Wave functions and compositeness for hadron resonances from hadron-hadron scattering amplitudes

Takayasu SEKIHARA

Contents

1. Introduction
   - Hadronic molecules are unique!
   - Since they are unique, we can use quantum mechanics:
     Wave function and its norm, scattering amplitude, ...

2. Two-body wave functions and compositeness
   - How to obtain the wave function of bound states (e.g. deuteron)?
     A. Solve the Schrödinger equation.
     B. Solve the Lippmann-Schwinger equation and extract it from the scattering amplitude. <-- Our approach!

3. Applications: compositeness of hadronic resonances
   - The Λ(1405) resonance is a KN molecular state?
   - Large pion clouds for the Δ(1232) resonance?
   - Meson-baryon components in N(1535) and N(1650)?

4. Conclusion
1. Introduction
1. Introduction

++ Exotic hadrons and their structure ++

- Exotic hadrons --- not same quark component as ordinary hadrons
  \[= \text{not } qqq \text{ nor } q\bar{q}.\]

- Actually some hadrons cannot be described by the quark model.

  - Do exotic hadrons really exist?
  - If they do exist, how are their properties?
  - Re-confirmation of quark models.
  - Constituent quarks in multi-quarks? "Constituent" gluons?
  - If they do not exist, what mechanism forbids their existence?

 <-- We know very few about hadrons (and dynamics of QCD).
1. Introduction

++ Uniqueness of hadronic molecules ++

- Hadronic molecules should be unique, because they are composed of hadrons themselves, which are color singlet.

Hadronic molecules (cf. deuteron)

--> Various quantitative/qualitative diff. from other compact hadrons.

- Large spatial size due to the absence of strong confining force.
- Hadron masses are “observable”, in contrast to quark masses.
  --> Expectation of the existence around two-body threshold.
- Treat them without complicated calculations of QCD.
  --- We can use quantum mechanics with appropriate interactions.
1. Introduction

++ Hadronic molecules and quantum mechanics ++
- An example of hadronic molecule: deuteron.

Deuteron is a proton-neutron bound state. <-- Who proved this?
--- Weinberg proved this by using general wave equations in quantum mechanics in the weak binding limit ($B_{E} \ll E_{typical}$).
<-- Without using QCD!

Introduce field renormalization constant $Z$:

-> “Bare” component $|B_{0}\rangle$ in the total wave function $|B\rangle$.

\[
a = \frac{2(1-Z)}{2-Z} R + O(m_{\pi}^{-1}), \quad r_{e} = -\frac{Z}{1-Z} R + O(m_{\pi}^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}
\]
\[
a = 5.419 \pm 0.007 \text{ fm}, \quad r_{e} = 1.7513 \pm 0.008 \text{ fm}
\]

Consistent with $Z \approx 0$!
1. Introduction

++ Hadronic molecules and quantum mechanics ++

☐ An example of hadronic molecule: deuteron.

Lesson: In a similar manner, we can study the structure of general hadronic molecules.

--- We can use quantum mechanics to investigate them:
Two-body wave function, its norm = compositeness (複合性), scattering amplitude, ...

<--> For hadrons of other configurations, we have to treat color multiplet states explicitly and appropriately.
++ How to clarify their structure ? ++

- How can we use quantum mechanics to clarify the structure of hadronic molecule candidates ?

- We evaluate the wave function of hadron-hadron composite contribution.
  --- cf. Wave function for relative motion of two nucleons inside deuteron.

- How to evaluate the wave function ?
  <-- We employ a fact that the two-body wave function appears in the residue of the scattering amplitude of the two particles at the resonance pole.

  --- The wave function from the residue is automatically normalized !

  --> Calculating the norm of the two-body wave function = compositeness, we may measure the fraction of the composite component and conclude the composite structure !
1. Introduction

++ Purpose and strategy of this study ++

- In this study we evaluate the hadron-hadron two-body wave functions and their norms = compositeness for hadron resonances from the hadron-hadron scattering amplitudes.

- We have to use precise scattering amplitudes for the evaluation.

--> Employ the chiral unitary approach.

Kaiser-Siegel-Weise ('95); Oset-Ramos ('98); Oller-Meissner ('01); Lutz-Kolomeitsev ('02);
Oset-Ramos-Bennhold ('02); Jido-Oller-Oset-Ramos-Meissner ('03); ...

\[ T = V + VGT \]

- Interaction kernel \( V \) from the chiral perturbation theory:
  Leading order (LO) + next-to-leading order (NLO) (+ bare \( \Delta \)).

- Loop function \( G \) calculated with the dispersion relation in a covariant way.

