A Novel Concept of Hadron form factors

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In 1954 right after the Korean war, Inha University was founded by the fund raised by Korean expatriates living in Hawaii. The University was then called as Inha Institute of Technology (IIT). The land was provided by the government (President Seungman Lee) at that time. After his exile, Inha University underwent harsh years.

It is now owned by the Korean Air. Ina University has one of the strongest engineering schools in Korea.
Form factors
What is a form factor?

Form factors tell you how the corresponding particle looks like in various aspects.

**Historical example: Rutherford scattering**

- Target is so heavy that the recoil effects are negligible.
- Elastic scattering

If \( Z\alpha \ll 1 \) (\( \alpha \approx 1/137 \))

\[ \psi_i = \frac{1}{\sqrt{V}} e^{i\mathbf{k} \cdot \mathbf{r}/\hbar} \]

Incoming wave for the electron

\[ \psi_f = \frac{1}{\sqrt{V}} e^{i\mathbf{k}' \cdot \mathbf{r}/\hbar} \]

Outgoing wave for the electron

with One-Photon Exchange assumed
Interpretation of the Form factors

Non-Relativistic picture of the EM form factors
Interpretation of the Form factors

Non-Relativistic picture of the EM form factors

Schroedinger Eq. & Wave functions
Interpretation of the Form factors

Non-Relativistic picture of the EM form factors

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions
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Non-Relativistic picture of the EM form factors

3-D Fourier Transform

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions
Interpretation of the Form factors

Non-Relativistic picture of the EM form factors

Electromagnetic form factors

3-D Fourier Transform

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions
**EM Form factors of the nucleon**

**ep scattering (Rosenbluth formula)**

\[
\frac{d\sigma_{ep}}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ \frac{1}{(1 + \tau)} \left( G_E^2 + \tau G_M^2 \right) + 2 \tan^2 \frac{\theta}{2} \tau G_M^2 \right]
\]

\[\tau = \frac{Q^2}{4M_N^2}\]

- **Magnetic Sachs form factor**
- **Electric Sachs form factor**

\[p = e^2 \bar{u}(k', \lambda') \gamma^\mu u(k, \lambda) \frac{1}{q^2} \langle p', s' | J_\mu(0) | p, s \rangle\]

Nucleon Matrix element of the EM current
EM Form factors of the nucleon

$ep$ scattering (Rosenbluth formula)

$$\frac{d\sigma_{ep}}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ \frac{1}{(1 + \tau)} \left( G_E^2 + \tau G_M^2 \right) + 2 \tan^2 \frac{\theta}{2} \tau G_M^2 \right]$$
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As $Q^2$ increases, this term with the electric form factor becomes smaller.
**EM Form factors of the nucleon**

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As $Q^2$ increases, this term with the electric form factor becomes smaller.

The magnetic form factor is measured rather precisely.
Recoil Polarisation

\[
\frac{G_E}{G_M} = -\frac{P_T}{P_L} \frac{E_e + E_{e'}}{2M} \tan \left( \frac{\theta_e}{2} \right)
\]

JLAB Hall-A Experiments
EM Form factors of the nucleon

Recoil Polarisation

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\]

JLAB Hall-A Experiments

\[\mu_{P_G}/G_M\] vs. \[Q^2 [GeV^2]\]
Recoil Polarisation

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\frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{E_e + E_e'}{2M} \tan\left(\frac{\theta_e}{2}\right)
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M. K. Jones et al., PRL 84, 1398 (2000)
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M. K. Jones et al., PRL 84, 1398 (2000)

Importance of two-photon exchange
Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

\[ F_1(Q^2) = \int d^3x e^{iQ \cdot x} \rho(r) \]

\[ \rho(r) = \sum \psi^\dagger(r) \psi(r) \]

The valid range of the momentum transfer: \( |q| \ll M_N \)

However, the initial and final momenta are different in a relativistic case. Thus, the initial and final wave functions are different.

Probability interpretation is wrong in a relativistic case!

We need a correct interpretation of the form factors

It was pointed out already long time ago.

Yennie, Levy, & Ravenhall, Rev. Mod. Phys. 29, 144 (1957)
Interpretation of the EMFFs

Non-Relativistic description

\[ M_{\text{atom}} R_{\text{atom}} = M_{\text{atom}} / (m_e \alpha) \sim 10^5 \]

\[ \|Q\| \ll M_{\text{atom}} \quad \frac{1}{\|Q\|} \leq R \]

Form factors can be measured and well interpreted (almost no recoil effect).

Relativistic description

\[ M_N R_N \sim 4 \]

Particle creation & annihilation

Initial and final momenta are different!

\[ \|Q\| \geq M_N \]

Nucleon cannot be treated non-relativistically!

Interpretation of the EMFFs

Modern understanding of the form factors

Transverse Charge densities $\rho_1(b)$

Lorentz invariant: independent of any observer.

Infinite momentum framework

$$\rho_1(b) = \sum_q e_q^2 \int dx f_{q\bar{q}}(x, b)$$

GPDs

Dirac & Pauli form factors

$$F_{1,2}(\Delta) = \int d^2 b e^{i\Delta \cdot b} \rho_{1,2}(r)$$

Nucleon Tomography

Figure 2.1: Aspects of different phenomenological observables and their interpretation in the infinite momentum frame:

a) The form factor as a charge density in the perpendicular plane (after a Fourier transform, Sec. 1.2).

b) A probabilistic interpretation for GPDs in the case of vanishing longitudinal momentum transfer, \( \xi = 0 \), with a resolution \( \sim \frac{1}{Q^2} \).

c) A parton distribution for the forward momentum case (Sec. 1.3).

