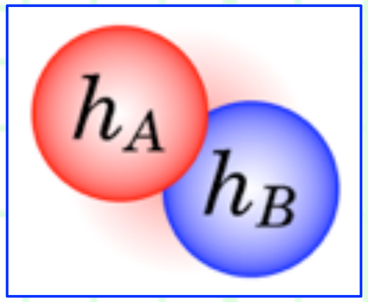
Wave functions and compositeness for hadron resonances from hadron-hadron scattering amplitudes

Takayasu SEKIHARA

[1] T. S. , T. Hyodo and D. Jido, *Prog. Theor. Exp. Phys.* 2015, 063D04.

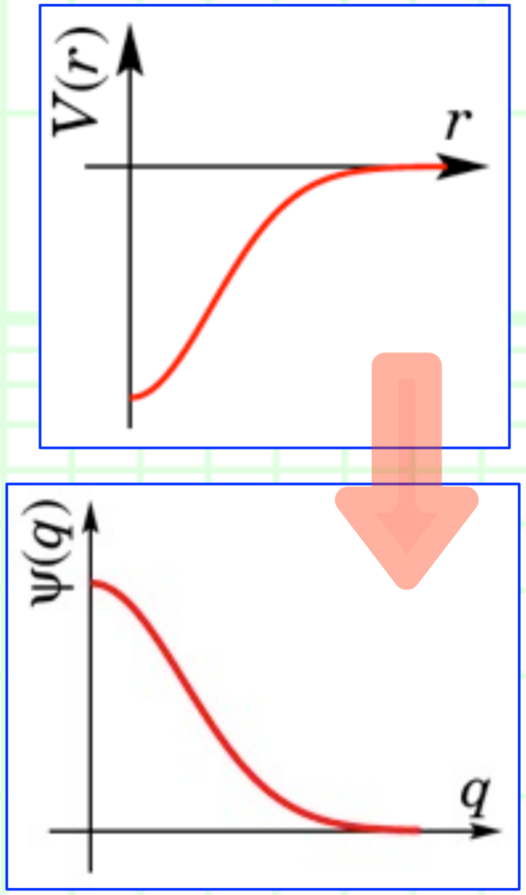
[2] T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2016) 035204.

Contents



1. Introduction

- Hadronic molecules are unique !
- Since they are unique, we can use quantum mechanics:
  Wave function and its norm, scattering amplitude, ...

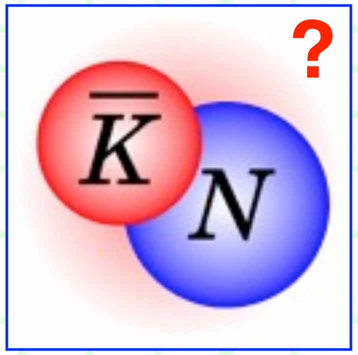


2. Two-body wave functions and compositeness

- How to obtain the wave function of bound states (*e.g.* deuteron) ?
  - A. Solve the Schrödinger equation.
  - B. Solve the Lippmann-Schwinger equation and extract it from the scattering amplitude. <-- Our approach !

3. Applications: compositeness of hadronic resonances

- The $\Lambda(1405)$ resonance is a $\bar{K}N$ molecular state ?
- Large pion clouds for the $\Delta(1232)$ resonance ?
- Meson-baryon components in $N(1535)$ and $N(1650)$?



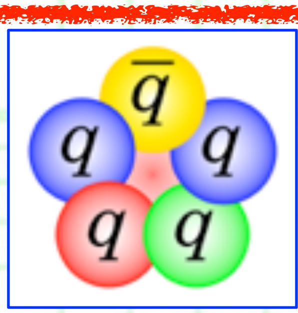
4. Conclusion

1. Introduction

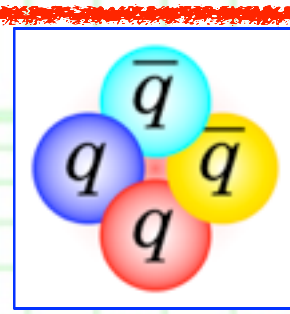
1. Introduction

++ Exotic hadrons and their structure ++

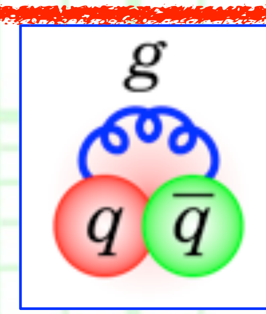
- **Exotic hadrons** --- not same quark component as ordinary hadrons
  = not qqq nor $q\bar{q}$.



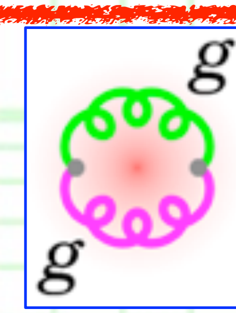
Penta-quarks



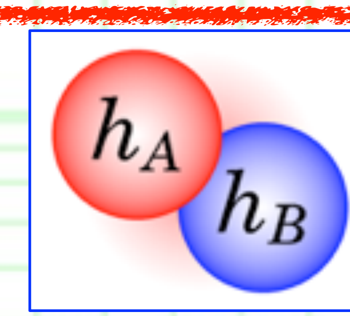
Tetra-quarks



Hybrids



Glueballs

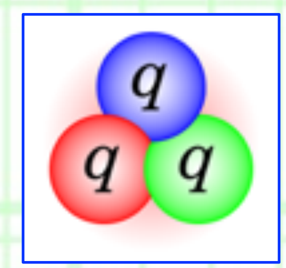


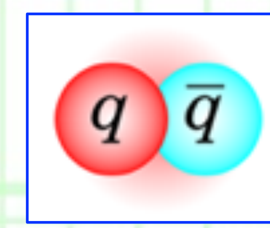
Hadronic molecules

...

--- Actually some hadrons cannot be described by the quark model.

- □ Do exotic hadrons really exist ?
- □ If they do exist, how are their properties ?
  --- Re-confirmation of quark models.
  --- Constituent quarks in multi-quarks ? "Constituent" gluons ?
- □ If they do not exist, what mechanism forbids their existence ?





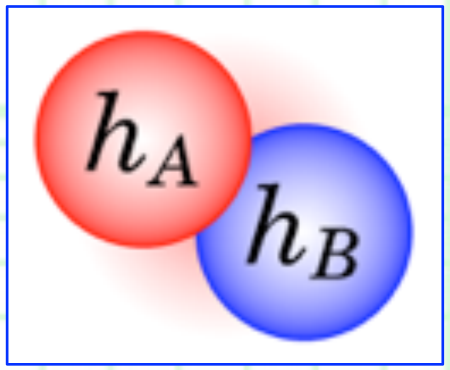
Ordinary hadrons

<-- We know very few about hadrons (and dynamics of QCD).

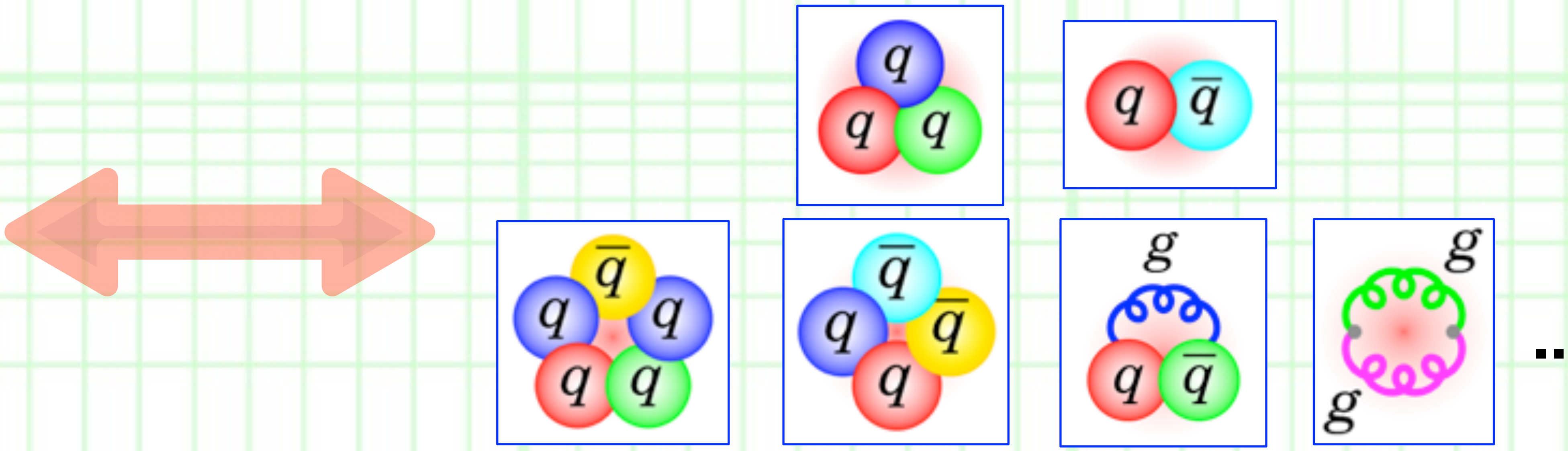
1. Introduction

++ Uniqueness of hadronic molecules ++

- **Hadronic molecules should be unique, because they are composed of hadrons themselves, which are color singlet.**



Hadronic molecules
(cf. deuteron)



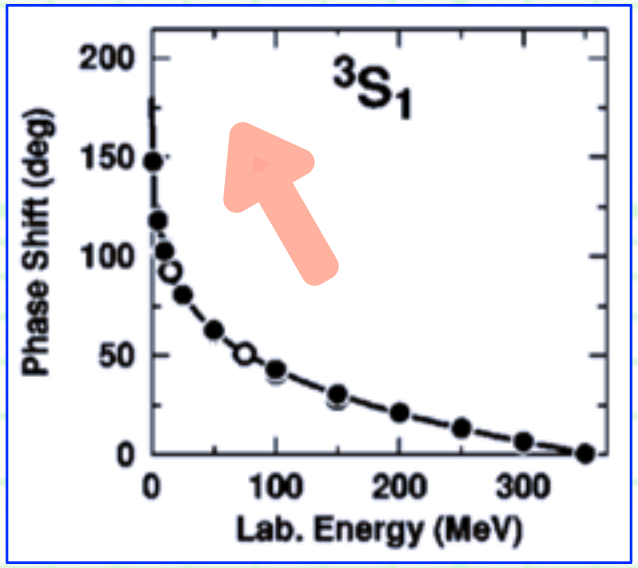
--> Various quantitative/qualitative diff. from other compact hadrons.

- **Large spatial size due to the absence of strong confining force.**
- **Hadron masses are "observable", in contrast to quark masses.**
  --> Expectation of the existence around two-body threshold.
- **Treat them without complicated calculations of QCD.**
  --- We can use quantum mechanics with appropriate interactions.

1. Introduction

++ Hadronic molecules and quantum mechanics ++

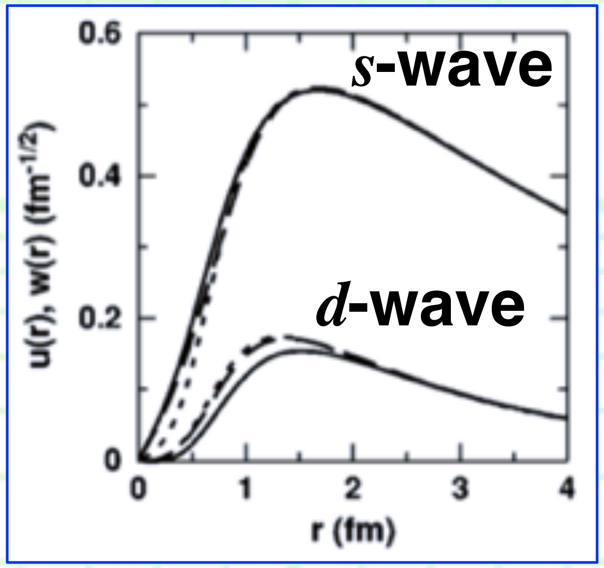
- An example of hadronic molecule: deuteron.

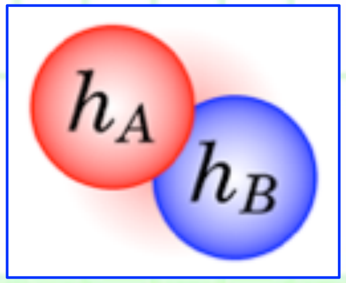


NN phase shift

Machleidt, *Phys. Rev.* C63 (2001) 024001.

Wave function of deuteron





□ Deuteron is a proton-neutron bound state. <-- Who proved this ?

--- Weinberg proved this by using general wave equations in quantum mechanics in the weak binding limit ($B_E << E_{typical}$).

<-- Without using QCD !

Weinberg (1965).

□ Introduce field renormalization constant Z: $Z \equiv \langle B|B_0\rangle\langle B_0|B\rangle$

--> "Bare" component | B_0 > in the total wave function | B >.

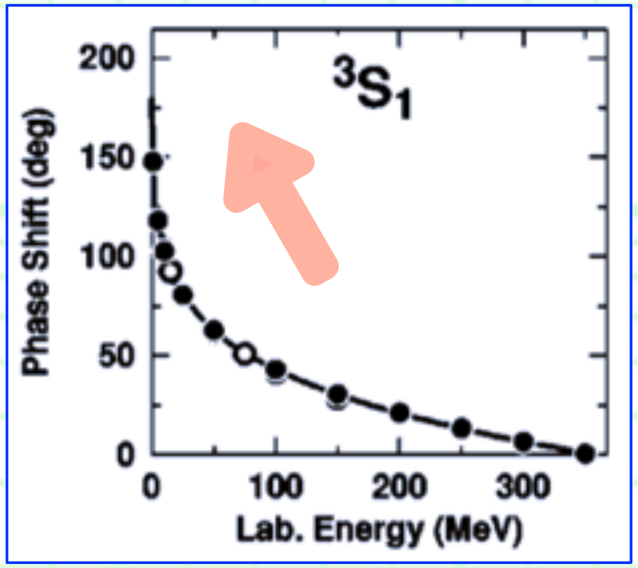
$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(m_\pi^{-1}), \quad r_e = -\frac{Z}{1-Z}R + \mathcal{O}(m_\pi^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

$a = 5.419 \pm 0.007$ fm, $\quad r_e = 1.7513 \pm 0.008$ fm --> Consistent with $Z \approx 0$!

1. Introduction

++ Hadronic molecules and quantum mechanics ++

- An example of hadronic molecule: deuteron.



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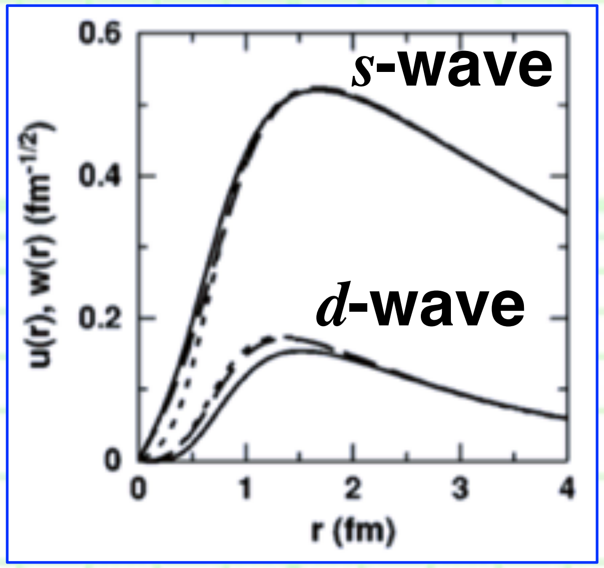
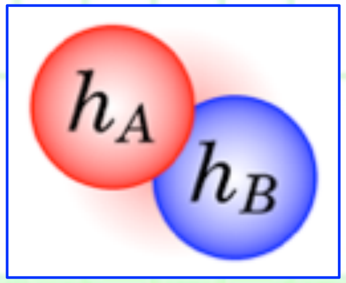




□ Lesson: In a similar manner, we can study the structure of general hadronic molecules.

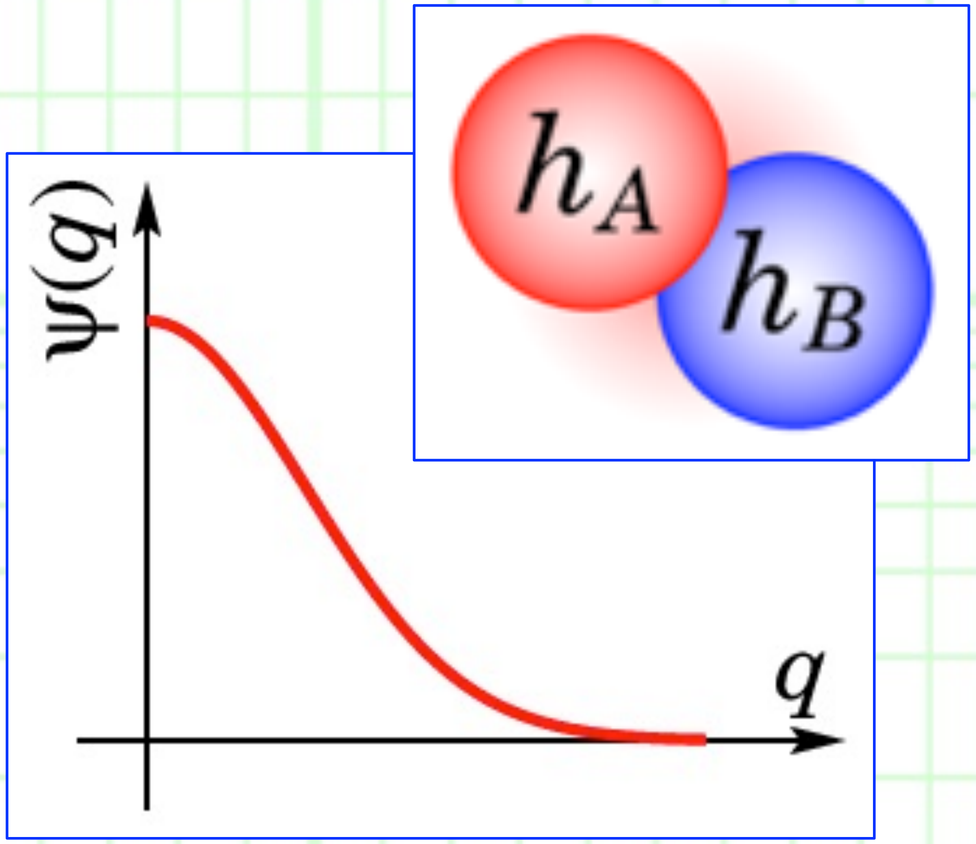
--- We can use quantum mechanics to investigate them:
Two-body wave function, its norm = compositeness (複合性),
scattering amplitude, ...

