Chiral symmetry for hadrons

Atsushi Hosaka
RCNP/ASRC
Lecture at TPI/ASRC, JAEA
May 10, 2017

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1. Introduction and Basics
   Hadrons, chiral symmetry and SSB
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1. Introduction

General references
S. Coleman, Aspects of symmetry, Cambridge, 1985

A system is chiral if it is distinguishable from its mirror image
= with degenerate energies
Hadrons: *Composites* of quarks and gluons of QCD

\[
\mathcal{L} = \sum_f \overline{q}_f (i \mathcal{D} - m_f) q_f - \frac{1}{2} \text{tr} G_{\mu \nu} G^{\mu \nu}
\]

- **Color SU(3) gauge theory** of the standard model
- **6 flavors**, u, d, s (~ light) // c, b, t (~ heavy)
- **Non-perturbative** (large coupling) at low energies, for u, d, s
- Confinement, **SSB** and mass generation → *Millennium problem*
- Hadrons emerge as **baryons** (qqq) and **mesons** (q\(\bar{q}\))
- **Lattice QCD** is the first principle method
- Successful for **static properties**; masses and FF
- But not yet for **dynamic properties**, resonances and reactions
- Important ideas and methods are explored by **effective theories**
- **Chiral symmetry** and **SSB** explain many hadron properties
Hadrons of opposite parities should degenerate
BUT NOT
Indicating SSB

Are hadrons $qqq$ or $q\bar{q}$?

<table>
<thead>
<tr>
<th>Mesons</th>
<th>Particle Data Group</th>
<th>Baryons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qq$</td>
<td>$qqq$</td>
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<tr>
<td>$cq$</td>
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</tbody>
</table>

25 kinds

22 kinds
# Superconductor vs Hadron

<table>
<thead>
<tr>
<th></th>
<th>Super conductor</th>
<th>Hadron systems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>System</strong></td>
<td>Metal, conductor</td>
<td>Vacuum</td>
</tr>
<tr>
<td></td>
<td>Many electrons</td>
<td>Many quarks and anti-quarks</td>
</tr>
<tr>
<td><strong>Particles and Interaction</strong></td>
<td>Electron, Phonons</td>
<td>Quark, Gluons, Anti-quark</td>
</tr>
<tr>
<td><strong>Order parameter</strong></td>
<td>$\langle ee \rangle$</td>
<td>$\langle \bar{q}q \rangle$</td>
</tr>
<tr>
<td><strong>Mass Gap</strong></td>
<td>Majorana mass</td>
<td>Dirac mass</td>
</tr>
<tr>
<td><strong>Zero mode</strong></td>
<td>Super fluid</td>
<td>Pions</td>
</tr>
<tr>
<td><strong>Two phases</strong></td>
<td>Magnetic, Normal</td>
<td>Super, Color, Normal</td>
</tr>
</tbody>
</table>
Chirality of fermions

Related to the representations of the Lorentz group: Relativity

Rotation and Boost, SL(2,C) Generators, $R$ and $B$

$(R+iB)$: Right handed fermion $\psi = \psi_R + \psi_L$

$(R-iB)$: Left handed fermion $\psi_{R,L} \equiv \frac{1 \pm \gamma_5}{2} \psi$

Two comp. each Four comp.

- $R$ and $L$ numbers are conserved separately $\rightarrow$ chiral symmetry
- Fermion number conserving mass term is possible by mixing the $R$ and $L$ components $\Rightarrow$ breaks Chiral symmetry

$L = \bar{\psi} \left( i \slashed{\partial} - m \right) \psi = \bar{\psi}_L i \slashed{\partial} \psi_L + \bar{\psi}_R i \slashed{\partial} \psi_R - m \bar{\psi}_L \psi_R - m \bar{\psi}_R \psi_L$

- This is the reason that Dirac equation needs four components
Physically,

- $L$ and $R$ components correspond to **helicity** states
- For a particle moving at the speed of light, $L$ and $R$ are conserved

**Left** helicity

\[ h = -\frac{1}{2} \]
\[ \nu = c \]
\[ s = +\frac{1}{2} \]

**Right** helicity

\[ h = +\frac{1}{2} \]
\[ \nu = c \]
\[ s = +\frac{1}{2} \]