For a detailed explanation see text. [Pictures inspired by [17]]

We will later argue that this infinite momentum frame is necessary for the GPDs. For the moment, we thus think of a two-dimensional distribution with respect to \( b_\perp \) in the transverse plane, sketched in Fig. 2.1.a. The \( z \)-direction is also suppressed in favour of the fractional (longitudinal) momentum \( x \) of the partons.

The second process led to parton distribution functions (PDFs) \( q(x) \) with the momentum fraction \( x \) carried by the parton. They give the probability of finding the parton \( q \) with this momentum inside the hadron and they are sketched in Fig. 2.1.c. One can also give a resolution \( \sim \frac{1}{Q^2} \) that can be resolved inside the hadron. So for different \( Q^2 \) partons of a 'different size' can be probed, consequently the parton content of the hadron changes.

To achieve a deeper understanding of the distribution of the quarks inside the hadron, it would be nice to combine the two cases, i.e. know the distribution in the transverse plane for quarks with a given momentum fraction. This is exactly in the interpretation of GPDs.

During the discussion of the form factor and the PDFs, we already mentioned the similarity of the matrix elements appearing in Eqs. (1.5) and (1.10). Initially and finally states of the two processes differed only in their momenta (after applying the optical theorem).

There are indeed processes with different asymptotic states that can be related to the two aforementioned, thus coining the term generalised distributions. We will later consider the problems arising from the complete freedom of the two momenta. For the moment, note that a density interpretation is possible if the longitudinal momentum transfer \( \xi \) vanishes. A Fourier transform of the remaining transverse momentum transfer then yields neglecting relativistic corrections, this would not be necessary for the form factor where we have elastic scattering with momenta down to zero.
Probes are unknown for Tensor form factors and the Energy-Momentum Tensor form factors!
Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors**!

Form factors as Mellin moments of the GPDs
Model
Merits of the chiral quark-soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalisation scale is naturally given.
  \( 1/\rho \approx 600 \text{ MeV} \)
- All relevant parameters were fixed already.

\[
Z_\chi_{\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})
\]

\[
S_{\text{eff}} = -N_c \text{Tr} \ln D(U)
\]
Chiral quark–soliton model

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\[ D(U) = \partial_4 + H(U) + \hat{m} \]
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\begin{align*}
Z_{\chi QSM} &= \int D U \exp(-S_{\text{eff}}) \\
S_{\text{eff}} &= -N_c \text{Tr} \ln D(U) \\
H(U) &= -i \gamma_4 \gamma_i \partial_i + \gamma_4 MU\gamma_5 \\
D(U) &= \partial_4 + H(U) + \hat{m}
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H(U) = -i\gamma_4 \gamma_i \partial_i + \gamma_4 MU^\gamma_5 \\
D(U) = \partial_4 + H(U) + \hat{m} \\
\hat{m} = \text{diag}(m_u, m_d, m_s) \gamma_4
\]
Classical solitons

\[ \langle J_N(\vec{x}, T)\bar{J}^i_N(\vec{y}, -T)\rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\text{val}} + E_{\text{sea}})T]} \]

\[
\frac{\delta}{\delta U} (N_c E_{\text{val}} + E_{\text{sea}}) = 0 \quad \Rightarrow \quad M_{\text{cl}} = N_c E_{\text{val}}(U_c) + E_{\text{sea}}(U_c)
\]

Hedgehog Ansatz:

\[
U_{SU(2)} = \exp [i \gamma_5 \mathbf{n} \cdot \boldsymbol{\tau} P(r)]
\]

Collective (Zero-mode) quantisation

\[ U_0 = \begin{bmatrix} e^{i\mathbf{n} \cdot \tau P(r)} & 0 \\ 0 & 1 \end{bmatrix} \]

Zero-mode quantisation

\[
U(\mathbf{x}, t) = R(t)U_c(\mathbf{x} - \mathbf{Z}(t))R^\dagger(t)
\]

\[
\int D U[\cdots] \rightarrow \int D A D Z[\cdots]
\]

\[
\mathcal{L} = -M_{sol} + \frac{I_1}{2} \sum_{i=1}^{3} \Omega_i^2 + \frac{I_2}{2} \sum_{i=4}^{7} \Omega_i^2 + \frac{N_c}{2\sqrt{3}} \Omega_8
\]

Observables

Valence part

Sea part
Transverse Charge Densities (EM Form factors)
Transverse charge densities

Why transverse charge densities?

\[
\langle P', S' | \bar{\psi}(0) \gamma_\mu \hat{Q} \psi(0) | P, S \rangle \\
= \bar{u}(p', s') \left( \gamma_\mu F_1(t) + i \frac{\sigma^{\mu \nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s)
\]
Transverse charge densities

Why transverse charge densities?

Electromagnetic form factors:

\[
\langle P', S' | \bar{\psi}(0) \gamma_\mu \hat{Q} \psi(0) | P, S \rangle
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\]

GPDs

\[
\int \frac{dx^-}{4\pi} \langle P', S'|\bar{q}(-\frac{x^-}{2}, 0_\perp)\gamma^+ q(\frac{x^-}{2}, 0_\perp)|P, S\rangle
\]

\[
= \frac{1}{2p^+} \bar{u}(p', s') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s)
\]
Transverse charge densities

Why transverse charge densities?