<--> For hadrons of other configurations, we have to treat color multiplet states explicitly and appropriately.

1. Introduction

++ How to clarify their structure ? ++

- How can we use quantum mechanics to clarify the structure of hadronic molecule candidates ?



- We evaluate the wave function of hadron-hadron composite contribution.

--- *cf.* Wave function for relative motion of two nucleons inside deuteron.

- How to evaluate the wave function ?

<-- We employ a fact that the two-body wave function appears in the residue of the scattering amplitude of the two particles at the resonance pole.

--- The wave function from the residue is automatically normalized !

--> Calculating the norm of the two-body wave function = compositeness, we may measure the fraction of the composite component and conclude the composite structure !

1. Introduction

++ Purpose and strategy of this study ++

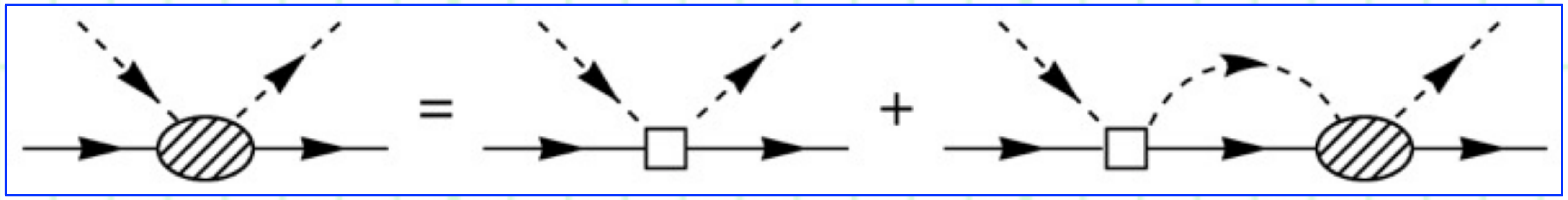
- In this study we evaluate the hadron-hadron two-body wave functions and their norms = compositeness for hadron resonances from the hadron-hadron scattering amplitudes.

- We have to use precise scattering amplitudes for the evaluation.

--> Employ the chiral unitary approach.

Kaiser-Siegel-Weise ('95); Oset-Ramos ('98); Oller-Meissner ('01); Lutz-Kolomeitsev ('02); Oset-Ramos-Bennhold ('02); Jido-Oller-Oset-Ramos-Meissner ('03); ...

$$\boxed{T = V + VGT}$$



- Interaction kernel V from the chiral perturbation theory: Leading order (LO) + next-to-leading order (NLO) (+ bare Δ).

- Loop function G calculated with the dispersion relation in a covariant way.

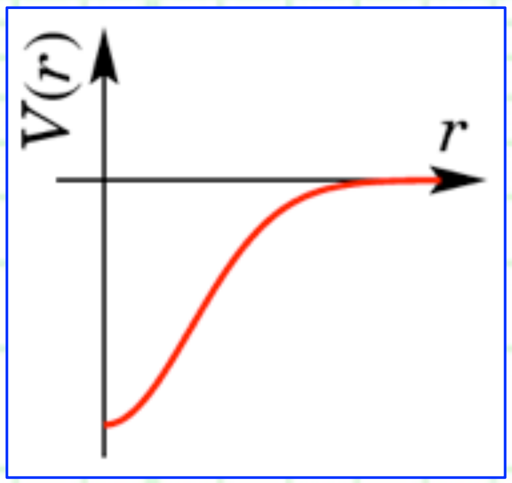
- We discuss the structure of $\Lambda(1405)$, $N(1535)$, $N(1650)$, and $\Delta(1232)$.

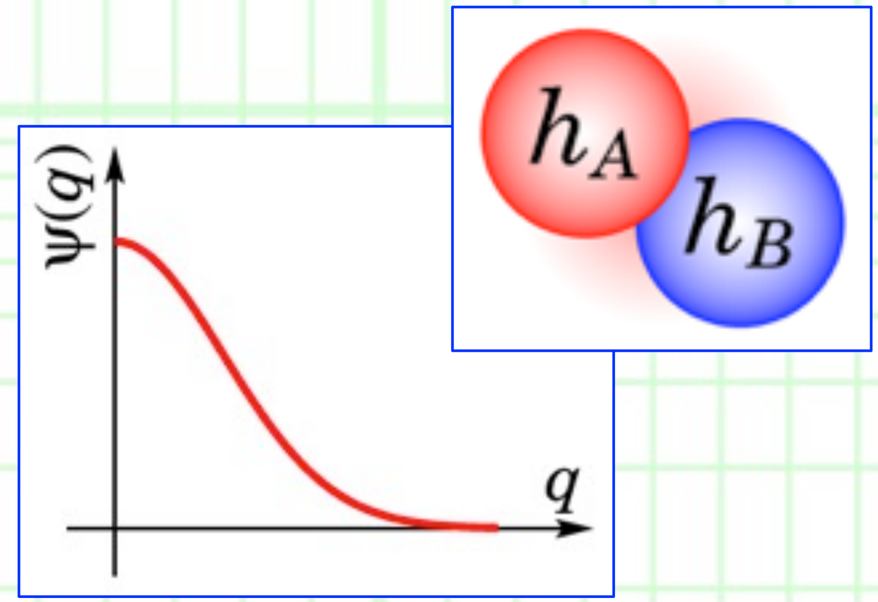
2. Two-body wave functions and compositeness

2. Wave functions and compositeness

++ Setup of the quantum system ++

- **Problem: Calculate the wave function of a bound state, both in the stable and unstable cases.**
  - Interaction V is known.





  - **Full Hamiltonian**: $\hat{H} = \hat{H}_0 + \hat{V}$

--- Free part has eigenstates of scattering states: $\hat{H}_0 |\mathbf{q}\rangle = E(q) |\mathbf{q}\rangle$ $\quad E(q) = M_{\text{th}} + \dfrac{q^2}{2\mu}$

  - **Wave function** (momentum space): $\langle \mathbf{q} | \Psi \rangle = \tilde{\psi}(q)$

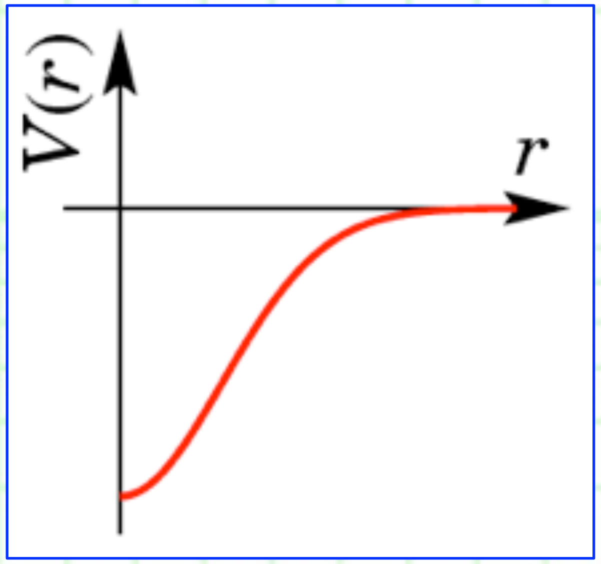
  - **The Schrödinger equation**: $\hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\text{pole}}|\Psi\rangle$

2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are **several approaches to calculate the wave function**.

Ex.) A bound state in a NR single-channel problem.



□ Usual approach: Solve the Schrödinger equation.

$$\hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\rm pole}|\Psi\rangle$$

--- Wave function in coordinate / momentum space:

$$\langle \mathbf{r}|\Psi\rangle = \psi(r) \qquad \langle \mathbf{q}|\Psi\rangle = \tilde{\psi}(q)$$

$$\left[M_{\rm th} - \frac{\nabla^2}{2\mu} + V(r) \right] \psi(r) = E_{\rm pole}\psi(r)$$

--- | q > is an eigenstate of free Hamiltonian H_0:

$$\hat{H}_0|\mathbf{q}\rangle = E(q)|\mathbf{q}\rangle \qquad E(q) = M_{\rm th} + \frac{q^2}{2\mu}$$

--> After solving the Schrödinger equation, we have to **normalize the wave function by hand**.

$$\int d^3 r \, [\psi(r)]^2 = 1 \quad \text{or} \quad \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(q) \right]^2 = 1$$

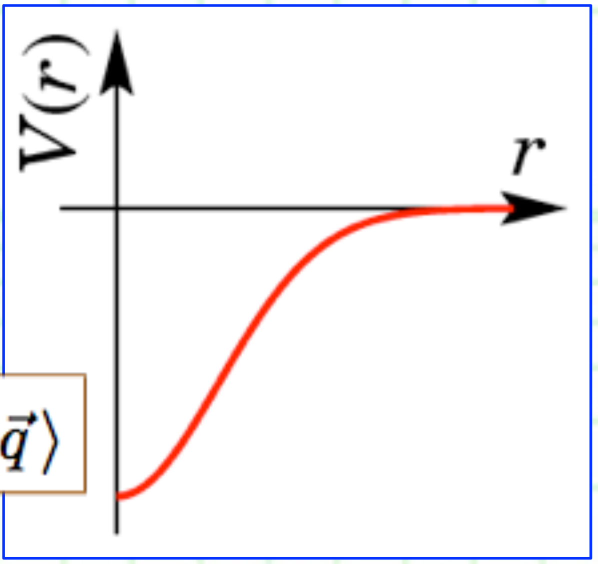
<-- We require !

2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are **several approaches to calculate the wave function**.

Ex.) A bound state in a NR single-channel problem.



- Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.

$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E - \hat{H}_0}\hat{T} = \hat{V} + \hat{V}\frac{1}{E - \hat{H}}\hat{V}$$

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle$$

--- Near the resonance pole position E_{pole}, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \langle \vec{q}\,' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\mathrm{pole}}} \langle \Psi^* | \hat{V} | \vec{q} \rangle$$

$|\Psi\rangle, |\vec{q}_{\mathrm{full}}\rangle, \ldots$ $\quad$ $\langle \Psi^* |, \langle \vec{q}_{\mathrm{full}} |, \ldots$

$$\Rightarrow \quad \mathbb{1} = |\Psi\rangle\langle\Psi^*| + \cdots$$

--- The residue of the amplitude at the pole position has information on the wave function !

$$\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \vec{q} | (\hat{H} - \hat{H}_0) | \Psi \rangle = [E_{\mathrm{pole}} - E(q)]\tilde{\psi}(q)$$

$$\langle \Psi^* | \hat{V} | \vec{q} \rangle = [E_{\mathrm{pole}} - E(q)]\tilde{\psi}(q)$$

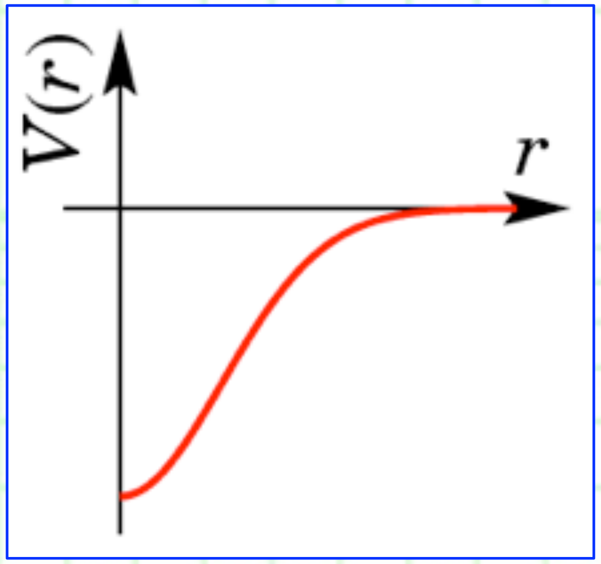
$$E(q) = M_{\mathrm{th}} + \frac{q^2}{2\mu}$$

2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function.

Ex.) A bound state in a NR single-channel problem.



□ Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.

--- The wave function can be extracted from the residue of the amplitude at the pole position:

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

$$\gamma(q) \equiv \langle \vec{q} | \hat{V} | \Psi \rangle = [E_{\text{pole}} - E(q)] \tilde{\psi}(q)$$

--> Because the scattering amplitude cannot be freely scaled due to the optical theorem, the wave function from the residue of the amplitude is automatically normalized !

If purely molecule --> $$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\text{pole}} - E(q)} \right]^2 = 1$$ <-- We obtain !

E. Hernandez and A. Mondragon, *Phys. Rev.* C 29 (1984) 722.

--> Therefore, from hadron-hadron scattering amplitudes with resonance poles, we can calculate their two-body wave function.

2. Wave functions and compositeness

++ Example 1: Stable bound state ++

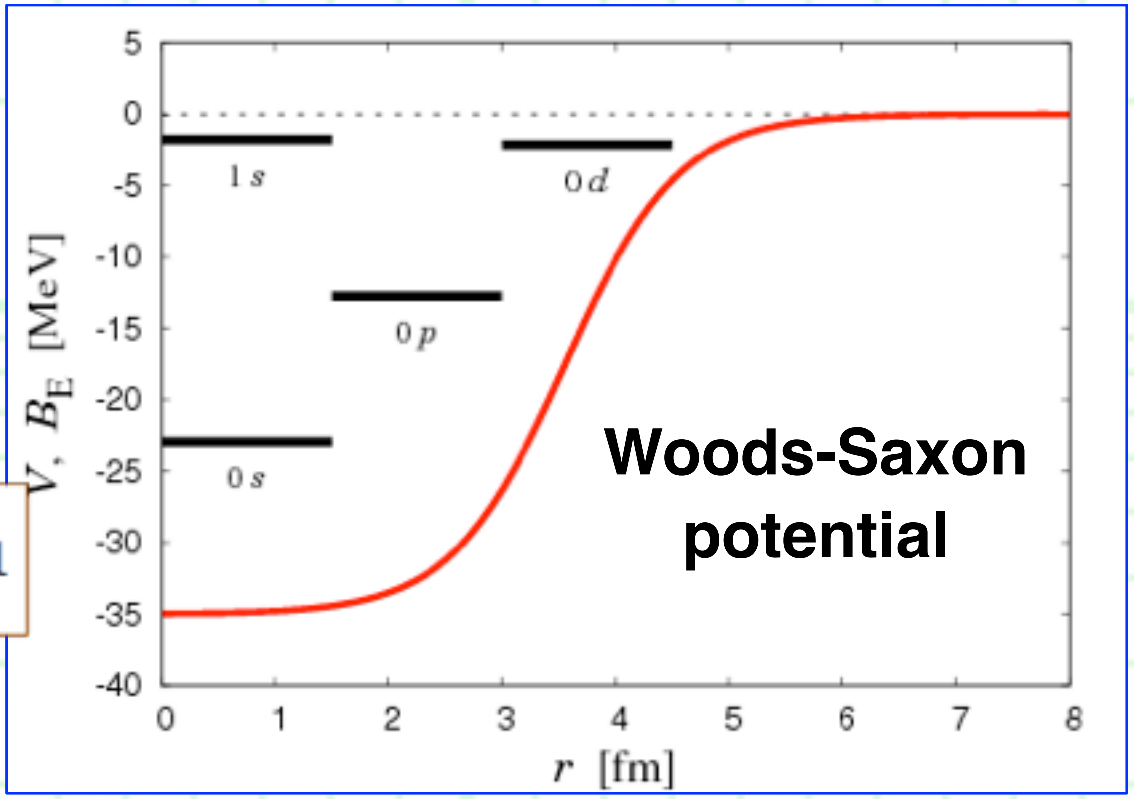
- A Λ hyperon in $A = 40$ nucleus.

--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$E(q)\tilde{\psi}(\mathbf{q}) + \int \frac{d^3q'}{(2\pi)^3} \tilde{V}(\mathbf{q},\mathbf{q}')\tilde{\psi}(\mathbf{q}') = E_{\text{pole}}\tilde{\psi}(\mathbf{q})$$

--> Normalize ψ by hand ! $\int \frac{d^3q}{(2\pi)^3}\left[\tilde{\psi}(\mathbf{q})\right]^2 = 1$



2. Solve Lippmann-Schwinger equation:

$$T(\mathbf{q}',\mathbf{q};E) = \tilde{V}(\mathbf{q}',\mathbf{q}) + \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{V}(\mathbf{q}',\mathbf{k})T(\mathbf{k},\mathbf{q};E)}{E - E(k)}$$

--> Extract WF from the residue:

$$T(\mathbf{q}',\mathbf{q};E) \approx \frac{\gamma(\mathbf{q}')\gamma(\mathbf{q})}{E - E_{\text{pole}}}$$ --> $$\tilde{\psi}(\mathbf{q}) = \frac{\gamma(\mathbf{q})}{E_{\text{pole}} - E(q)}$$

--- Without normalizing by hand !