- Left transformation:
  \[ \psi_L \rightarrow e^{i\theta_L} \psi_L \xrightarrow{\theta_L=\pi} -\psi_L \]
- Parity eigenstates (under $L \Leftrightarrow R$)

\[
\begin{aligned}
\psi_+ &= \psi_L + \psi_R \\
\psi_- &= -\psi_L + \psi_R
\end{aligned}
\]

\[ \psi_+ \xrightarrow{L\ \text{transformation}} \psi_- \]
2. The NJL Model

The NJL Model

- A model for relativistic and \textit{massless fermion}
- Assign particle numbers of an internal symmetry for \( L \) and \( R \) separately

\[ SU(N_f)_L \times SU(N_f)_R \rightarrow \left( \frac{1}{2}, 0 \right) + \left( 0, \frac{1}{2} \right) = \psi \]

- With a chiral invariant \textit{four point (zero range) interaction}

\[ L_{NJL} = \bar{\psi} i \not{\partial} \psi + \frac{g}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right] \]

\((\bar{\psi} \psi, \bar{\psi} i \gamma_5 \tau \psi)\) behaves as an \( O(4) \) vector of \( SU(2_f)_L \times SU(2_f)_R \)

\[(\bar{\psi} \psi)^2 \rightarrow \text{Gap equation for mass generation} \]

\[ (\bar{\psi} i \gamma_5 \psi)^2 \rightarrow \text{Equation for massless pion, NG boson} \]
Mean field in **scalar channel** - Mass generation -

\[ L_{NJL} = \bar{\psi} i \not{\partial} \psi - \frac{g}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] \]

\[ = \bar{\psi} i \not{\partial} \psi - \frac{g}{2} \bar{\psi} \left[ (\bar{\psi} \psi) + (\bar{\psi} i \gamma_5 \psi) i \gamma_5 \right] \psi \]

\[ \rightarrow \bar{\psi} i \not{\partial} \psi - \bar{\psi} g \left\{ \frac{\langle \bar{\psi} \psi \rangle}{m} \right\} \psi \]

Symmetric and \( m = 0 \)

\[ \text{Gap equation} \]

\[ g \langle \bar{\psi} \psi \rangle = m \]

\[ m = 2g \int_0^\Lambda \frac{d^3p}{(2\pi)^3} \frac{m}{\sqrt{p^2 + m^2}} \]

\[ \text{Broken and } m \neq 0 \]

- There is always a **trivial solution** \( m = 0 \)
- **Nontrivial one of** \( m > 0 \) **is possible for large** \( g \) **and** \( \Lambda \)
  \[ \rightarrow \text{Spontaneous breaking of symmetry } SSB \]
Schroedinger eq. in pion channel

• Possible to solve the equation for the pion channel

\[ L_{NJL} = \bar{\psi} i \not{\partial} \psi - \frac{g}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] \]

• One can write down the Schrodinger equation

\[ 2\sqrt{p^2} + m^2 \Psi(p) - 4g \int \frac{d^3q}{(2\pi)^3} \Psi(q) = E \Psi(p) \]

• For \( E = 0 \), this leads to the same eq. as the gap equation

**Massless**

• The equivalence is due to chiral symmetry

• One can write down potential and see SSB explicitly

NJL model is a simple model of SSB with mass generation and NG mode
3. Pion (NG boson) interaction
Low energy theorems

• **PCAC (~ CAC)**
  Partially conserved axial current

• **Goldberger-Treiman relation**
  Two terms of the neutron beta decay relates $f_\pi$ and $g_A$
  \[
  n \quad \rightarrow \quad p \quad e \quad v_e \quad = \quad \gamma \quad \rightarrow \quad p \quad \pi^- \quad \rightarrow \quad n \quad \pi^- \quad \rightarrow \quad p \quad \pi^- \quad \rightarrow \quad p \quad g_A \quad g_{\pi NN} \]

• **Kroll-Ruderman theorem**
  Determines the photoproduction of the pion

• **Tomozawa-Weinberg theorem**
  Determines the pion-matter interaction

  This is one of main issues in this talk
Picture for deriving the Tomozawa-Weinberg int.

