OSAKA UNIVERSITY

A Novel Concept of Hadron form factors

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Department of Physics, Inha University

仁荷大學校

Incheon Hawaii

In 1954 right after the Korean war, Inha University was founded by the fund raised by Korean expatriates living in Hawaii. The University was then called as Inha Institute of Technology (IIT). The land was provided by the government (President Seungman Lee) at that time. After his exile, Inha University underwent harsh years. ,

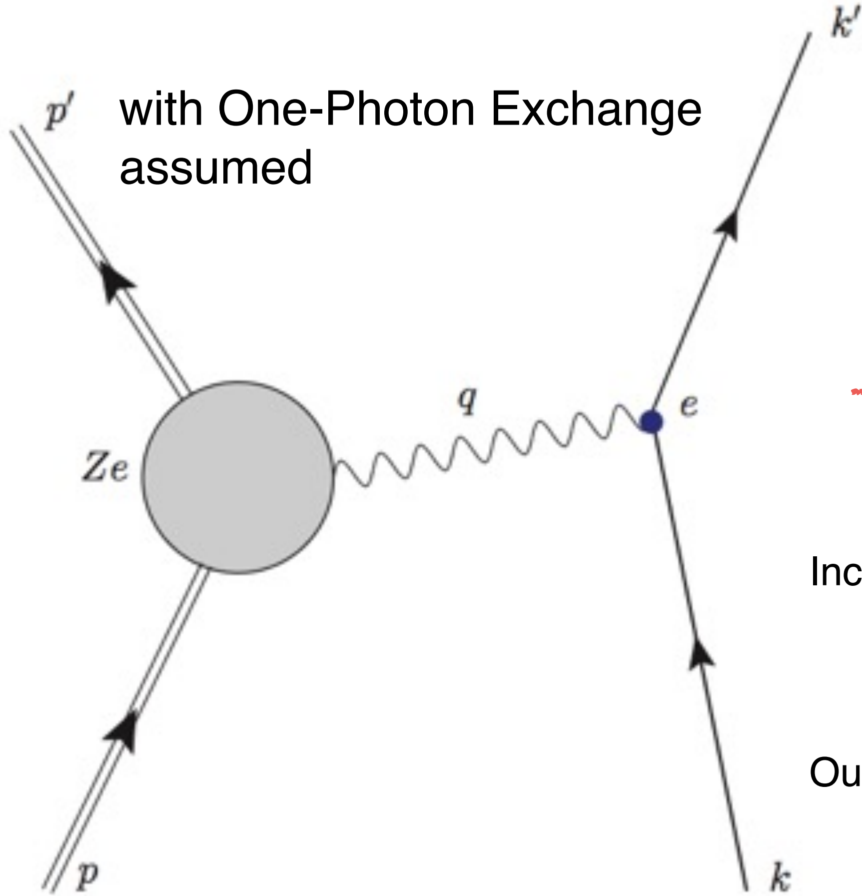
It is now owned by the Korean Air. Ina University has one of the strongest engineering schools in Korea.

Form factors

What is a form factor?

Form factors tell you how the corresponding particle looks like in various aspects.

Historical example: Rutherford scattering



with One-Photon Exchange assumed

- Target is so heavy that the recoil effects are negligible.
- Elastic scattering

If $Z\alpha \ll 1\,(\alpha \approx 1/137)$

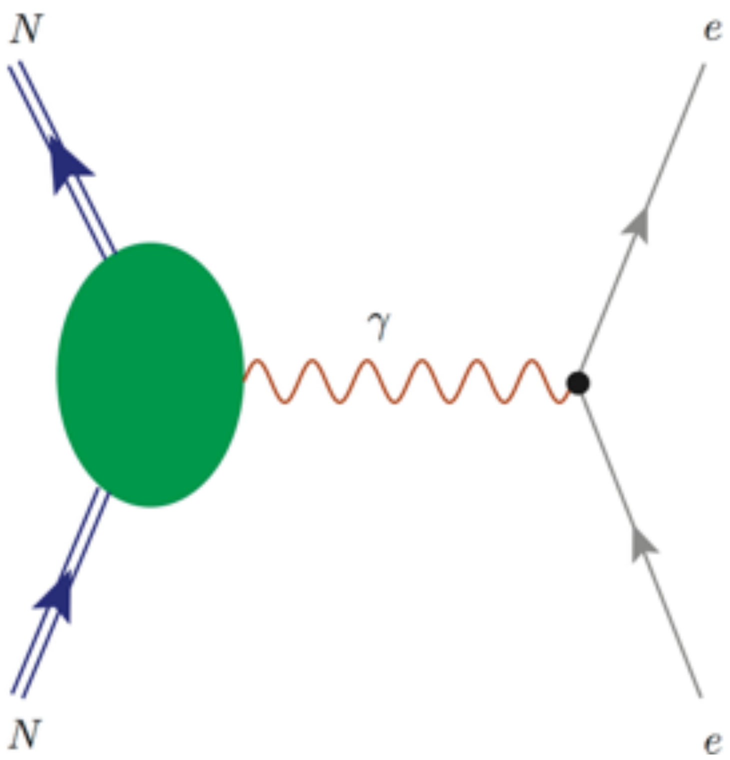
→ Born approximation can be used.

Incoming wave for the electron $\psi_i = \dfrac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}/\hbar}$

Outgoing wave for the electron $\psi_f = \dfrac{1}{\sqrt{V}} e^{i\mathbf{k}'\cdot\mathbf{r}/\hbar}$

Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



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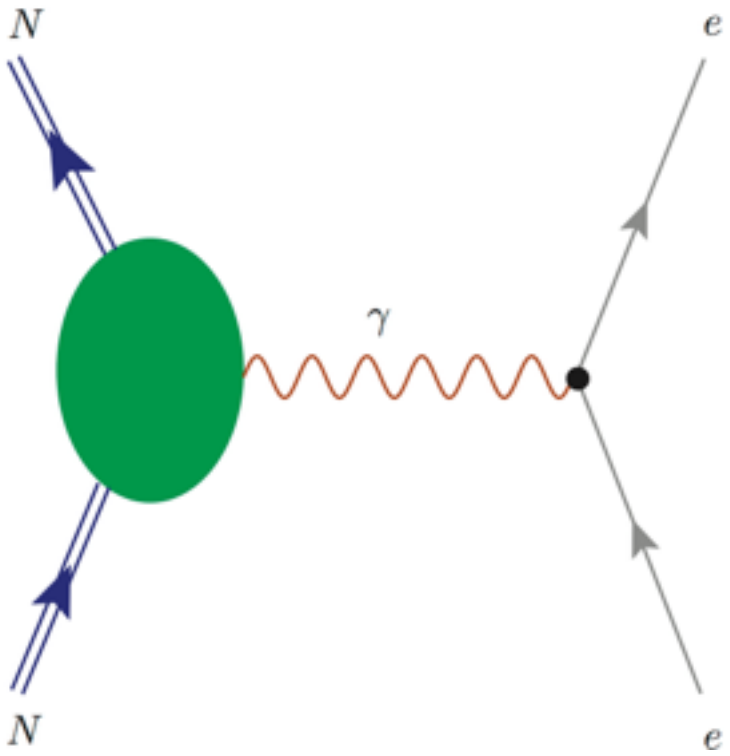

