Chiral symmetry for hadrons

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Lecture at TPI/ASRC, JAEA
May 10, 2017

Contents

1. Introduction

General references

S. Weinberg, The Quantum Theory of Fields II, Cambridge, 1995
S. Coleman, Aspects of symmetry, Cambridge, 1985
Y. Nambu and Jona-Lasinio, Phys. Rev. 122, 345; 124, 246 (1961)
A. Hosaka and H. Toki, Quarks, Baryons and Chiral Symmetry, World Scientific, 2001.

A system is chiral if it is distinguishable from its mirror image
= with degenerate energies

Hadrons: ***Composites*** of quarks and gluons of QCD

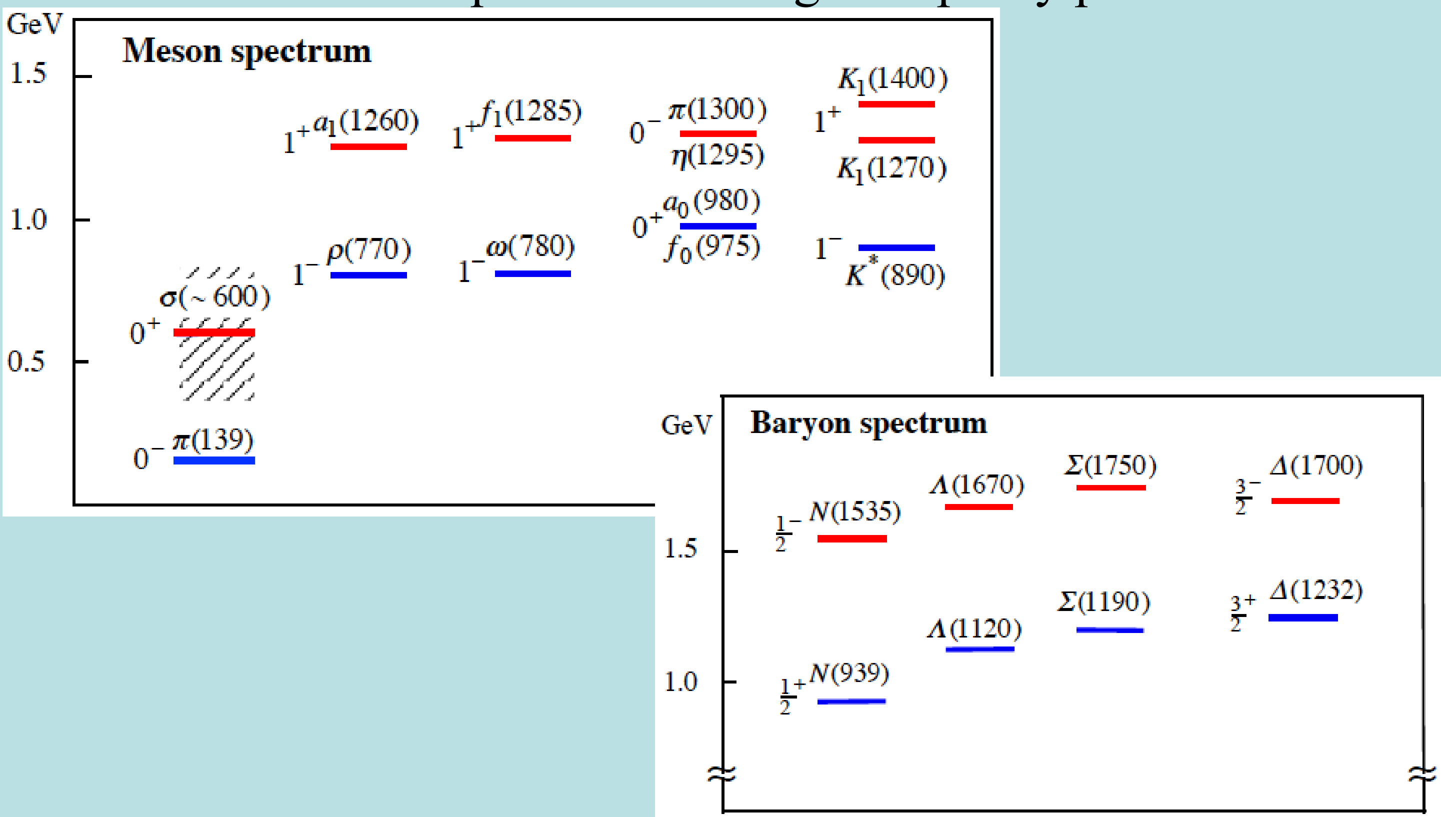
$$\mathcal{L} = \sum_f \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{2} \mathrm{tr}\, G_{\mu\nu} G^{\mu\nu}$$

- ***Color SU(3) gauge theory*** of the standard model
- ***6 flavors***, **u, d, s (~ light)** // c, b, t (~ heavy)
- ***Non-perturbative*** (large coupling) at low energies, for u, d, s
- Confinement, ***SSB*** and mass generation →***Millennium problem***
- Hadrons emerge as ***baryons*** (***qqq***) and ***mesons*** ($q\bar{q}$)
- ***Lattice QCD*** is the first principle method
- Successful for ***static properties***; masses and FF
- But not yet for ***dynamic properties***, resonances and reactions
- Important ideas and methods are explored by ***effective theories***
- ***Chiral symmetry*** and ***SSB*** explain many hadron properties

Hadrons of opposite parities should degenerate
BUT NOT
Indicating SSB

Jido,Oka,Hosaka, PTP106 (2001) 823

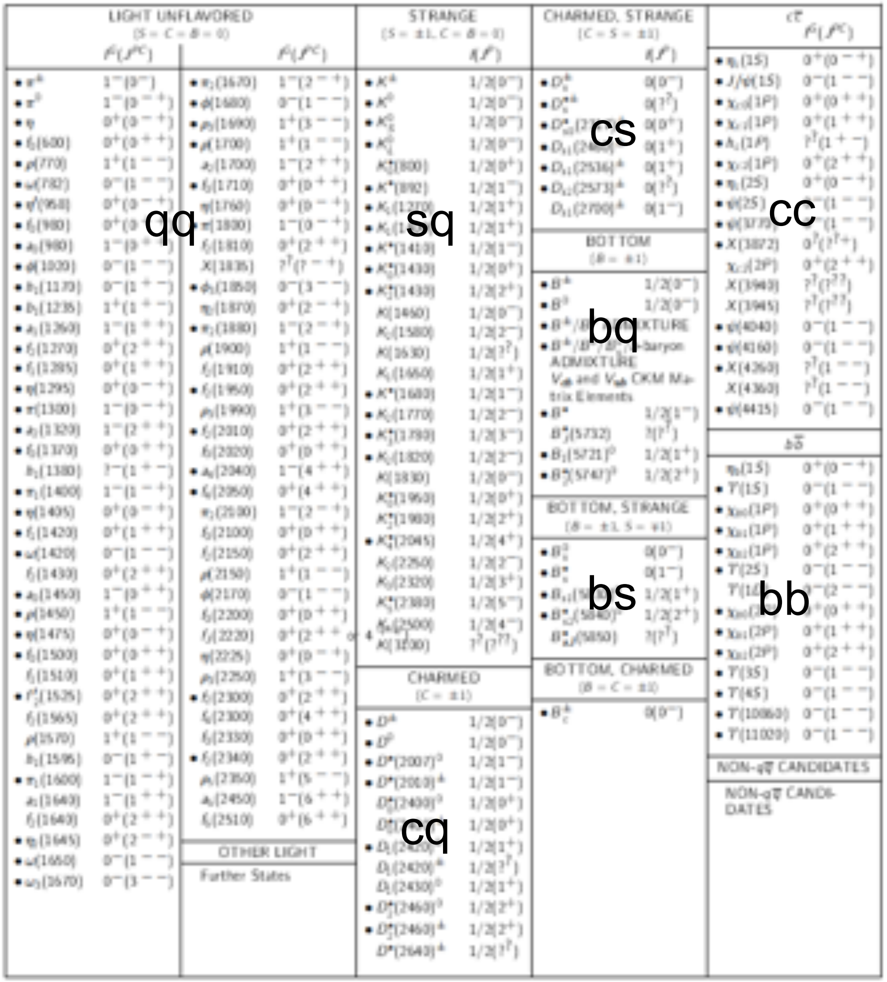
Masses of positive and negative parity particles