\[ \pi N \text{ scattering} \]

\[ I \sim \int d^4x d^4y \ e^{-i\vec{q}_1 y} e^{+i\vec{q}_2 x} \ \langle N(p_2)|T \partial^\mu A^b_\mu(x) \partial^\nu A^a_\nu(y)|N(p_1)\rangle \]

\[ I = I_1 + I_2 + I_3 , \]

\[ I_1 = - \int d^4x d^4y \ e^{+i\vec{q}_2 y} e^{-i\vec{q}_1 x} \ \delta(x_0 - y_0) \langle N(p_2)| \left[A^b_0(y), \partial^\nu A^a_\nu(x)\right]|N(p_1)\rangle , \]

\[ I_2 = - \int d^4x d^4y \ e^{+i\vec{q}_2 y} e^{-i\vec{q}_1 x} \ \partial^\mu \partial^\nu \langle N(p_2)| \left[A^b_\mu(y), A^a_\nu(x)\right]|N(p_1)\rangle , \]

\[ I_3 = -i \int d^4x d^4y \ e^{+i\vec{q}_2 y} e^{-i\vec{q}_1 x} \ q^\mu \delta(x_0 - y_0) \langle N(p_2)| \left[A^b_0(y), A^a_\mu(x)\right]|N(p_1)\rangle . \]

\[ T = -i \left( \frac{2m_\pi}{f^2_\pi} \right)^2 \vec{I}_\pi \cdot \vec{I}_N , \quad (I^a_\pi)_{bc} = i \epsilon_{abc} , \quad \vec{I}_N = \frac{\vec{\tau}}{2} \]

\[ a_{2I} = - \frac{m_\pi}{8\pi f^2_\pi} \left(1 + \frac{m_\pi}{M}\right)^{-1} \ 2\vec{I}_\pi \cdot \vec{I}_N \]

\[ = - \frac{m_\pi}{8\pi f^2_\pi} \left(1 + \frac{m_\pi}{M}\right)^{-1} \ (I(I+1) - I_N(I_N + 1) - 2) \]

Isospin current \[ \rightarrow \] \[ \rho\text{-exchange} \]

\[ a_1 = 0.2 m_\pi^{-1} , a_3 = -0.1 m_\pi^{-1} \]

\[ a^{\text{exp}}_1 = 0.173 m_\pi^{-1} , a^{\text{exp}}_3 = -0.101 m_\pi^{-1} \]
Practical method ~ Effective Lagrangians

- **Non-linear sigma model** for the pion

\[ L = \frac{f_\pi^2}{4} \text{tr} \left( U^\dagger \partial_\mu U \right) \left( U^\dagger \partial^\mu U \right) \quad U = \exp \left( i \vec{\tau} \cdot \vec{\pi} / f_\pi \right) \]

The TW interaction by expanding in powers of \( \pi \)
Practical method ~ Effective Lagrangians

- **Non-linear sigma model** for the pion and nucleon

\[
\begin{align*}
L_{\text{sigma}} &= \bar{q} i \partial q - gf_\pi \bar{q}(\sigma + i \bar{n} \cdot \pi \gamma_5) / f_\pi q \quad (\sigma + i \bar{n} \cdot \pi \gamma_5) / f_\pi = U \\
&= \bar{q} i \partial q - gf_\pi \bar{q}U^{1/2}U^{1/2} q \\
&\rightarrow \bar{\psi} i \bar{D} \psi - gf_\pi \bar{\psi} \psi
\end{align*}
\]

\[\nu_\mu = -\frac{i}{2}((\partial_\mu U^{-1/2})U^{1/2} + (\partial_\mu U^{1/2})U^{-1/2}) \sim [\partial_\mu \pi, \pi]\]

\[L_{\text{int}} \sim \bar{\psi} i \bar{D} \psi \sim \bar{\psi} [\partial_\mu \pi, \pi] \gamma^\mu \psi\]

The leading term is \(\mu = 0\), proportional to the mass of the pion

- Extended SU(3) (Kaon) interaction is \(\sim m_K\), and with **CG coeff.**
- Strong attraction for \(\bar{K}\) (not for \(K\))
- Accommodates the hadronic molecule \(\sim \Lambda(1405) \sim \bar{K}N\)
- **Problem:** point-like interaction (s-wave only)
POSSIBLE RESONANT STATE IN PION-HYPERON SCATTERING

R. H. Dalitz and S. F. Tuan
Enrico Fermi Institute for Nuclear Studies and Department of Physics,
University of Chicago, Chicago, Illinois
(Received April 27, 1959) PhysRevLett.2.425

will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the $K^-\rho$ (c.m.) threshold energy. In the present discussion, charge-