Schroedinger Eq. & Wave functions

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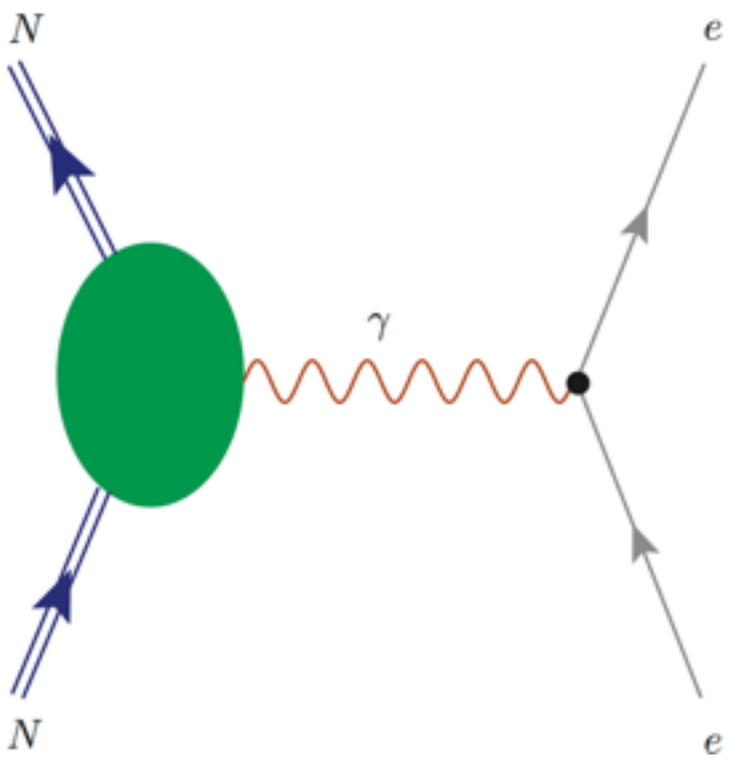

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions

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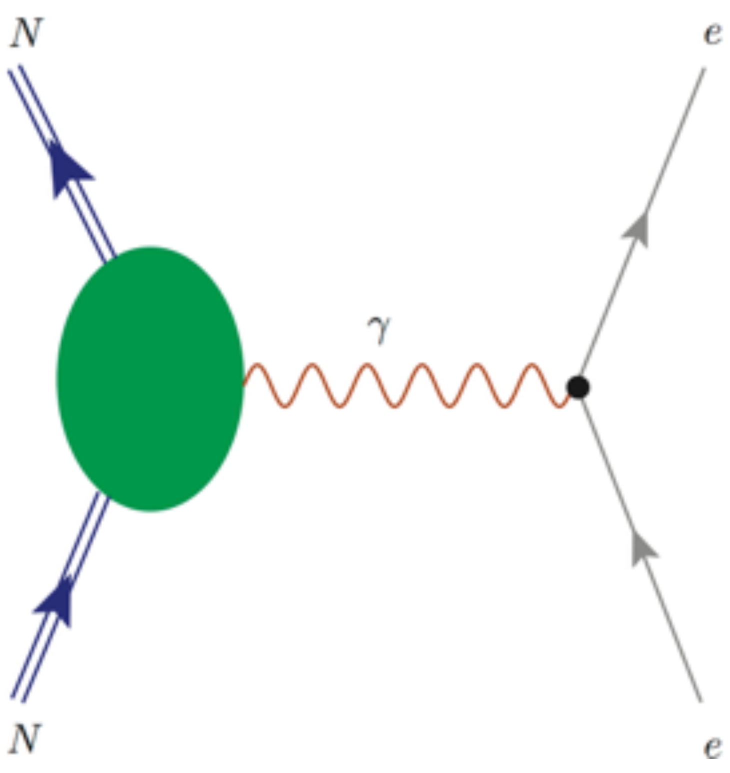

3-D Fourier Transform

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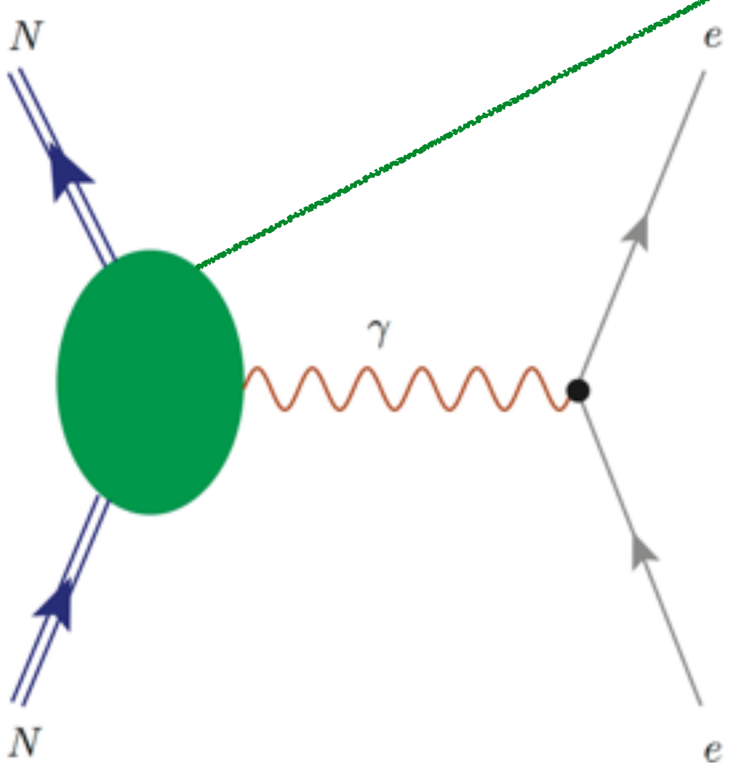


