# Chiral symmetry for hadrons

Atsushi Hosaka RCNP/ASRC Lecture at TPI/ASRC, JAEA May 10, 2017

Contents

- 1. Introduction and Basics Hadrons, chiral symmetry and SSB
- 2. The NJL model

Gap equation and pion eq. of motion

- 3. Pion (NG boson) interactions
- 4. Skyrmions and KN interactions

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# 1. Introduction

General references

S. Weinberg, The Quantum Theory of Fields II, Cambridge, 1995S. Coleman, Aspects of symmetry, Cambridge, 1985Y. Nambu and Jona-Lasinio, Phys. Rev. 122, 345; 124, 246 (1961)

A. Hosaka and H. Toki, Quarks, Baryons and Chiral Symmetry, World Scientific, 2001.

A system is chiral if it is distinguishable from its mirror image = with degenerate energies

## Hadrons: Composites of quarks and gluons of QCD

$$\mathcal{L} = \sum_{f} \bar{q}_{f} (i \not\!\!D - m_{f}) q_{f} - \frac{1}{2} \operatorname{tr} G_{\mu\nu} G^{\mu\nu}$$

- *Color SU(3) gauge theory* of the standard model
- *6 flavors*, **u**, **d**, **s** (~ light) // c, b, t (~ heavy)
- *Non-perturbative* (large coupling) at low energies, for u, d, s
- Confinement, *SSB* and mass generation  $\rightarrow$ *Millennium problem*
- Hadrons emerge as *baryons* (qqq) and *mesons* ( $q\overline{q}$ )
- *Lattice QCD* is the first principle method
- Successful for *static properties*; masses and FF
- But not yet for *dynamic properties*, resonances and reactions
- Important ideas and methods are explored by *effective theories*
- *Chiral symmetry* and *SSB* explain many hadron properties