- We discuss the structure of \( \Lambda(1405) \), \( N(1535) \), \( N(1650) \), and \( \Delta(1232) \).
2. Two-body wave functions and compositeness
++ Setup of the quantum system ++

- Problem: Calculate the wave function of a bound state, both in the stable and unstable cases.
  - Interaction $V$ is known.

- Full Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

--- Free part has eigenstates of scattering states: $\hat{H}_0 |q\rangle = E(q) |q\rangle$

- Wave function (momentum space): $\langle q | \Psi \rangle = \tilde{\psi}(q)$

- The Schrödinger equation: $\hat{H} |\Psi\rangle = (\hat{H}_0 + \hat{V}) |\Psi\rangle = E_{\text{pole}} |\Psi\rangle$
2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function. Ex.) A bound state in a NR single-channel problem.
  - Usual approach: Solve the Schrödinger equation.

\[ \hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\text{pole}}|\Psi\rangle \]

--- Wave function in coordinate / momentum space:

\[ \langle r|\Psi\rangle = \psi(r) \quad \langle q|\Psi\rangle = \tilde{\psi}(q) \]

\[ \left[ M_{\text{th}} - \frac{\nabla^2}{2\mu} + V(r) \right] \psi(r) = E_{\text{pole}}\psi(r) \]

\[ \hat{H}_0|q\rangle = E(q)|q\rangle \quad E(q) = M_{\text{th}} + \frac{q^2}{2\mu} \]

--> After solving the Schrödinger equation, we have to normalize the wave function by hand.

\[ \int d^3r [\psi(r)]^2 = 1 \quad \text{or} \quad \int \frac{d^3q}{(2\pi)^3} [\tilde{\psi}(q)]^2 = 1 \]

\[ \Rightarrow \text{We require!} \]
2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function. Ex.) A bound state in a NR single-channel problem.
- **Our approach:** Solve the Lippmann-Schwinger equation at the pole position of the bound state.

\[
\hat{T}(E) = \hat{V} + \hat{V} \frac{1}{E - \hat{H}_0} \hat{T} = \hat{V} + \hat{V} \frac{1}{E - \hat{H}} \hat{V}
\]

\[
T(q', q; E) = \langle q' | \hat{T}(E) | q \rangle
\]

--- Near the resonance pole position \( E_{\text{pole}} \), amplitude is dominated by the pole term in the expansion by the eigenstates of \( H \) as

\[
\langle q' | \hat{T}(E) | q \rangle \approx \langle q' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\text{pole}}} \langle \Psi^* | \hat{V} | q \rangle
\]

--- The residue of the amplitude at the pole position has information on the wave function!

\[
\langle q | \hat{V} | \Psi \rangle = \langle q | (\hat{H} - \hat{H}_0) | \Psi \rangle = (E_{\text{pole}} - E(q)) \tilde{\psi}(q)
\]

\[
\langle \Psi^* | \hat{V} | q \rangle = [E_{\text{pole}} - E(q)] \tilde{\psi}(q)
\]

\[
E(q) = M_{\text{th}} + \frac{q^2}{2\mu}
\]

| \[ |\Psi\rangle, |\overline{q}_{\text{full}}\rangle, \ldots \] | \[ \langle \Psi^* |, \langle \overline{q}_{\text{full}} |, \ldots \] | \[ 1 = |\Psi\rangle\langle \Psi^* | + \ldots \] |
2. Wave functions and compositeness

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function.
  Ex.) A bound state in a NR single-channel problem.
- Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.

--- The wave function can be extracted from the residue of the amplitude at the pole position:

\[
T(q', q; E) = \langle q' | \hat{T}(E) | q \rangle \approx \frac{\gamma(q') \gamma(q)}{E - E_{\text{pole}}} 
\]

\[
\gamma(q) \equiv \langle q | \hat{V} | \Psi \rangle = [E_{\text{pole}} - E(q)]\bar{\psi}(q)
\]

--> Because the scattering amplitude cannot be freely scaled due to the optical theorem, the wave function from the residue of the amplitude is automatically normalized!

If purely molecule -->

\[
\int \frac{d^3q}{(2\pi)^3} \left[ \frac{\gamma(q)}{E_{\text{pole}} - E(q)} \right]^2 = 1
\]

--> Therefore, from hadron-hadron scattering amplitudes with resonance poles, we can calculate their two-body wave function.

---

2. Wave functions and compositeness

++ Example 1: Stable bound state ++
- A Λ hyperon in $A = 40$ nucleus.