Are hadrons qqq or $q\bar{q}$?

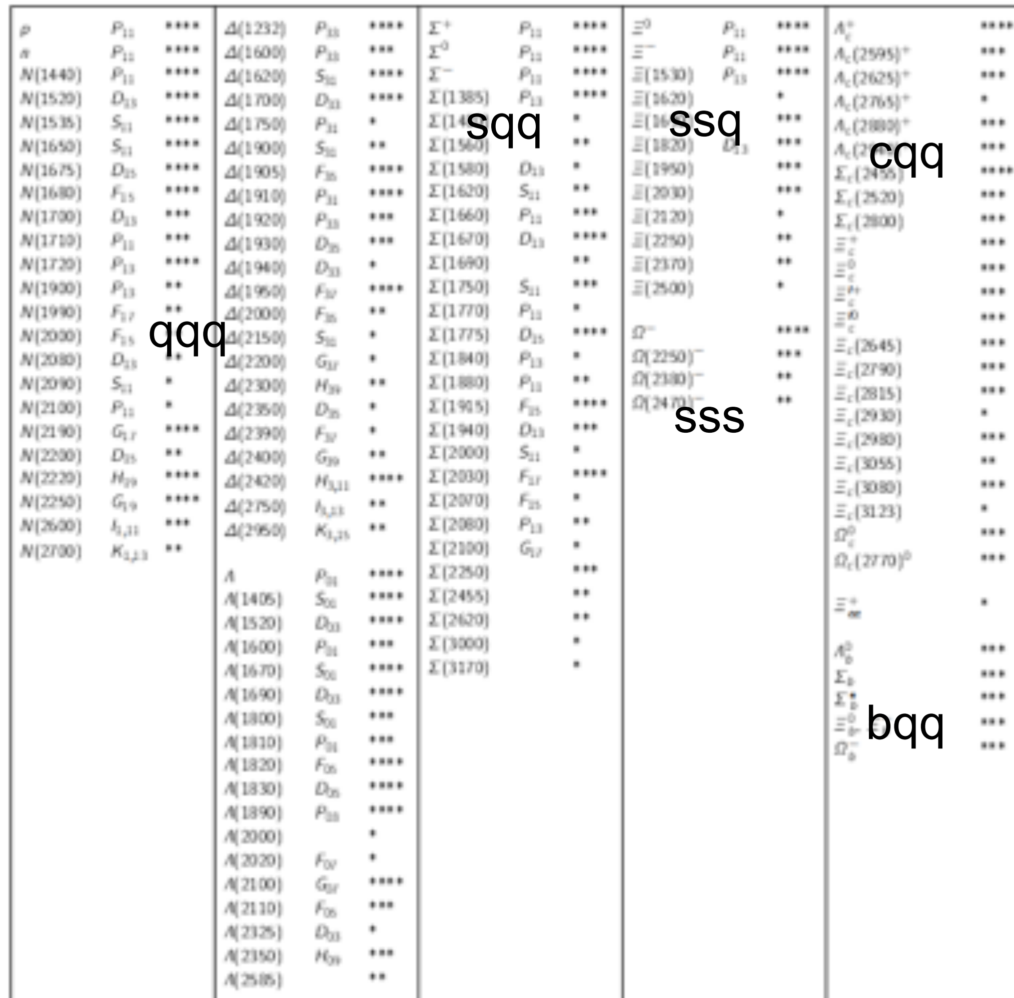
Particle Data Group

Mesons



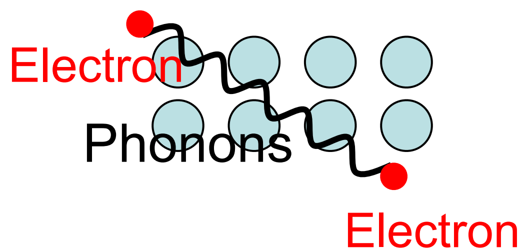
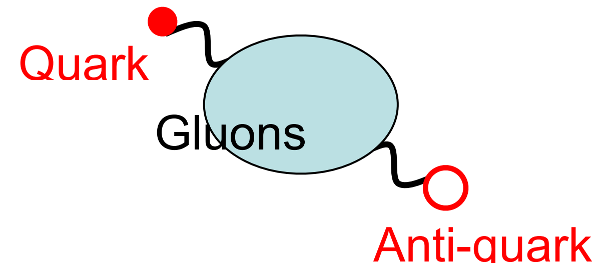
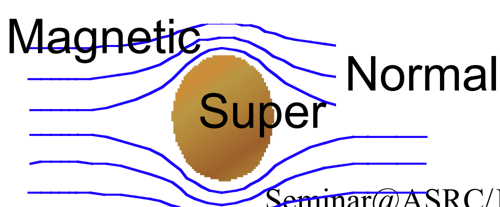
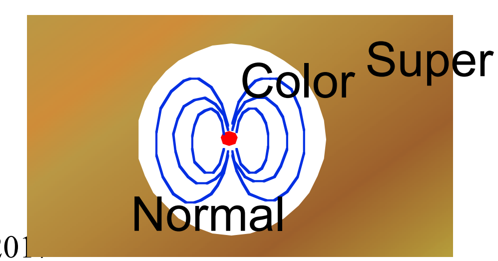
25 kinds

Baryons



22 kinds

Superconductor vs Hadron

<table>
<tr><th></th><th>Super conductor</th><th>Hadron systems</th></tr>
<tr><td>System</td><td>Metal, conductor
Many electrons</td><td>Vacuum
Many quarks and anti-quarks</td></tr>
<tr><td>Particles and Interaction</td><td>
</td><td>
</td></tr>
<tr><td>Order parameter</td><td>$\langle ee \rangle$</td><td>$\langle \bar{q}q \rangle$</td></tr>
<tr><td>Mass Gap</td><td>Majorana mass</td><td>Dirac mass</td></tr>
<tr><td>Zero mode</td><td>Super fluid</td><td>Pions</td></tr>
<tr><td>Two phases</td><td>
</td><td>
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Chirality of fermions

Related to the representations of the Lorentz group: Relativity

Rotation and **B**oost, SL(2,C) Generators, R and B

($R+iB$): Right handed fermion

($R-iB$): Left handed fermion

$$\psi = \psi_R + \psi_L$$

$$\psi_{R,L} \equiv \frac{1 \pm \gamma_5}{2} \psi$$

Two comp. each Four comp.