### Hadrons of opposite parities should degenerate BUT NOT Indicating SSB

#### Jido, Oka, Hosaka, PTP106 (2001) 823



## Are hadrons qqq or $q\bar{q}$ ?

#### Particle Data Group

Baryons

A PROFESSION AND A PROF			THE CHARGE TRANSF									_											
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• **	1-(0-)	<ul> <li>m<sub>2</sub>(1670)</li> </ul>	1-(2-+)	<ul> <li>K<sup>±</sup></li> </ul>	1/2(0**)	<ul> <li>D<sup>±</sup></li> </ul>	0(0")	<ul> <li>J/ψ(15)</li> </ul>	0-(1)	N(1440)	$P_{11}$		∆(1620)	S <sub>30</sub>		2 -	$P_{11}$		=(1530)	$P_{13}$		A <sub>c</sub> (2625)*	
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• 17	0+(0-+)	<ul> <li>p<sub>1</sub>(1690)</li> </ul>	$1^{+}(3^{-})$	<ul> <li>K<sup>0</sup><sub>S</sub></li> </ul>	$1/2(0^{})$	• D. 127 NO	$0(0^+)$	<ul> <li>χ<sub>c1</sub>(1P)</li> </ul>	$0^{+}(1^{++})$	N(1535)	S11		$\Delta(1750)$	$P_{11}$	•	Σ[14]Ω]	n	•	E[16:2]	SCI -		A. (2885)+	
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<ul> <li>p(770)</li> </ul>	1+(1)	a <sub>2</sub> (1700)	$1^{-}(2^{+})$	AC(800)	$1/2(0^+)$	<ul> <li>D<sub>11</sub>(2536)<sup>±</sup></li> </ul>	$0(1^+)$	<ul> <li>\$\cappa_c(1P)\$</li> </ul>	0+(2++)	M(1625)	Du		4(1905)	F		£(1580)	D11		E(1950)			UUU	
<ul> <li>u(782)</li> </ul>	0-(1)	<ul> <li>§(1710)</li> </ul>	$0^{+}(0^{+}+)$	<ul> <li>K*(892)</li> </ul>	1/2(1")	<ul> <li>D<sub>12</sub>(2573)<sup>±</sup></li> </ul>	0(22)	<ul> <li>n<sub>1</sub>(25)</li> </ul>	$0^+(0^{-+})$	M(1680)	6.		4(100.0)	1.25		£(1620)	S.,		E(2030)			£ (2100)	
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<ul> <li>f<sub>0</sub>(980)</li> </ul>	0+(0-(-))	<b>=</b> [1800]	$1^{-}(0^{-+})$	<ul> <li>K<sub>1</sub>(1-5)</li> </ul>	$1/2(1^+)$			• #(W75)	2-(1)	N[1/00]	D13		∆(1920)	$P_{33}$		2 (1060)	P11		=[2120]			$\Sigma_{c}(2800)$	
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<ul> <li>\$\$\phi(1029)\$</li> </ul>	0-(1)	X[1835]	51(5-+)	<ul> <li>K<sup>*</sup><sub>2</sub>(1430)</li> </ul>	$1/2(0^+)$	(8 - 8)	42	$\chi_{cl}(2P)$	0+(2++)	N(1720)	$P_{13}$		山(1940)	$D_{33}$	•	$\Sigma(1690)$			$\Xi(2370)$		••	=0	
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<ul> <li>n(1295)</li> </ul>	0+(0-+)	<ul> <li>6(1950)</li> </ul>	0+(2++)	<ul> <li>K*(1680)</li> </ul>	$1/2(1^{-})$	trix Elements		X(4360)	P <sup>1</sup> (1)	N(2090)	511		∆(2300)	H39		2 [1880]	211		0(24/20)-			$\Xi_{c}(2815)$	
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<ul> <li>§(1370)</li> </ul>	0*(0 * *)	6(2020)	0+(0++)	<ul> <li>K<sub>2</sub>(1820)</li> </ul>	$1/2(2^{-})$	<ul> <li>B<sub>1</sub>[5721]<sup>0</sup></li> </ul>	$1/2(1^+)$	- (16)	a+ia = +1	N(2200)	$D_{25}$	••	∆(2400)	$G_{39}$	••	Σ(2000)	S11	•				= (3055)	
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6(152.0)	0+11+1	12200	1+(1)	CHART	65.0	BOTTOM, CH	<b>WRMED</b>	• 7(35)	0-(1)	1			A(1670)	Sec		Σ[3170]						5	
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(1570)	3+633	6(2330)	0+(0++1	• 0*	1/200 1			• 7(11020)	0-(1)	1			74(1000)	-701								- ĐQQ	
A-(1595)	0-0+-1	• 6(2340)	0+12++1	• D*C200202	1/2(0 )					1			7(1810)	P11								$\Omega_{b}^{-}$	
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6(1640)	0+(2++)	6(2510)	0+(6++)	01000	1/308+3			DATES		1			A(1890)	$P_{00}$									
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<ul> <li>un (1670)</li> </ul>	0-(3)	Further St	ates	D (2430) <sup>0</sup>	1/2(1+)					1			A(2100)	Gu									
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				OPCHARM+	1/2020					1			7(2325)	6.03									
				The (1994)	1.46.1					1			7(2350)	14,99									
										1			A(2585)										

 $25 \ \text{kinds}$ 

Mesons

#### 22 kinds

Seminar at Vladivostok, March 28,29, 2016

# Superconductor vs Hadron

	L	
	Super conductor	Hadron systems
System	Metal, conductor Many electrons	Vacuum Many quarks and anti-quarks
Particles and Interaction	Electron Phonons	Quark Gluons
	Electron	
Order parameter	$\langle ee \rangle$	$\langle \overline{q}q  angle$
Mass Gap	Majorana mass	Dirac mass
Zero mode	Super fluid	Pions
Two phases	Magnetic Super Normal Seminar@ASRC/JAEA, May 10,	201 Color Super

# Chirality of fermions

Related to the representations of the Lorentz group: Relativity

<b>Rotation and Boost, SL(2,C)</b>	Generators, $R$ and $B$
( <i>R</i> + <i>iB</i> ): Right handed fermion	$\psi = \psi_R + \psi_L$
( <i>R–iB</i> ): Left handed fermion	$\psi_{R,L} \equiv \frac{1 \pm \gamma_5}{2} \psi$
Two	o comp. each Four comp.