Electromagnetic form factors

↑

3-D Fourier Transform

↑

Charge & Magnetisation Densities

↑

Schroedinger Eq. & Wave functions

EM Form factors of the nucleon

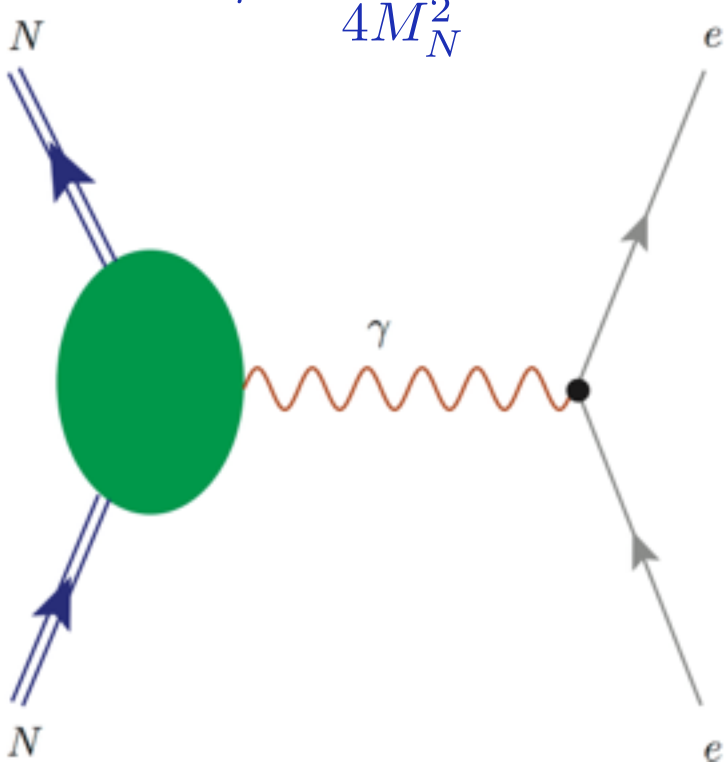
ep scattering (Rosenbluth formula)

$$\frac{d\sigma_{ep}}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{1}{(1+\tau)}\left(\boxed{G_E^2} + \tau \boxed{G_M^2}\right) + 2\tan^2\frac{\theta}{2}\tau \boxed{G_M^2}\right]$$

$$\tau = \frac{Q^2}{4M_N^2}$$

Magnetic Sachs form factor

Electric Sachs form factor



$${}_{p} = e^2 \bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda) \frac{1}{q^2} \boxed{\langle p', s' | J_\mu(0) | p, s\rangle}$$

cleon Matrix element of the EM current

EM Form factors of the nucleon

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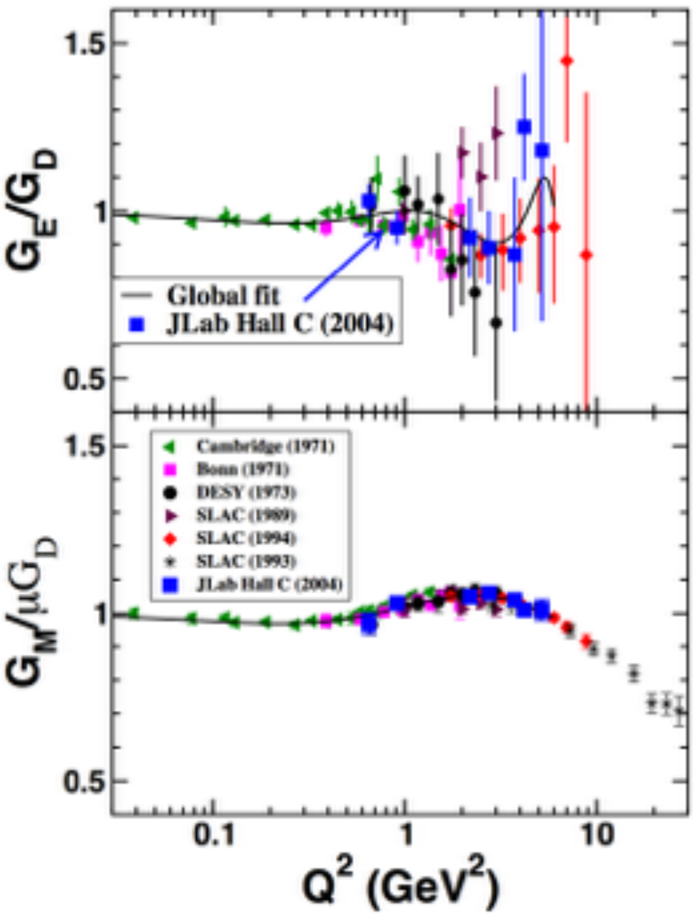

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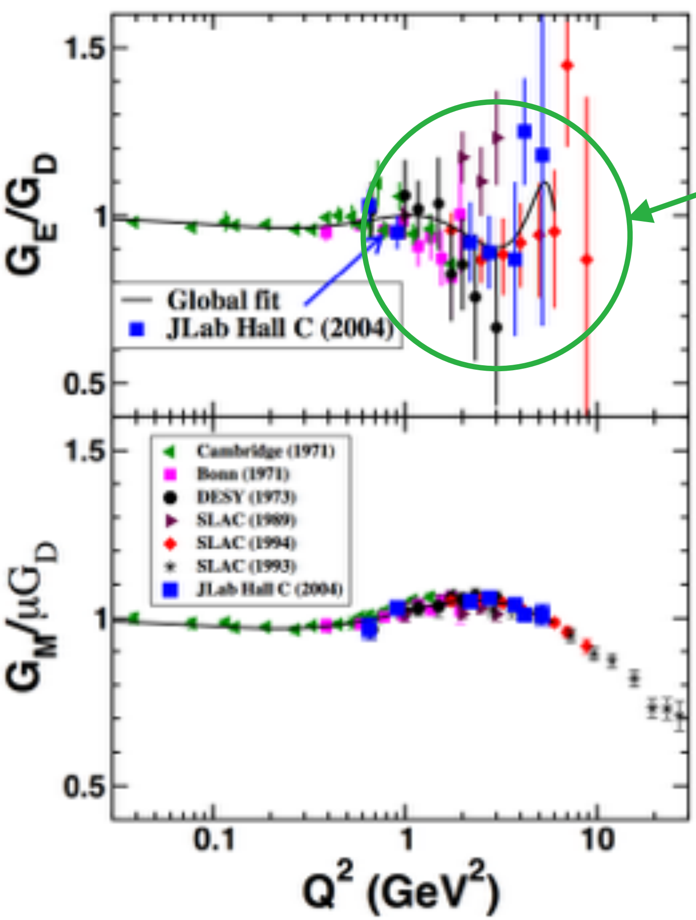

As Q^2 increases, this term with
the electric form factor becomes smaller.