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GPDs

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\]

\[
F_1(t) = \sum_q e_q \int dx H_q(x, 0, t), \\
F_2(t) = \sum_q e_q \int dx E_q(x, 0, t)
\]

The EM form factors as the first moments of the vector GPDs
Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

\[ q(x, b) = \int \frac{d^2 q}{(2\pi)^2} e^{iq \cdot b} H_q(x, -q^2) \]
Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

\[ q(x, b) = \int \frac{d^2 q}{(2\pi)^2} e^{i q \cdot b} H_q(x, -q^2) \]

It can be interpreted as the probability distribution of a quark in the transverse plane.


\[ \rho(b) := \sum_q e_q \int dx q(x, b) \]

\[ = \int \frac{d^2 q}{(2\pi)^2} F_1(Q^2) e^{i q \cdot b} \]

Moving direction of the nucleon
Transverse charge densities

Inside an unpolarized nucleon


\[
\rho_{\text{ch}}^{\chi}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) F_1^\chi(Q^2)
\]

Inside a polarized nucleon

Carlson and Vanderhaeghen, PRL 100, 032004

\[
\rho_T^{\chi}(b) = \rho_{\text{ch}}^{\chi}(b) - \sin(\phi_b - \phi_S) \frac{1}{2M_N} \int_0^\infty \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^\chi(Q^2)
\]
Dirac & Pauli Form factors


\[ F_1(Q^2) \sim \frac{1}{Q^4}, \quad Q^2 \to \infty \]
\[ F_2(Q^2) \sim \frac{1}{Q^6}, \quad Q^2 \to \infty \]

Silva, Urbano, HChK, hep-ph/1305.6373
Dirac & Pauli Form factors

Up quark FFs

Down quark FFs

Strange quark FFs
Results

Transverse charge densities inside an **unpolarized** proton

![Graph showing transverse charge densities](graph.png)
Results

Transverse charge densities inside an unpolarized proton

- Long positive tail: Possible positive pion cloud
  - Centered positive charge distribution

Silva, Urbano, HChK, hep-ph/1305.6373
Transverse charge densities inside an **unpolarized** neutron

\[ \rho(b) = \int_0^\infty \frac{dQ^2}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau} \]

\[ \tau = \frac{Q^2}{4M_N^2} \]

**Results**

Silva, Urbano, HChK, hep-ph/1305.6373
Transverse charge densities inside an **unpolarized** neutron

\[ \rho(b) = \int_0^\infty \frac{dQ}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau} = F_1(Q^2) \]

\[ \tau = \frac{Q^2}{4M_N^2} \]

---

Silva, Urbano, HChK, hep-ph/1305.6373
Surprisingly, negative charge distribution in the center of the neutron!

Old 3-D charge densities inside an unpolarized neutron

\[ G_E(Q^2) \sim \int drr^2 j_0(Qr) \rho_{ch}(r) \]
Old 3-D charge densities inside an unpolarized neutron

\[ G_E(Q^2) \sim \int drr^2 j_0(Qr)\rho_{ch}(r) \]

Results

Centered positive charge distribution
Long positive tail:
Positive pion cloud

Centered positive charge distribution
Long negative tail

Yennie, Levy, & Ravenhall, Rev. Mod. Phys. 29, 144 (1957)
Results

Transverse charge densities inside an polarized nucleon

Silva, Urbano, HChK, hep-ph/1305.6373
Discussion

Carlson, Vanderhaeghen, PRL 100, 032004

Silva, Urbano, HChK, hep-ph/1305.6373
Nucleon polarization along the $x$ axis: Magnetic dipole field $B$

Carlson, Vanderhaeghen, PRL 100, 032004

Silva, Urbano, HChK, hep-ph/1305.6373
Nucleon polarization along the $x$ axis:
Magnetic dipole field $B$

$\vec{E}' = -\gamma (\vec{v} \times \vec{B})$

Induced electric dipole field along the negative $y$ axis: Relativistic effects
Nucleon polarization along the \( \mathbf{x} \) axis: Magnetic dipole field \( \mathbf{B} \)

\[
\mathbf{E}' = -\gamma (\mathbf{\hat{v}} \times \mathbf{B})
\]

Induced electric dipole field along the negative \( \mathbf{y} \) axis: Relativistic effects
Discussion

Nucleon polarization along the $x$ axis:
Magnetic dipole field $B$

$\mathbf{E}' = -\gamma (\mathbf{\vec{v}} \times \mathbf{\vec{B}})$

Induced electric dipole field along the negative $y$ axis: Relativistic effects

Note that the neutron anomalous magnetic moment is negative!
Flavor-decomposed Transverse charge densities inside a polarized nucleon
Transverse Spin Densities (Tensor form factors)
\[
\langle N_s'(p')|\bar{\psi}(0)i\sigma^{\mu\nu}\lambda^\chi\psi(0)|N_s(p)\rangle = \bar{u}_{s'}(p') \left[ H_T^\chi(Q^2)i\sigma^{\mu\nu} + E_T^\chi(Q^2)\gamma^\mu q^\nu - q^\mu \gamma^\nu \right. \\
+ \left. \tilde{H}_T^\chi(Q^2)\frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2} \right] u_s(p)
\]

\[
\int_{-1}^{1} dx H_T^\chi(x, \xi = 0, t) = H_T^\chi(q^2),
\]

\[
\int_{-1}^{1} dx E_T^\chi(x, \xi = 0, t) = E_T^\chi(q^2),
\]

\[
\int_{-1}^{1} dx \tilde{H}_T^\chi(x, \xi = 0, t) = \tilde{H}_T^\chi(q^2)
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\[
H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s
\]

\[
H_T^3(0) = g_T^3 = \delta u - \delta d
\]

\[
H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)
\]
\[ \langle N_{s'}(p')|\bar{\psi}(0)i\sigma^{\mu\nu}\lambda^x\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p') \left[ H_T^X(Q^2)i\sigma^{\mu\nu} + E_T^X(Q^2)\frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} ight. \\
+ \left. \tilde{H}_T^X(Q^2)\frac{\eta^{\mu\nu} q^\nu - q^\mu \eta^{\mu\nu}}{2M^2} \right] u_s(p) \]