- *R* and *L* ***numbers*** are conserved separately → ***chiral symmetry***
- Fermion number conserving *mass term* is possible by mixing the *R* and *L* components => breaks Chiral symmetry

$$L = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi = \bar{\psi}_L i\partial\!\!\!/\psi_L + \bar{\psi}_R i\partial\!\!\!/\psi_R - m\bar{\psi}_L\psi_R - m\bar{\psi}_R\psi_L$$

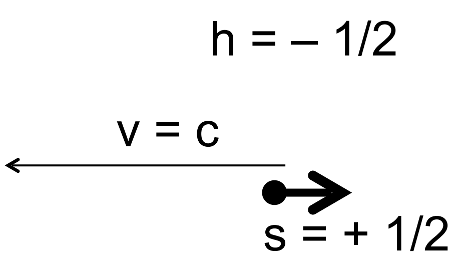
- This is the reason that Dirac equation needs four components

Physically,

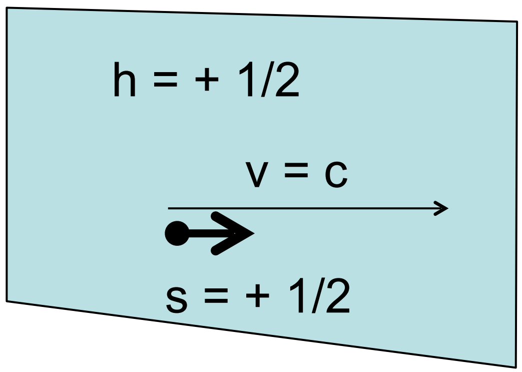
- *L* and *R* components correspond to ***helicity*** states
- For a particle moving at the speed of light, *L* and *R* are conserved

massless

Left helicity



Right helicity



- Left transformation: $\psi_L \to e^{i\theta_L}\psi_L \xrightarrow{\theta_L=\pi} -\psi_L$
- Parity eigenstates (under $L \rightleftarrows R$)

$$\left.\begin{aligned} \psi_+ &= \psi_L + \psi_R \\ \psi_- &= -\psi_L + \psi_R \end{aligned}\right\} \qquad \psi_+ \xrightarrow{L\ trensformation} \psi_-$$

2. The NJL Model

Y. Nambu and Jona-Lasinio, Phys. Rev. 122, 345; 124, 246 (1961)

The NJL Model

- A model for relativistic and ***massless fermion***
- Assign particle numbers of an internal symmetry for ***L*** and ***R*** separately

$$SU(N_f)_L \times SU(N_f)_R \xrightarrow{\text{Fundamental representation}} \left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)=\psi$$

- With a chiral invariant *four point (zero range) interaction*

$$L_{NJL} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{g}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i\gamma_5 \vec{\tau}\psi\right)^2\right]$$

$\left(\bar{\psi}\psi, \bar{\psi}i\gamma_5\vec{\tau}\psi\right)$ behaves as an $O(4)$ vector of $SU(2_f)_L$ x $SU(2_f)_R$

$\left(\bar{\psi}\psi\right)^2 \rightarrow$ ***Gap equation*** for mass generation

↕ ***Equivalent*** due to chiral symmetry

$\left(\bar{\psi}i\gamma_5\psi\right)^2 \rightarrow$ ***Equation for massless pion***, **NG boson**

Mean field in scalar channel - Mass generation -

$$L_{NJL} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{g}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right]$$

$$= \bar{\psi} i \partial\!\!\!/ \psi - \frac{g}{2} \bar{\psi} \left[(\bar{\psi}\psi) + (\bar{\psi} i \gamma_5 \psi) i \gamma_5 \right] \psi$$

$$\rightarrow \bar{\psi} i \partial\!\!\!/ \psi - \bar{\psi} g \langle \bar{\psi}\psi \rangle \psi$$

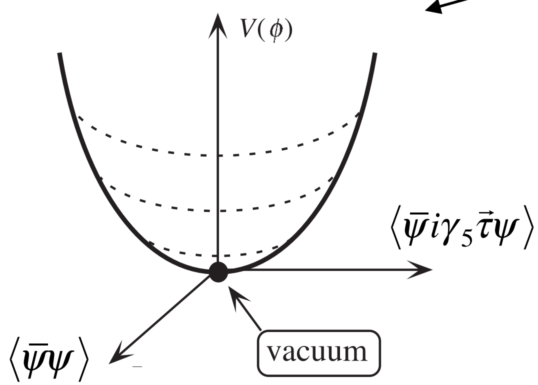
m

Gap equation

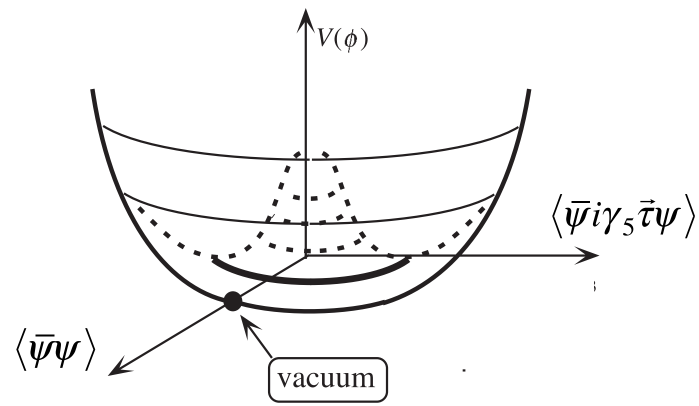
$$g \langle \bar{\psi}\psi \rangle = m$$

$$\rightarrow \quad m = 2g \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{m}{\sqrt{\vec{p}^2 + m^2}}$$

Symmetric and $m = 0$



Broken and $m \neq 0$



- There is always a trivial solution $m = 0$
- Nontrivial one of $m > 0$ is possible for large g and Λ

  → Spontaneous breaking of symmetry ***SSB***

Schrodinger eq. in pion channel

土岐 、保坂、相対論的他体系としての原子核 、大阪大学出版会 (2011)

- Possible to solve the equation for the pion channel

$$L_{NJL} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{g}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right]$$

- One can write down the Schrodinger equation

$$2\sqrt{\vec{p}^2 + m^2}\Psi(\vec{p}) - 4g \int \frac{d^3q}{(2\pi)^3}\Psi(\vec{q}) = E\Psi(\vec{p})$$