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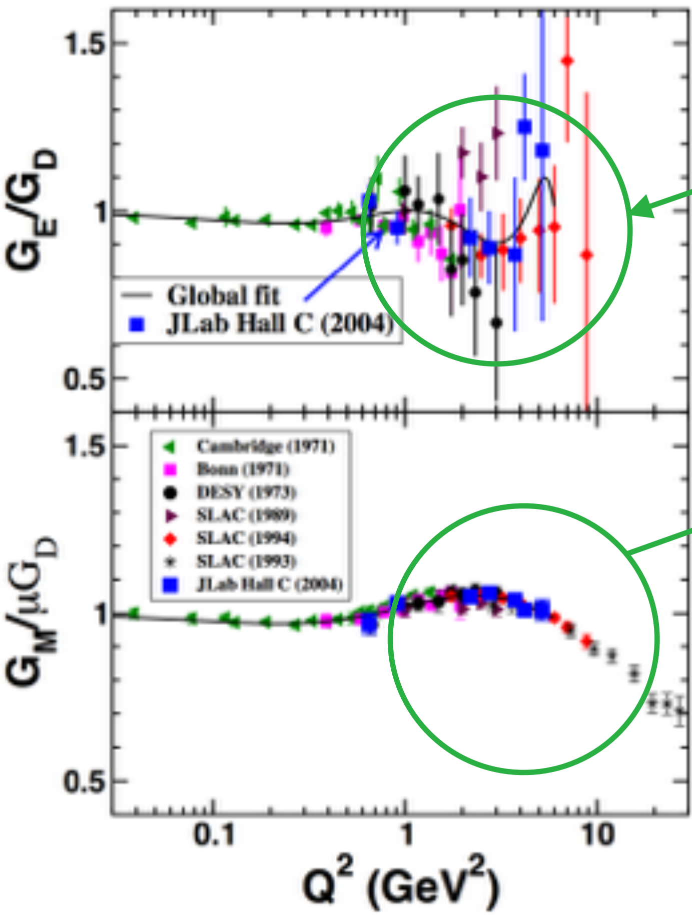


As Q^2 increases, this term with the electric form factor becomes smaller.

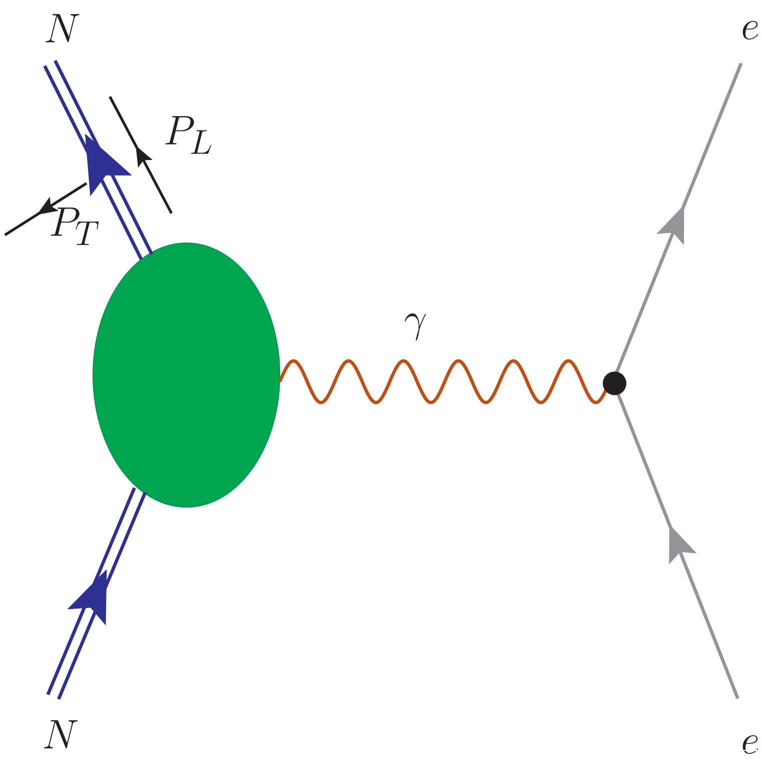
The magnetic form factor is measured rather precisely.

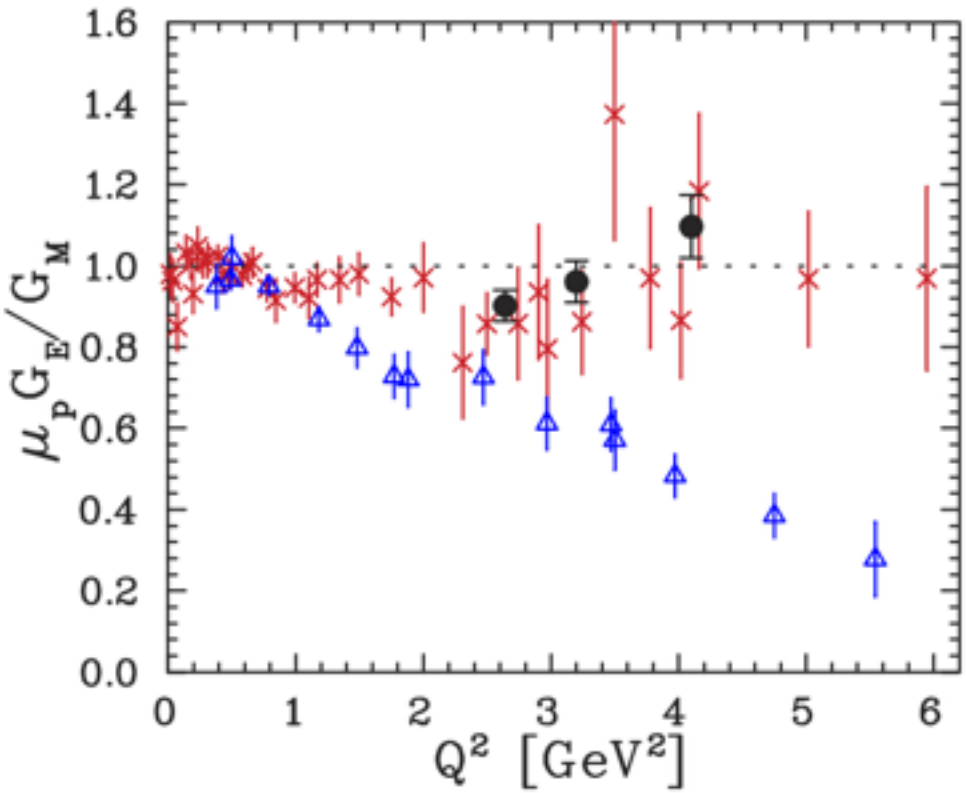
EM Form factors of the nucleon

Recoil Polarisation

$$\frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{E_e + E_{e'}}{2M}\tan\left(\frac{\theta_e}{2}\right)$$