--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

\[
E(q)\tilde{\psi}(q) + \int \frac{d^3q'}{(2\pi)^3} \tilde{V}(q, q')\tilde{\psi}(q') = E_{pole} \tilde{\psi}(q)
\]

--> Normalize $\psi$ by hand!

\[
\int \frac{d^3q}{(2\pi)^3} [\tilde{\psi}(q)]^2 = 1
\]

2. Solve Lippmann-Schwinger equation:

\[
T(q', q; E) = \tilde{V}(q', q) + \int \frac{d^3k}{(2\pi)^3} \tilde{V}(q', k)T(k, q; E) \frac{E - E(k)}{E - E_{pole}}
\]

--> Extract WF from the residue:

\[
T(q', q; E) \approx \frac{\gamma(q')\gamma(q)}{E - E_{pole}} \quad \Rightarrow \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{pole} - E(q)}
\]

--- Without normalizing by hand!
2. Wave functions and compositeness

++ Example 1: Stable bound state ++

- A Λ hyperon in $A = 40$ nucleus.

--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$E(q)\tilde{\psi}(q) + \int \frac{d^3q'}{(2\pi)^3} \tilde{V}(q, q')\tilde{\psi}(q') = E_{pole}\tilde{\psi}(q)$$

--> Normalize $\psi$ by hand!

2. Solve Lippmann-Schwinger equation:

$$T(q', q; E) = \tilde{V}(q', q) + \int \frac{d^3k}{(2\pi)^3} \tilde{V}(q', k)\tilde{\psi}(k)$$

--> Extract WF from the residue:

$$T(q', q; E) \approx \frac{\gamma(q')\gamma(q)}{E - E_{pole}} \quad \Rightarrow \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{pole} - E(q)}$$

--- Without normalizing by hand!

□ In 1st way: Points.

2nd way: Lines.

□ Exact coincidence!

--- We obtain automatically normalizedWF from the Amp.!
2. Wave functions and compositeness

++ Example 2: Unstable resonance state ++

- Unstable resonance in $\bar{K}N-\pi\Sigma$ system.

--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$E_j(q)\tilde{\psi}_j(q) + \sum_k \int \frac{d^3 q'}{(2\pi)^3} \tilde{V}_{jk}(q, q')\tilde{\psi}_k(q') = E_{\text{pole}}\tilde{\psi}_j(q)$$

--> Normalize $\psi_j$ by hand!

$$X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}_j(q)\right]^2$$

2. Solve Lippmann-Schwinger equation:

$$T_{jk}(q', q; E) = \tilde{V}_{jk}(q, q) + \sum_l \int \frac{d^3 k}{(2\pi)^3} \tilde{V}_{jl}(q', k)T_{lk}(k, q; E)$$

--> Extract WF from the residue:

$$T_{jk}(q', q; E) \approx \frac{\gamma_j(q')\gamma_k(q)}{E - E_{\text{pole}}}$$

--- Without normalizing by hand!

Gaussian potential
Coupling strength is controlled by $x$. 
2. Wave functions and compositeness

++ Example 2: Unstable resonance ++

- Unstable resonance in $\overline{K}N$-$\pi\Sigma$ system.

--> Calculate wave functions

1. Solve Schrödinger equation:

\[ E_j(q)\tilde{\psi}_j(q) + \sum_k \int \frac{d^3q'}{(2\pi)^3} \tilde{V}_{jk}(q, q')\tilde{\psi}_k(q') = 0 \]

--> Normalize $\psi_j$ by hand!

\[ X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3q}{(2\pi)^3} \left[ \tilde{\psi}_j(q) \right]^2 \]

2. Solve Lippmann-Schwinger equation:

\[ T_{jk}(q', q; E) \approx \frac{\gamma_j(q')\gamma_k(q)}{E - E_{\text{pole}}} \]

--> Extract wave functions:

\[ \tilde{\psi}_j(q) = \frac{\gamma_j(q)}{E - E_j(q)} \]

\[ T_{jk}(q', q; E) \approx \frac{\gamma_j(q')\gamma_k(q)}{E - E_{\text{pole}}} \]

- In 1st way: **Points.**
- 2nd way: **Lines.**
- Coincidence again!
- --- Our method is valid even for resonances!
3. Applications: compositeness of hadronic resonances
3. Applications

++ Wave functions for hadrons ++

- We can obtain normalized WF from the Scatt. Amp. even for resonances.
- However, if the interaction depends on energy, the norm deviates from unity.

<-- Interpreted as the contribution from a missing channel |ψ₀⟩.