2. Wave functions and compositeness

++ Example 1: Stable boun

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$

- A Λ hyperon in A = 40 nucleus

--> Calculate wave functions in

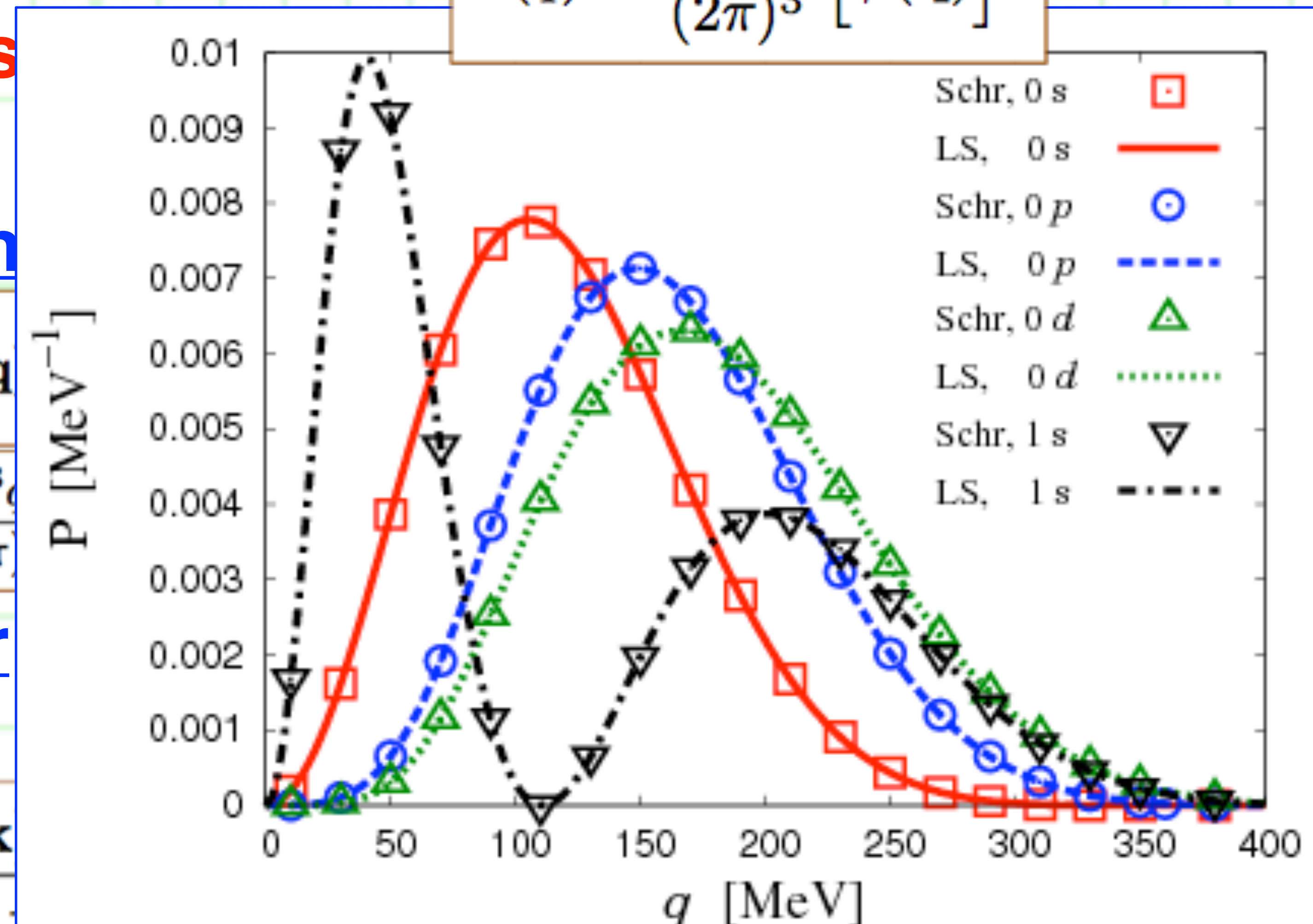
1. Solve Schrödinger equation

$$E(q)\tilde{\psi}(\mathbf{q}) + \int \frac{d^3 q'}{(2\pi)^3} \tilde{V}(\mathbf{q}, \mathbf{q}')\tilde{\psi}(\mathbf{q}') = E_{\text{pole}}\tilde{\psi}(\mathbf{q}$$

--> Normalize ψ by hand ! $\int \frac{d^3}{(2\pi}$

2. Solve Lippmann-Schwinger equation:

$$T(\mathbf{q}', \mathbf{q}; E) = \tilde{V}(\mathbf{q}', \mathbf{q}) + \int \frac{d^3 k}{(2\pi)^3} \frac{\tilde{V}(\mathbf{q}', \mathbf{k}}{E}$$



--> Extract WF from the residue:

$$T(\mathbf{q}', \mathbf{q}; E) \approx \frac{\gamma(\mathbf{q}')\gamma(\mathbf{q})}{E - E_{\text{pole}}} \quad \text{-->} \quad \tilde{\psi}(\mathbf{q}) = \frac{\gamma(\mathbf{q})}{E_{\text{pole}} - E(q)}$$

--- Without normalizing by hand !

- In 1st way: Points.
  2nd way: Lines.
- Exact coincidence !

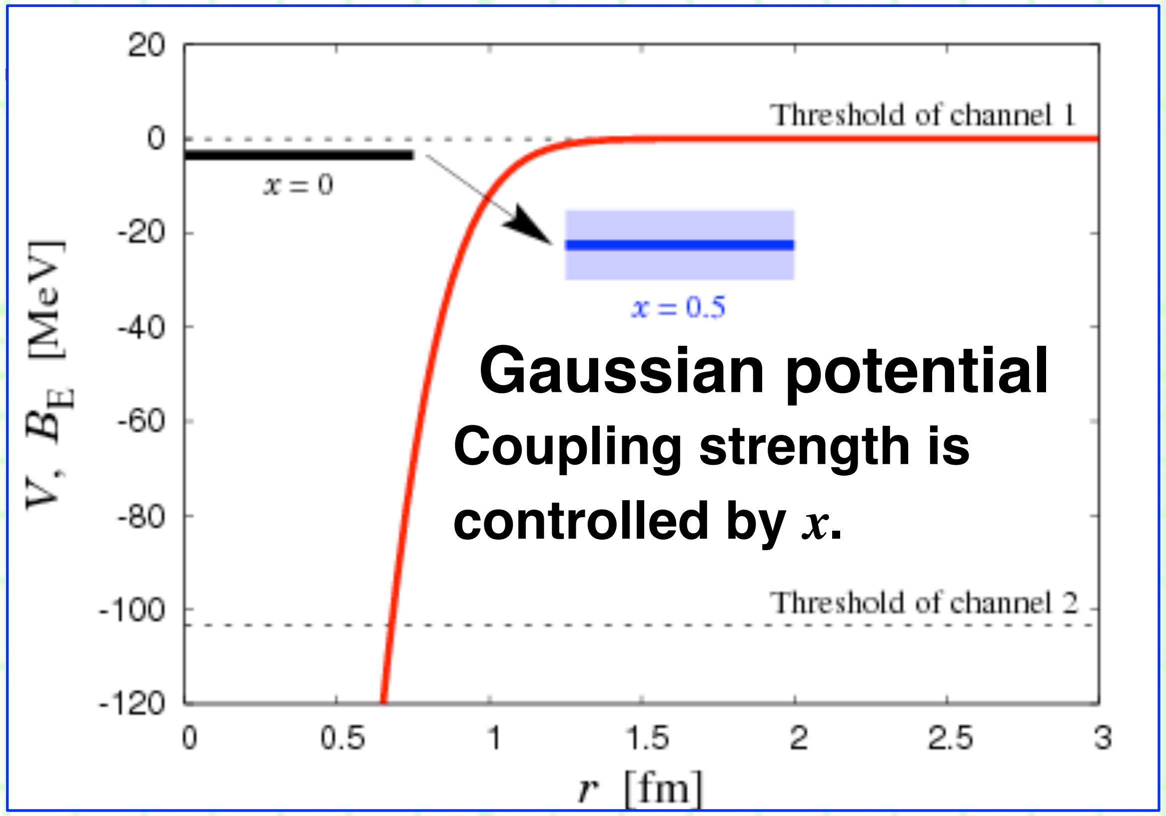
--- We obtain auto-matically normalized WF from the Amp. !

2. Wave functions and compositeness

++ Example 2: Unstable resonance state ++

- **Unstable resonance in $\bar{K}N$-$\pi\Sigma$ system**

--> Calculate wave functions in 2 ways.



1. Solve Schrödinger equation:

$$E_j(q)\tilde{\psi}_j(\mathbf{q}) + \sum_k \int \frac{d^3q'}{(2\pi)^3} \tilde{V}_{jk}(\mathbf{q}, \mathbf{q}')\tilde{\psi}_k(\mathbf{q}') = E_{\text{pole}}\tilde{\psi}_j(\mathbf{q})$$

--> Normalize ψ_j by hand !

$$X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3q}{(2\pi)^3} \left[\tilde{\psi}_j(\mathbf{q})\right]^2$$

2. Solve Lippmann-Schwinger equation:

$$T_{jk}(\mathbf{q}', \mathbf{q}; E) = \tilde{V}_{jk}(\mathbf{q}', \mathbf{q}) + \sum_l \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{V}_{jl}(\mathbf{q}', \mathbf{k})T_{lk}(\mathbf{k}, \mathbf{q}; E)}{E - E_l(k)}$$

--> Extract WF from the residue:

$$T_{jk}(\mathbf{q}', \mathbf{q}; E) \approx \frac{\gamma_j(\mathbf{q}')\gamma_k(\mathbf{q})}{E - E_{\text{pole}}} \quad \text{-->} \quad \tilde{\psi}_j(\mathbf{q}) = \frac{\gamma_j(\mathbf{q})}{E_{\text{pole}} - E_j(q)}$$

--- Without normalizing by hand !

2. Wave functions and compositeness

++ Example 2: Unstable resonan

$$\mathrm{P}_j(q) = \frac{4\pi q^2}{(2\pi)^3}\left[\tilde{\psi}_j(\mathbf{q})\right]^2$$

- **Unstable resonance in $\bar{K}N$-π**

--> **Calculate wave functions**

1. **Solve Schrödinger equati**

$$E_j(q)\tilde{\psi}_j(\mathbf{q}) + \sum_k \int \frac{d^3q'}{(2\pi)^3}\tilde{V}_{jk}(\mathbf{q},\,\mathbf{q}')\tilde{\psi}_k(\mathbf{q}') =$$

--> **Normalize ψ_j by hand !**

$$X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3q}{(2\pi)^3}\left[\tilde{\psi}_j(\mathbf{q})\right]^2$$

2. **Solve**
equati

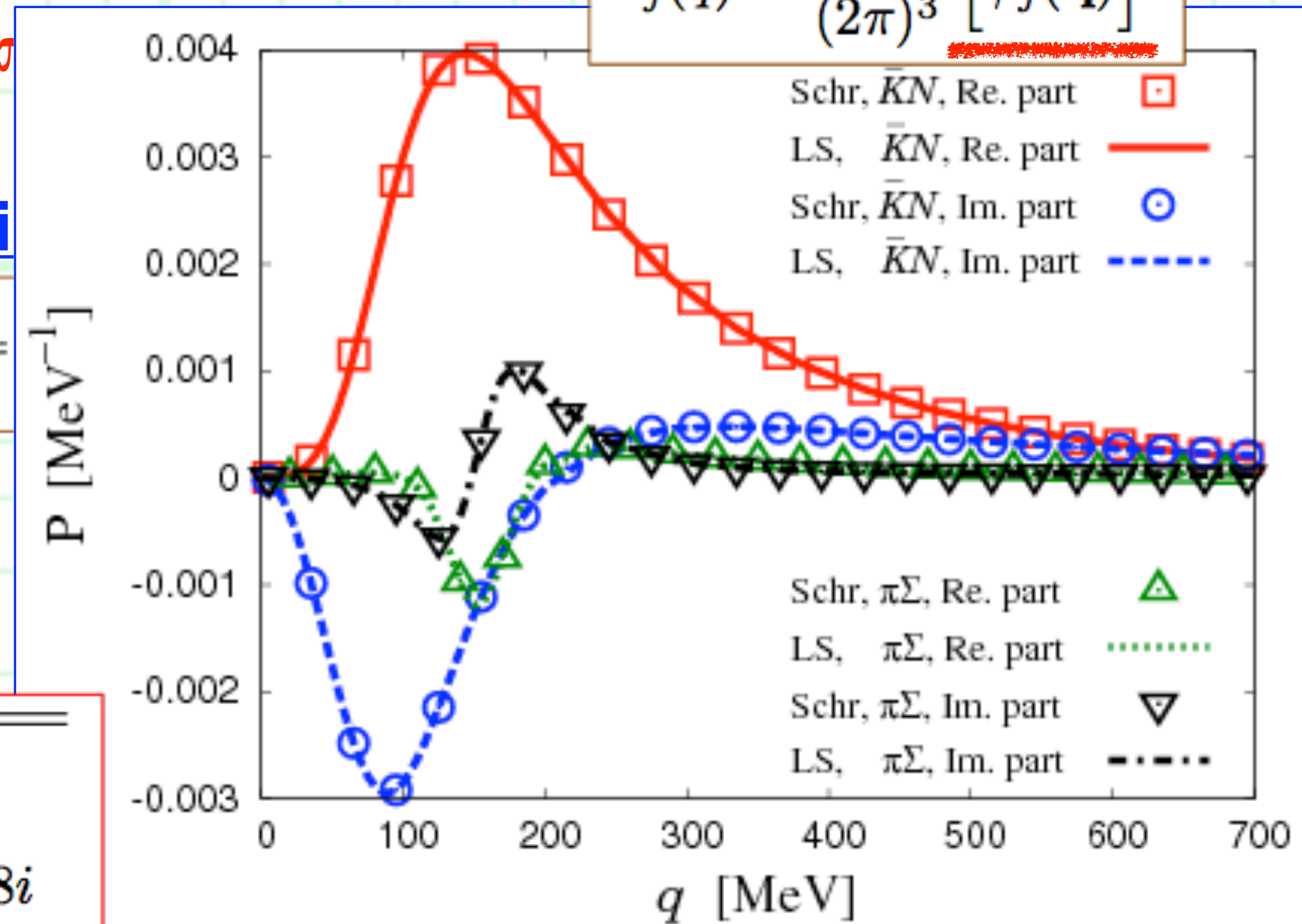
$$T_{jk}(\mathbf{q}',\,\mathbf{q};\,E \quad\quad E - E_l(k)$$

B_{E} [MeV]	22.6
Γ [MeV]	14.7
X_1	$0.99 - 0.08i$
X_2	$0.01 + 0.08i$
$X_1 + X_2$	$1.00 + 0.00i$

--> **Extra** **e:**

$$T_{jk}(\mathbf{q}',\,\mathbf{q};\,E) \approx \frac{\gamma_j(\mathbf{q}')\gamma_k(\mathbf{q})}{E - E_{\mathrm{pole}}} \quad \text{-->} \quad \tilde{\psi}_j(\mathbf{q}) = \frac{\gamma_j(\mathbf{q})}{E_{\mathrm{pole}} - E_j(q)}$$

--- **Without normalizing by hand !**



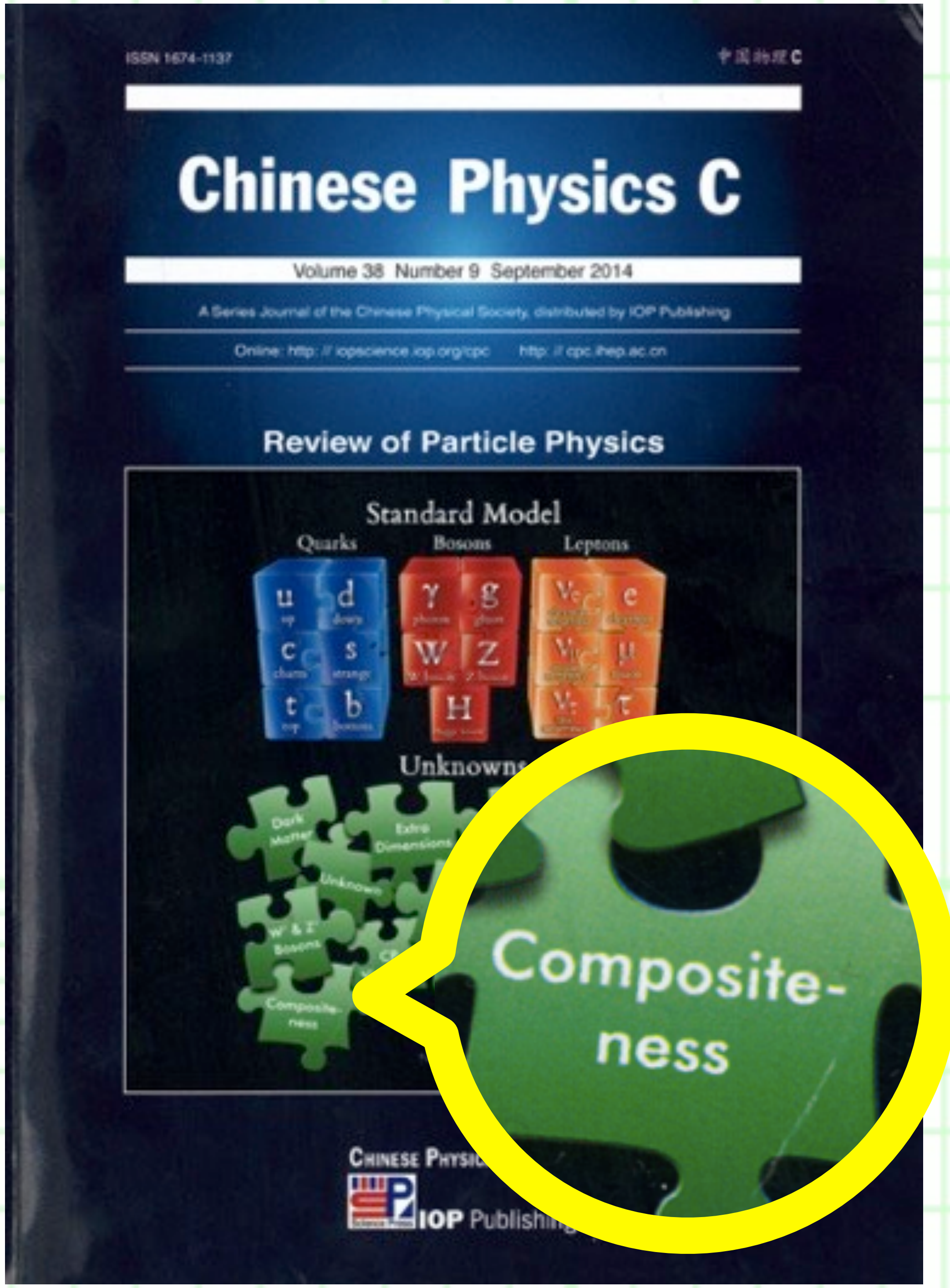
□ **In 1st way: Points.**
2nd way: Lines.

□ **Coincidence again !**
--- **Our method is valid**
even for resonances !

3. Applications: compositeness of hadronic resonances

3. Applications

++ Wave functions for hadrons ++



Particle Data Group (2014).
(similar but not same as our compositeness)

- We can obtain normalized WF from the Scatt. Amp. even for resonances.
- However, if the interaction depends on energy, the norm deviates from unity.

<-- Interpreted as the contribution from a missing channel | ψ_0 >. T. S. , Hyodo and Jido, *PTEP* 2015, 063D04.

$$1 = \int \frac{d^3 q}{(2\pi)^3} |\vec{q}\rangle\langle\vec{q}| + |\psi_0\rangle\langle\psi_0|$$

- By using this fact, we can interpret the norm = compositeness (X) of the wave function from the Amp. as the "fraction" of the two-body state for a resonance in general interaction.

$$\langle \Psi^* | \Psi \rangle = \sum_j X_j + Z = 1$$

$$X_j = \int \frac{d^3 q}{(2\pi)^3} \langle \Psi^* | \vec{q}_j \rangle \langle \vec{q}_j | \Psi \rangle$$

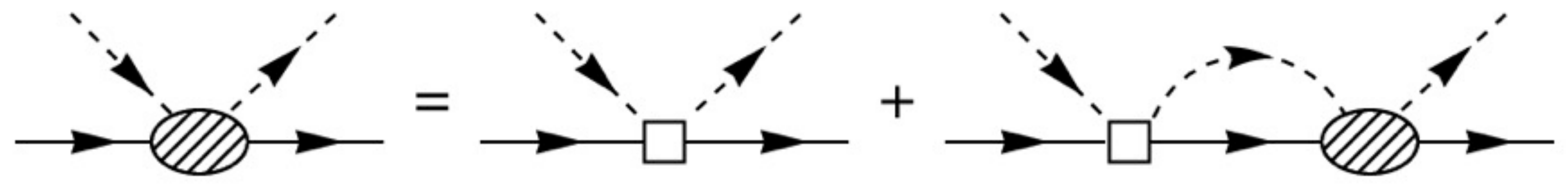
3. Applications

++ Our strategy ++

- In this study we **investigate the structure of hadronic molecule candidates** in the following strategy.