JLAB Hall-A Experiments





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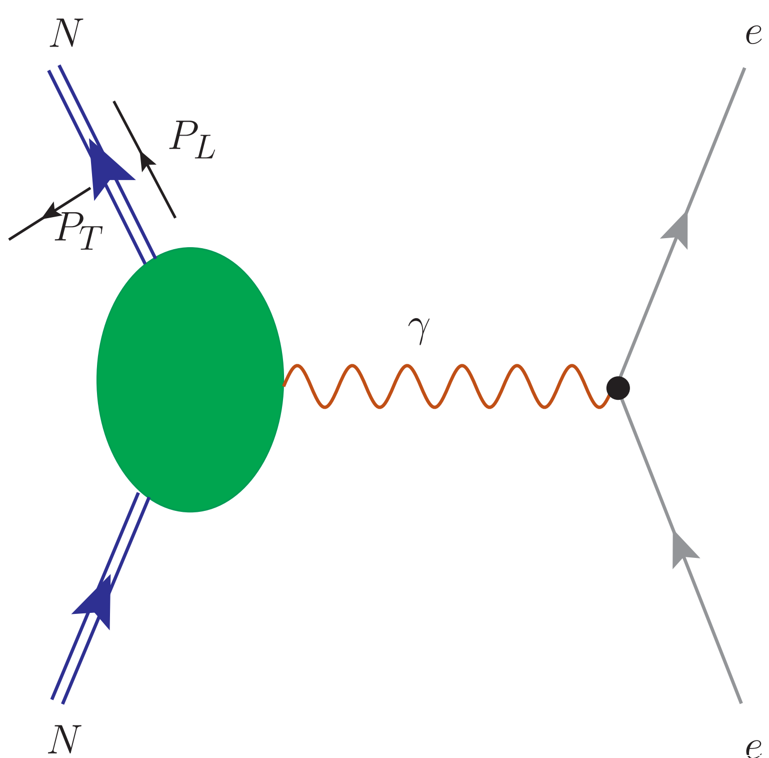

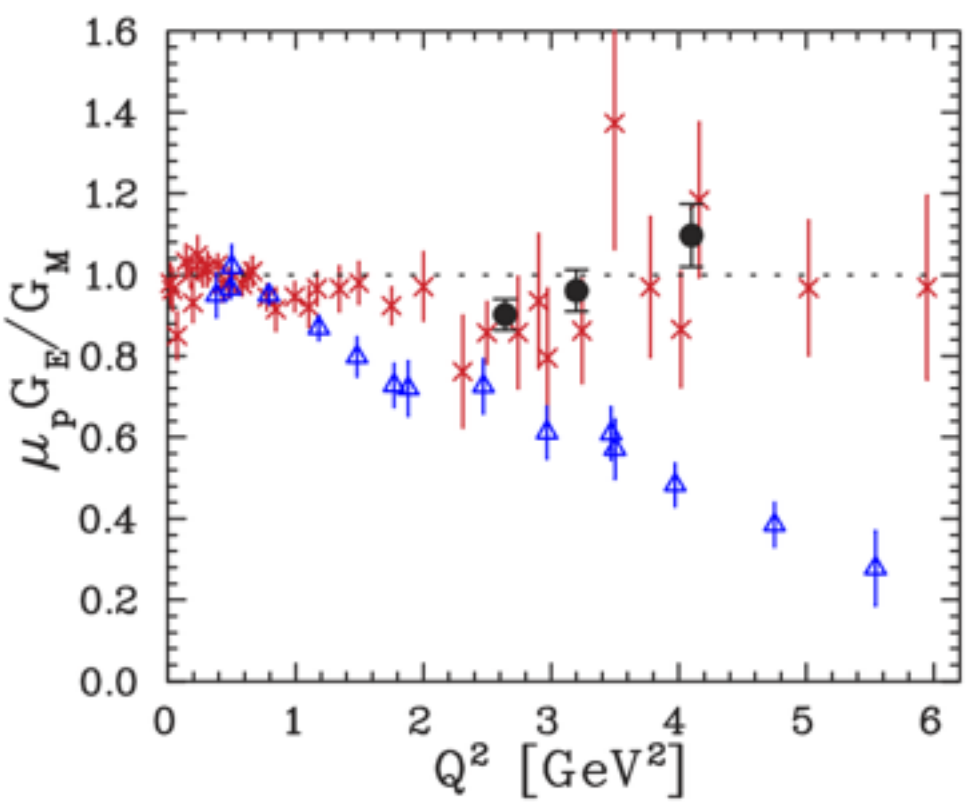

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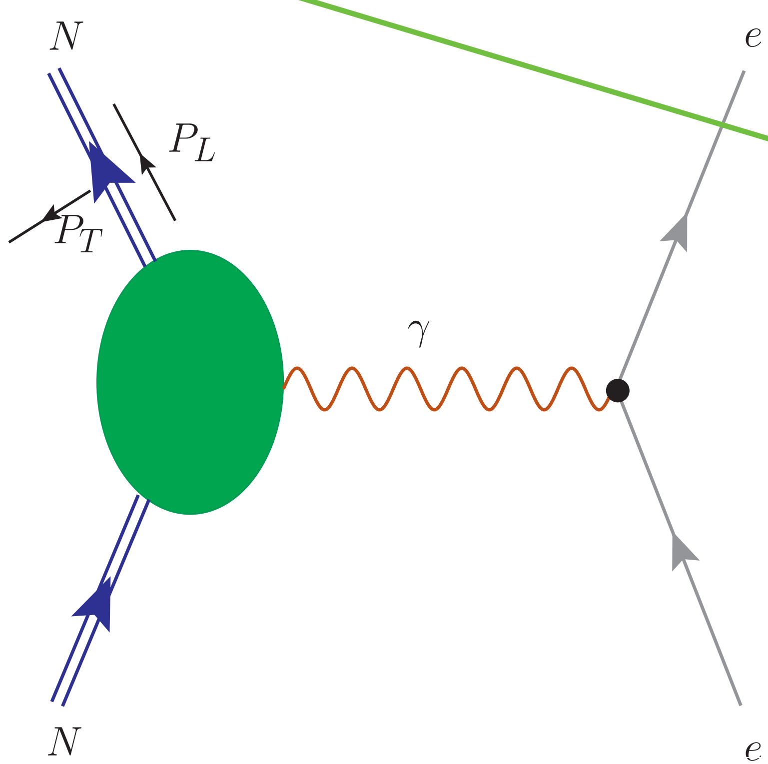