- For $\boldsymbol{E = 0}$, this leads to the same eq. as the gap equation

  Massless
- The equivalence is due to chiral symmetry
- One can write down ***potential*** and see ***SSB*** explicitly

NJL model is a simple model of SSB with mass generation and NG mode

3. Pion (NG boson) interaction

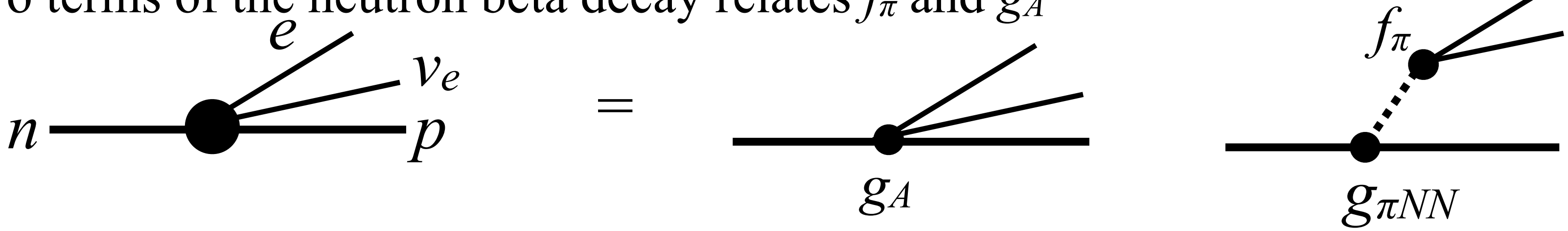
Low energy theorems

- **PCAC (~ CAC)**

  Partially conserved axial current

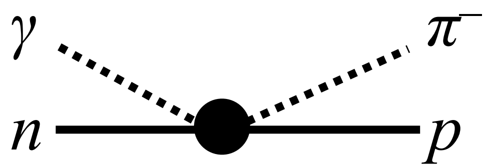
- **Goldberger-Treiman relation**

  Two terms of the neutron beta decay relates f_π and g_A



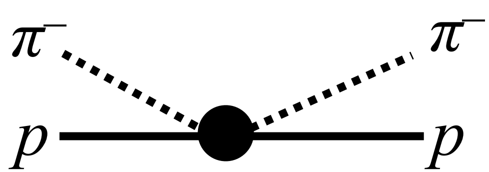
- **Kroll-Ruderman theorem**

  Determines the photoproduction of the pion



- **Tomozawa-Weinberg theorem**

  Determines the pion-matter interaction

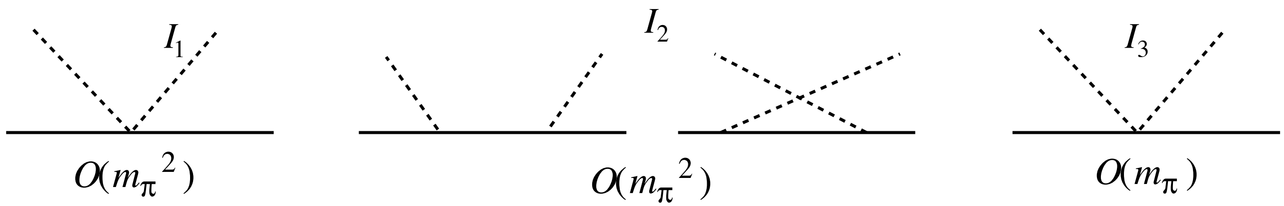


This is one of main issues in this talk

Picture for deriving the Tomozawa-Weinberg int.

πN scattering

$$I \sim \int d^4x d^4y\ e^{-iq_1 y} e^{+iq_2 x}\ \langle N(p_2)|\mathrm{T}\partial^\mu A^b_\mu(x)\partial^\nu A^a_\nu(y)|N(p_1)\rangle$$



$$\begin{aligned}
I &= I_1 + I_2 + I_3\,, \\
I_1 &= -\int d^4x d^4y\ e^{+iq_2 y} e^{-iq_1 x}\ \delta(x_0 - y_0)\langle N(p_2)|\left[A^b_0(y), \partial^\nu A^a_\nu(x)\right]|N(p_1)\rangle\,, \\
I_2 &= -\int d^4x d^4y\ e^{+iq_2 y} e^{-iq_1 x}\ \partial^\mu_x \partial^\nu_y \langle N(p_2)|\left[A^b_\mu(y), A^a_\nu(x)\right]|N(p_1)\rangle\,, \\
I_3 &= -i\int d^4x d^4y\ e^{+iq_2 y} e^{-iq_1 x}\ q^\mu \delta(x_0 - y_0)\langle N(p_2)|\left[A^b_0(y), A^a_\mu(x)\right]|N(p_1)\rangle\,.
\end{aligned}$$

Isospin current
→
ρ-exchange

$$T = -i\frac{2m_\pi}{f_\pi^2}\vec{I}_\pi \cdot \vec{I}_N\,, \quad (I^a_\pi)_{bc} = i\epsilon_{abc}\,, \quad \vec{I}_N = \frac{\vec{\tau}}{2}$$

$$\begin{aligned}
a_{2I} &= -\frac{m_\pi}{8\pi f_\pi^2}\left(1 + \frac{m_\pi}{M}\right)^{-1}\ 2\vec{I}_\pi \cdot \vec{I}_N \\
&= -\frac{m_\pi}{8\pi f_\pi^2}\left(1 + \frac{m_\pi}{M}\right)^{-1}\ (I(I+1) - I_N(I_N+1) - 2)
\end{aligned}$$

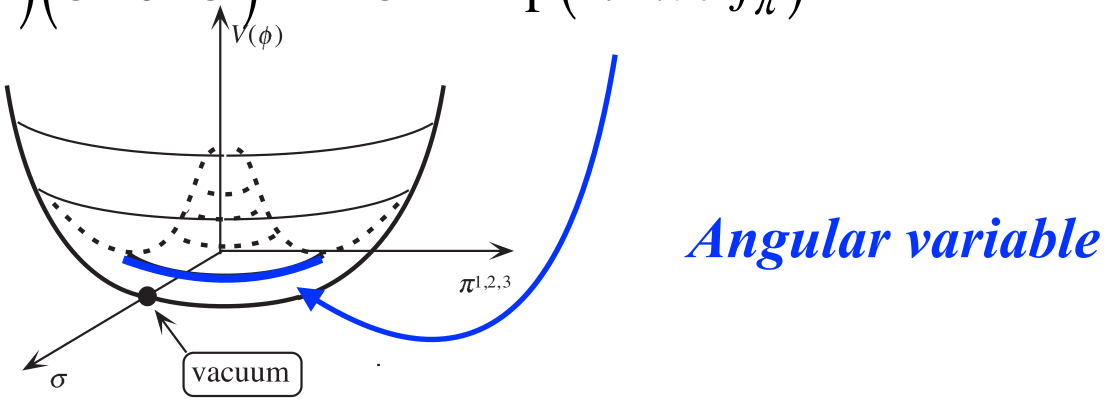
$$a_1 = 0.2 m_\pi^{-1}, a_3 = -0.1 m_\pi^{-1}$$

$$a_1^{\rm exp} = 0.173 m_\pi^{-1}, a_3^{\rm exp} = -0.101 m_\pi^{-1}$$

Practical method ~ Effective Lagrangians

- ***Non-linear sigma model*** for the pion

$$L = \frac{f_\pi^2}{4} \mathrm{tr}\left(U^\dagger \partial_\mu U\right)\left(U^\dagger \partial^\mu U\right) \qquad U = \exp\left(i\vec{\tau}\cdot\vec{\pi}/f_\pi\right)$$