- *R* and *L numbers* are conserved separately  $\rightarrow$  *chiral symmetry*
- Fermion number conserving *mass term* is possible by mixing the *R* and *L* components => breaks Chiral symmetry  $L = \overline{\psi} (i \partial - m) \psi = \overline{\psi}_L i \partial \psi_L + \overline{\psi}_R i \partial \psi_R - m \overline{\psi}_L \psi_R - m \overline{\psi}_R \psi_L$
- This is the reason that Dirac equation needs four components

## Physically,

- *L* and *R* components correspond to *helicity* states
- For a particle moving at the speed of light, *L* and *R* are conserved *massless*



**Right** helicity



- Left transformation:  $\Psi_L \rightarrow e^{i\theta_L} \Psi_L \xrightarrow{\theta_L = \pi} \Psi_L$
- Parity eigenstates (under  $L \rightleftharpoons R$ )

# 2. The NJL Model

Y. Nambu and Jona-Lasinio, Phys. Rev. 122, 345; 124, 246 (1961)

# The NJL Model

- A model for relativistic and *massless fermion*
- Assign particle numbers of an internal symmetry for *L* and *R* separately Fundamental

 $SU(N_f)_L \times SU(N_f)_R \xrightarrow{\text{representation}} \left(\frac{1}{2}, 0\right) + \left(0, \frac{1}{2}\right) = \psi$ 

• With a chiral invariant four point (zero range) interaction

$$L_{NJL} = \overline{\psi}i\,\partial\psi\psi + \frac{g}{2}\left[\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

 $(\bar{\psi}\psi,\bar{\psi}i\gamma_5\vec{\tau}\psi)$  behaves as an O(4) vector of  $SU(2_f)_L \ge SU(2_f)_R$ 

$$(\bar{\psi}\psi)^2 \rightarrow Gap \ equation$$
 for mass generation  
*Equivalent* due to chiral symmetry  
 $(\bar{\psi}i\gamma_5\psi)^2 \rightarrow Equation \ for \ massless \ pion, \ NG \ boson$   
Seminar@ASRC/JAEA, May 10, 2017

## Mean field in scalar channel - Mass generation -



- There is always a trivial solution m = 0
- Nontrivial one of m > 0 is possible for large g and  $\Lambda$  $\rightarrow$  Spontaneous breaking of symmetry *SSB*

# Schrodinger eq. in pion channel

土岐、保坂、相対論的他体系としての原子核、大阪大学出版会(2011)

• Possible to solve the equation for the pion channel

$$L_{NJL} = \overline{\psi} i \, \partial \psi - \frac{g}{2} \Big[ \left( \overline{\psi} \psi \right)^2 + \left( \overline{\psi} i \gamma_5 \psi \right)^2 \Big]$$

• One can write down the Schrodinger equation

$$2\sqrt{\vec{p}^{\,2}+m^{2}}\Psi(\vec{p})-4g\int\frac{d^{3}q}{(2\pi)^{3}}\Psi(\vec{q})=E\Psi(\vec{p})$$

- For E = 0, this leads to the same eq. as the gap equation *Massless*
- The equivalence is due to chiral symmetry
- One can write down *potential* and see *SSB* explicitly

NJL model is a simple model of SSB with mass generation and NG mode

# 3. Pion (NG boson) interaction

# Low energy theorems

### • **PCAC** (~ **CAC**)

Partially conserved axial current

### • Goldberger-Treiman relation



### Kroll-Ruderman theorem

Determines the photoproduction of the pion



• Tomozawa-Weinberg theorem Determines the pion-matter interaction



This is one of main issues in this talk

### Picture for deriving the Tomozawa-Weinberg int.

 $\pi N$  scattering

 $I \sim \int d^4x d^4y \ e^{-iq_1y} e^{+iq_2x} \ \langle N(p_2) | \mathrm{T}\partial^{\mu} A^b_{\mu}(x) \partial^{\nu} A^a_{\nu}(y) | N(p_1) \rangle$ 



$$I = I_{1} + I_{2} + I_{3},$$

$$I_{1} I_{1} = I_{3} - \int d^{4}x d^{4}y \ e^{+iq_{2}y} e^{-iq_{1}x} \ \delta(x_{0} - y_{0}) \langle N(p_{2})| \left[A_{0}^{b}(y), \partial^{\nu}A_{\nu}^{a}(x)\right] I_{2}N(p_{1}) \rangle,$$