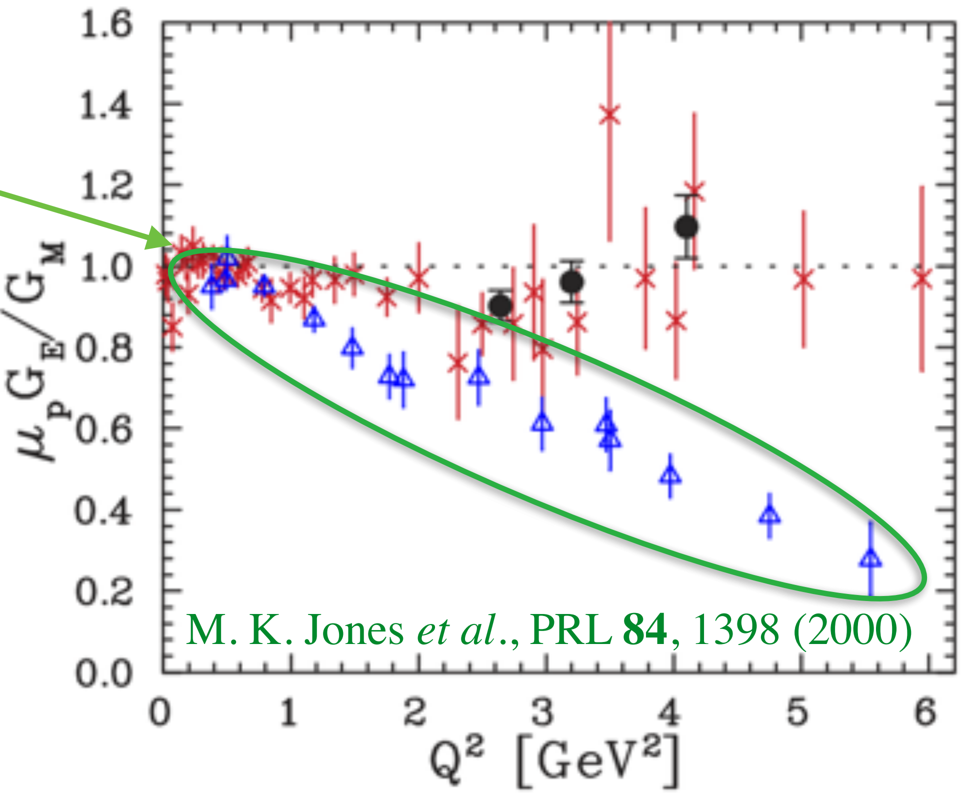

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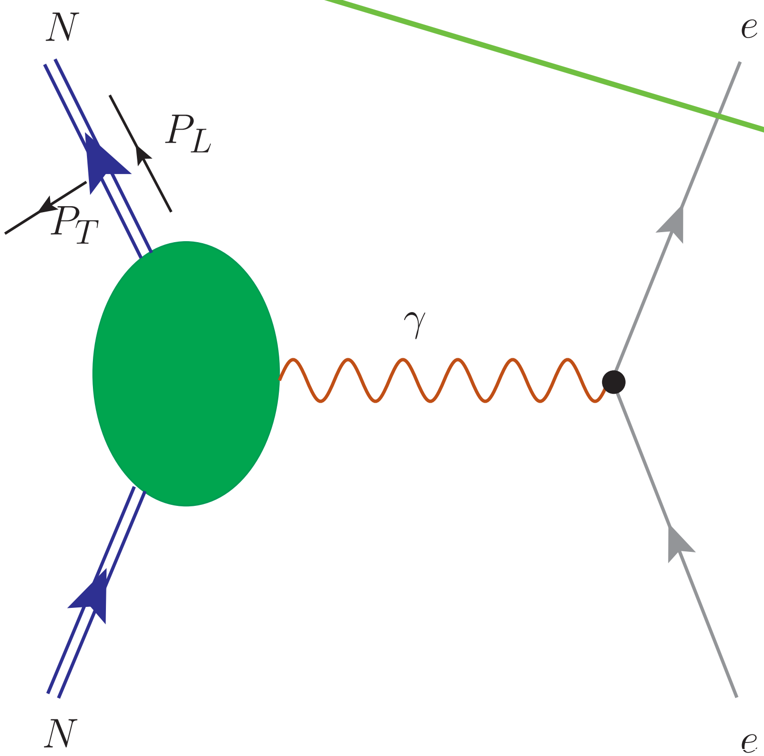
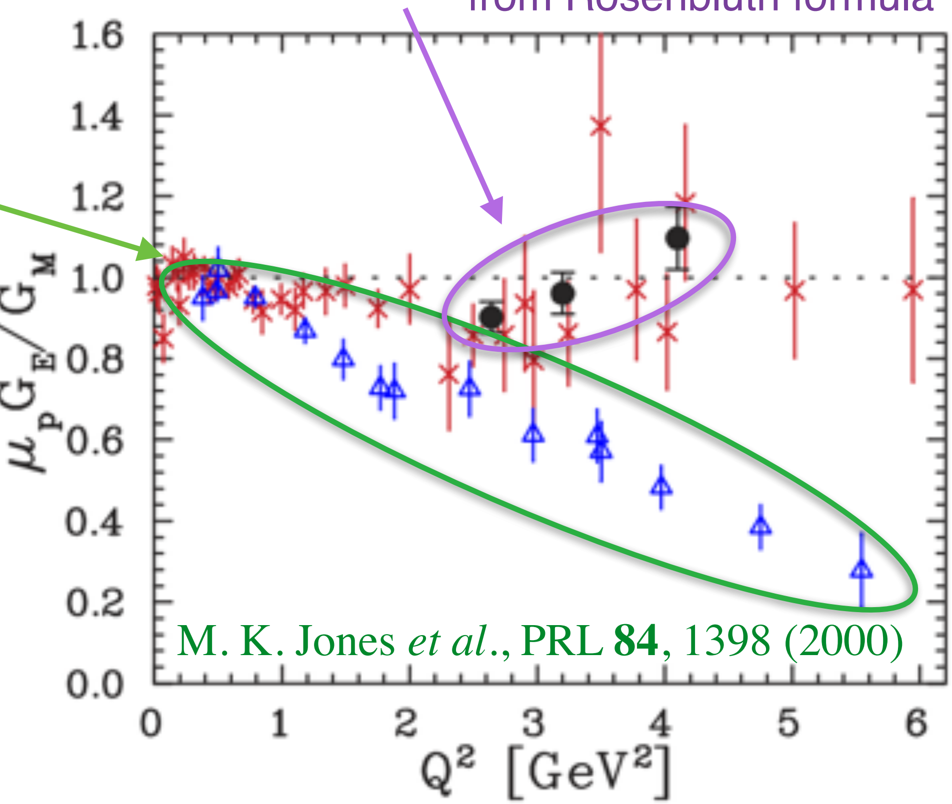