By using this fact, we can interpret the norm = compositeness (X) of the wave function from the Amp. as the “fraction” of the two-body state for a resonance in general interaction.

\[
1 = \int \frac{d^3q}{(2\pi)^3} |\vec{q}'\rangle\langle \vec{q}'| + |\psi_0\rangle\langle \psi_0|
\]

\[
\langle \Psi^* | \Psi \rangle = \sum_j X_j + Z = 1
\]

\[
X_j = \int \frac{d^3q}{(2\pi)^3} \langle \Psi^* | \hat{q}_j \rangle \langle \hat{q}_j | \Psi \rangle
\]

Particle Data Group (2014).
(similar but not same as our compositeness)

3. Applications

++ Our strategy ++

- In this study we investigate the structure of hadronic molecule candidates in the following strategy.

1. Construct hadron-hadron scattering amplitude, which precisely reproduces experimental data and contains resonance poles for hadronic molecule candidates, in appropriate effective models.

\[
T_{jk}(\vec{q}', \vec{q}; s) = V_{jk}(\vec{q}', \vec{q}; s) + i \sum_l \int \frac{d^4q''}{(2\pi)^4} \frac{V_{jl}(\vec{q}', \vec{q}''; s) T_{lk}(\vec{q}'', \vec{q}; s)}{(q''^2 - m_l^2)((P - q)^2 - M_l^2)}
\]

2. Extract the two-body wave function from the residue of the amplitude at the resonance pole.

\[
T_{jk}(\vec{q}', \vec{q}; s) = \frac{\gamma_j(q')\gamma_k(q)}{s - s_{pole}} + (\text{regular at } s = s_{pole})
\]

\[
\gamma_j(q) = \langle \bar{q}_j|\hat{V}|\Psi \rangle = [s_{pole} - s_j(q)]\bar{\psi}_j(q)
\]
3. Applications

++ Our strategy ++

- In this study we investigate the structure of hadronic molecule candidates in the following strategy.

\[ T_{jk}(q', q; s) = \frac{\gamma_j(q') \gamma_k(q)}{s - s_{\text{pole}}} + \text{(regular at } s = s_{\text{pole}}) \]

\[ \gamma_j(q) \equiv \langle q_j | \hat{V} | \Psi \rangle = [s_{\text{pole}} - s_j(q)] \tilde{\psi}_j(q) \]

3. Calculate the compositeness \( X_j \) = norm of the two-body wave function in channel \( j \), from Amp. and compare it with unity.

\[ X_j = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[ \tilde{\psi}(q) \right]^2 = \int \frac{d^3 q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \left[ \frac{\gamma_j(q)}{s_{\text{pole}} - s_j(q)} \right]^2 \]

- The sum of \( X_j \) will exactly unity for a purely molecular state.
  - The interaction does not have energy dependence.

- On the other hand, if the interaction has energy dependence, which can be interpreted as the contribution from a missing channel, the sum of \( X_j \) deviates from unity.

\[ \sum_j X_j = 1 - Z \]

---> Fraction of a missing channel is expressed by \( Z \):

--- Same as the Weinberg's Z.


3. Applications

++ Observable and model (in)dependence ++

- Here we comment on the observables and non-observables.

- **Observables:**
  - Cross section.
  - Its partial-wave decomposition.
  - On-shell Scatt. amplitude via the optical theorem.
  - Mass of bound states.

- **NOT observables:**
  - Wave function and potential.
  - Resonance pole position.
  - Residue at pole.
  - Off-shell amplitude.

--> Since the residue of the amplitude at the resonance pole is NOT observable, the wave function and its norm = compositeness are also not observable and model dependent.

--- Exception: Compositeness for near-threshold poles.
++ Observable and model (in)dependence ++

- **Special case**: Compositeness for near-threshold poles.

--- Compositeness can be expressed with threshold parameters such as scattering length and effective range.

- **Deuteron**.
  - Weinberg (’65).
- **$f_0(980)$ and $a_0(980)$**.
- **$\Lambda(1405)$**.
- ...

- **General case**: Compositeness are model dependent quantity.

--> Therefore, we have to employ **appropriate effective models** ($V$) to construct **precise** hadron-hadron scattering amplitude, in order to discuss the structure of hadronic molecule candidates!
3. Applications

++ List of hadron resonances in our analysis ++

- In this talk, we discuss the structure of candidates of hadronic molecules listed as follows in terms of the compositeness:

1. $\Lambda(1405)$.
   - One of classical examples of the exotic hadron candidates.

2. $N(1535)$ and $N(1650)$.
   - Expected to be usual $qqq$ states, but can be described also in meson-baryon d.o.f.

3. $\Delta(1232)$.
   - Established as a member of the decuplet in the flavor $SU(3)$ symmetry, together with $\Sigma(1385)$, $\Xi(1530)$, and $\Omega$, in the quark model, but ...