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2015) 035204.

1. Construct hadron-hadron scattering amplitude, which precisely reproduces experimental data and contains **resonance poles for hadronic molecule candidates**, in appropriate effective models.



$$T_{jk}(\vec{q}\,', \vec{q}\,; s) = V_{jk}(\vec{q}\,', \vec{q}\,; s) + i \sum_{l} \int \frac{d^4 q''}{(2\pi)^4} \frac{V_{jl}(\vec{q}\,', \vec{q}\,''; s) T_{lk}(\vec{q}\,'', \vec{q}\,; s)}{(q^2 - m_l^2)[(P-q)^2 - M_l^2]}$$

2. Extract the two-body wave function from **the residue** of the amplitude at the resonance pole.

$$T_{jk}(\vec{q}\,', \vec{q}\,; s) = \frac{\gamma_j(q')\gamma_k(q)}{s - s_{\rm pole}} + (\text{regular at } s = s_{\rm pole})$$

$$\gamma_j(q) \equiv \langle \vec{q}_j | \hat{V} | \Psi \rangle = [s_{\rm pole} - s_j(q)] \tilde{\psi}_j(q)$$

3. Applications

++ Our strategy ++

- In this study we investigate the structure of hadronic molecule candidates in the following strategy.

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2015) 035204.

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$$\gamma_j(q) \equiv \langle \vec{q}_j | \hat{V} | \Psi \rangle = [s_{\rm pole} - s_j(q)] \tilde{\psi}_j(q)$$

3. Calculate the compositeness X_j = norm of the two-body wave function in channel j, from Amp. and compare it with unity.

$$X_j = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[\tilde{\psi}(q) \right]^2 = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[\frac{\gamma_j(q)}{s_{\rm pole} - s_j(q)} \right]^2$$

- The sum of X_j will exactly unity for a purely molecular state.
  <= The interaction does not have energy dependence.
  E. Hernandez and A. Mondragon (1984).
- On the other hand, if the interaction has energy dependence, which can be interpreted as the contribution from a missing channel, the sum of X_j deviates from unity.

--> Fraction of a missing channel is expressed by Z:

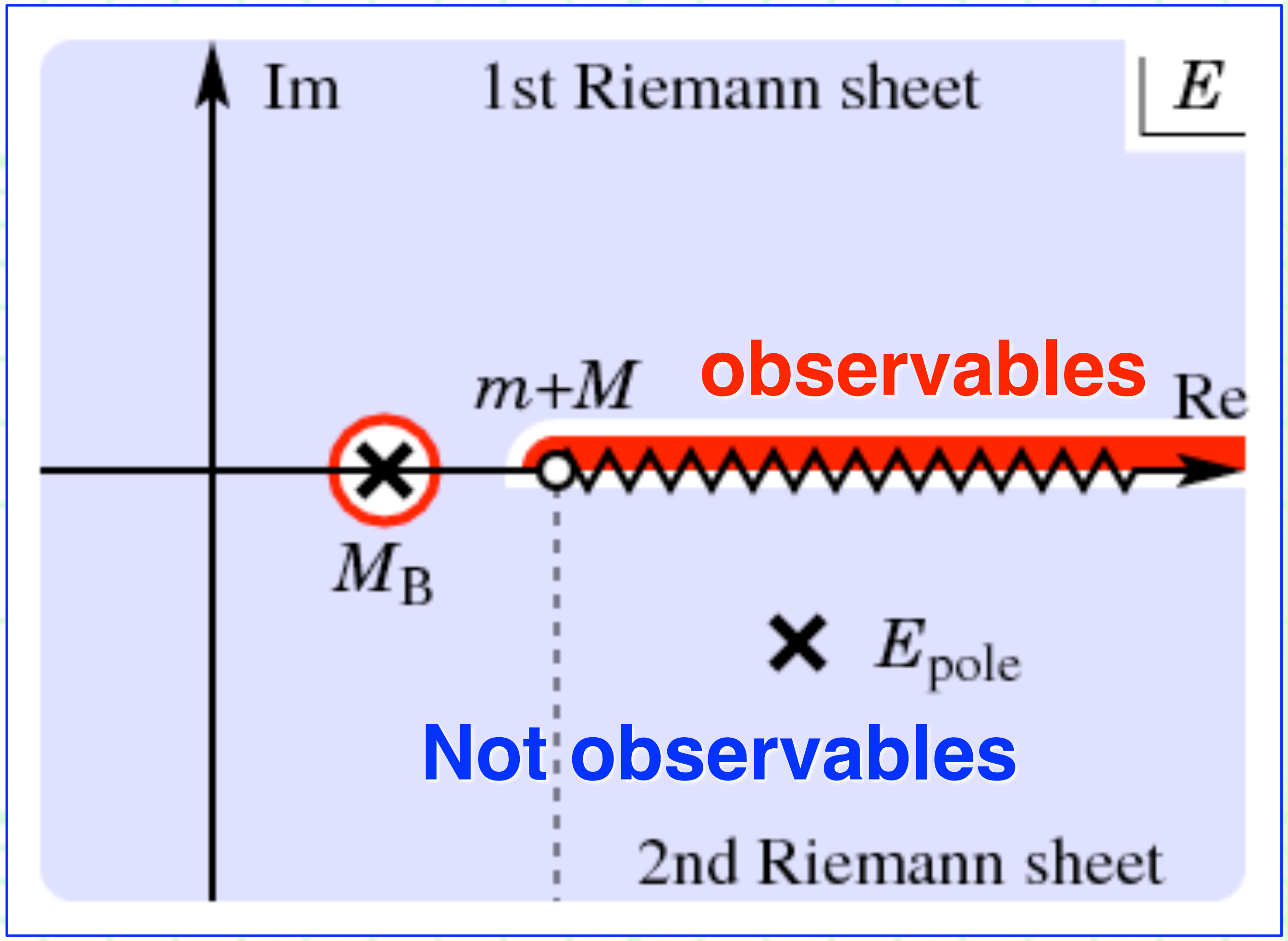
$$\sum_j X_j = 1 - Z$$

--- Same as the Weinberg's Z.

3. Applications

++ Observable and model (in)dependence ++

- Here we comment on the observables and non-observables.
  - Observables:
    Cross section.
    Its partial-wave decomposition.
    --> On-shell Scatt. amplitude
    via the optical theorem.
    Mass of bound states.
  - NOT observables:
    Wave function and potential.
    Resonance pole position.
    Residue at pole.
    Off-shell amplitude.



--> Since the residue of the amplitude at the resonance pole is NOT observable, the wave function and its norm = compositeness are also not observable and model dependent.

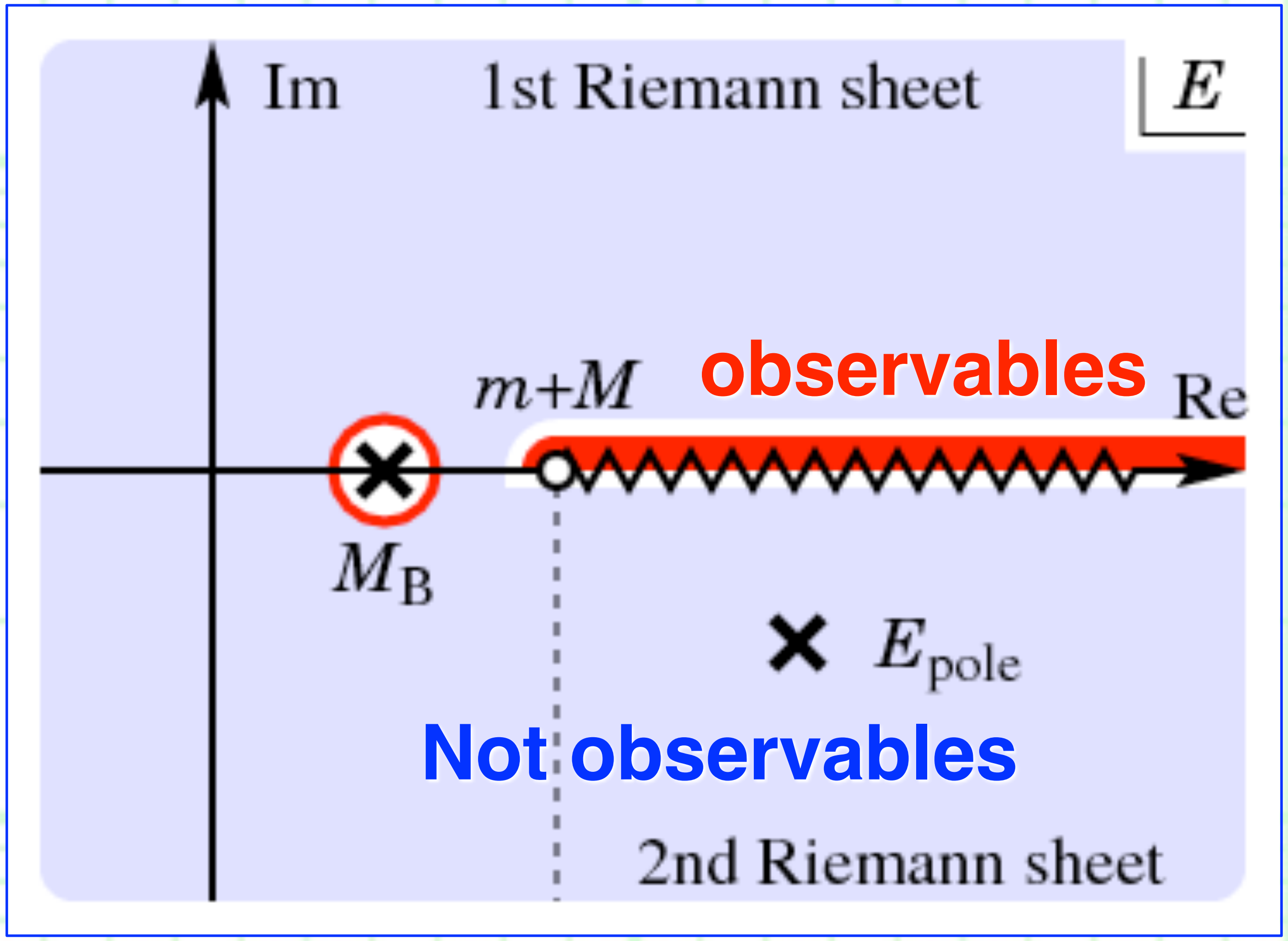
--- Exception: Compositeness for near-threshold poles.

3. Applications

++ Observable and model (in)dependence ++

- **Special case**: **Compositeness for near-threshold poles.**

--- Compositeness can be **expressed with threshold parameters** such as scattering length and effective range.



- □ **Deuteron.**
  Weinberg ('65).
- □ **f_0(980) and a_0(980).**
  Baru et al. ('04),
  Kamiya-Hyodo, arXiv:1509.00146.
- □ **Λ(1405).**
  Kamiya-Hyodo, arXiv:1509.00146.
- □ ...

- **General case**: **Compositeness are model dependent quantity.**

--> Therefore, we have to employ appropriate effective models (V) to construct precise hadron-hadron scattering amplitude, in order to discuss the structure of hadronic molecule candidates !

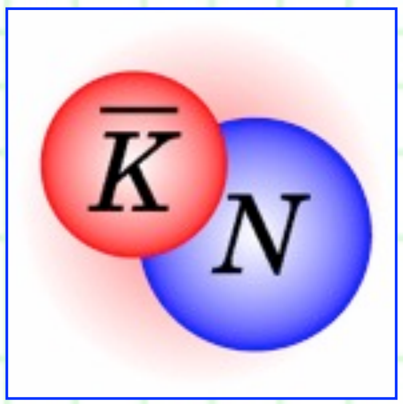
3. Applications

++ List of hadron resonances in our analysis ++

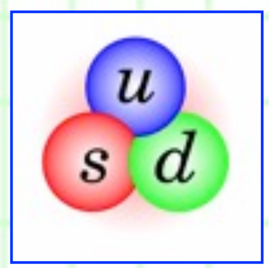
- In this talk, we discuss the structure of candidates of hadronic molecules listed as follows in terms of the compositeness:

1. $\Lambda(1405)$.

--- One of classical examples of the exotic hadron candidates.

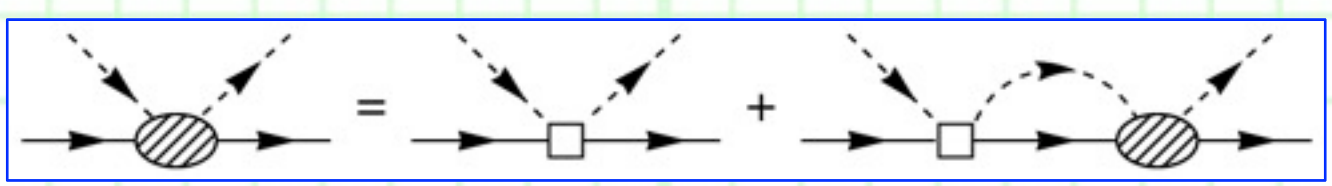


???



2. $N(1535)$ and $N(1650)$.

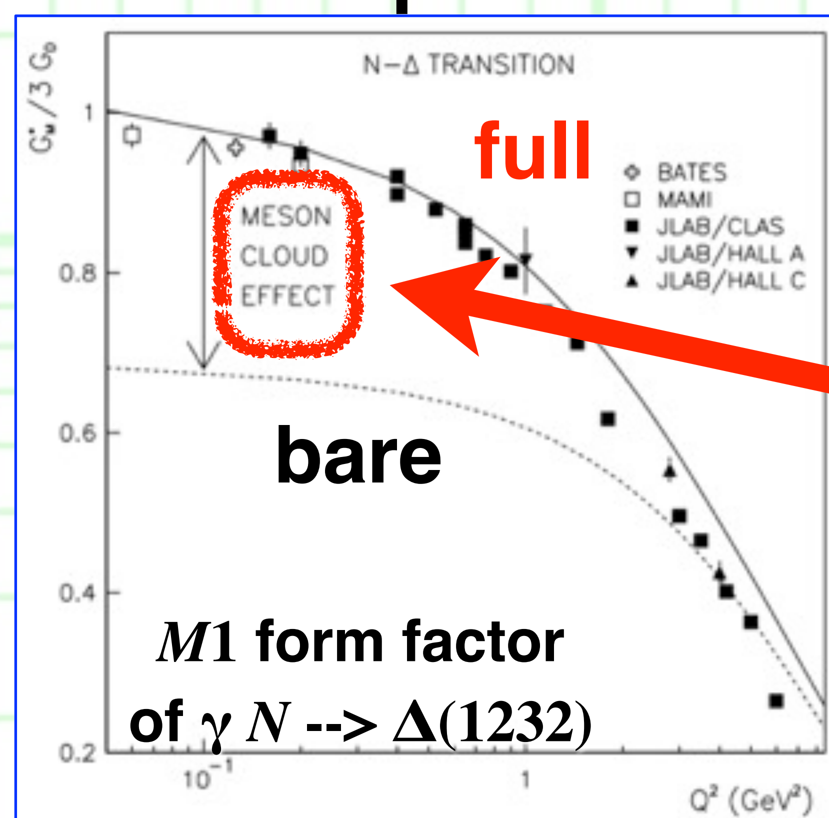
--- Expected to be usual qqq states, but can be described also in meson-baryon d.o.f.



Kaiser-Siegel-Weise ('95), Bruns-Mai-Meissner ('11), ...

3. $\Delta(1232)$.

--- Established as a member of the decuplet in the flavor $SU(3)$ symmetry, together with $\Sigma(1385)$, $\Xi(1530)$, and Ω, in the quark model, but ...



Meson cloud effect ~ 30 % !

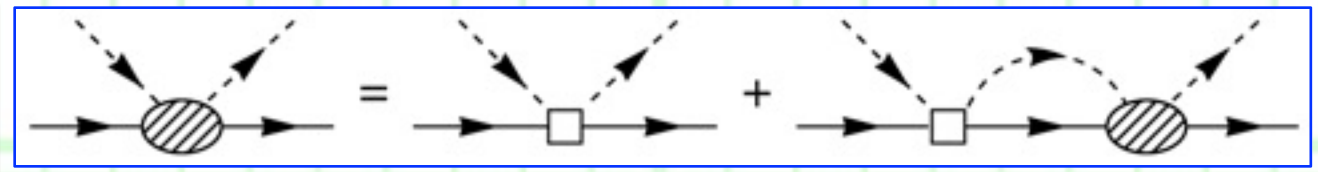
Sato and Lee ('09).

3. Applications

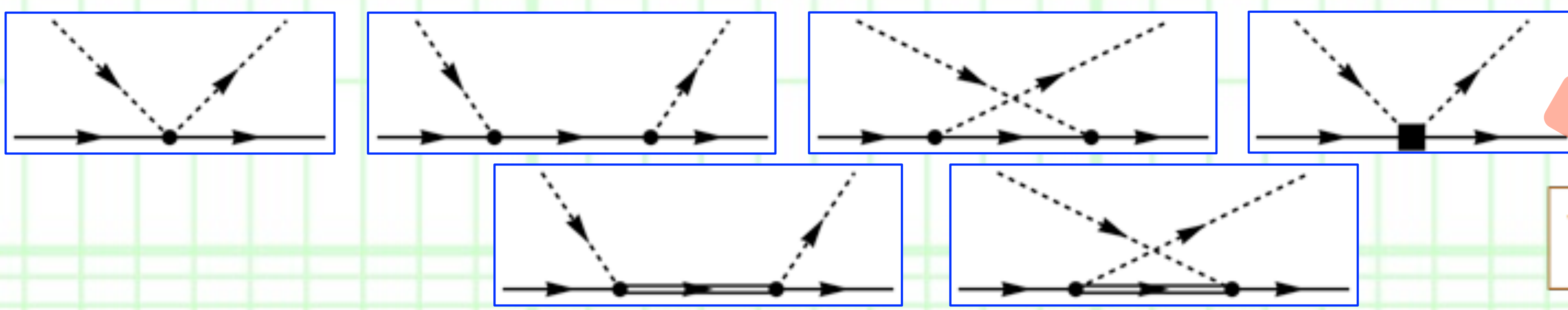
++ Chiral unitary approach ++

- We employ **chiral unitary approach for meson-baryon scatterings**.

$$T'_{\alpha L,jk}(s) = V'_{\alpha L,jk}(s) + \sum_{l} V'_{\alpha L,jl}(s) G_{L,l}(s) T'_{\alpha L,lk}(s)$$



□ For the interaction kernel V we take **LO + NLO** (+ bare Δ) of **chiral perturbation theory** and project it to partial wave L and quantum number α to **construct a separable interaction**. --> V^{prime}.



$$V_{\alpha L,jk} = |\vec{q}|^{2L} \times V'_{\alpha L,jk}(s)$$

□ The loop function G_L is obtained with **the dispersion relation**:

$$G_{L,j}(s) = \int_{s_{\mathrm{th},j}}^{\infty} \frac{ds'}{2\pi} \frac{\rho_j(s') q_j(s')^{2L}}{s' - s - i0} = i \int \frac{d^4 q}{(2\pi)^4} \frac{|\vec{q}|^{2L}}{[(P-q)^2 - m_j^2](q^2 - M_j^2)}$$

$\rho_j(s)$: phase space in channel j.