This is perhaps the first article which implied a hadronic molecule, $K^{\text{bar}}_N$
Well established light baryon resonances measured from the corresponding ground states


Negative parity baryons
Quark excitations near **thresholds** reform molecules

\[ \sim 500 \text{ MeV} \]

sufficient to create \( q\bar{q} \)

More in the presence of **heavy quarks**

\[ \implies \]

Recent findings of exotic hadrons, \( X, Y, Z \)

For molecular states, **hadron-hadron interaction** is the key inputs

NN: many **experiments** and recent **lattice simulations**

\( \pi(K)N \): Low energy theorems, but limited applicability

Three days lectures at U. Tokyo
Neutral $X$, $Y$, $Z_0$ states

Figure 1: Level diagram for the neutral $c\bar{c}$ sector. Conventional, observed $c\bar{c}$ states are solid (black) lines labeled by Greek letters, the lowest predicted but not yet observed conventional $c\bar{c}$ states are labeled with dashed (blue) lines (the clusters indicating predictions of several different model calculations), and the solid (red) lines labeled by $X$, $Y$, or $Z$ indicate exotic charmoniumlike candidates. Each measured state mass, including its central value and uncertainty, is presented as a rectangle (lines simply indicating very thin rectangles). Relevant thresholds are given by gray dashed lines; if a gray dotted line is nearby, it indicates the threshold isospin partner to the labeled dashed line. In some cases, likely quantum numbers have been assigned to states for which some uncertainty remains; this is the case, for example, for the $X(3940)$ and $X(4160)$ candidates. The actual known quantum numbers are listed in Table 2.
Charged Z states

Figure 2: Charged charmoniumlike states, both bosonic and fermionic. Each measured state mass, including its central value and uncertainty, is presented as a rectangle. Relevant thresholds are given by gray dashed lines; if a gray dotted line is nearby, it indicates the threshold isospin partner to the labeled dashed line.
4. Skyrmions and $\bar{K}N$ interactions

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

Skyrmions

Baryons as *pionic solitons with structure of finite size*

How fermions emerge from bosons (the pions)?
How they can be quantized as nucleons?
How they (nucleons) interact with mesons (kaons)?

Relation with QCD  E. Witten, Nucl. Phys. B 223, 422; 433 (1983)

QCD becomes a theory of weakly interacting mesons for large $N_c$
Nucleons emerge as solitons of a pion field theory
A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME
A.E.R.E., Harwell, England

Received 29 September 1961

fields interacting through various types of coupling. The objective is the construction of a theory of self-interacting (boson) meson fields, which will admit states that have the phenomenological properties of (fermion) particles, interacting with mesons.

This programme is the obverse of the more fashionable endeavour to reduce the truly elementary particles to a set of spinor fields, out of which everything can be built by simple conjunction. It is a priori much less reasonable because, in particular, it is more difficult to construct half-integral representations of rotation groups out of integral than conversely; indeed it is patently impossible to do this within the limitations of a polynomial expansion. The hope that remains is that the particle-like states will be of a kind that cannot be reached by perturbation theory, and which cannot necessarily be discounted by general arguments. In the type of theory we are using the particle-like solutions have this character, arising from the fact that periodic, transcendental, functions

References

1) T. H. R. Skyrme, Nuclear Physics 31 (1962) 550
Recipe 1: To make a nucleon

1. Prepare a **two-term Lagrangian** of the pion
   Positive and negative pressure terms \( \sim \) balance
   \[
   L = \frac{f^2_\pi}{4} \text{tr} R_\mu R^\mu + \frac{1}{32e^2} \text{tr} \left[ R_\mu, R^\nu \right]^2
   \]

2. Find a **classical hedgehog solution** of \( B = 1 \) (winding number)
   Minimize the spin and isospin energy, \( \text{spin+isospin} = 0 \)
   \[
   U = \exp(i \vec{\tau} \cdot \vec{\pi} / f_\pi) \quad \rightarrow \quad U_H = \exp(i \vec{\tau} \cdot \hat{r} F(r) / f_\pi)
   \]