The TW interaction by expanding in powers of π

Practical method ~ Effective Lagrangians

- ***Non-linear sigma model*** for the pion and nucleon

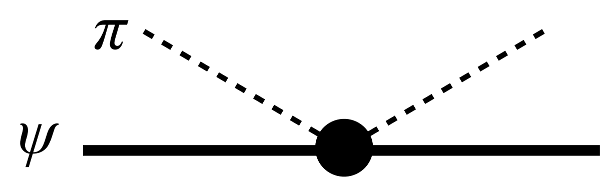
$$L_{sigma} = \bar{q} i \partial q - g f_\pi \bar{q} (\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)/f_\pi q \qquad (\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)/f_\pi = U$$

$$= \bar{q} i \partial q - g f_\pi \bar{q} U^{1/2} U^{1/2} q \qquad \psi = U^{1/2} q$$

$$\to \bar{\psi} i \not{D} \psi - g f_\pi \bar{\psi}\psi \qquad D_\mu = \partial_\mu - i v_\mu$$

$$v_\mu = -\frac{i}{2}\left(\left(\partial_\mu U^{-1/2}\right)U^{1/2} + \left(\partial_\mu U^{1/2}\right)U^{-1/2}\right) \sim \left[\partial_\mu \pi, \pi\right]$$

$$L_{\text{int}} \sim \bar{\psi} i \not{D} \psi \sim \bar{\psi}\left[\partial_\mu \pi, \pi\right]\gamma^\mu \psi$$



The leading term is $\mu = 0$, proportional to the mass of the pion

- Extended SU(3) (Kaon) interaction is ~ $\boldsymbol{m_K}$, and with ***CG coeff.***
- Strong attraction for $\bar{K}$ (not for K)
- Accommodates the hadronic molecule ~ **Λ(1405) ~ $\bar{K}N$**
- **Problem**: point-like interaction (s-wave only)

Λ(1405)

POSSIBLE RESONANT STATE IN PION-HYPERON SCATTERING*

R. H. Dalitz and S. F. Tuan
Enrico Fermi Institute for Nuclear Studies and Department of Physics,
University of Chicago, Chicago, Illinois
(Received April 27, 1959) PhysRevLett.2.425

....

will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the $K^- - p$ (c.m.) threshold energy. In the present discussion, charge-

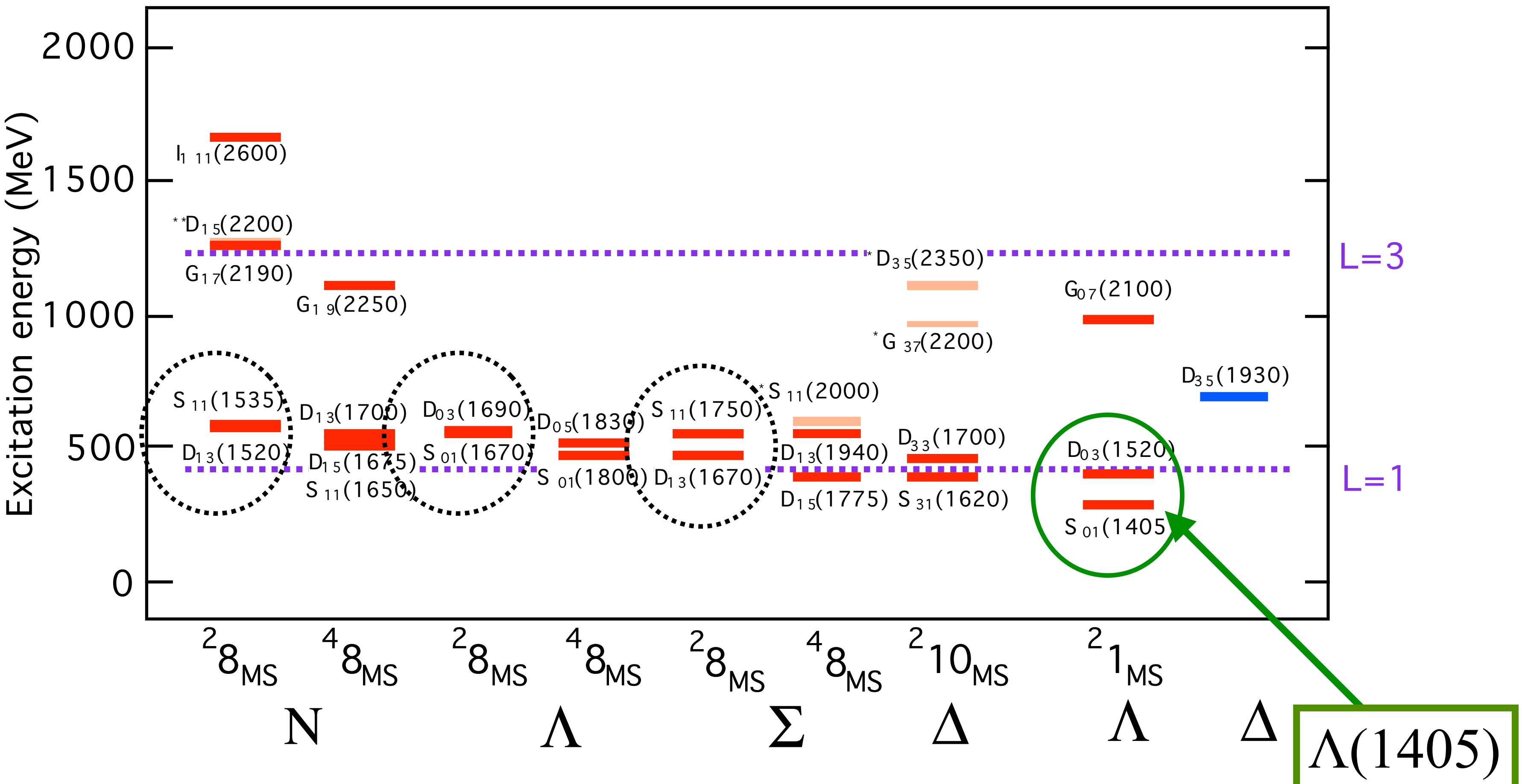
....