--- We need **one subtraction** for s wave / **two subtractions** for p wave which are fixed as discussed below.

3. Applications

++ Compositeness with separable interaction ++

- For the separable interaction, which we employ in this study, we can calculate the residue at the resonance pole as:

$$\langle \vec{q}\,' | \hat{T}(s) | \vec{q}\,\rangle \approx \langle \vec{q}\,' | \hat{V}(s_{\rm pole}) | \Psi \rangle \frac{1}{s - s_{\rm pole}} \langle \Psi^* | \hat{V}(s_{\rm pole}) | \vec{q}\,\rangle$$

$$\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \Psi^* | \hat{V} | \vec{q} \rangle = g \times |\vec{q}\,|^L$$

Aceti and Oset, *Phys. Rev.* D86 (2012) 014012;
T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2015) 035204.

  - For resonances in L wave, g is the coupling constant.
  - This form is necessary for the correct behavior of the wave function at small q region: $\tilde{\psi}(q) = \mathcal{O}(q^L) \quad \text{for small } q$

- As a result, the norm of the two-body wave function is written as

$$X_j = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \langle \Psi^* | \vec{q}_j \rangle \langle \vec{q}_j | \Psi \rangle = -g_j^2 \left[\frac{dG_{L,j}}{ds} \right]_{s=s_{\rm pole}}$$

--- G_L is the loop function in L wave.

T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2015) 035204.

<=> Elementariness Z with separable interaction:

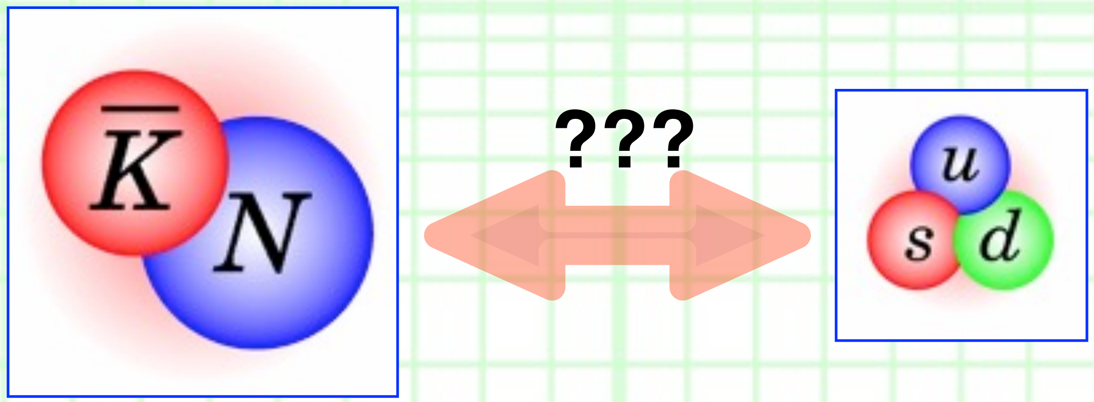
$$Z = -\sum_{j,k} g_k g_j \left[G_{L,j} \frac{dV'_{\alpha L,jk}}{ds} G_{L,k} \right]_{s=s_{\rm pole}}$$

3. Applications

++ Compositeness for $\Lambda(1405)$ ++

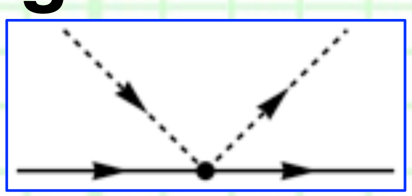
- $\Lambda(1405)$ --- The lightest excited baryon with $J^P = 1/2^{-}$, Why ??
  - Strongly attractive $\bar{K}N$ interaction in the $I = 0$ channel.

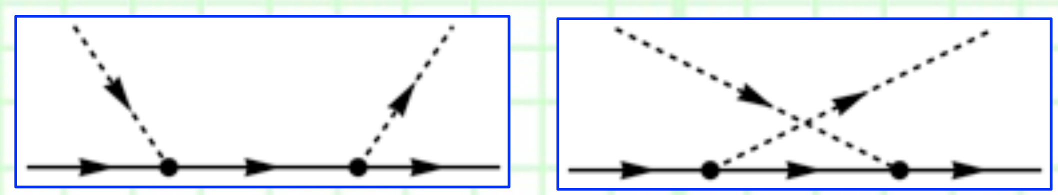
  --> $\Lambda(1405)$ is a $\bar{K}N$ quasi-bound state ??? Dalitz and Tuan ('60), ...



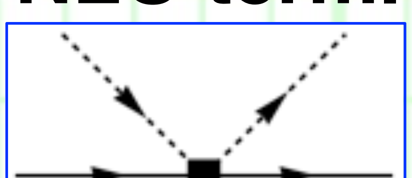
- We use the Ikeda-Hyodo-Weise amplitude for $\Lambda(1405)$ in chiral unitary approach, which was constrained by the recent data of the $1s$ shift and width of kaonic hydrogen. Ikeda, Hyodo, and Weise ('11), ('12).

--- V: Weinberg-Tomozawa term + s- and u-channel Born term





+ NLO term.



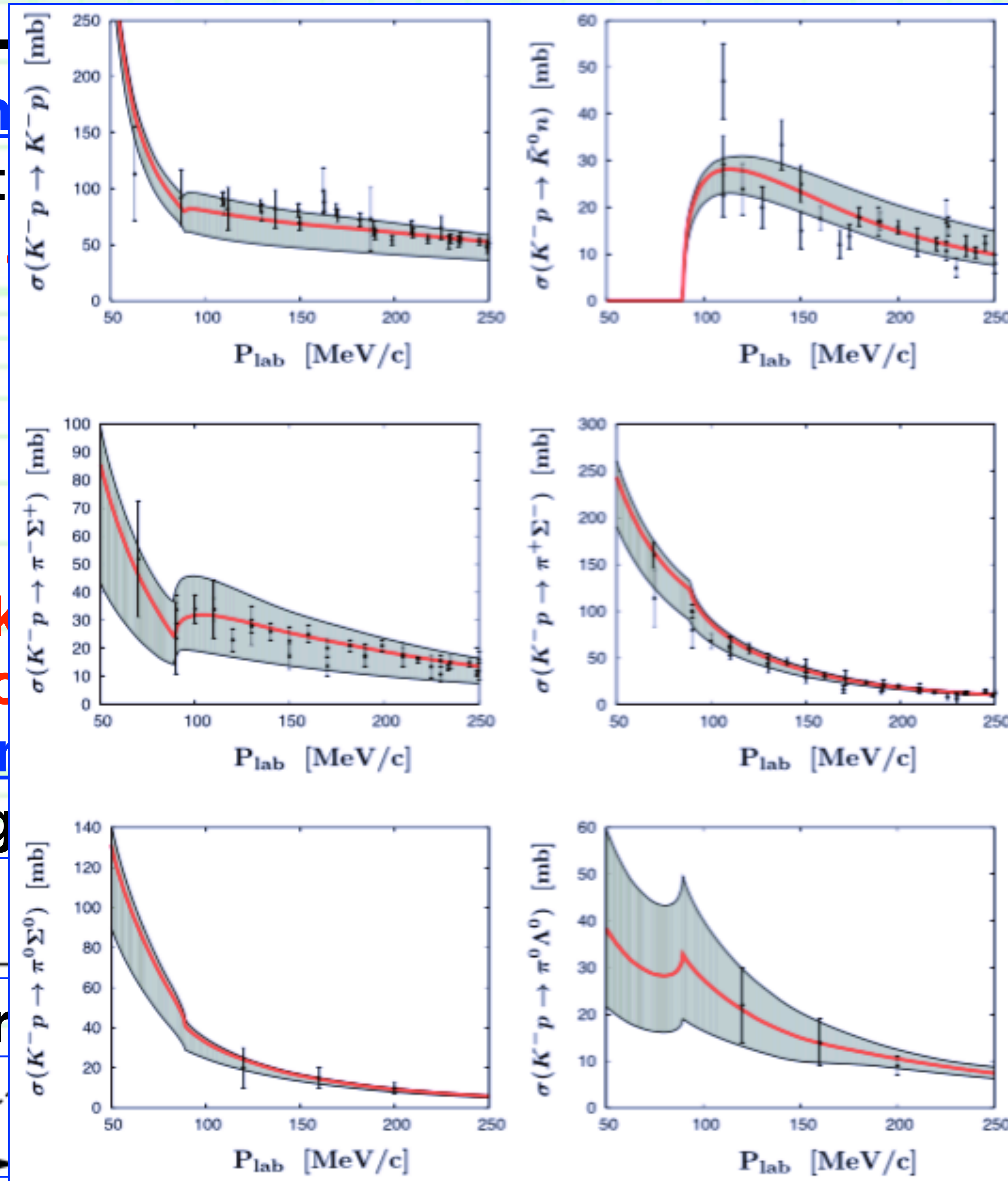
3. Applications

++ ++

▪ Λ(1405) --- Th, Why ??

▫ Strongly att nel.

--> Λ(1405) is Yuan ('60), ...



▪ We use the Ik 5) in chiral unitary appro cent data of the 1s shift ar lo, and Weise ('11), ('12).

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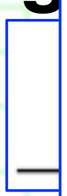
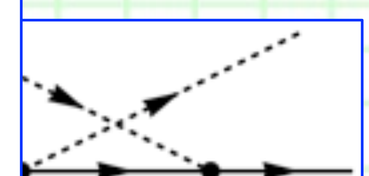
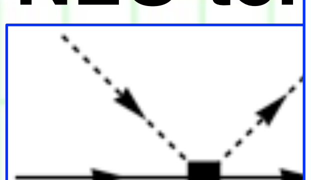




+ NLO ter



3. Applications

++ ... ++

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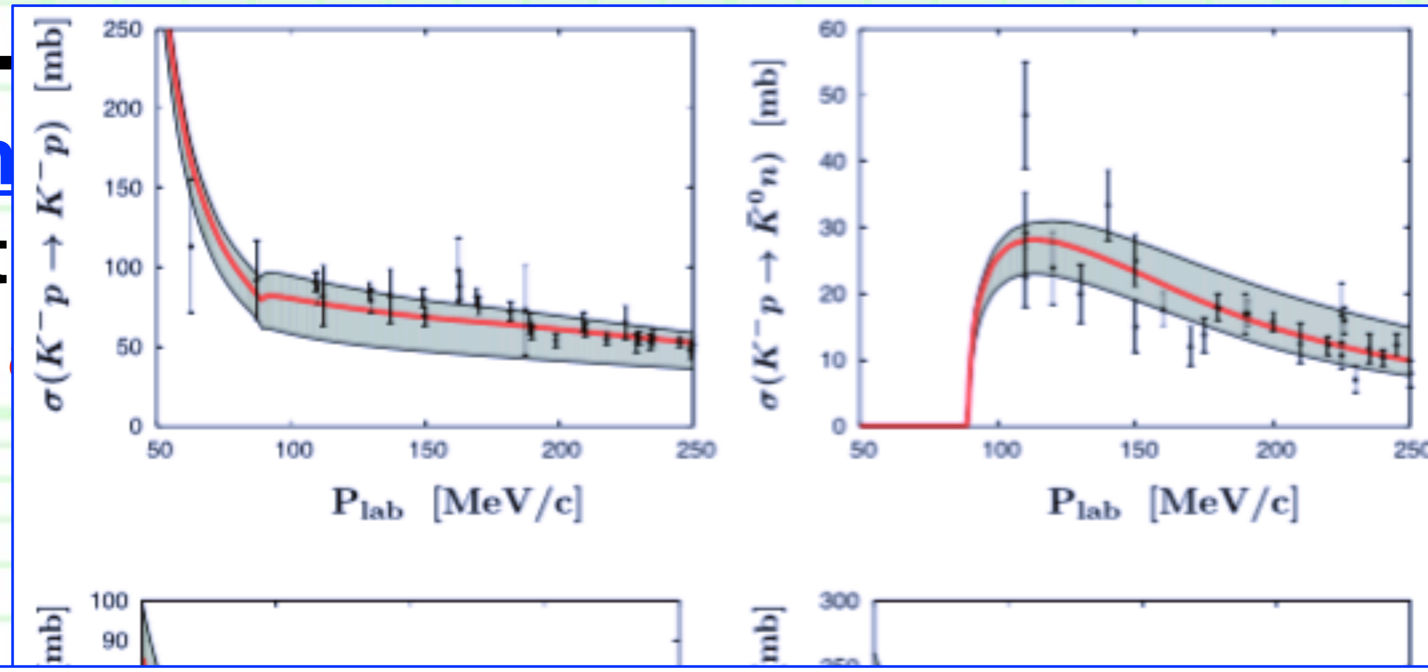
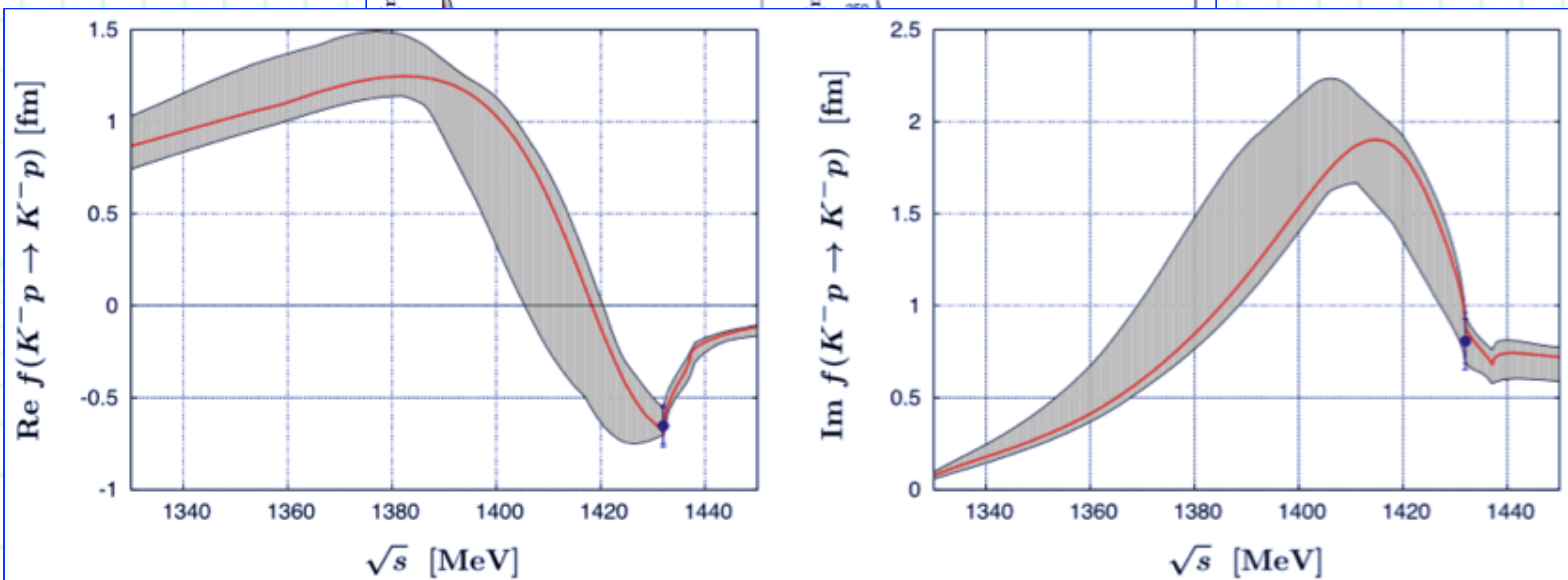




3. Applications

++ Compositeness for $\Lambda(1405)$ ++

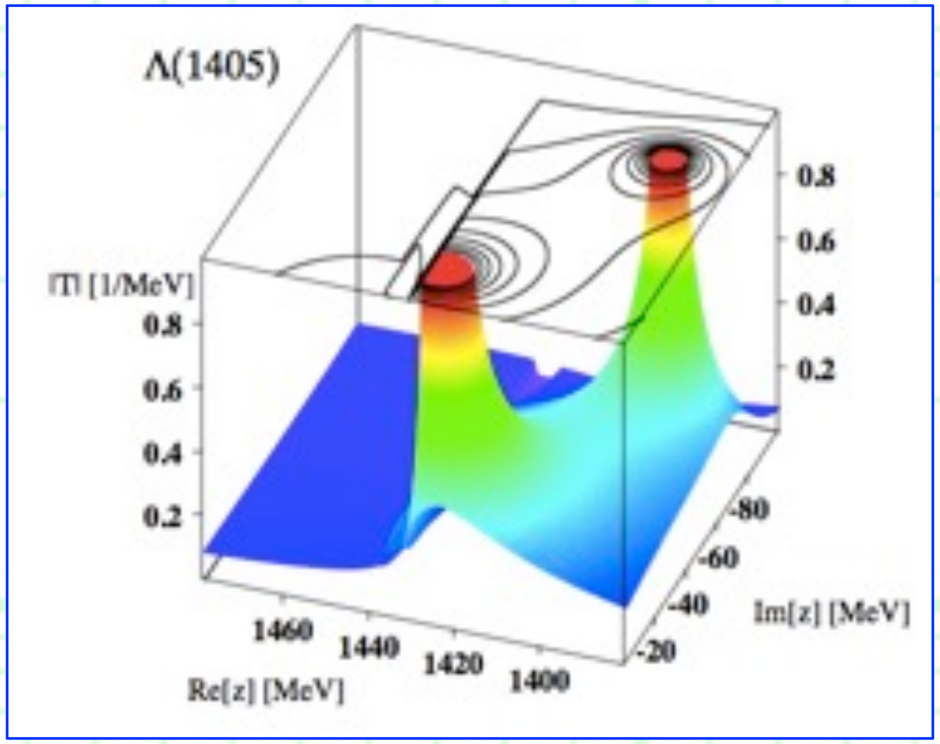
- **Compositeness X and elementariness Z for hadrons in the model.**

T. S. , Hyodo and Jido, *PTEP* 2015, 063D04.

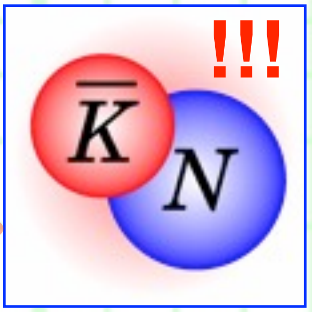
$$X_j = -g_j^2 \left[\frac{dG_j}{ds} \right]_{s=s_{\rm pole}} \qquad Z = -\sum_{j,k} g_k g_j \left[G_j \frac{dV_{jk}}{ds} G_k \right]_{s=s_{\rm pole}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_j X_j + Z = 1$$

□ $\Lambda(1405)$ (two poles!).