$$I_{2} = \partial^{\nu}A_{\mu} \left( \frac{d^{4}x}{(y)} \frac{d^{4}y}{(y)} \frac{e^{+iq_{2}y}}{m_{\pi}} \frac{m_{\pi}}{2} \frac{\partial^{\mu}\partial^{\nu}}{\partial x} \frac{\partial^{\mu}\partial^{\nu}}{\partial y} \langle N(p_{2})| \left[A_{\mu}^{b}(y), A_{\nu}^{a}(x)\right] |N(p_{1}) \rangle,$$

$$I_{3} = -i \int d^{4}x d^{4}y \pi e^{\frac{1}{2} iq_{2}y} e^{-iq_{1}x} \ q^{\mu}\delta(x_{0} - y_{0}) \langle N(p_{2})| \left[A_{0}^{b}(y), A_{\mu}^{a}(x)\right] |N(p_{1}) \rangle.$$

$$\rho$$
-exchange

$$T_{O}(\oplus -i\frac{2m_{\pi}}{f_{\pi}^{2}}\vec{I}_{\pi}\cdot\vec{I}_{N}, \quad (I_{\pi}^{a})_{bc} = i\epsilon_{abc}, \quad \vec{I}_{N} = \frac{1}{2^{3}} \qquad O(m_{\pi})$$

$$a_{2I} = -\frac{m_{\pi}}{8\pi f_{\pi}^{2}} \left(1 + \frac{m_{\pi}}{M}\right)^{-1} 2\vec{I}_{\pi} \cdot \vec{I}_{N} \qquad a_{1} = 0.2m_{\pi}^{-1}, a_{3} = -0.1m_{\pi}^{-1}$$

$$= -\frac{m_{\pi}}{8\pi f_{\pi}^{2}} \left(1 + \frac{m_{\pi}}{M}\right)^{-1}_{2} (I(I+1) - I_{N}(I_{N}+1) - 2) \qquad a_{1} = 0.173m_{\pi}^{-1}, a_{3}^{\exp} = -0.101m_{\pi}^{-1}$$

$$T = -i\frac{2}{f_{\pi}^{2}}I_{\pi} \cdot I_{N}, \qquad I_{N} = \frac{\tau}{\text{Thge}}, (I_{\pi}^{a})_{b \in I} = i\epsilon_{abc}$$

$$a_{1} = 0.2m_{\pi}^{-1}, a_{3} = -0.1m_{\pi}^{-1}$$

$$a_{1} = 0.173m_{\pi}^{-1}, a_{3} = -0.101m_{\pi}^{-1}$$

Practical method ~ Effective Lagrangians

• Non-linear sigma model for the pion



#### The TW interaction by expanding in powers of $\pi$

Practical method ~ Effective Lagrangians

• Non-linear sigma model for the pion and nucleon

$$\begin{split} L_{sigma} &= \overline{q}i\,\partial q - gf_{\pi}\overline{q}(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_{5}) / f_{\pi}q \qquad (\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_{5}) / f_{\pi} = U \\ &= \overline{q}i\,\partial q - gf_{\pi}\overline{q}U^{1/2}U^{1/2}q \qquad \qquad \psi = U^{1/2}q \\ &\to \overline{\psi}i \mathcal{D}\psi - gf_{\pi}\overline{\psi}\psi \qquad \qquad D_{\mu} = \partial_{\mu} - iv_{\mu} \\ &v_{\mu} &= -\frac{i}{2} \Big( \Big(\partial_{\mu}U^{-1/2}\Big)U^{1/2} + \Big(\partial_{\mu}U^{1/2}\Big)U^{-1/2} \Big) \sim \Big[\partial_{\mu}\pi,\pi\Big] \end{split}$$

The leading term is  $\mu = 0$ , proportional to the mass of the pion

- Extended SU(3) (Kaon) interaction is ~  $m_K$ , and with *CG coeff*.
- Strong attraction for  $\overline{K}$  (not for K)
- Accommodates the hadronic molecule ~  $\Lambda(1405) \sim \overline{K}N$
- **Problem**: point-like interaction (s-wave only)

# $\Lambda(1405)$

#### POSSIBLE RESONANT STATE IN PION-HYPERON SCATTERING\*

R. H. Dalitz and S. F. Tuan

Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois (Received April 27, 1959) PhysRevLett.2.425

. . . .

will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the  $K^- - p$  (c.m.) threshold energy. In the present discussion, charge-....