---

Kaiser-Siegel-Weise ('95), Bruns-Mai-Meissner ('11), ...

---

Sato and Lee ('09).

---

Meson cloud effect $\sim 30\%$!
3. Applications

++ Chiral unitary approach ++

- We employ chiral unitary approach for meson-baryon scatterings.

\[
T'_{\alpha L,jk}(s) = V'_{\alpha L,jk}(s) + \sum_l V'_{\alpha L,jl}(s)G_{L,l}(s)T'_{\alpha L,lk}(s)
\]

- For the interaction kernel \( V \) we take LO + NLO (+ bare \( \Delta \)) of chiral perturbation theory and project it to partial wave \( L \) and quantum number \( \alpha \) to construct a separable interaction. --> \( V_{\text{prime}} \).

\[
V_{\alpha L,jk} = |\vec{q}|^{2L} \times V'_{\alpha L,jk}(s)
\]

- The loop function \( G_L \) is obtained with the dispersion relation:

\[
G_{L,j}(s) = \int_{s_{\text{th},j}}^{\infty} \frac{ds'}{2\pi} \frac{\rho_j(s')q_j(s')^{2L}}{s' - s - i0} = i \int \frac{d^4q}{(2\pi)^4} \frac{|\vec{q}|^{2L}}{[(P - q)^2 - m_j^2](q^2 - M_j^2)}
\]

\( q_j(s) \): phase space in channel \( j \).

--- We need one subtraction for \( s \) wave / two subtractions for \( p \) wave which are fixed as discussed below.
3. Applications

++ Compositeness with separable interaction ++

- **For the separable interaction**, which we employ in this study, we can **calculate the residue at the resonance pole as**:

\[
\langle \tilde{q}' | \hat{T}(s) | \tilde{q} \rangle \approx \langle \tilde{q}' | \hat{V}(s_{\text{pole}}) | \Psi \rangle \frac{1}{s - s_{\text{pole}}} \langle \Psi^* | \hat{V}(s_{\text{pole}}) | \tilde{q} \rangle
\]

\[
\langle \tilde{q} | \hat{V} | \Psi \rangle = \langle \Psi^* | \hat{V} | \tilde{q} \rangle = g \times |\tilde{q}|^L
\]


- **For resonances in** $L$ **wave**, $g$ **is the coupling constant**.

- This form is **necessary for the correct behavior** of the wave function at small $q$ region:

\[
\tilde{\psi}(q) = O(q^L) \quad \text{for small } q
\]

- **As a result, the norm of the two-body wave function is written as**

\[
X_j = \int \frac{d^3q}{(2\pi)^3} \frac{\sqrt{s_j(q)}}{2\omega_j(q)\Omega_j(q)} \langle \Psi^* | \tilde{q}_j \rangle \langle \tilde{q}_j | \Psi \rangle = -g_j^2 \left[ \frac{dG_{L,j}}{ds} \right]_{s=s_{\text{pole}}}
\]

--- $G_L$ **is the loop function in** $L$ **wave**.

\[
Z = - \sum_{j,k} g_k g_j \left[ G_{L,j} \frac{dV_{\alpha L,jk}}{ds} G_{L,k} \right]_{s=s_{\text{pole}}}
\]

3. Applications

++ Compositeness for $\Lambda(1405)$ ++

- $\Lambda(1405)$ --- The lightest excited baryon with $J^P = 1/2^-$, Why ??
- Strongly attractive $\bar{K}N$ interaction in the $I = 0$ channel.

--> $\Lambda(1405)$ is a $\bar{K}N$ quasi-bound state ??

Dalitz and Tuan ('60), ...

We use the Ikeda-Hyodo-Weise amplitude for $\Lambda(1405)$ in chiral unitary approach, which was constrained by the recent data of the $1s$ shift and width of kaonic hydrogen. Ikeda, Hyodo, and Weise ('11), ('12).

--- $V$: Weinberg-Tomozawa term + $s$- and $u$-channel Born term

+ NLO term.

Weinberg-Tomozawa term + $s$- and $u$-channel Born term

Ikeda-Hyodo-Weise amplitude

Jaap van der Zande
3. Applications

++ The lightest excited baryon with \( J^P = \frac{1}{2}^- \), Why ??

- Strongly attractive \( K\Lambda \) interaction in the \( I = 0 \) channel.
  
  \( \Lambda(1405) \) is a \( K\Lambda \) quasi-bound state ???
  
  Dalitz and Tuan ('60), ...

- We use the Ikeda-Hyodo-Weise amplitude for \( \Lambda(1405) \) in chiral unitary approach, which was constrained by the recent data of the 1s shift and width of kaonic hydrogen.
  