3. Quantize its **spin-isospin rotations**
   **Nucleons as rotating hedgehogs** (originally by Pauli and Dankoff)
   \[
   U_H (\varphi) \rightarrow U_H (t, \varphi) = A (t) \exp \left[ i \tau_a R_{ab} (t) \hat{r}_b F(r) \right] A^\dagger (t)
   \]

Isospin rotation

Spin rotation
The Pseudoscalar Meson Field with Strong Coupling

W. PAULI, Institute for Advanced Study, Princeton, New Jersey

AND

S. M. DANCOFF, Institute for Advanced Study, Princeton, New Jersey and University of Illinois, Urbana, Illinois

(Received June 23, 1942)

The present paper treats the symmetrical and charged pseudoscalar theories of the meson field, using the strong coupling approximation; it restricts itself to the case of a single source. The energy levels of the excited states of the heavy particle and the scattering cross section for free mesons are computed by wave mechanical methods. An expression is also obtained for the magnetic moment of the proton or neutron. While the scattering cross section can, with reasonable assumptions, be brought into agreement with experimental values, the results for the magnetic moment are qualitatively at variance with the known values in that equal and opposite moments are predicted for proton and neutron.
Toward the Nucleon and Kaon interaction

**Recipe 2a: Callan-Klebaonov, 1/N_c, strong copying limit, PAV**


1. Prepare a hedgehog soliton

\[ U_H = \exp(i\tilde{\tau} \cdot \vec{r}F(r) / f_\pi) \]

2. Introduce a kaon around the hedgehog, find the wave equation and bound-state solutions (exists!)

\[ U_{HK} = \sqrt{U_H} U_K \sqrt{U_H} \]

\[ -\frac{1}{r^2} \frac{d}{dr} \left( r^2 h(r) \frac{dk_i^\alpha(r)}{dr} \right) - E^2 f(r) k_i^\alpha(r) + (m_K^2 + V(r)) k_i^\alpha(r) = 0 \]

3. Quantize its spin-isospin for hyperons, \( \Lambda, \Sigma, \ldots \)

\[ U_{HK} \rightarrow A(t)U_{HK}A(t)^\dagger \]

This recipe follows the 1/N_c expansion, kaon around the hedgehog → *transmutation of quantum numbers*

*The kaon (boson) behaves as a strange quark (fermion)*
Toward the Nucleon and Kaon interaction

Recipe 2b: Ezoe-Hosaka, violates $1/N_c$, weak copying limit, PBV
Ezoe and Hosaka, Phys. Rev. D 4, no. 3, 034022 (2016); arXive:1703.01004

1. Prepare a nucleon as a rotating hedgehog

$$U_N = A(t)\exp(i\vec{\tau} \cdot \vec{r}F(r)/f_\pi)A(t)$$

2. Introduce a kaon around 1, find the wave equation

$$U_{NK} = \sqrt{U_N} U_K \sqrt{U_N} \quad -\frac{1}{r^2}\frac{d}{dr}\left( r^2 h(r) \frac{dk_i^\alpha(r)}{dr} \right) - E^2 f(r) k_i^\alpha(r) + (m_K^2 + V(r)) k_i^\alpha(r) = 0$$

3. Find KN interactions from the wave equation

$$V(r) = V_{C}^{I=0}(r) + V_{C}^{I=1}(r) + V_{LS}^{I=0}(r) + V_{LS}^{I=1}(r)$$

No transmutation of quantum numbers
Physical kaon and nucleon interaction is obtained
\[ I = 0, \ L = 0, \ J = 1/2 \]

\[ I = 0, \ L = 1, \ J = 3/2 \]

- Qualitatively consistent with Chiral TW theorem
- Finite range reflecting the nucleon structure
- State dependent
- S-wave interaction has an attractive pocket
- Accommodates S-wave weakly bound state for \( \Lambda(1405) \)
- Repulsive core which is the finding in the Skyrme model

Ezoe and Hosaka, arXive:1703.01004
Summary and other issues

Chiral symmetry
• Important ingredient of QCD and hadron dynamics
• Relates the mass generation and the NG modes
• Describes the low energy hadron dynamics
• Kaon (NG boson) interaction has been derived
• May explain the exotic structure of Λ(1405) as molecule

What are not discussed
• Chiral multiples of different parities, mesons and baryons
• Recovery of the broken symmetry, Phase of QCD
• Property changes of hadrons
• Hadrons from Holography

Liu, Zahed, arXive:1704.03412