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\[ H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s) \]
Tensor form factors

\[ \langle N_s'(p') | \bar{\psi}(0) i \sigma^{\mu\nu} \lambda^x \psi(0) | N_s(p) \rangle = \bar{u}_{s'}(p') \left[ H_T^X(Q^2) i \sigma^{\mu\nu} + E_T^X(Q^2) \frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} \right. \]

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\[ \int_{-1}^{1} dx \tilde{H}_T^X(x, \xi = 0, t) = \tilde{H}_T^X(q^2) \]

\[ H_T^X(Q^2) = \frac{2M}{q^2} \int \frac{d\Omega}{4\pi} \langle N_{1/2}^z(p') | \psi^\dagger \gamma^k q^k \lambda^x \psi | N_{1/2}^z(p) \rangle \]

\[ \kappa_T^X = -H_T^X(0) - H_T^{*X}(0) \]

Together with the anomalous magnetic moment, this will allow us to describe the **transverse spin quark densities** inside the nucleon.
Tensor form factors

Tensor charges and anomalous tensor magnetic moments are scale-dependent.

\[
\delta q(\mu^2) = \left( \frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)} \right)^{4/27} \left[ 1 - \frac{337}{486\pi} (\alpha_S(\mu_i^2) - \alpha_S(\mu^2)) \right] \delta q(\mu_i^2),
\]

\[
\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9 \ln(\mu^2/\Lambda_{QCD}^2)} \left[ 1 - \frac{64}{81} \frac{\ln \ln(\mu^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)} \right]
\]

\[\Lambda_{QCD} = 0.248 \text{ GeV}\]

## Results

<table>
<thead>
<tr>
<th>Proton</th>
<th>This work</th>
<th>SU(2)</th>
<th>Lattice</th>
<th>SIDIS</th>
<th>NR</th>
</tr>
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<tbody>
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</table>

SIDIS [16] (0.80 GeV$^2$): $\delta u = 0.54^{+0.09}_{-0.22}$, $\delta d = -0.231^{+0.09}_{-0.16}$, $\delta d = -0.26^{+0.1}_{-0.18}$

SIDIS [16] (0.36 GeV$^2$): $\delta u = 0.60^{+0.10}_{-0.24}$, $\delta d = -0.21 \pm 0.005$

Lattice [21] (4.00 GeV$^2$): $\delta u = 0.86 \pm 0.13$, $\delta d = -0.26 \pm 0.01$

Lattice [21] (0.36 GeV$^2$): $\delta u = 1.05 \pm 0.16$, $\delta d = -0.32$

$\chi$QSM (0.36 GeV$^2$): $\delta u = 1.08$, $\delta d = -0.32$

---


$$\mu^2 = 0.36 \text{ GeV}^2$$

<table>
<thead>
<tr>
<th>$\kappa_T^u$</th>
<th>Present work SU(3)</th>
<th>Present work SU(2)</th>
<th>Lattice</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_T^d$</td>
<td>3.56</td>
<td>3.72</td>
<td>3.00 (3.70)</td>
</tr>
<tr>
<td>$\kappa_T^s$</td>
<td>1.83</td>
<td>1.83</td>
<td>1.90 (2.35)</td>
</tr>
<tr>
<td>$\kappa_T^u / \kappa_T^d$</td>
<td>0.2 ~ −0.2</td>
<td>2.02</td>
<td>1.58</td>
</tr>
</tbody>
</table>

The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)
\[
\rho(b, S, s) = \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \\
+ s^i S^i \left\{ H_T(b^2) - \frac{1}{4M_N^2} \nabla^2 \tilde{H}_T(b^2) \right\} \\
+ s^i \left( 2b^i b^j - b^2 \delta^{ij} \right) S^j \frac{1}{M_N^2} \left( \frac{\partial}{\partial b^2} \right)^2 \tilde{H}_T(b^2) \right],
\]

Transverse spin density

\[
\rho(b, S, s) = \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]
\]

\[
[S, s] = [(1, 0), (0, 0)], \quad [S, s] = [(0, 0), (1, 0)]
\]

Transverse spin density

\[
\rho(b, S, s) = \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial K_T(b^2)}{\partial b^2} \right]
\]

\[
[S, s] = [(1, 0), (0, 0)], \quad [S, s] = [(0, 0), (1, 0)]
\]

\[
\mathcal{F}^\chi(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^\chi(Q^2)
\]

\[
H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)
\]

Results

Up quark transverse spin density inside a nucleon

FIG. 4. Transverse up and down quark spin densities of the nucleon from the QSM with the lowest moment. In the upper left panel, the density of unpolarized quarks in a transversely polarized nucleon $z([S,S],s) = [z_1,0, z_0,0]$ is drawn and in the upper right panel, that of transversely polarized quarks in a unpolarized nucleon $z([S,S],s) = [z_0,0, z_1,0]$). In the lower panel, we plot the down quark densities.

$[S,S], s = [m_v, un, m_u, un]$ it is shown from Figs y that the down quark density is more distorted in the negative direction of $b_y$ compared to the up quark densities. The reason can be found in the fact that firstly the down anomalous magnetic moment $m_v$ is negative and its absolute value is larger than the up one $m_u$. Secondly, the down Pauli form factor falls off more slowly than that of the up quarks. As for the $[S,S], s = [m_v, un, m_u, un]$ the tensor anomalous magnetic moments come into play. Since both $u_T$ and $d_T$ are positive, the both transverse spin densities are deformed in the direction of the positive $b_y$. Moreover, the form factor $d_T$ falls off rather more slowly than $u_T$ as shown in Figs $w$, the density for the down quark is more strongly deformed. Note that the $Q^2$ dependence of the form factors play a more important role in governing the deformation of the densities. The results for the transverse up and down quark spin densities of the nucleon with the lowest moment are very similar to those from the lattice calculation [v7] and as a result we can draw similar conclusions.