  Ikeda, Hyodo, and Weise ('11), ('12).

--- \( V \): Weinberg-Tomozawa term + \( s \)- and \( u \)-channel Born term + NLO term
3. Applications

- $\Lambda(1405)$ --- The lightest excited baryon with $J^P = \frac{1}{2}^-$, Why??
- Strongly attractive $\Lambda N$ interaction in the $I = 0$ channel.

$\Lambda(1405)$ is a $\Lambda N$ quasi-bound state??

- Dalitz and Tuan ('60), ...

- We employ the Ikeda-Hyodo-Weise amplitude for $\Lambda(1405)$ in chiral unitary approach, which was constrained by the recent data of the $1_s$ shift and width of kaonic hydrogen. Ikeda, Hyodo, and Weise ('11), ('12).

- $V$: Weinberg-Tomozawa term + $s$- and $u$-channel Born term + NLO term.

- ++ Compositeness for $\Lambda(1405)$ --- The lightest excited baryon with $J^P = \frac{1}{2}^-$, Why??
- Strongly attractive $\Lambda N$ interaction in the $I = 0$ channel.

$\Lambda(1405)$ is a $\Lambda N$ quasi-bound state??

- Dalitz and Tuan ('60), ...

- We employ the Ikeda-Hyodo-Weise amplitude for $\Lambda(1405)$ in chiral unitary approach, which was constrained by the recent data of the $1_s$ shift and width of kaonic hydrogen. Ikeda, Hyodo, and Weise ('11), ('12).

- $V$: Weinberg-Tomozawa term + $s$- and $u$-channel Born term + NLO term.
3. Applications

++ Compositeness for $\Lambda(1405)$ ++

- Compositeness $X$ and elementariness $Z$ for hadrons in the model.


- $\Lambda(1405)$ (two poles!).

$\Lambda(1405)$, higher pole

<table>
<thead>
<tr>
<th>$\sqrt{s_{\text{pole}}}$</th>
<th>$1424 - 26i$ MeV</th>
<th>$1381 - 81i$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{KN}$</td>
<td>$1.14 + 0.01i$</td>
<td>$-0.39 - 0.07i$</td>
</tr>
<tr>
<td>$X_{\pi \Sigma}$</td>
<td>$-0.19 - 0.22i$</td>
<td>$0.66 + 0.52i$</td>
</tr>
<tr>
<td>$X_{\eta \Lambda}$</td>
<td>$0.13 + 0.02i$</td>
<td>$-0.04 + 0.01i$</td>
</tr>
<tr>
<td>$X_{K \Xi}$</td>
<td>$0.00 + 0.00i$</td>
<td>$-0.00 + 0.00i$</td>
</tr>
<tr>
<td>$Z$</td>
<td>$-0.08 + 0.19i$</td>
<td>$0.77 - 0.46i$</td>
</tr>
</tbody>
</table>

[Image of graph and table]

--- Large $\bar{K}N$ component for (higher) $\Lambda(1405)$, since $X_{KN}$ is almost unity with small imaginary parts.

Hyodo and Jido ('12).

--- Large $\bar{K}N$ component for (higher) $\Lambda(1405)$, since $X_{KN}$ is almost unity with small imaginary parts.
3. Applications

++ Compositeness for \( N(1535) \) and \( N(1650) \) ++

- \( N(1535) \) and \( N(1650) \) --- Nucleon resonances with \( J^P = 1/2^- \).

- We naively expect that they are conventional \( qqq \) states, but there are several studies that they can be dynamically generated from the meson-baryon degrees of freedom without explicit resonance poles, especially in the chiral unitary approach.

Kaiser-Siegel-Weise (’95); Nieves-Ruiz Ariola (’01); Inoue-Oset-Vicente Vacas (’02); Bruns-Mai-Meissner (’11); ...

- For example:
  --- \( V \): Weinberg-Tomozawa term

\[ \pi N \text{ Amp. } S_{11} \]

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^-$. We naively expect that they are conventional $qar{q}q$ states, but there are several studies that they can be dynamically generated from the meson-baryon degrees of freedom without explicit resonance poles, especially in the chiral unitary approach. (Kaiser-Siegel-Weise ('95); Nieves-Ruiz Ariola ('01); Inoue-Oset-Vicente Vacas ('02); Bruns-Mai-Meissner ('11); ...)

- For example:
  --- $V$: Weinberg-Tomozawa term
    $+$ NLO term.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- $N(1535)$ and $N(1650)$ --- Nucleon resonances with $J^P = 1/2^-$.  
- We construct our own $s$-wave $\pi N - \eta N - K \Lambda - K \Sigma$ scattering amplitude in the chiral unitary approach.
  - $V$: Weinberg-Tomozawa term $+$ NLO term.
  - $G$: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in $G$.