Hyodo and Jido ('12).



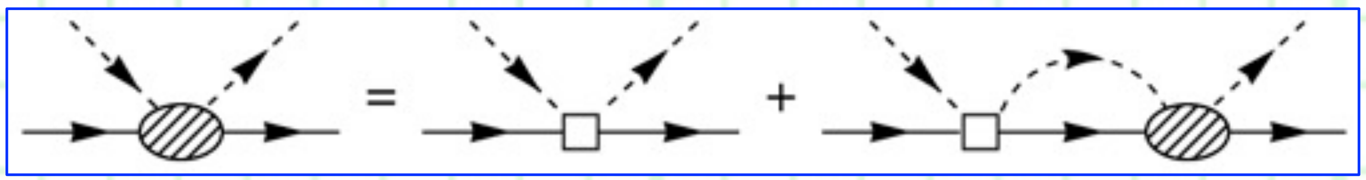
	$\Lambda(1405)$, higher pole	$\Lambda(1405)$, lower pole
$\sqrt{s_{\rm pole}}$	$1424 - 26i$ MeV	$1381 - 81i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$	$-0.39 - 0.07i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$	$0.66 + 0.52i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$	$-0.04 + 0.01i$
$X_{K\Xi}$	$0.00 + 0.00i$	$-0.00 + 0.00i$
Z	$-0.08 + 0.19i$	$0.77 - 0.46i$

--- Large $\bar{K}N$ component for (higher) $\Lambda(1405)$, since $X_{\bar{K}N}$ is almost unity with small imaginary parts.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

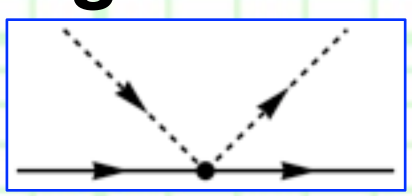
- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^-$.
  - We naively expect that they are conventional qqq states, but there are several studies that they can be dynamically generated from the meson-baryon degrees of freedom without explicit resonance poles, especially in the chiral unitary approach.



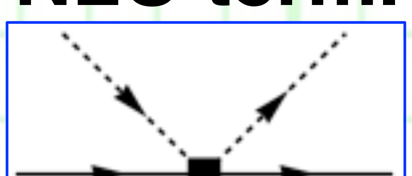
Kaiser-Siegel-Weise ('95); Nieves-Ruiz Ariola ('01); Inoue-Oset-Vicente Vacas ('02); Bruns-Mai-Meissner ('11); ...

  - For example:

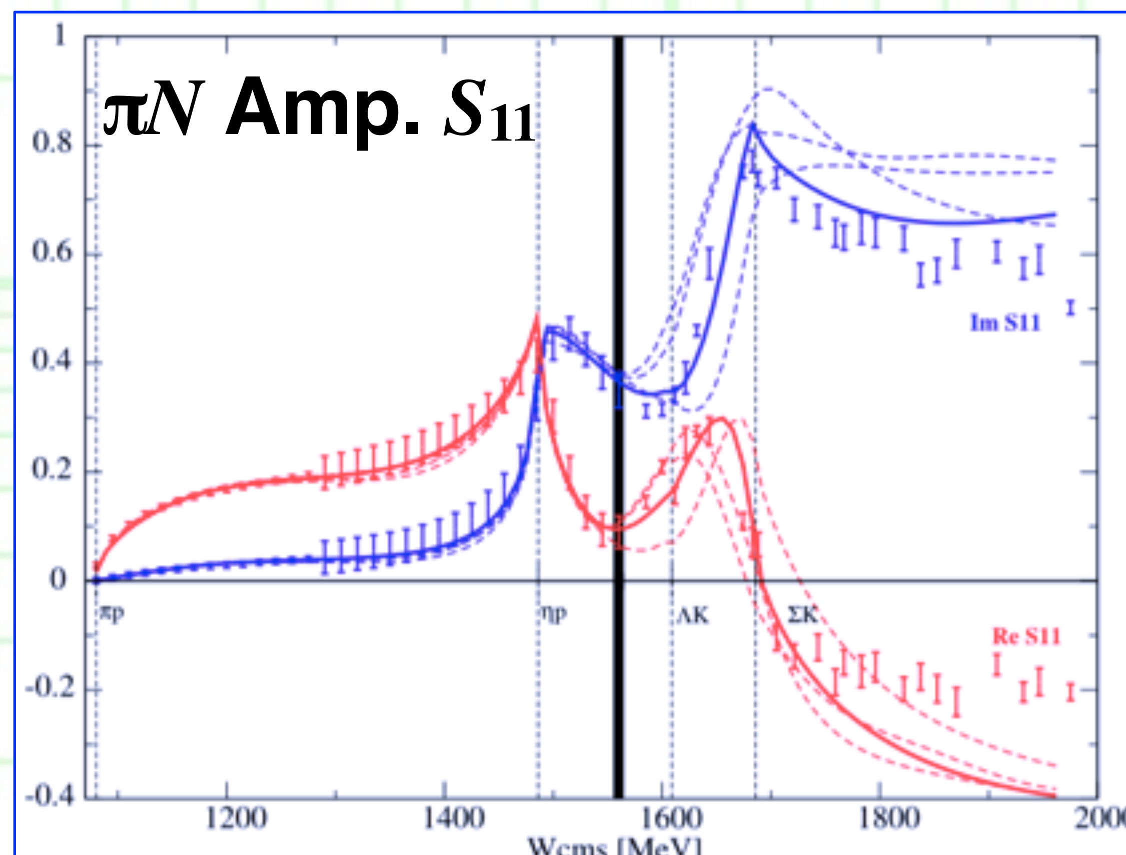
--- V: Weinberg-Tomozawa term



+ NLO term.



Bruns, Mai and Meissner, *Phys. Lett.* B 697 (2011) 254.



3. Applications

++ Compositeness for N(1535) and N(1650) ++

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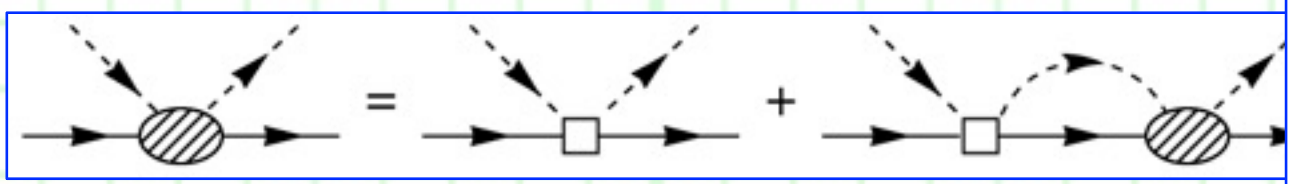

  - For example:

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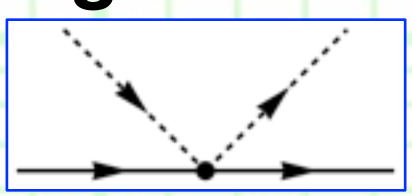

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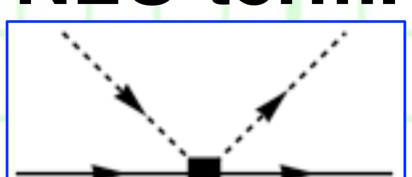
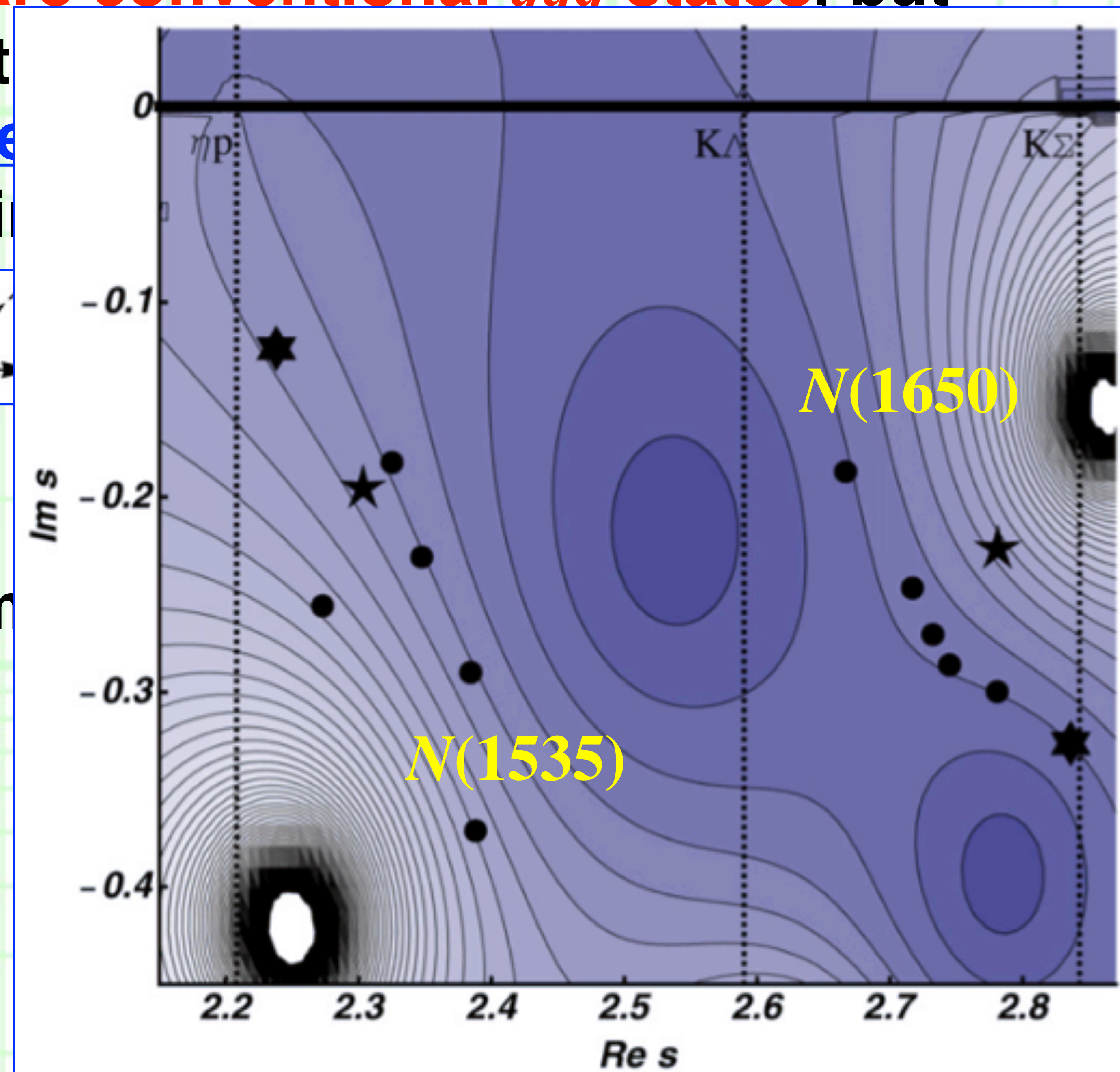




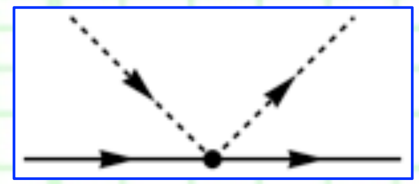
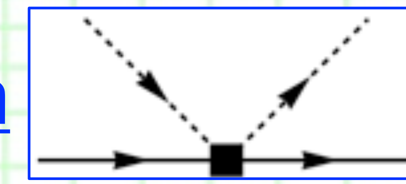
Bruns, Mai and Meissner,
Phys. Lett. B 697 (2011) 254.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^{-}$.

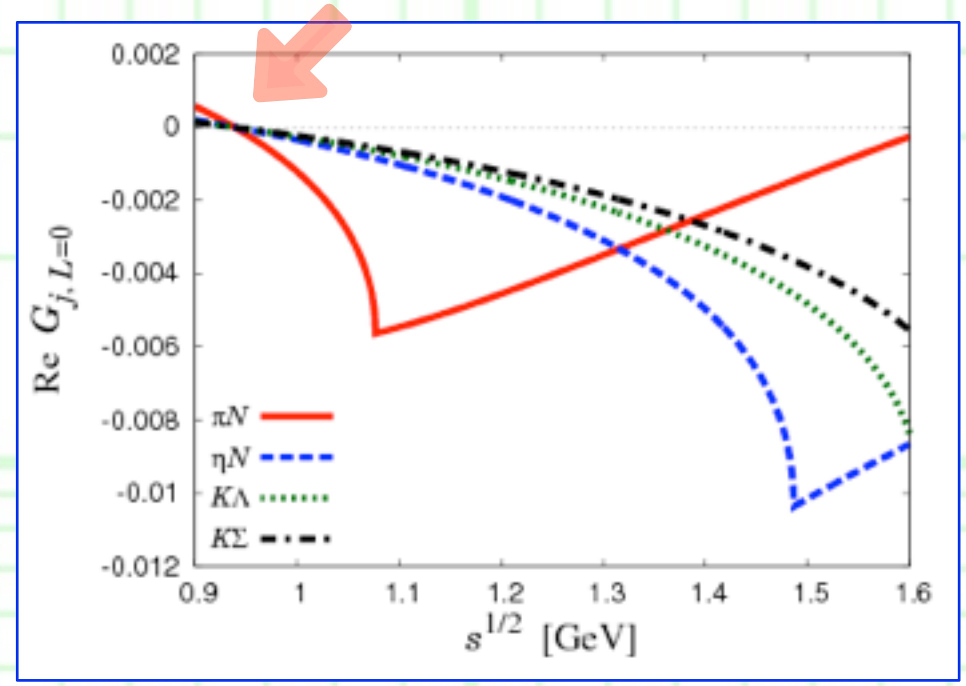
- We construct our own s-wave πN-ηN-$K\Lambda$-$K\Sigma$ scattering amplitude in the chiral unitary approach.

  - V: Weinberg-Tomozawa term  + NLO term  .

  - G: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in G.

$$G_{j,L=0}(s = M_N^2) = 0$$

Hyodo, Jido and Hosaka, *Phys. Rev.* C78 (2008) 025203.



  - Parameters: The low-energy constants in NLO term.

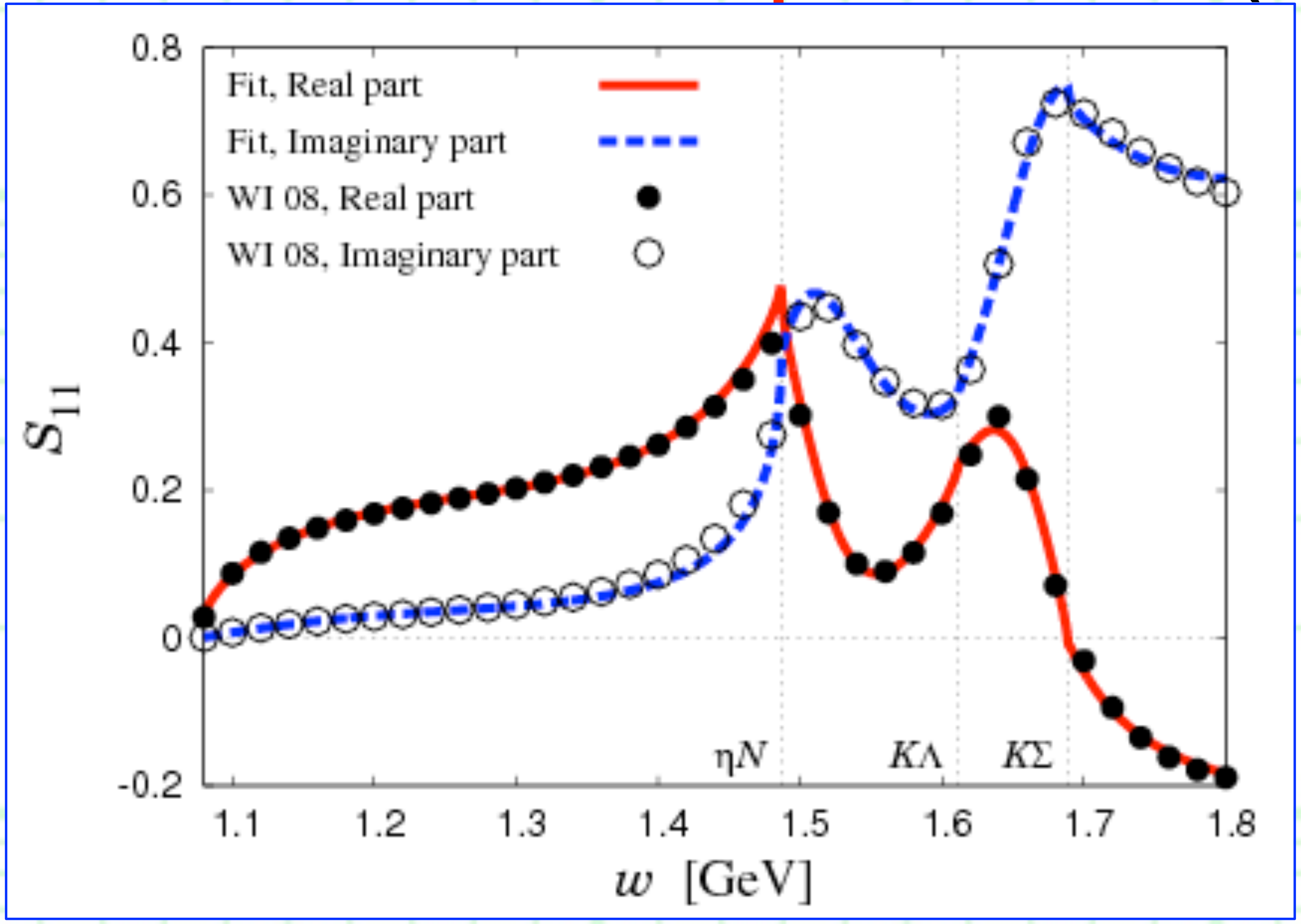
--> Parameters are fixed so as to reproduce the πN scattering amplitude S_{11} as a PWA solution "WI 08" up to $\sqrt{s} = 1.8$ GeV.

Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- **Fitted to the πN amplitude WI 08 (S_{11}).**



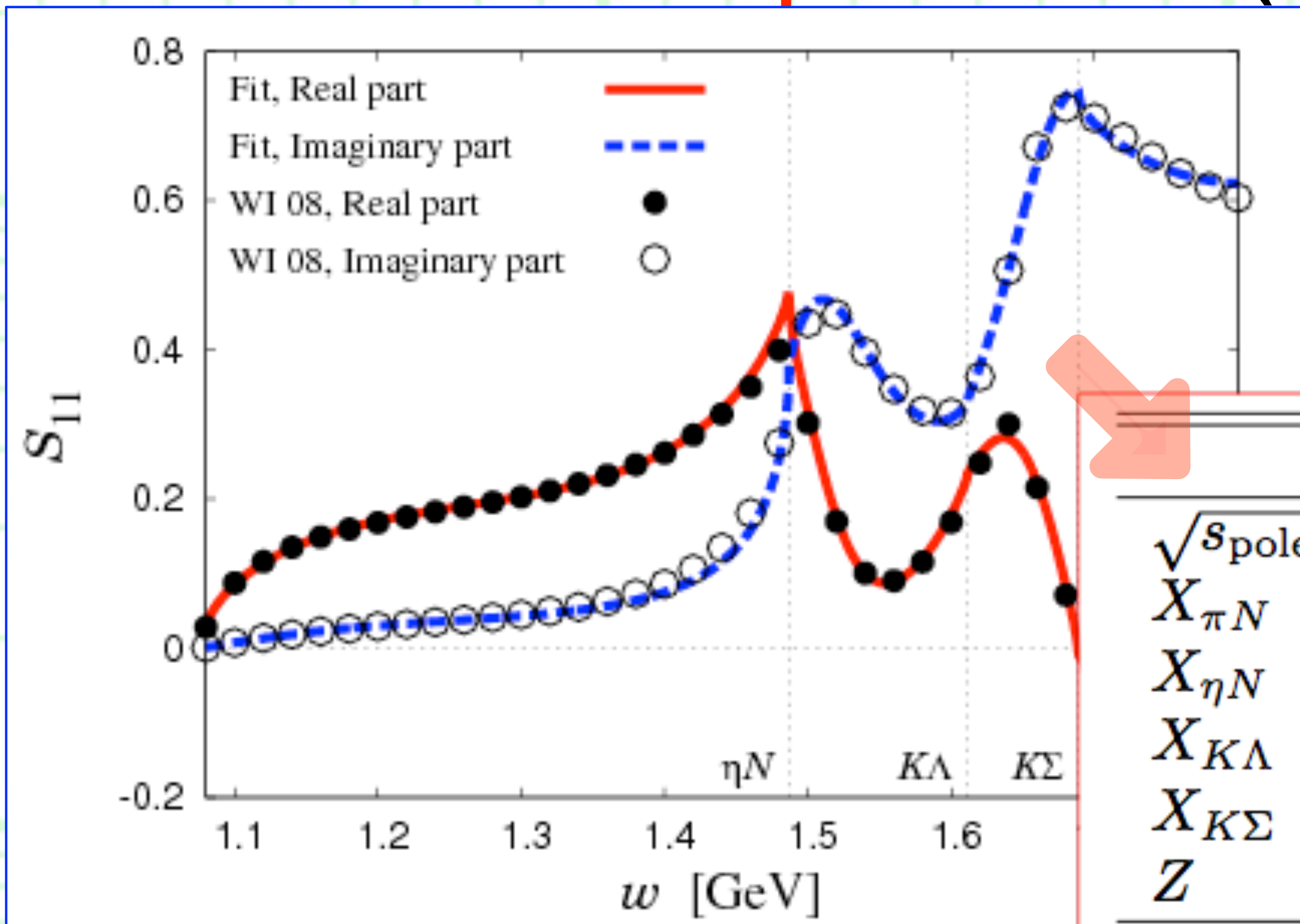
--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6$.

- **Chiral unitary approach reproduces the amplitude of PWA very well.**

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- Fitted to the πN amplitude WI 08 (S_{11}).



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6$.

- Chiral unitary approach reproduces the amplitude of PWA very well.

	$N(1535)$	$N(1650)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1496.4 - 58.7i$	$1660.7 - 70.0i$
$X_{\pi N}$	$-0.02 + 0.03i$	$0.00 + 0.04i$
$X_{\eta N}$	$0.04 + 0.37i$	$0.00 + 0.01i$
$X_{K\Lambda}$	$0.14 + 0.00i$	$0.08 + 0.05i$
$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$

- The pole positions of both $N(1535)$ and $N(1650)$ are consistent with the PDG value.

N(1535) 1/2⁻ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

Breit-Wigner mass = 1525 to 1545 (≈ 1535) Me
Breit-Wigner full width = 125 to 175 (≈ 150) M
Re(pole position) = 1490 to 1530 (≈ 1510) MeV
−2Im(pole position) = 90 to 250 (≈ 170) MeV

N(1650) 1/2⁻ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

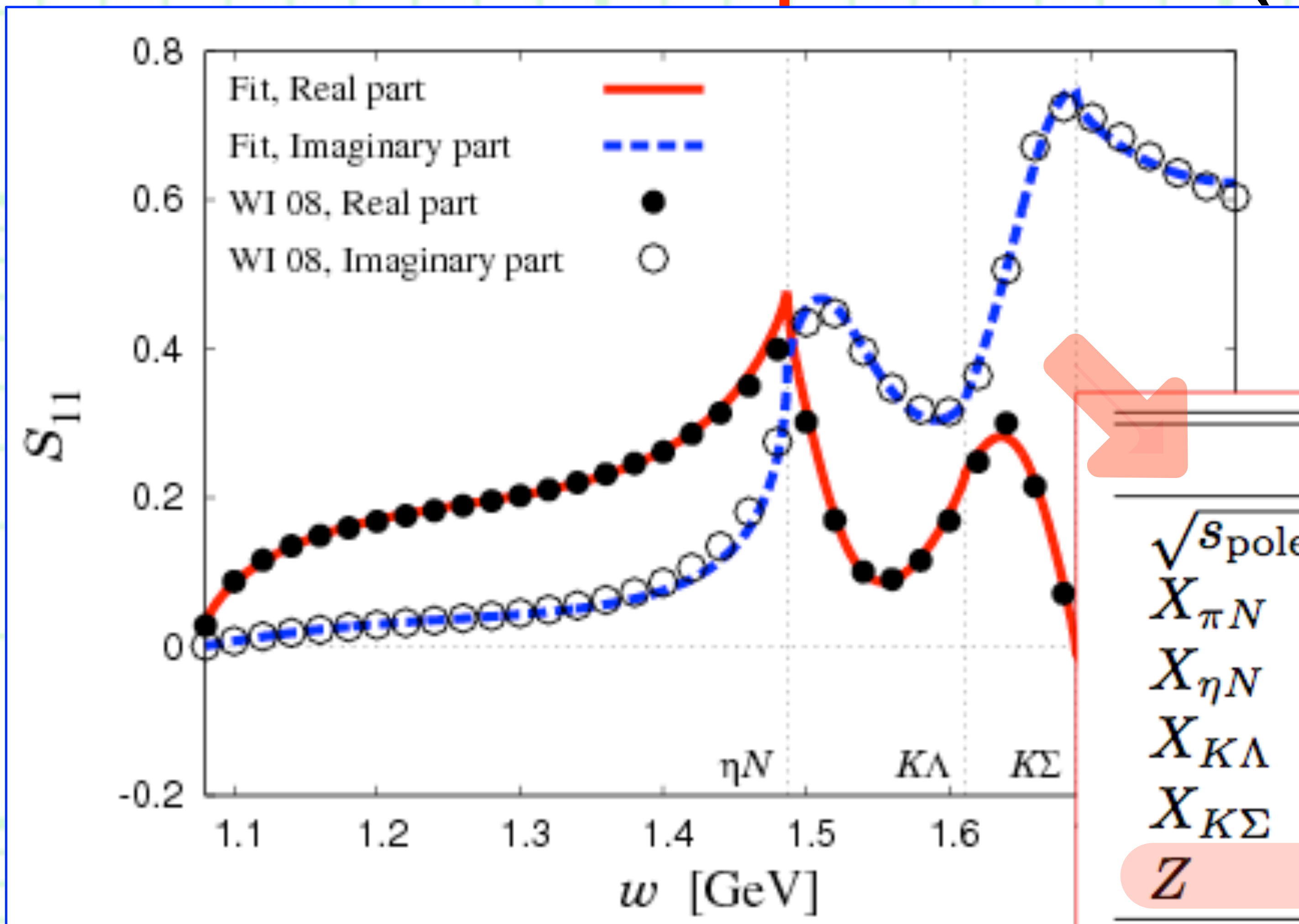
Breit-Wigner mass = 1645 to 1670 (≈ 1655) MeV
Breit-Wigner full width = 110 to 170 (≈ 140) MeV
Re(pole position) = 1640 to 1670 (≈ 1655) MeV
−2Im(pole position) = 100 to 170 (≈ 135) MeV

Particle Data Group.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- **Fitted to the πN amplitude WI 08 (S_{11}).**



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 94.6 / 167 \approx 0.6$.

□ Chiral unitary approach reproduces the amplitude of PWA very well.

	$N(1535)$	$N(1650)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1496.4 - 58.7i$	$1660.7 - 70.0i$
$X_{\pi N}$	$-0.02 + 0.03i$	$0.00 + 0.04i$
$X_{\eta N}$	$0.04 + 0.37i$	$0.00 + 0.01i$
$X_{K\Lambda}$	$0.14 + 0.00i$	$0.08 + 0.05i$
$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$

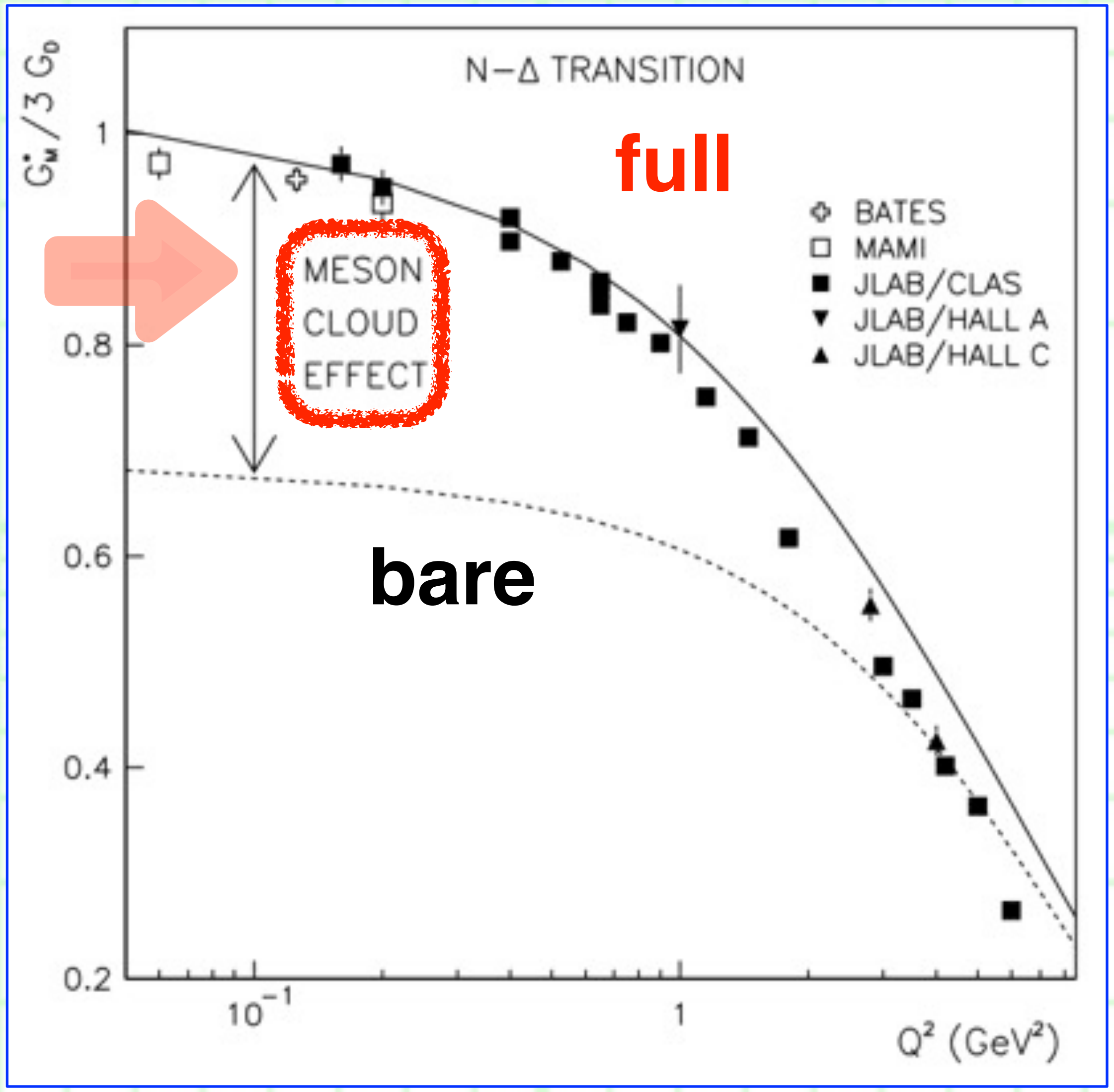
□ For both N^* resonances, the elementariness Z is dominant.

--> $N(1535)$ and $N(1650)$ have large components originating from contributions other than πN, ηN, $K\Lambda$, and $K\Sigma$. The missing channels should be encoded in the energy dep. of V and LEC.

3. Applications

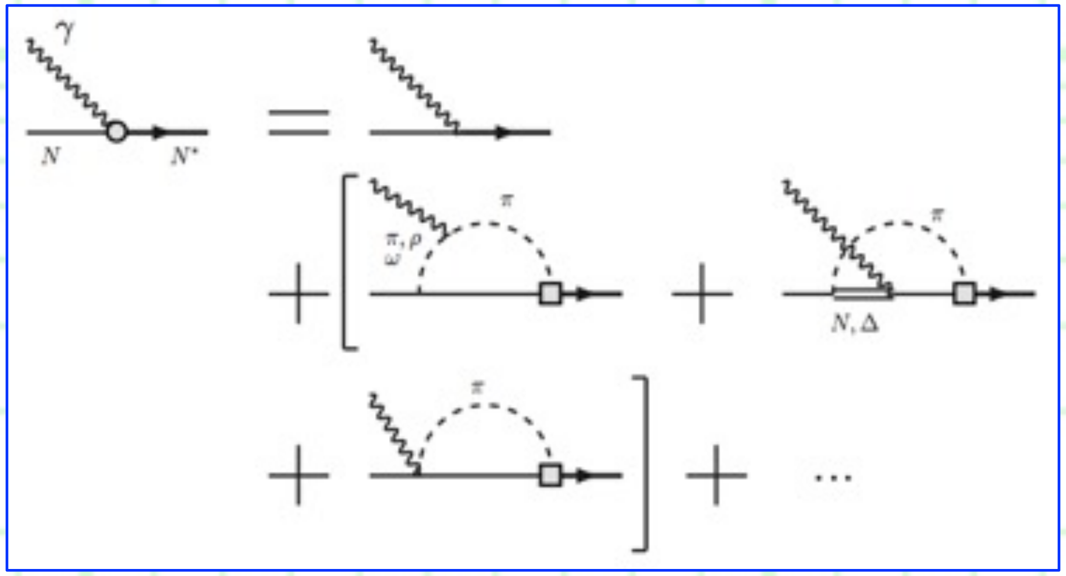
++ Compositeness for Δ(1232) ++

- **Δ(1232) --- The excellent successes of the quark model strongly indicate that Δ(1232) is described as genuine *qqq* states very well.**
- **However, effect of the meson-nucleon cloud for Δ(1232) seems to be "large".**



- **The magnetic $M1$ form factor of γN --> Δ(1232) shows that the meson cloud effect brings ~ 30 % of the form factor at $Q^2 = 0$.**

Sato and Lee, *J. Phys.* G36 (2009) 073001.



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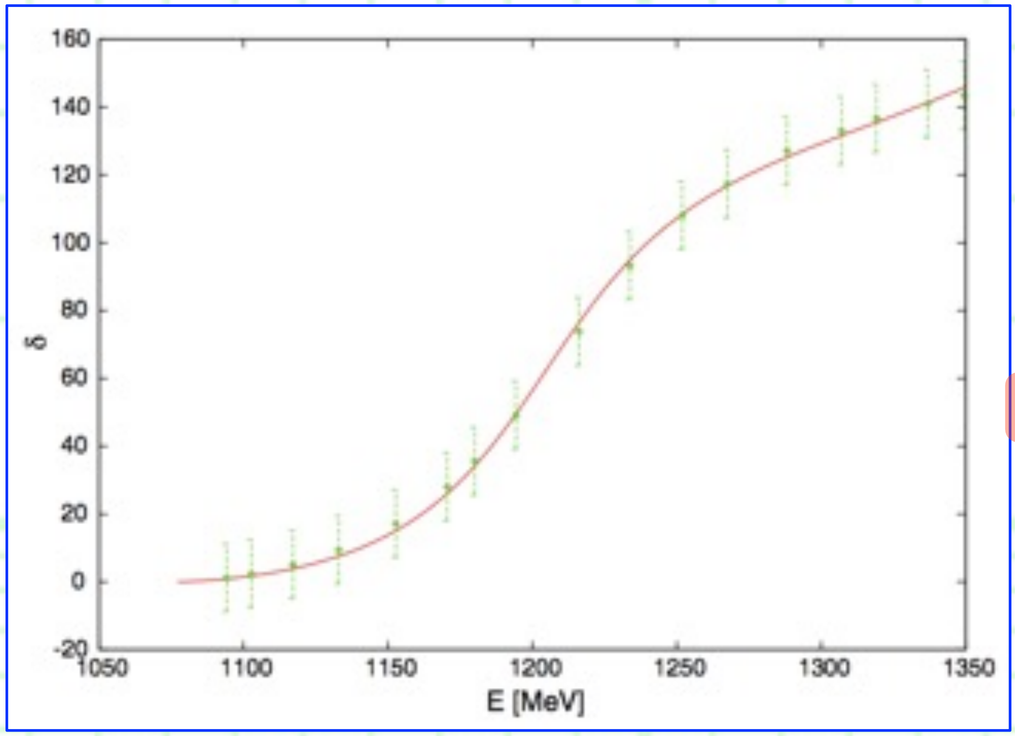


□ The π*N* compositeness for Δ(1232) is evaluated in a very simple model.