This is perhaps the first article which implied a hadronic molecule, K^{bar}-N

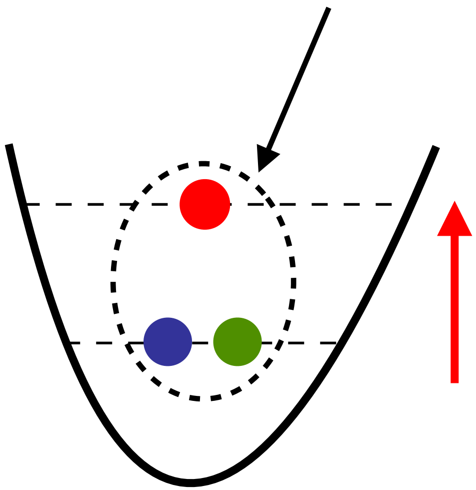
Well established light baryon resonances measured from the corresponding ground states

M. Takayama, H. Toki and A. Hosaka, PTP101 (1999) 1271

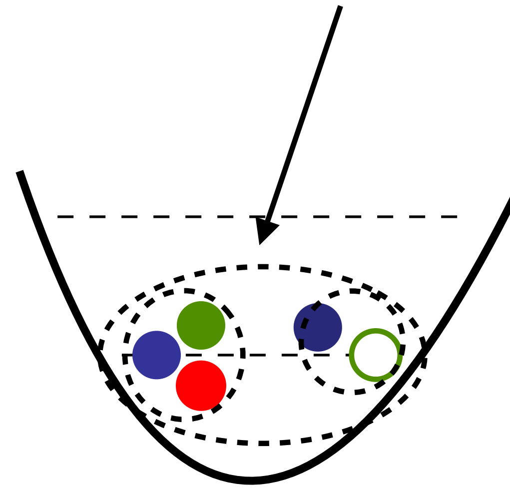
Negative parity baryons



Quark excitations near **thresholds** reform molecules



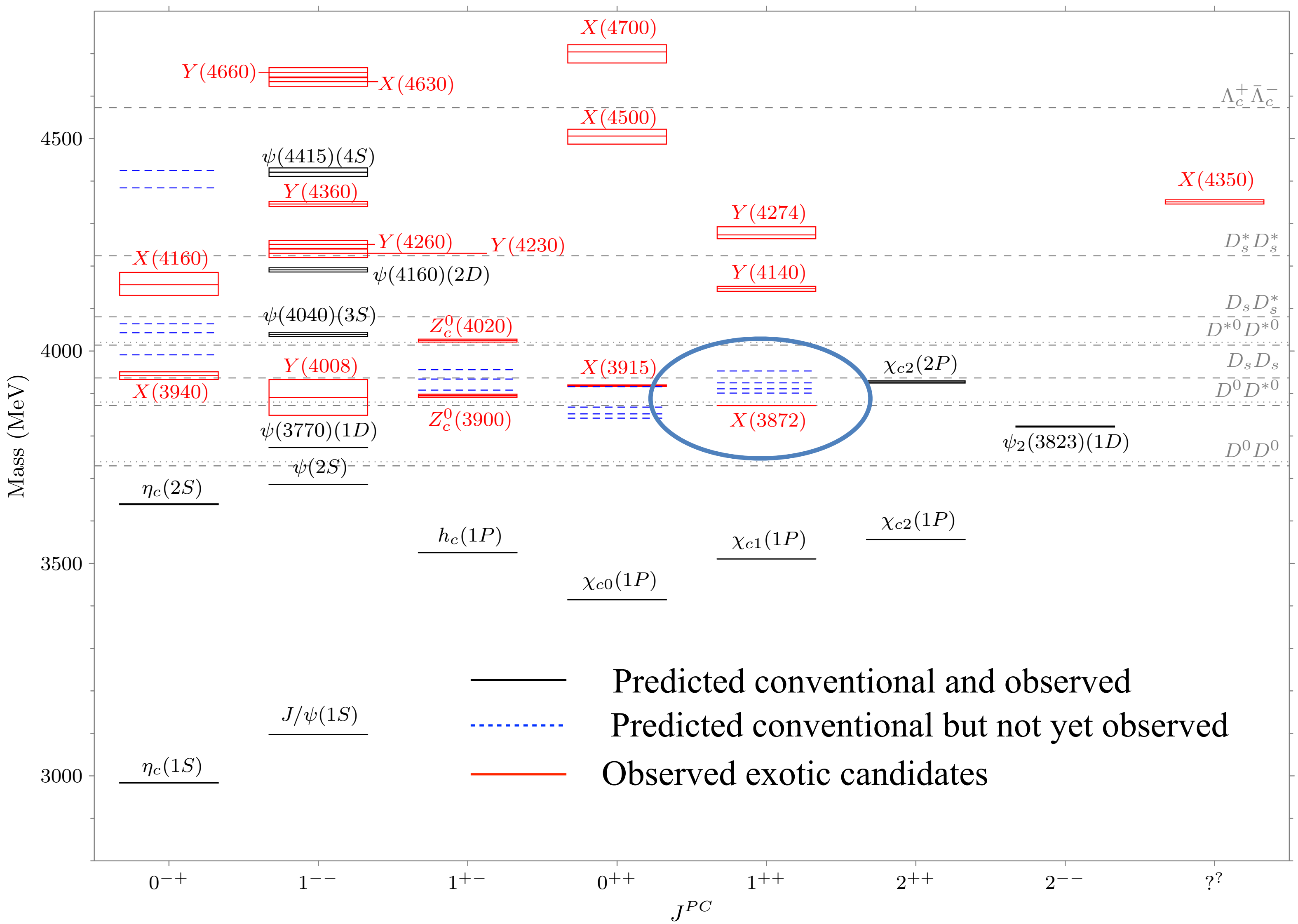
~ 500 MeV
sufficient to create $q\bar{q}$