- **Parameters**: The low-energy constants in NLO term.
  - Parameters are fixed so as to reproduce the $\pi N$ scattering amplitude $S_{11}$ as a PWA solution “WI 08” up to $\sqrt{s} = 1.8$ GeV.

3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- Fitted to the $\pi N$ amplitude WI 08 ($S_{11}$).

\[ \chi^2 / N_{\text{d.o.f.}} = 94.6 / 167 \approx 0.6. \]

- Chiral unitary approach reproduces the amplitude of PWA very well.
3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- Fitted to the $\pi N$ amplitude WI 08 ($S_{11}$).

-->$\chi^2 / N_{d.o.f.} = 94.6 / 167 \approx 0.6$.

- Chiral unitary approach reproduces the amplitude of PWA very well.

- The pole positions of both $N(1535)$ and $N(1650)$ are consistent with the PDG value.

<table>
<thead>
<tr>
<th>$N(1535)$</th>
<th>$N(1650)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}_{\text{pole}}$ [MeV]</td>
<td>$1496.4 - 58.7i$</td>
</tr>
<tr>
<td>$X_{\pi N}$</td>
<td>$-0.02 + 0.03i$</td>
</tr>
<tr>
<td>$X_{\eta N}$</td>
<td>$0.04 + 0.37i$</td>
</tr>
<tr>
<td>$X_{K\Lambda}$</td>
<td>$0.14 + 0.00i$</td>
</tr>
<tr>
<td>$X_{K\Sigma}$</td>
<td>$0.01 - 0.02i$</td>
</tr>
<tr>
<td>$Z$</td>
<td>$0.84 - 0.38i$</td>
</tr>
</tbody>
</table>

$N(1535) 1/2^-$: $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

$N(1650) 1/2^-$: $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$

Breit-Wigner mass = 1525 to 1545 ($\approx 1535$) MeV
Breit-Wigner full width = 125 to 175 ($\approx 150$) MeV
Re(pole position) = 1490 to 1530 ($\approx 1510$) MeV
Im(pole position) = 200 to 250 ($\approx 170$) MeV
3. Applications

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- Fitted to the $\pi N$ amplitude WI 08 ($S_{11}$).

\[ \chi^2 / N_{d.o.f.} = 94.6 / 167 \approx 0.6. \]

- Chiral unitary approach reproduces the amplitude of PWA very well.

- For both $N^*$ resonances, the elementariness $Z$ is dominant.

\[ N(1535) \quad \text{and} \quad N(1650) \] have large components originating from contributions other than $\pi N$, $\eta N$, $K\Lambda$, and $K\Sigma$. The missing channels should be encoded in the energy dep. of $V$ and LEC.
3. Applications

++ Compositeness for $\Delta(1232)$ ++

- $\Delta(1232)$ --- The excellent successes of the quark model strongly indicate that $\Delta(1232)$ is described as genuine $qqq$ states very well.

- However, **effect of the meson-nucleon cloud for $\Delta(1232)$** seems to be “large”.

- The magnetic $M1$ form factor of $\gamma N \to \Delta(1232)$ shows that the meson cloud effect brings $\sim 30\%$ of the form factor at $Q^2 = 0$.

3. Applications

++ Compositeness for $\Delta(1232)$ ++

- $\Delta(1232)$ --- The excellent successes of the quark model strongly indicate that $\Delta(1232)$ is described as genuine $qqq$ states very well.
- However, effect of the meson-nucleon cloud for $\Delta(1232)$ seems to be “large”.

- The $\pi N$ compositeness for $\Delta(1232)$ is evaluated in a very simple model.

- Large real part of the $\pi N$ compositeness, but imaginary part is non-negligible.
  --- The result implies large $\pi N$ contribution to, e.g., the transition form factor.

- However, this result was obtained in a very simple model.
  --> Need a more refined model!
3. Applications

++ Compositeness for $\Delta(1232)$ ++

- We construct our own $\pi N$ elastic scattering amplitude in the chiral unitary approach.

$$T''_{IL} = V''_{IL} + V''_{IL} G_L T''_{IL} = \frac{1}{1/V''_{IL} - G_L}$$

- $V$:  

--- We include an explicit $\Delta(1232)$ pole term.

- $G$: Subtraction constant is fixed in the natural renormalization scheme, which can exclude explicit pole contributions in $G$.

$$G_{j,L}(s = M_N^2) = 0$$

--- This makes the physical $N(940)$ mass in the full Amp. unchanged.