Since both the strange Pauli form factor and tensor anomalous magnetic form factor turn out to be rather small, we can expect that the strength of the strange densities will be rather small. Nevertheless, it is of great interest to see how much the transverse strange densities are distorted. Figure $z$ plots the transverse strange quark spin densities with the lowest moments. Note that the magnitudes of the densities are smaller than those of the up and down quarks by an order of magnitudes.

It is interesting to see that the density of unpolarized strange quarks in a polarized nucleon is negatively distorted. It is due to the fact that the strange Pauli form factor $F_s$ turns negative from $Q^2 \leftrightarrow u_w \text{GeV}^2$. This negative value strictly means...
Results

Up quark transverse spin density inside a nucleon

T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)
Results

Up quark transverse spin density inside a nucleon

![Graph showing up quark transverse spin density](image)
Results

Down quark transverse spin density inside a nucleon

T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)
Results

Down quark transverse spin density inside a nucleon

T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)
Results

Down quark transverse spin density inside a nucleon

![Graph showing down quark transverse spin density](image-url)
Strange quark transverse spin density inside a nucleon

This is the **first** result of the strange quark transverse spin density inside a nucleon

T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)
Results

Strange quark transverse spin density inside a nucleon

\[ [(0,0),(0,0)] \]
\[ [(1,0),(0,0)] \]

\[ b_x = 0 \]
Results

Strange quark transverse spin density inside a nucleon

Polarized to the negative direction in the b plane.
EMT form factors: Stability of the nucleon
Energy-momentum tensor form factors

\[ \langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p') \left[ M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} \right. \]

\[ + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu} \left. \right] u(p) \]
EMT form factors

Energy-momentum tensor form factors

\[
\langle N(p')|T^{Q,G}_{\mu\nu}(0)|N(p)\rangle = \bar{u}(p') \left[ M^Q_G(t) \frac{P_{\mu}P_{\nu}}{M_N} + J^{Q,G}(t) \frac{i(P_{\mu}\sigma_{\nu\rho} + P_{\nu}\sigma_{\mu\rho})\Delta^{\rho}}{2M_N} \right. \\
\left. + d_1^{Q,G}(t) \frac{\Delta_{\mu}\Delta_{\nu} - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu} \right] u(p)
\]

GPDs

\[
\int \frac{dx^-}{4\pi} \langle P', S'|\bar{q}(-\frac{x^-}{2}, 0_\perp)\gamma^+q(\frac{x^-}{2}, 0_\perp)|P, S\rangle \\
= \frac{1}{2\bar{p}^+} \bar{u}(p', s') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu}\Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s)
\]
EMT form factors

Energy-momentum tensor form factors

\[
\langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p') \left[ M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu} \right] u(p)
\]

GPDs

\[
\int \frac{dx^-}{4\pi} \langle P', S'|\bar{q}(-\frac{x^-}{2}, 0_\perp)\gamma^+ q(\frac{x^-}{2}, 0_\perp)|P, S\rangle = \frac{1}{2\bar{p}^+} \bar{u}(p', s') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s)
\]

The EMT form factors as the second moments of the isoscalar vector GPDs

\[
\int_{-1}^{1} dx \sum_f H_q(x, \xi, t) = M_2^Q(t) + \frac{4}{5} d_1^Q(t) \xi^2,
\]

\[
\int_{-1}^{1} dx \sum_f E_q(x, \xi, t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5} d_1^Q(t) \xi^2,
\]
EMT form factors

In the Breit frame,

\[
T_{\mu\nu}^Q(r, s) = \frac{1}{2E} \int \frac{d^3 \Delta}{(2\pi)^3} \exp(i\Delta \cdot r) \langle p', S' | T_{\mu\nu}^Q(0) | p, S \rangle
\]

\[
M_2(t) - \frac{t}{4M_N^2} \left( M_2(t) - 2J(t) + \frac{4}{5} d_1(t) \right) = \frac{1}{M_N} \int d^3r e^{-i\cdot\Delta} T_{00}(r, s)
\]

Momentum fractions carried by quarks and gluons

\[
M_2^Q(0) = \int_0^1 dx \sum_q x(f_1^q + f_1^{\bar{q}})(x),
\]

\[
M_2^G(0) = \int_0^1 dx x f_1^g(x),
\]
EMT form factors

In the Breit frame,

\[ T_{\mu\nu}^Q(r, s) = \frac{1}{2E} \int \frac{d^3 \Delta}{(2\pi)^3} \exp(i\Delta \cdot r) \langle p', S' | T_{\mu\nu}^Q(0) | p, S \rangle \]

\[ M_2(t) - \frac{t}{4M_N^2} \left( M_2(t) - 2J(t) + \frac{4}{5}d_1(t) \right) = \frac{1}{M_N} \int d^3 re^{-i\cdot\Delta} T_{00}(r, s) \]

Momentum fractions carried by quarks and gluons

\[ M_2^Q(0) = \int_0^1 dx \sum_q x(f_q^q + f_\bar{q}^\bar{q})(x), \]

Unpolarized parton distributions

\[ M_2^G(0) = \int_0^1 dx x f_1^g(x), \]
EMT form factors

\[ J^Q(t) + \frac{2t}{3} J^{Q'}(t) = \int d^3r e^{-ir\Delta} \epsilon^{ijk} s_i r_j T^{Q}_{0k}(r, s), \]

\[ d^Q_1(t) + \frac{4t}{3} d^{Q'}_1(t) + \frac{4t^2}{15} d^{Q''}_1(t) \]
\[ = - \frac{M_N}{2} \int d^3r e^{-ir\Delta} T^Q_{ij}(r) \left( r^i r^j - \frac{r^2}{3} \delta^{ij} \right), \]