Aceti *et al.*, *Eur. Phys. J.* A50 (2014) 57.

$$-\tilde{g}_{\Delta}^{2}\left[\frac{\mathrm{d}G^{II}(s)}{\mathrm{d}\sqrt{s}}\right]_{\sqrt{s}=\sqrt{s_0}} = (0.62 - i0.41),$$

□ Large real part of the π*N* compositeness, but imaginary part is non-negligible.

--- The result implies large π*N* contribution to, *e.g.*, the transition form factor.

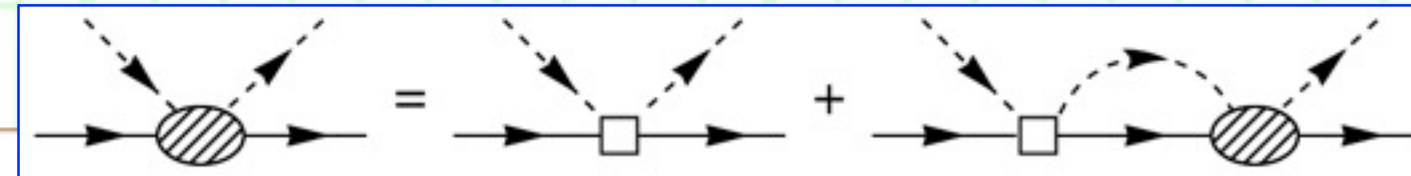
- However, this result was obtained in a very simple model.

--> Need a more refined model !

3. Applications

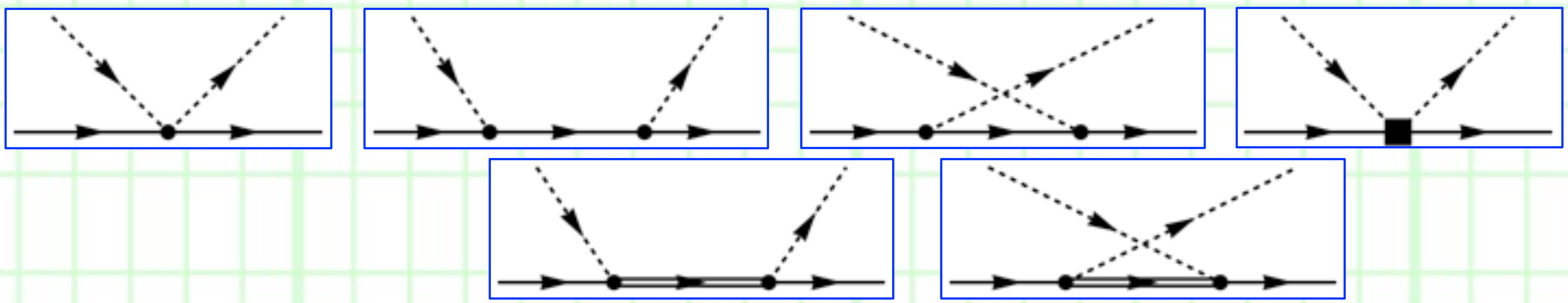
++ Compositeness for Δ(1232) ++

- We construct our own πN elastic scattering amplitude in the chiral unitary approach.



$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$

□ V:



--- We include an explicit Δ(1232) pole term.

□ G: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in G.

$$G_{j,L}(s = M_N^2) = 0$$

Hyodo, Jido and Hosaka, *Phys. Rev.* C78 (2008) 025203.

--- This makes the physical N(940) mass in the full Amp. unchanged.

--- In addition, we constrain G so as to exclude unphysical bare-state contributions to N(940):

$$\frac{dG_L}{ds}(s = M_N^2) \leq 0$$

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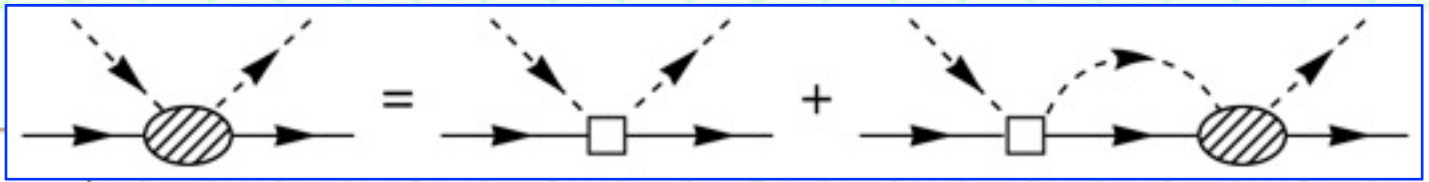


$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$

- □ We have model parameters of: LECs, bare Δ mass and coupling constant to πN, and a subtraction const.

--> Fitted to six πN scattering amplitudes ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$) obtained as a PWA solution "WI 08" up to √s = 1.35 GeV.

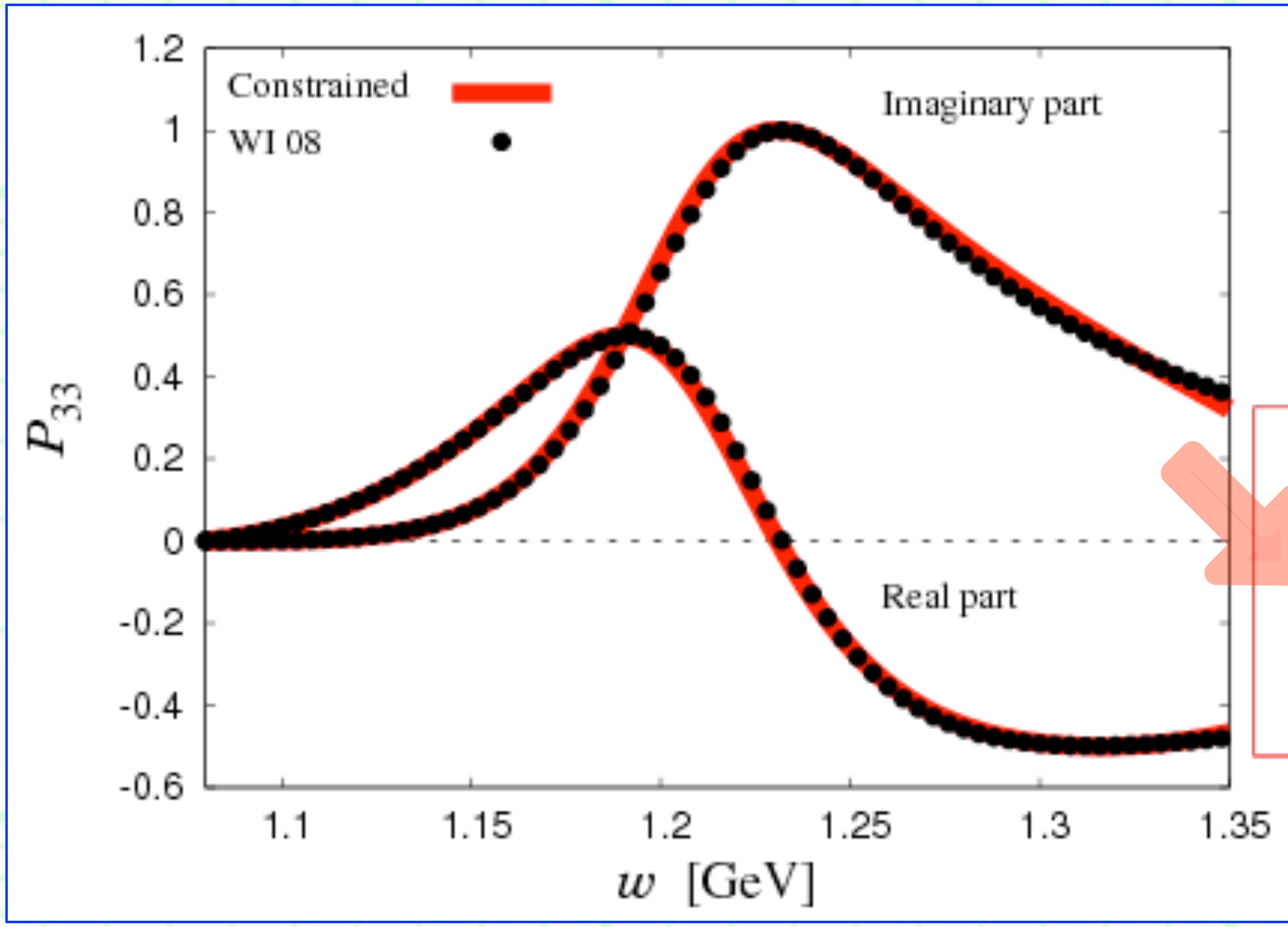
Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

- □ The P_{11} and P_{33} amplitude contain poles corresponding to the physical $N(940)$ and $\Delta(1232)$, respectively:

3. Applications

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude** WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 1240 / 809 \approx 1.5$.

- Chiral unitary approach reproduces the amplitude of PWA well.

Constrained	$\Delta(1232)$	$N(940)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1206.9 - 49.6i$	938.9
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

- For $\Delta(1232)$, its pole position is very similar to the PDG value.

$$\mathrm{Re(pole\ position)} = 1209 \text{ to } 1211 \ (\approx 1210) \text{ MeV}$$
$$-2\mathrm{Im(pole\ position)} = 98 \text{ to } 102 \ (\approx 100) \text{ MeV}$$

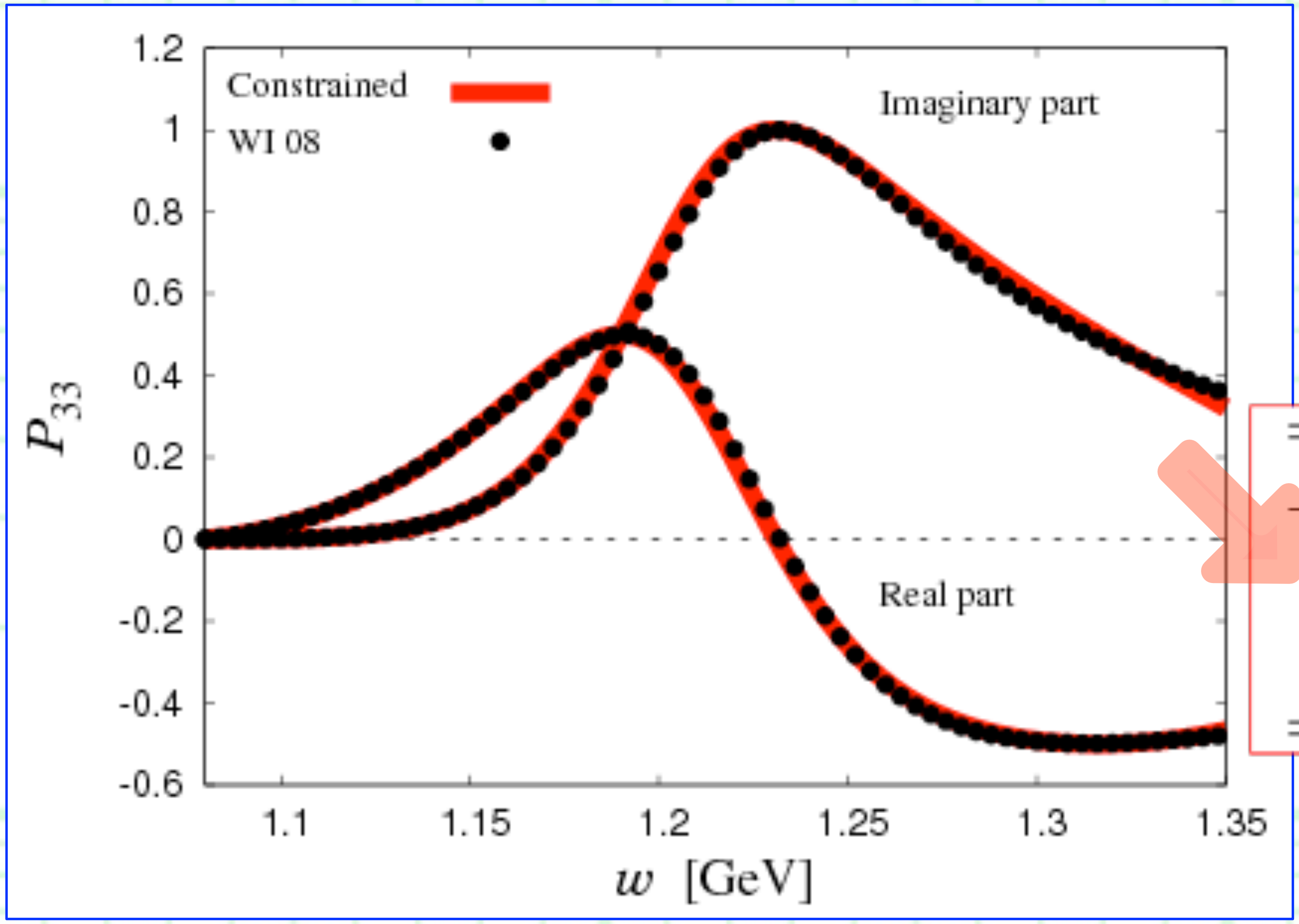
- The πN compositeness $X_{\pi N}$ takes large real part ! But non-negligible imaginary part as well.

--> Our refined model reconfirms the result in the previous study.

3. Applications

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).**



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Z	$0.13 - 0.35i$	1.00

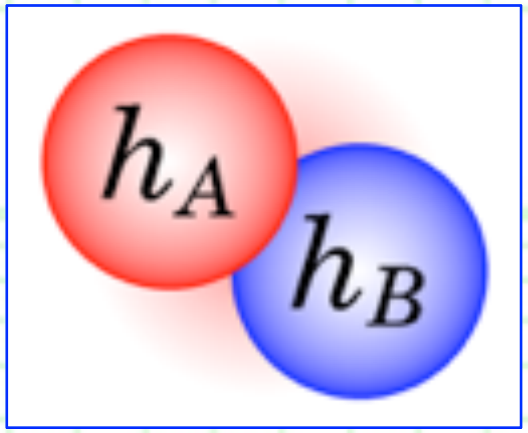
□ **For $N(940)$, $X_{\pi N}$ is non-negative and zero.**

--> Implies that $N(940)$ is not described by the πN molecular picture.

4. Conclusion

4. Conclusion

- **Hadronic molecules are unique, because they are composed of color singlet states, which can be observed as asymptotic states.**
  - **We can use quantum mechanics in a usual manner.**
  - **In particular, we can investigate their structure of composites by the two-body wave functions and their norms = compositeness.**



- **The two-body wave functions can be extracted from the hadron-hadron scattering amplitude, although they are model dependent.**

$$T(\vec{q}\,', \vec{q}; E) = \langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

$$\gamma(q) \equiv \langle \vec{q} | \hat{V} | \Psi \rangle = [E_{\text{pole}} - E(q)] \tilde{\psi}(q)$$

--- The residue at the pole position contains information on the two-body wave function, which is automatically normalized.

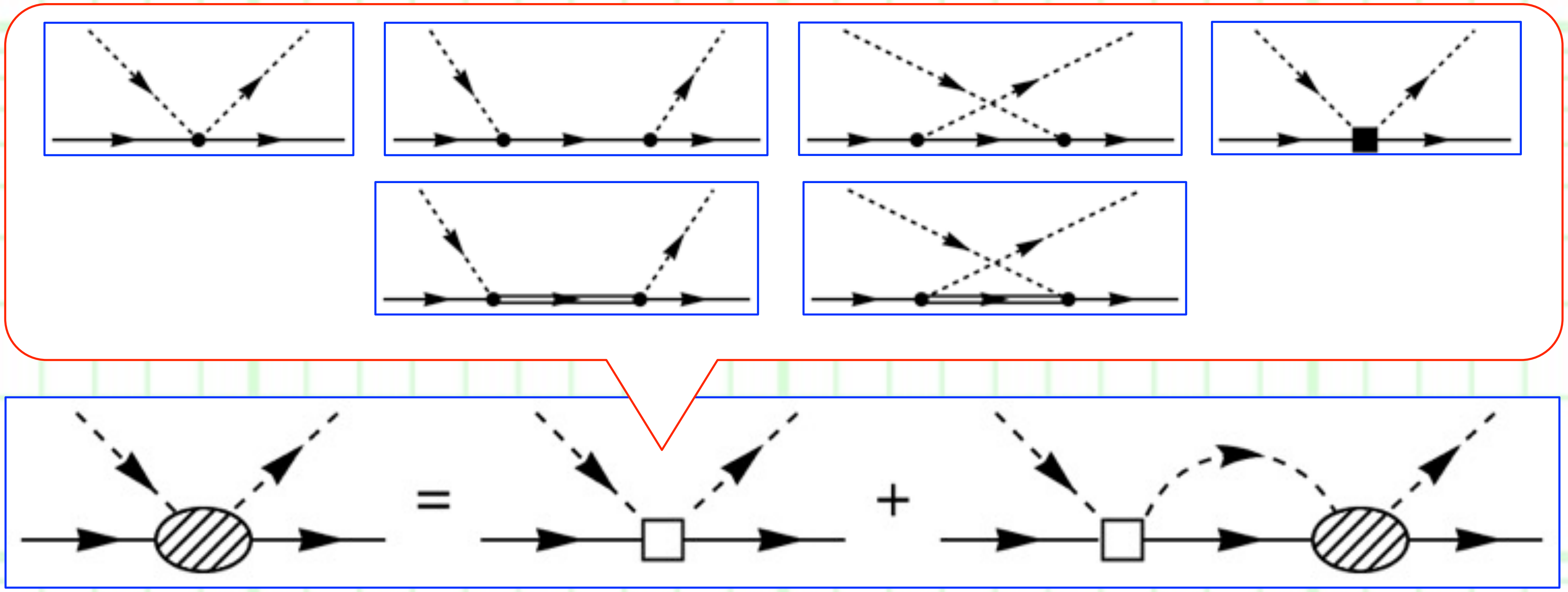
$$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\text{pole}} - E(q)} \right]^2 = 1$$

<-- If the state is purely molecule !

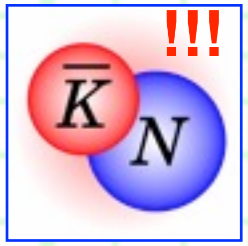
--> Comparing the norm = compositeness with unity, we may able to conclude the structure of hadronic molecule candidates.

4. Conclusion

- We apply this scheme to $\Lambda(1405)$, $N(1535)$, $N(1650)$, and $\Delta(1232)$ in an effective model, chiral unitary approach, with a separable interaction of LO + NLO (+ bare Δ) taken from chiral perturbation theory.



--- In this model, we find that ...



- □ $\Lambda(1405)$ (higher pole) is indeed a $\bar{K}N$ molecule.
- □ $N(1535)$ and $N(1650)$ have small πN, ηN, $K\Lambda$ and $K\Sigma$ components.
- □ $\Delta(1232)$ has a non-negligible πN component.

Thank you very much for your kind attention !