JLAB Hall-A Experiments

Qattan et al., PRL **94**, 142301 (2005)

from Rosenbluth formula



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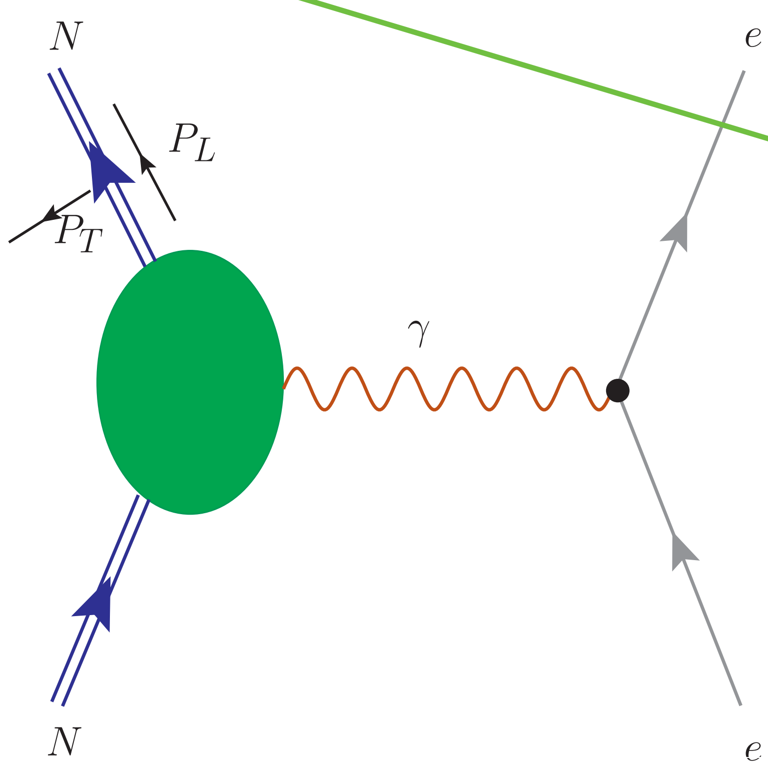
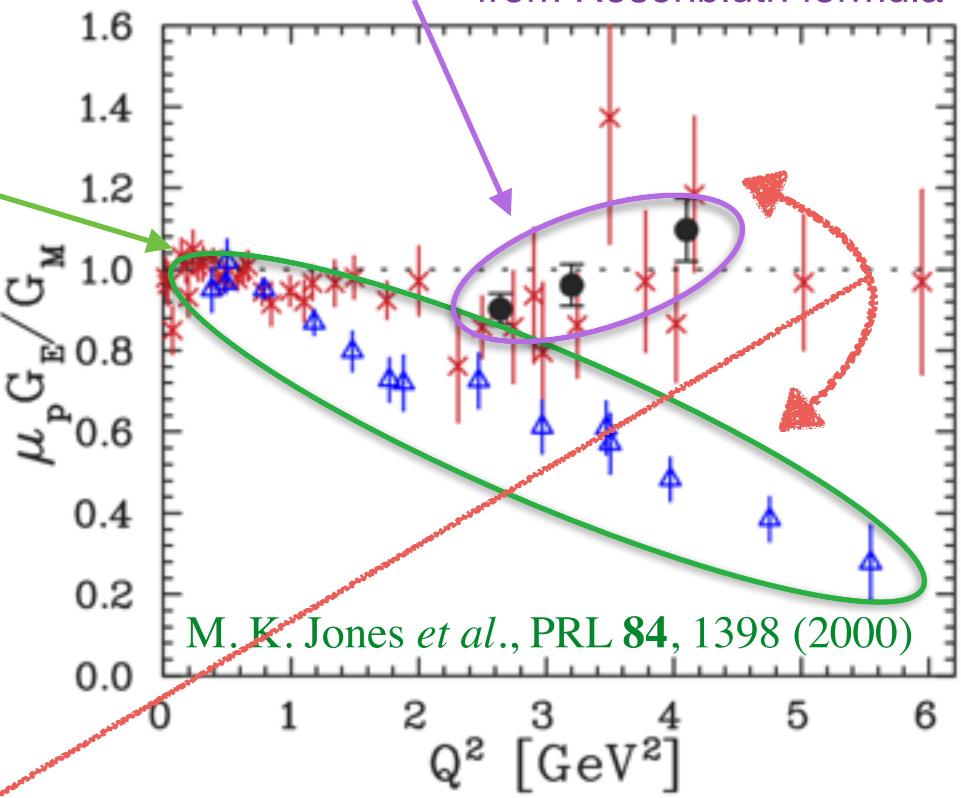


Importance of two-photon exchange

JLAB Hall-A Experiments

Qattan et al., PRL **94**, 142301 (2005)

from Rosenbluth formula



M. K. Jones *et al.*, PRL **84**, 1398 (2000)

Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \boxed{\rho(\mathbf{r})} \longrightarrow \qquad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

The valid range of the momentum transfer: $|\mathbf{q}| \ll M_N$

However, the initial and final momenta are different in a relativistic case.
Thus, the initial and final wave functions are different.

→ **Probability interpretation is wrong in a relativistic case!**

↓

We need a correct interpretation of the form factors

It was pointed out already long time ago.

Yennie, Levy, & Ravenhall, Rev. Mod. Phys. **29**, 144 (1957)

Interpretation of the EMFFs

R: Size of the system
M: Mass of the system

Non-Relativistic description

$$M_{\rm atom} R_{\rm atom} = M_{\rm atom}/(m_e \alpha) \sim 10^5$$

$$\rho(\mathbf{r}) = \sum \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$$

$\|Q\| \ll M_{\rm atom}$ $\quad 1/\|Q\| \leq R$

Particle number fixed.

Form factors can be measured and well interpreted (almost no recoil effect).