Constraints

\[ M_2(0) = \frac{1}{M_N} \int d^3r T_{00}(r, s) = 1, \]

\[ J(0) = \int d^3r \epsilon^{ijk} s_i r_j T_{0k}(r, s) = \frac{1}{2}, \]

\[ d_1(0) = - \frac{M_N}{2} \int d^3r T_{ij}(r) \left( r^i r^j - \frac{r^2}{3} \delta^{ij} \right) = d_1 \]

Nucleon Mass

Nucleon Spin

D-term
Stability of the nucleon

$$T_{ij}(r) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$
Stability of the nucleon

\[ T_{ij}(\mathbf{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij} \]

\[ \int_0^\infty dr \ r^2 p(r) = 0 \]

: Stability condition of the nucleon
Stability of the nucleon

Any model for the nucleon should satisfy this condition!
Stability of the nucleon: XQSM

\[ r^2 p(r) \text{ in GeV fm}^{-1} \]

- quark core
- pion cloud
- total

Goeke et al, PRD 75, 094021
Stability of the nucleon: \( \pi \)-\( \rho \)-\( \omega \) model

\[ 4\pi r^2 p(r) \left[ \text{GeVfm}^{-1} \right] \]

\( \rho \) and \( \omega \)

\[ r \left[ \text{fm} \right] \]

J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)
Stability of the nucleon: Skyrme

It is quite nontrivial to satisfy the stability of the nucleon!

HChK et al, PLB 718, 625 (2012)
EMT form factors: Results

Mass form factors

\[ M_2(t) = \frac{1}{M_{\text{sol}}} \int d^3r \, T_{00}(r) \, j_0(r \sqrt{-t}) - \frac{t}{5M_{\text{sol}}^2} \, d_1(t) \]

J. Jung, U. Yakhshiev, HChK et al, JGP 41, 055107 (2014)
EMT form factors: Results

Spin form factors

\[ J(t) = 3 \int d^3 r \rho_J(r) \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \quad T^{0i}(\vec{r},\vec{s}) = \frac{e^{ilm} r^l s^m}{(\vec{s} \times \vec{r})^2} \rho_J(r) \]

J. Jung, U. Yakhshiev, HChł
d1 form factors

\[ d_1(t) = \frac{15M_{\text{sol}}}{2} \int d^3r \ p(r) \ \frac{j_0(r\sqrt{-t})}{t} \]

\[ T^{ij}(r) = s(r) \left( \frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij} \]

J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)
EMT form factors: Results

\[ d_1(t) = \frac{15M_{\text{sol}}}{2} \int d^3r \ p(r) \ \frac{j_0(r\sqrt{-t})}{t} \]

\[ T^{ij}(r) = s(r) \left( \frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij} \]

\[ d_1 < 0 \]

To secure the stability of a particle

J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)
Transverse spin structure of the pion
The spin structure of the Pion

Vector & Tensor Form factors of the pion

Pion: Spin $S=0$

Longitudinal spin structure is trivial.

$$\langle \pi(p') | \bar{\psi} \gamma_3 \gamma^5 \psi | \pi(p) \rangle = 0$$

What about the transversely polarized quarks inside a pion?

Internal spin structure of the pion
The spin distribution of the quark

\[ \rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx \, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2} \left[ A_{n0}(b_\perp^2) - \frac{s_\perp \epsilon^{ij} b^j_\perp}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2} \right] \]

Spin probability densities in the transverse plane

A

\[ A_{n0}: \text{Vector densities of the pion,} \quad B_{n0}: \text{Tensor densities of the pion} \]

\[ \int_{-1}^{1} dx \, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx \, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2) \]
The spin distribution of the quark

\[ \rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx \ x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2} \left[ A_{n0}(b_\perp^2) - \frac{s^i_\perp \epsilon^{ij} b^j_\perp}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2} \right] \]

Spin probability densities in the transverse plane

A_{n0}: Vector densities of the pion, \quad B_{n0}: Tensor densities of the pion

\[ \int_{-1}^{1} dx \ x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx \ x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2) \]
The spin distribution of the quark

Spin probability densities in the transverse plane

\[ \rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx \, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2} \left[ A_{n0}(b_\perp^2) \right. \left. - \frac{s^i \epsilon^{ij} b^j_\perp}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b^2_\perp} \right] \]

**A\textsubscript{n0}: Vector densities of the pion, **  \qquad **B\textsubscript{n0}: Tensor densities of the pion**

\[ \int_{-1}^{1} dx \, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx \, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2) \]

**Vector and Tensor form factors of the pion**

\[ \langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2) \]

\[ \langle \pi^+(p_f) | O^{\mu \nu \mu_1 \cdots \mu_{n-1}}_T | \pi^+(p_i) \rangle = AS \left[ \frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\text{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} \right] B_{ni}(Q^2) \]
Nonlocal chiral quark model

Gauged Effective Nonlocal Chiral Action

\[ S_{\text{eff}} = -N_c \text{Tr} \ln \left[ i\bar{\psi} + im + i\sqrt{M(iD, m)U^\gamma_5} \sqrt{M(iD, m)} \right] \]

\[ D_\mu = \partial_\mu - i\gamma_\mu V_\mu \]

The nonlocal chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- “Derived” from QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter!

\[ \rho \approx 0.3 \text{ fm, } R \approx 1 \text{ fm} \]
\[ \mu \approx 600 \text{ MeV} \]