--- In addition, we constrain $G$ so as to exclude unphysical bare-state contributions to $N(940)$: $\frac{dG_L}{ds}(s = M_N^2) \leq 0$
3. Applications

++ Compositeness for $\Delta(1232)$ ++

- We construct our own $\pi N$ elastic scattering amplitude in the chiral unitary approach.

\[
T'_{IL}^{\pm} = V'_{IL}^{\pm} + V'_{IL}^{\pm} G_L T'_{IL}^{\pm} = \frac{1}{1/V'_{IL}^{\pm} - G_L}
\]

- We have model parameters of: LECs, bare $\Delta$ mass and coupling constant to $\pi N$, and a subtraction const.

---> Fitted to six $\pi N$ scattering amplitudes ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$) obtained as a PWA solution “WI 08” up to $\sqrt{s} = 1.35$ GeV.


- The $P_{11}$ and $P_{33}$ amplitude contain poles corresponding to the physical $N(940)$ and $\Delta(1232)$, respectively:
++ Compositeness from fitted amplitude ++

- Fitted to the $\pi N$ amplitude WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).

-->$\chi^2 / N_{d.o.f.} = 1240 / 809 \approx 1.5$.

- Chiral unitary approach reproduces the amplitude of PWA well.

<table>
<thead>
<tr>
<th>Constrained</th>
<th>$\Delta(1232)$</th>
<th>$N(940)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}_{\text{pole}}$ [MeV]</td>
<td>$1206.9 - 49.6i$</td>
<td>$938.9$</td>
</tr>
<tr>
<td>$X_{\pi N}$</td>
<td>$0.87 + 0.35i$</td>
<td>$0.00$</td>
</tr>
<tr>
<td>$Z$</td>
<td>$0.13 - 0.35i$</td>
<td>$1.00$</td>
</tr>
</tbody>
</table>

- For $\Delta(1232)$, its pole position is very similar to the PDG value.

- The $\pi N$ compositeness $X_{\pi N}$ takes large real part! But non-negligible imaginary part as well.

-->$\text{Our refined model reconfirms the result in the previous study.}$
3. Applications

++ Compositeness from fitted amplitude ++

- Fitted to the $\pi N$ amplitude WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).

\[ \chi^2 / N_{d.o.f.} = \frac{1240}{809} \approx 1.5. \]

- Chiral unitary approach reproduces the amplitude of PWA well.

For $N(940)$, $X_{\pi N}$ is non-negative and zero.

\[ X_{\pi N} = 0.87 + 0.35i \]

\[ Z = 0.13 - 0.35i \]

\hline
Constrained & $\Delta(1232)$ & $N(940)$ \\
$\sqrt{s}_{\text{pole}}$ [MeV] & $1206.9 - 49.6i$ & $938.9$ \\
$X_{\pi N}$ & $0.87 + 0.35i$ & $0.00$ \\
$Z$ & $0.13 - 0.35i$ & $1.00$ \\
\hline

- For $N(940)$ is not described by the $\pi N$ molecular picture.
4. Conclusion
4. Conclusion

- Hadronic molecules are unique, because they are composed of color singlet states, which can be observed as asymptotic states.
  - We can use quantum mechanics in a usual manner.
  - In particular, we can investigate their structure of composites by the two-body wave functions and their norms = compositeness.

- The two-body wave functions can be extracted from the hadron-hadron scattering amplitude, although they are model dependent.

\[ T(q', q; E) = \langle q'|\hat{T}(E)|q\rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{pole}} \]

\[ \gamma(q) \equiv \langle q'|\hat{V}|\Psi\rangle = [E_{pole} - E(q)]\tilde{\psi}(q) \]

--- The residue at the pole position contains information on the two-body wave function, which is automatically normalized.

\[ \int \frac{d^3q}{(2\pi)^3} \left[ \frac{\gamma(q)}{E_{pole} - E(q)} \right]^2 = 1 \]

<-- If the state is purely molecule!

--> Comparing the norm = compositeness with unity, we may able to conclude the structure of hadronic molecule candidates.
4. Conclusion

- We apply this scheme to $\Lambda(1405)$, $N(1535)$, $N(1650)$, and $\Delta(1232)$ in an effective model, chiral unitary approach, with a separable interaction of $\text{LO} + \text{NLO} (+ \text{bare } \Delta)$ taken from chiral perturbation theory.

--- In this model, we find that ...

- $\Lambda(1405)$ (higher pole) is indeed a $\bar{K}N$ molecule.
- $N(1535)$ and $N(1650)$ have small $\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$ components.
- $\Delta(1232)$ has a non-negligible $\pi N$ component.
Thank you very much for your kind attention!