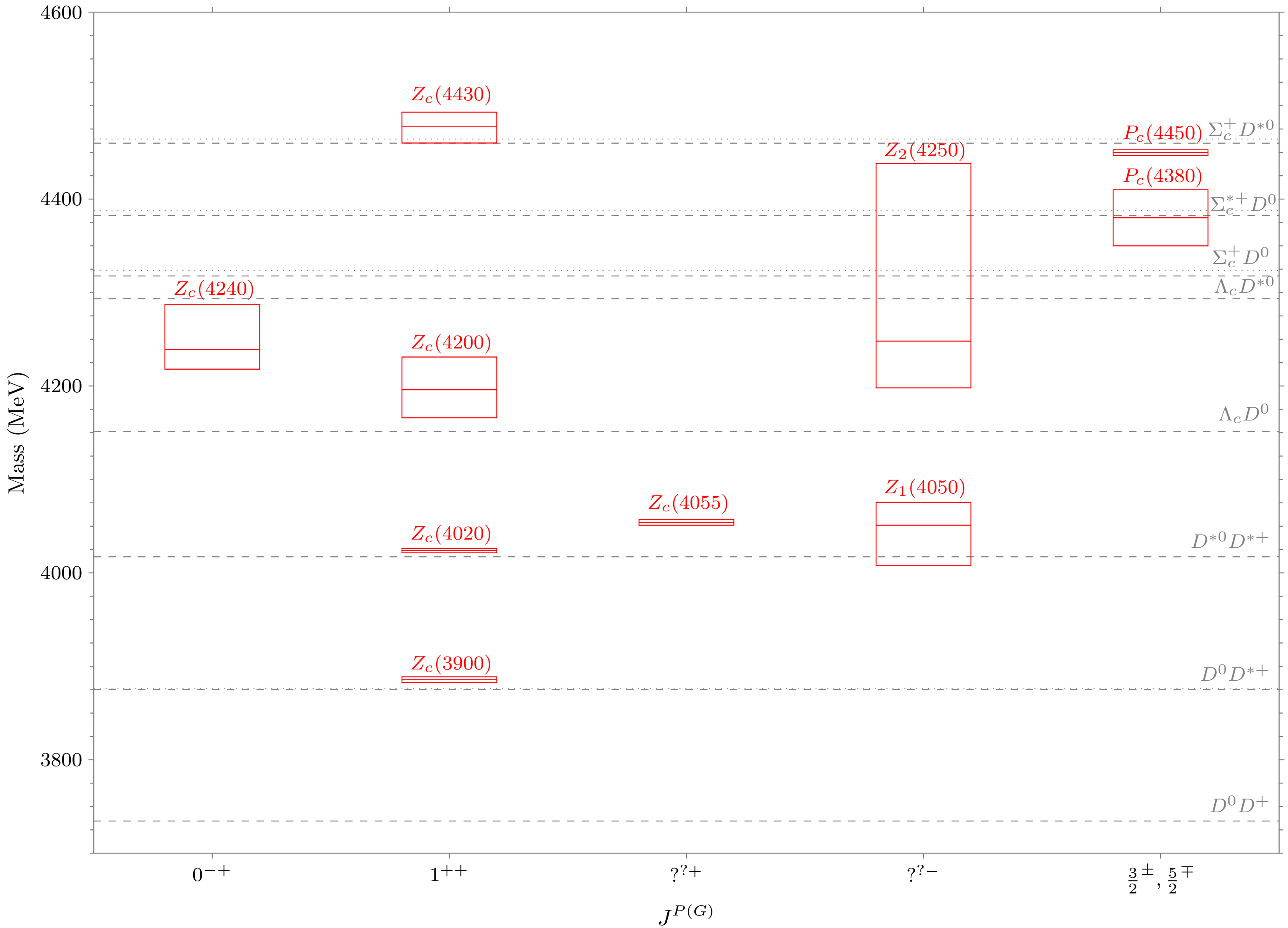
More in the presence of ***heavy quarks***
==>
Recent findings of exotic hadrons, ***X, Y, Z***

For molecular states, ***hadron-hadron interaction*** is the key inputs
NN: many experiments and recent lattice simulations
π(K)N: Low energy theorems, but limited applicability

Neutral X, Y, Z_0 states



Charged Z states



4. Skyrmions and $\bar{K}N$ interactions

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

Ezoe and Hosaka, Phys. Rev. D 4, no. 3, 034022 (2016), arXive:1703.01004

Skyrmions

Baryons as ***pionic solitons with structure of finite size***

How fermions emerge from bosons (the pions)?
How they can be quantized as nucleons?
How they (nucleons) interact with mesons (kaons)?

Relation with QCD

E. Witten, Nucl. Phys. B 223, 422; 433 (1983)

QCD becomes a theory of weakly interacting mesons for large N_c
Nucleons emerge as solitons of a pion field theory

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †
A.E.R.E., Harwell, England

Nucl. Phys. 31, 556 (1962)

Received 29 September 1961

fields interacting through various types of coupling. The objective is the construction of a theory of self-interacting (boson) meson fields, which will admit states that have the phenomenological properties of (fermion) particles, interacting with mesons.

This programme is the obverse of the more fashionable endeavour to reduce the truly elementary particles to a set of spinor fields, out of which everything can be built by simple conjunction. It is a priori much less reasonable because, in particular, it is more difficult to construct half-integral representations of rotation groups out of integral than conversely; indeed it is patently impossible to do this within the limitations of a polynomial expansion. The hope that remains is that the particle-like states will be of a kind that cannot be reached by perturbation theory, and which cannot necessarily be discounted by general arguments. In the type of theory we are using the particle-like solutions have this character, arising from the fact that periodic, transcendental, functions

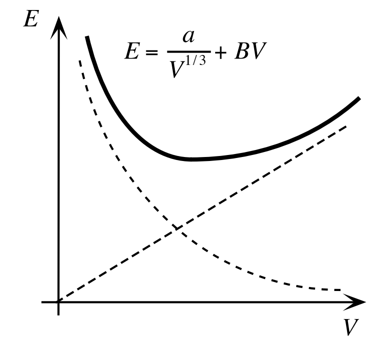
References

1) T. H. R. Skyrme, Nuclear Physics **31** (1962) 550
2) T. H. R. Skyrme, Proc. Roy. Soc. **262** (1961) 237
3) T. H. R. Skyrme, Proc. Roy. Soc. **260** (1961) 127

Recipe 1: To make a nucleon

1. Prepare a **two-term Lagrangian** of the pion

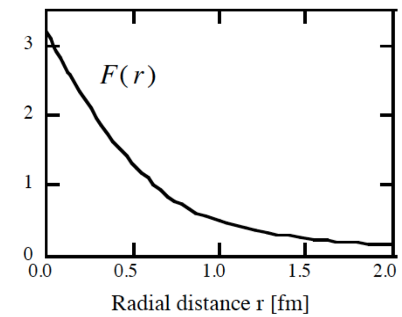
   Positive and negative pressure terms ~ balance



$$L = \frac{f_\pi^2}{4}\,\mathrm{tr}\,R_\mu R^\mu + \frac{1}{32e^2}\,\mathrm{tr}\left[R_\mu, R^\nu\right]^2$$

2. Find **a classical hedgehog solution** of $B = 1$ (winding number)

   Minimize the spin and isospin energy, spin+isospin = 0

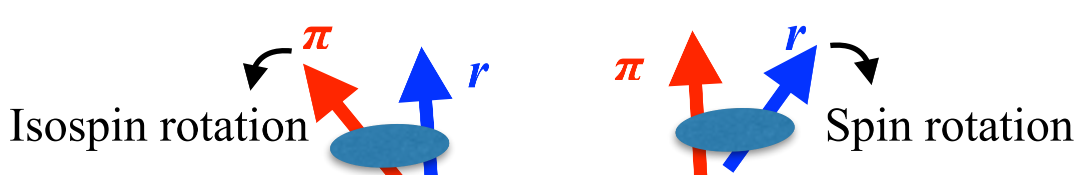


$$U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi) \quad \rightarrow \quad U_H = \exp(i\vec{\tau}\cdot\hat{r}F(r)/f_\pi)$$

3. Quantize its **spin-isospin rotations**

   Nucleons as rotating hedgehogs (originally by **Pauli and Dankoff**)

$$U_H(\boldsymbol{x}) \rightarrow U_H(t,\boldsymbol{x}) = A(t)\exp\left[i\tau_a R_{ab}(t)\hat{r}_b F(r)\right] A^\dagger(t)$$



Physical Review

Vol. 62, Nos. 3 and 4 AUGUST 1 and 15, 1942 Second Series

The Pseudoscalar Meson Field with Strong Coupling

W. Pauli, *Institute for Advanced Study, Princeton, New Jersey*

AND

S. M. Dancoff, *Institute for Advanced Study, Princeton, New Jersey and University of Illinois, Urbana, Illinois*

(Received June 23, 1942)

The present paper treats the symmetrical and charged pseudoscalar theories of the meson field, using the strong coupling approximation; it restricts itself to the case of a single source. The energy levels of the excited states of the heavy particle and the scattering cross section for free mesons are computed by wave mechanical methods. An expression is also obtained for the magnetic moment of the proton or neutron. While the scattering cross section can, with reasonable assumptions, be brought into agreement with experimental values, the results for the magnetic moment are qualitatively at variance with the known values in that equal and opposite moments are predicted for proton and neutron.