Dilute instanton liquid ensemble

EM Form factor of the pion

EM form factor \( (A_{10}) \)

\[
\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)
\]

\[
\sqrt{\langle r^2 \rangle} = 0.675 \text{ fm}
\]

\[
\sqrt{\langle r^2 \rangle} = 0.672 \pm 0.008 \text{ fm (Exp)}
\]

\[
F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1 + Q^2/M^2}
\]

\[M(\text{Phen.}): 0.714 \text{ GeV}\]

\[M(\text{Lattice}): 0.727 \text{ GeV}\]

\[M(\text{XQM}): 0.738 \text{ GeV}\]

Tensor Form factor of the pion


\[
B_{10}(Q^2, \mu) = B_{10}(0) \left[ \frac{\alpha(\mu)}{\alpha(\mu_0)} \right]^{\gamma/2\beta_0}
\]
\[
\gamma_1 = 8/3, \quad \gamma_2 = 8, \quad \beta_0 = 11N_c/3 - 2N_f/3
\]

RG equation for the tensor form factor

\[
B_{10}(Q^2) = B_{10}(0) \left[ 1 + \frac{Q^2}{pm_p^2} \right]^{-p}
\]


Spin density of the quark

\[
\rho_1 \left( b_{\perp}, s_x = \pm \frac{1}{2} \right) = \frac{1}{2} \left[ A_{10}(b^2) \mp \frac{b \sin \theta}{m_\pi} B'_{10}(b^2) \right]
\]
Spin density of the quark

Significant distortion appears for the polarized quark!

<table>
<thead>
<tr>
<th>$m_\pi$ = 140 MeV</th>
<th>$B_{10}(0)$</th>
<th>$m_{p_1}$ [GeV]</th>
<th>$\langle b_y \rangle$ [fm]</th>
<th>$B_{20}(0)$</th>
<th>$m_{p_2}$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>0.216</td>
<td>0.762</td>
<td>0.152</td>
<td>0.032</td>
<td>0.864</td>
</tr>
<tr>
<td>Lattice QCD [7]</td>
<td>0.216 ± 0.034</td>
<td>0.756 ± 0.095</td>
<td>0.151</td>
<td>0.039 ± 0.099</td>
<td>1.130 ± 0.265</td>
</tr>
</tbody>
</table>

Results are in a good agreement with the lattice calculation!
EMT form factors
stability of the pion
Stability of the pion

Isoscalar vector GPDs of the pion

\[ 2\delta^{ab} H_{\pi}^{I=0}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p') | \overline{\psi}(-\lambda n/2) \gamma[\lambda n/2, \lambda n/2] \psi(\lambda n/2) | \pi^b(p) \rangle \]

The second moment of the GPD

\[ \int dx \, x H_{\pi}^{I=0}(x, \xi, t) = A_{20}(t) + 4\xi^2 A_{22}(t) \]
Stability of the pion

Isoscalar vector GPDs of the pion

\[ 2\delta^{ab} H_{\pi}^{I=0}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p') | \bar{\psi}(-\lambda n/2) \gamma[\lambda n/2] \psi(\lambda n/2) | \pi^b(p) \rangle \]

The second moment of the GPD

\[ \int dx \, x H_{\pi}^{I=0}(x, \xi, t) = [A_{20}(t)] + 4\xi^2 [A_{22}(t)] \] : Generalized form factors of the pion
Stability of the pion

Isoscalar vector GPDs of the pion

\[ 2\delta^{ab} H^{I=0}_{\pi}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P \cdot n)} \langle \pi^a(p') | \bar{\psi}(-\lambda n/2) \gamma[ -\lambda n/2, \lambda n/2] \psi(\lambda n/2) | \pi^b(p) \rangle \]

The second moment of the GPD

\[ \int dx \, x \, H^{I=0}_{\pi}(x, \xi, t) = A_{20}(t) + 4\xi^2 A_{22}(t) \] : Generalized form factors of the pion

Energy-momentum Tensor Form factors (Pagels, 1966)

\[ \langle \pi^a(p') | T_{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} \left[ (tg_{\mu\nu} - q_\mu q_\nu u) \Theta_1(t) + 2P_\mu P_\nu \Theta_2(t) \right] \]

\[ T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x) \gamma\{ i \overset{\leftarrow}{\partial_\nu} \} \psi(x) \] : QCD EMT operator for the quark part
Stability of the pion

Isoscalar vector GPDs of the pion

\[ 2\delta^{ab} H_{\pi}^{I=0}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P \cdot n)} \langle \pi^a(p') | \bar{\psi}(-\lambda n/2) \gamma\lambda \psi(-\lambda n/2, \lambda n/2) \psi(\lambda n/2) | \pi^b(p) \rangle \]

The second moment of the GPD

\[ \int dx x H_{\pi}^{I=0}(x, \xi, t) = \boxed{A_{20}(t)} + 4\xi^2 \boxed{A_{22}(t)} : \text{Generalized form factors of the pion} \]

Energy-momentum Tensor Form factors (Pagels, 1966)

\[ \langle \pi^a(p') | T_{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} \left[ (tg_{\mu\nu} - q_\mu q_n u) \Theta_1(t) + 2P_\mu P_\nu \Theta_2(t) \right] \]

\[ T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x) \gamma\{i \overset{\leftrightarrow}{\partial}_\nu\} \psi(x) : \text{QCD EMT operator for the quark part} \]
Stability of the pion

Isoscalar vector GPDs of the pion

\[ 2 \delta^{ab} H^{I=0}_\pi (x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P \cdot n)} \langle \pi^a (p') | \bar{\psi} (-\lambda n/2) \gamma^I [ -\lambda n/2, \lambda n/2] \psi (\lambda n/2) | \pi^b (p) \rangle \]