This is perhaps the first article which implied a hadronic molecule, K<sup>bar</sup>-N

### Well established light baryon resonances measured from the corresponding ground states

M. Takayama, H. Toki and A. Hosaka, PTP101 (1999) 1271

Negative parity baryons



Three days lectures at U. Tokyo



More in the presence of *heavy quarks* ==> Recent findings of exotic hadrons, *X*, *Y*, *Z* 

For molecular states, *hadron-hadron interaction* is the key inputs NN: many experiments and recent lattice simulations  $\pi(K)N$ : Low energy theorems, but limited applicability

### Neutral X, Y, Z<sub>0</sub> states



Seminar@YITP Molecule. 2016, Nov. 22

### Charged Z states



Seminar@YITP Molecule. 2016, Nov. 22

# 4. Skyrmions and $\overline{KN}$ interactions

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

Ezoe and Hosaka, Phys. Rev. D 4, no. 3, 034022 (2016), arXive:1703.01004

# Skyrmions

Baryons as *pionic solitons with structure of finite size* 

How fermions emerge from bosons (the pions)? How they can be quantized as nucleons? How they (nucleons) interact with mesons (kaons)?

Relation with QCD E. Witten, Nucl. Phys. B 223, 422; 433 (1983)

QCD becomes a theory of weakly interacting mesons for large  $N_c$ Nucleons emerge as solitons of a pion field theory

#### A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME + Nucl. Phys. 31, 556 (1962)

A.E.R.E., Harwell, England

Received 29 September 1961

fields interacting through various types of coupling. The objective is the construction of a theory of self-interacting (boson) meson fields, which will admit states that have the phenomenological properties of (fermion) particles, interacting with mesons.

This programme is the obverse of the more fashionable endeavour to reduce the truly elementary particles to a set of spinor fields, out of which everything can be built by simple conjunction. It is a priori much less reasonable because, in particular, it is more difficult to construct half-integral representations of rotation groups out of integral than conversely; indeed it is patently impossible to do this within the limitations of a polynomial expansion. The hope that remains is that the particle-like states will be of a kind that cannot be reached by perturbation theory, and which cannot necessarily be discounted by general arguments. In the type of theory we are using the particle-like solutions have this character, arising from the fact that periodic, transcendental, functions

#### References

- 1) T. H. R. Skyrme, Nuclear Physics 31 (1962) 550
- 2) T. H. R. Skyrme, Proc. Roy. Soc. 262 (1961) 237
- 3) T. H. R. Skyrme, Proc. Roy. Soc. 260 (1961) 127

# **Recipe 1: To make a nucleon**

1. Prepare a **two-term Lagrangian** of the pion Positive and negative pressure terms ~ balance

$$L = \frac{f_{\pi}^{2}}{4} \operatorname{tr} R_{\mu} R^{\mu} + \frac{1}{32e^{2}} \operatorname{tr} \left[ R_{\mu}, R^{\nu} \right]^{2}$$



F(r)

2. Find a classical hedgehog solution of B = 1 (winding number) Minimize the spin and isospin energy, spin+isospin = 0

$$U = \exp(i\vec{\tau} \cdot \vec{\pi} / f_{\pi}) \quad \to \quad U_H = \exp(i\vec{\tau} \cdot \vec{r}F(r) / f_{\pi})$$

3. Quantize its spin-isospin rotations Nucleons as rotating hedgehogs (originally by Pauli and Dankoff)

$$U_H(\boldsymbol{x}) \to U_H(t, \boldsymbol{x}) = \boldsymbol{A}(t) \exp\left[i\tau_a R_{ab}(t)\hat{r}_b F(r)\right] \boldsymbol{A}^{\dagger}(t)$$