Toward the Nucleon and Kaon interaction

Recipe 2a: Callan-Klebaonov, $1/N_c$, strong copying limit, PAV

C. G. Callan, Jr. and I. R. Klebanov, Nucl. Phys. B 262, 365 (1985)

1. Prepare a hedgehog soliton

$$U_H = \exp(i\vec{\tau}\cdot\hat{r}F(r)/f_\pi)$$

2. Introduce a kaon around the hedgehog,
   find the wave equation and bound-state solutions (exists!)

$$U_{HK} = \sqrt{U_H}U_K\sqrt{U_H} \qquad -\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r)k_l^\alpha(r) + \left(m_K^2 + V(r)\right)k_l^\alpha(r) = 0$$

3. Quantize its spin-isospin for hyperons, Λ, Σ, …

$$U_{HK} \to A(t)U_{HK}A(t)^\dagger$$

This recipe follows the $1/N_c$ expansion, kaon around the hedgehog
→ *transmutation of quantum numbers*
The kaon (boson) behaves as a strange quark (fermion)

Toward the Nucleon and Kaon interaction

Recipe 2b: Ezoe-Hosaka, violates 1/Nc, weak copying limit, PBV

Ezoe and Hosaka, Phys. Rev. D 4, no. 3, 034022 (2016); arXive:1703.01004

1. Prepare a nucleon as a rotating hedgehog

$$U_N = A(t)\exp(i\vec{\tau}\cdot\hat{r}F(r)/f_\pi)A(t)^\dagger$$

2. Introduce a kaon around 1, find the wave equation

$$U_{NK} = \sqrt{U_N}U_K\sqrt{U_N} \qquad -\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r)k_l^\alpha(r) + \left(m_K^2 + V(r)\right)k_l^\alpha(r) = 0$$

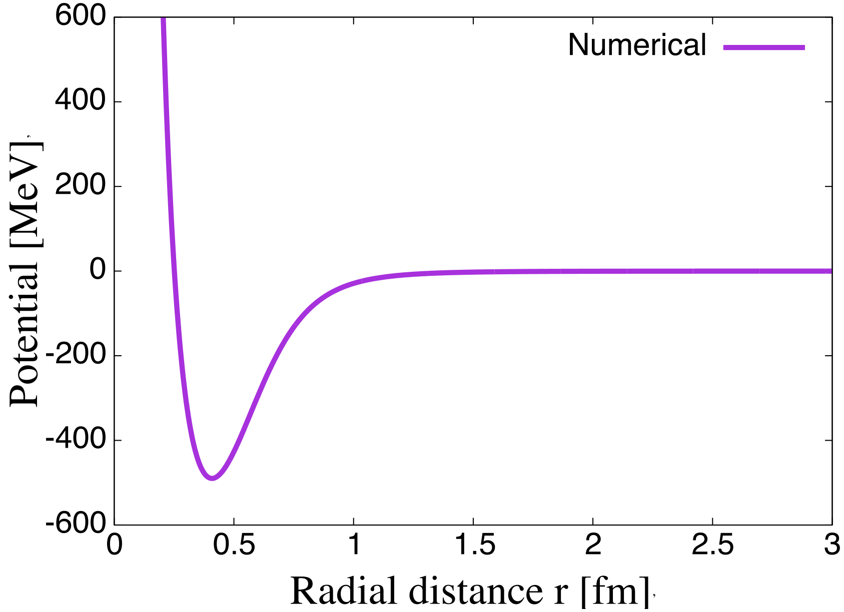
3. Find KN interactions from the wave equation

$$V(r) = V_C^{I=0}(r) + V_C^{I=1}(r) + V_{LS}^{I=0}(r) + V_{LS}^{I=1}(r)$$

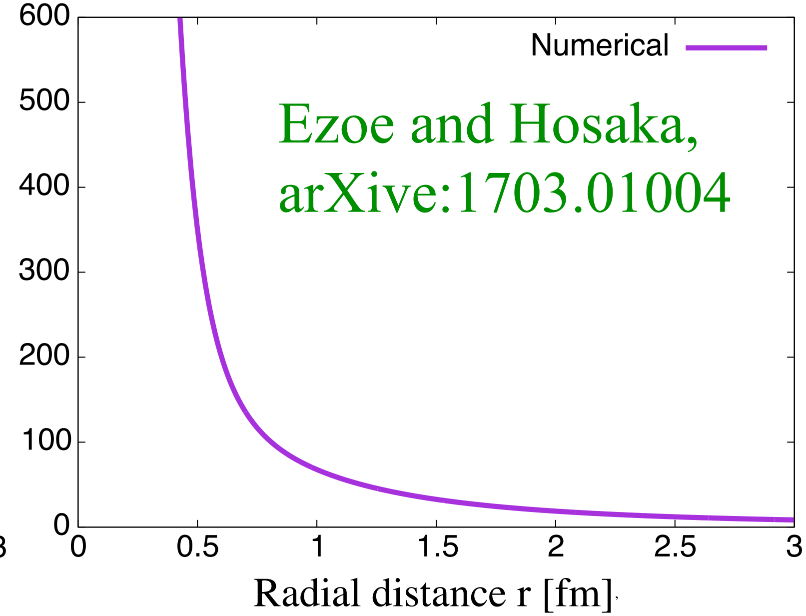
No transmutation of quantum numbers
Physical kaon and nucleon interaction is obtained

KN potential

$I = 0, L = 0, J = 1/2$



$I = 0, L = 1, J = 3/2$



- Qualitatively consistent with Chiral TW theorem
- Finite range reflecting the nucleon structure
- State dependent
- S-wave interaction has an attractive pocket
- Accommodates S-wave weakly bound state for $\Lambda(1405)$
- Repulsive core which is the finding in the Skyrme model

Summary and other issues

Chiral symmetry

- Important ingredient of QCD and hadron dynamics
- Relates the mass generation and the NG modes
- Describes the low energy hadron dynamics
- Kaon (NG boson) interaction has been derived
- May explain the exotic structure of $\Lambda(1405)$ as molecule

What are not discussed

- Chiral multiples of different parities, mesons and baryons
- Recovery of the broken symmetry, Phase of QCD
- Property changes of hadrons
- Hadrons from Holography

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