2)C_1(q)\Big(2C_1(q) - \frac{13}{3}\Big) - \sum\Big(\frac{3}{6}\delta^{ii} d^{\alpha\alpha\gamma} F_k^\gamma \Big($$

When we take the matrix element, the last term vanishes. Furthermore, for color singlet states, $C_1(N) = 0$, and also, the color state of quark and that of antiquark should be the same as the total is a color singlet. Therefore, we have

$$T_2 = -(N_1 - N_2)C_1(q)\Big(2C_1(q) - \frac{13}{3}\Big). \qquad (68)$$

Therefore, for pentaqurk, it is the same as the baryon. For tetraquark, it is zero.

Summary - from quark picture

1] Compact multiquark states can be understood from color spin flavor wave function:

- A strong attractive short range interaction is needed in the SU(3) broken limit
- Heavy quarks are needed to reduce extra Kinetic energy

→ Pc, d*, X(3872), Zc: unlikely to be compact multiquark states as there are no strong attraction in these channels at compact configurations

→ Tcc could be strongly bound $\mathrm{T}_{cc}^{1}(\mathrm{ud}\bar{\mathrm{c}}\bar{\mathrm{c}}) \rightarrow (\bar{D}^{0} + D^{*-}) \rightarrow K^{+}\pi^{-} + K^{+}\pi^{-}\pi^{-}$

threshold	decay mode	lifetime
$M_{T_{cc}} > M_{D^*} + M_D$	$D^{*-}\bar{D}^0$	hadronic decay
$2M_D + M_\pi < M_{T_{cc}} < M_{D^*} + M_D$	$\bar{D}^0\bar{D}^0\pi^-$	hadronic decay
$M_{T_{cc}} < 2M_D + M_\pi$	$D^{*-}K^+\pi^-$, $D^{*-}K^+\pi^+\pi^-\pi^-$	0.41×10^{-12} sec.

2] Thee body nuclear force must be repulsive at short distance

→ Magnitude of repulsion for given quantum number can be calculated

→ Related to nuclear matter, Hyperon puzzle in neutron star

Summary : constituent quark model and compact Multiquark states

- Recently discovered Multiquark states, D*, Pc, X(3872), Z are most likely molecular states

- A compact multiquark candidate: Tcc (cc ubar dbar)

- Nuclear two-body and three-body repulsion can be understood from Pauli Principle and color spin interaction

- Exotica measurements from heavy ion collision could discriminate the structure between a compact multiquark and a molecular configuration