2.0

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#### AUGUST 1 AND 15, 1942

SECOND SERIES

#### The Pseudoscalar Meson Field with Strong Coupling

W. PAULI, Institute for Advanced Study, Princeton, New Jersey

AND

S. M. DANCOFF, Institute for Advanced Study, Princeton, New Jersey and University of Illinois, Urbana, Illinois (Received June 23, 1942)

The present paper treats the symmetrical and charged pseudoscalar theories of the meson field, using the strong coupling approximation; it restricts itself to the case of a single source. The energy levels of the excited states of the heavy particle and the scattering cross section for free mesons are computed by wave mechanical methods. An expression is also obtained for the magnetic moment of the proton or neutron. While the scattering cross section can, with reasonable assumptions, be brought into agreement with experimental values, the results for the magnetic moment are qualitatively at variance with the known values in that equal and opposite moments are predicted for proton and neutron. Toward the Nucleon and Kaon interaction

**Recipe 2a: Callan-Klebaonov**, 1/*N<sub>c</sub>*, strong copying limit, PAV C. G. Callan, Jr. and I. R. Klebanov, Nucl. Phys. B 262, 365 (1985)

1. Prepare a hedgehog soliton

 $U_H = \exp(i\vec{\tau} \cdot \hat{\mathbf{r}} F(r) / f_{\pi})$ 

2. Introduce a kaon around the hedgehog, find the wave equation and bound-state solutions (exists!)

$$U_{HK} = \sqrt{U_H} U_K \sqrt{U_H} \qquad -\frac{1}{r^2} \frac{d}{dr} \left( r^2 h(r) \frac{dk_l^{\alpha}(r)}{dr} \right) - E^2 f(r) k_l^{\alpha}(r) + \left( m_K^2 + V(r) \right) k_l^{\alpha}(r) = 0$$

3. Quantize its spin-isospin for hyperons,  $\Lambda$ ,  $\Sigma$ , ...

 $U_{HK} \to A(t) U_{HK} A(t)^{\dagger}$ 

This recipe follows the  $1/N_c$  expansion, kaon around the hedgehog  $\rightarrow$  transmutation of quantum numbers The kaon (boson) behaves as a strange quark (fermion)

### Toward the Nucleon and Kaon interaction

**Recipe 2b: Ezoe-Hosaka**, violates 1/Nc, weak copying limit, PBV Ezoe and Hosaka, Phys. Rev. D 4, no. 3, 034022 (2016); arXive:1703.01004

1. Prepare a nucleon as a rotating hedgehog  $U_N = A(t) \exp(i\vec{\tau} \cdot \hat{r}F(r) / f_{\pi}) A(t)^{\dagger}$ 

2. Introduce a kaon around 1, find the wave equation

$$U_{NK} = \sqrt{U_N} U_K \sqrt{U_N} \qquad -\frac{1}{r^2} \frac{d}{dr} \left( r^2 h(r) \frac{dk_l^{\alpha}(r)}{dr} \right) - E^2 f(r) k_l^{\alpha}(r) + \left( m_K^2 + V(r) \right) k_l^{\alpha}(r) = 0$$

3. Find KN interactions from the wave equation

$$V(r) = V_C^{I=0}(r) + V_C^{I=1}(r) + V_{LS}^{I=0}(r) + V_{LS}^{I=1}(r)$$

No transmutation of quantum numbers Physical kaon and nucleon interaction is obtained



- Qualitatively consistent with Chiral TW theorem
- Finite range reflecting the nucleon structure
- State dependent
- S-wave interaction has an attractive pocket
- Accommodates S-wave weakly bound state for  $\Lambda(1405)$
- Repulsive core which is the finding in the Skyrme model

## Summary and other issues

Chiral symmetry

- Important ingredient of QCD and hadron dynamics
- Relates the mass generation and the NG modes
- Describes the low energy hadron dynamics
- Kaon (NG boson) interaction has been derived
- May explain the exotic structure of  $\Lambda(1405)$  as molecule

## What are not discussed

- Chiral multiples of different parities, mesons and baryons
- Recovery of the broken symmetry, Phase of QCD
- Property changes of hadrons
- Hadrons from Holography

Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843; 114 (2006), 1083 Liu, Zahed, arXive:1704.03412