The second moment of the GPD

\[ \int dx \, x H^{I=0}_\pi (x, \xi, t) = \boxed{A_{20}(t) + 4\xi^2 A_{22}(t)} : \text{Generalized form factors of the pion} \]

Energy-momentum Tensor Form factors (Pagels, 1966)

\[ \langle \pi^a (p') | T_{\mu \nu} (0) | \pi^b (p) \rangle = \frac{\delta^{ab}}{2} \left[ (tg_{\mu \nu} - q_{\mu} q_{\nu} u) \Theta_1 (t) + 2P_{\mu} P_{\nu} \Theta_2 (t) \right] \]

\[ T_{\mu \nu} (x) = \frac{1}{2} \bar{\psi} (x) \gamma^I \{ i \nabla_\nu \} \psi (x) : \text{QCD EMT operator for the quark part} \]

\[ \Theta_1 = -4A_{22}^{I=0} , \ \Theta_2 = A_{20}^{I=0} \]

EMTFFs (Gravitational FFs)
Stability of the pion

The time component of the EMT matrix element gives the pion mass:

\[ \langle \pi^a(p) | T_{44}(0) | \pi^b(p) \rangle \bigg|_{t=0} = -2m^2_\pi \Theta_2(0) \delta^{ab} \]

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

\[ \langle \pi^a(p) | T_{ii}(0) | \pi^b(p) \rangle \bigg|_{t=0} = \frac{3}{2} \delta^{ab} t \Theta_1(t) \bigg|_{t=0} \]

\[ P = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle \]

\[ = \frac{12N_c m M}{f^2_\pi} \int d\vec{l} \frac{-l^2}{[l^2 + M^2]^2} + \frac{12N_c M^2}{f^2_\pi} \int d\vec{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + M^2]^3} \]

(Based on the local model)

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[
\mathcal{P} = \frac{12N_c m M}{f^2_\pi} \int d\tilde{l} \frac{-l^2}{[l^2 + \overline{M}^2]^2} + \frac{12N_c M^2}{f^2_\pi} \int d\tilde{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1 - x)p^2 + \overline{M}^2]^3}
\]

H.D. Son & H.-Ch.K, PRD 90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[
\mathcal{P} = \frac{12N_c m M}{f_\pi^2} \int d\tilde{\ell} \frac{-\ell^2}{[\ell^2 + M^2]^2} + \frac{12N_c M^2}{f_\pi^2} \int d\tilde{\ell} \int_0^1 dx \frac{-p^2 \ell^2}{[\ell^2 + x(1-x)p^2 + M^2]^3}
\]

\[
i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{\ell} \frac{M}{[\ell^2 + M^2]}
\]

Quark condensate

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[ \mathcal{P} = \frac{12N_c m M}{f_\pi^2} \int d\tilde{l} \frac{-l^2}{[l^2 + M^2]^2} + \frac{12N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1 - x)p^2 + M^2]^3} \]

Quark condensate

\[ i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{l} \frac{M}{[l^2 + M^2]} \]

Pion decay constant

\[ f_\pi^2 = 4N_c \int_0^1 dx \int d\tilde{l} \frac{M \bar{M}}{[l^2 + \bar{M}^2 + x(1 - x)p^2]^2} \]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[
\mathcal{P} = \frac{12N_c m M}{f_\pi^2} \int d\tilde{\ell} \frac{-l^2}{[l^2 + M^2]^2} + \frac{12N_c M^2}{f_\pi^2} \int d\tilde{\ell} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + M^2]^3}
\]

\[
i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{\ell} \frac{M}{[l^2 + M^2]}
\]

\[
f_\pi^2 = 4N_c \int_0^1 dx \int d\tilde{\ell} \frac{M \overline{M}}{[l^2 + \overline{M}^2 + x(1-x)p^2]^2}
\]

\[
\mathcal{P} = \frac{3M}{f_\pi^2 \overline{M}} \left( m \langle \bar{\psi}\psi \rangle + m_\pi^2 f_\pi^2 \right) = 0 !
\]

by the Gell-Mann-Oakes-Renner relation to linear m order

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Energy–momentum Tensor FFs

$\Theta_1 = \Theta_2$

in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
The difference arises from the explicit chiral symmetry breaking.

\[ \Theta_1 = \Theta_2 \]

in the chiral limit
Transverse charge density of the pion

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Transverse charge density of the pion

\[ \rho_{20}(b) \]

\[ \rho_{20}(b) \text{[fm}^{-2}] \]

\[ b_x \text{[fm]} \]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
The transverse charge density is divergent at $b=0$. 

$\rho_{20}(b)$
Transverse charge density of the pion

\[ \rho_{\pi}(b_y, b_x = 0) \]

\[ b_y \text{ [fm]} \]

\[ \rho_{10} \]

\[ \rho_{20} \]
Transverse charge density of the pion

\[ \rho_{10}(b) \]
The transverse charge density is divergent at $b=0$. 

$$\rho_{10}(b)$$
Summary & Conclusion
Summary

- We have reviewed recent investigations on the charge and spin structures of the nucleon and the pion, based on the chiral quark(-soliton) model.

- We have derived the EM and tensor form factors of the nucleon, from which we have obtained its transverse charge & spin densities. The results are compared with the lattice and “experimental” data.

- We also showed the EMT form factors of the nucleon and the pion. The stabilities of the nucleon and the pion, which are quite nontrivial, were also discussed.
Outlook

- Weak GPDs and generalized form factors are under investigation (A relevant work will be published soon).

- The excited states for the nucleon and the hyperon can be investigated (Extension of the XQSM is under way).

- Internal structure of Heavy-light quark systems (Derivation of the Partition function is close to the final result.)

- New perspective on hadron tomography
Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!