Relativistic description

$$M_N R_N \sim 4$$

Particle creation & annihilation

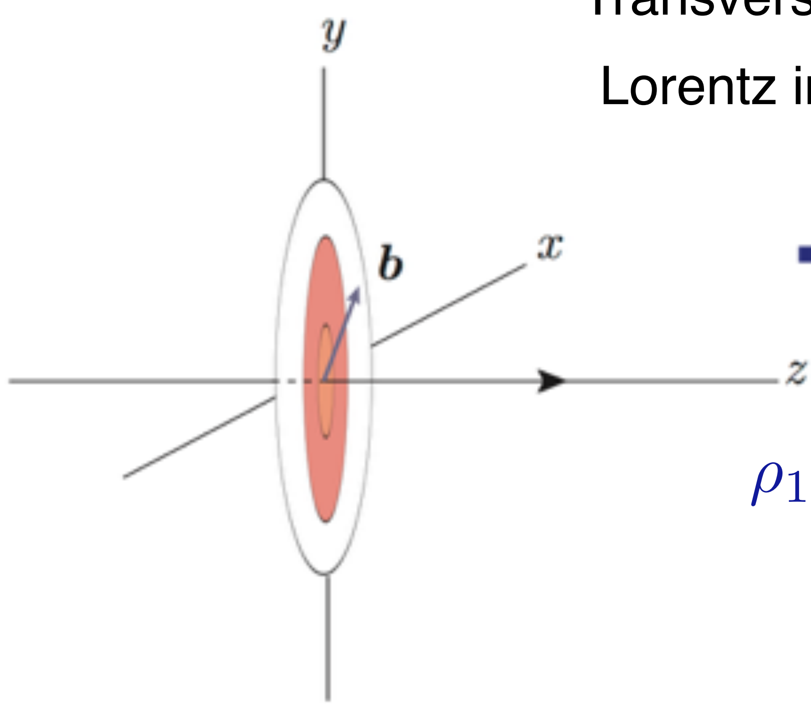
Initial and final momenta are different! $\|Q\| \geq M_N$

⇒ Nucleon cannot be treated non-relativistically!

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

Interpretation of the EMFFs

Modern understanding of the form factors



Transverse Charge densities $\rho_1(\mathbf{b})$

Lorentz invariant: independent of any observer.

$\vec{p}$ Infinite momentum framework

$$\rho_1(b) = \sum_q e_q^2 \int dx \boxed{f_{q-\bar{q}}(x, \mathbf{b})}$$

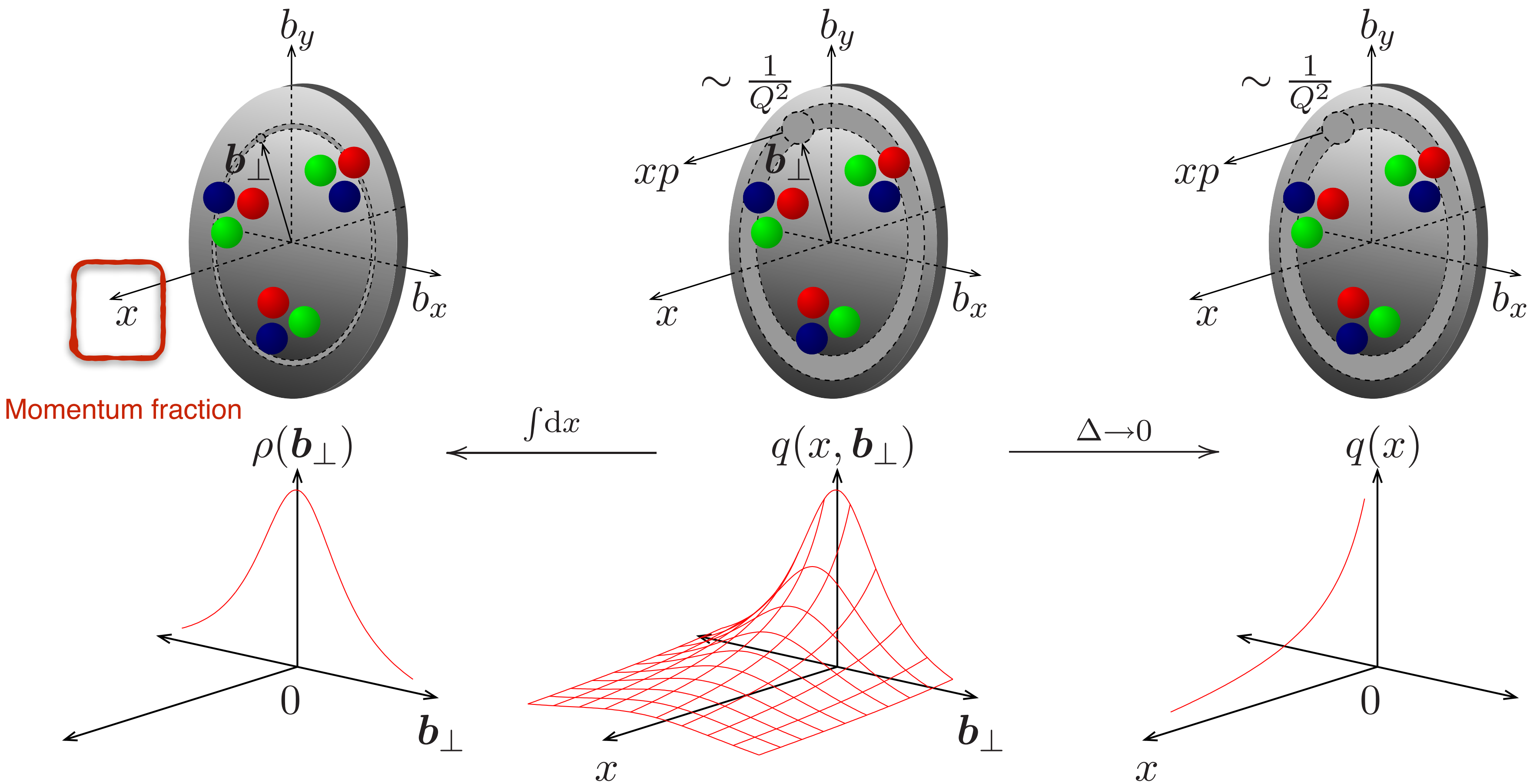
GPDs

Dirac & Pauli form factors

$$F_{1,2}(\boldsymbol{\Delta}) = \int d^2 b e^{i\boldsymbol{\Delta}_\perp \cdot \mathbf{b}} \rho_{1,2}(\mathbf{r})$$

G.A. Miller, PRL **99,** 112001 (2007)

Nucleon Tomography



Transverse densities
of Form factors

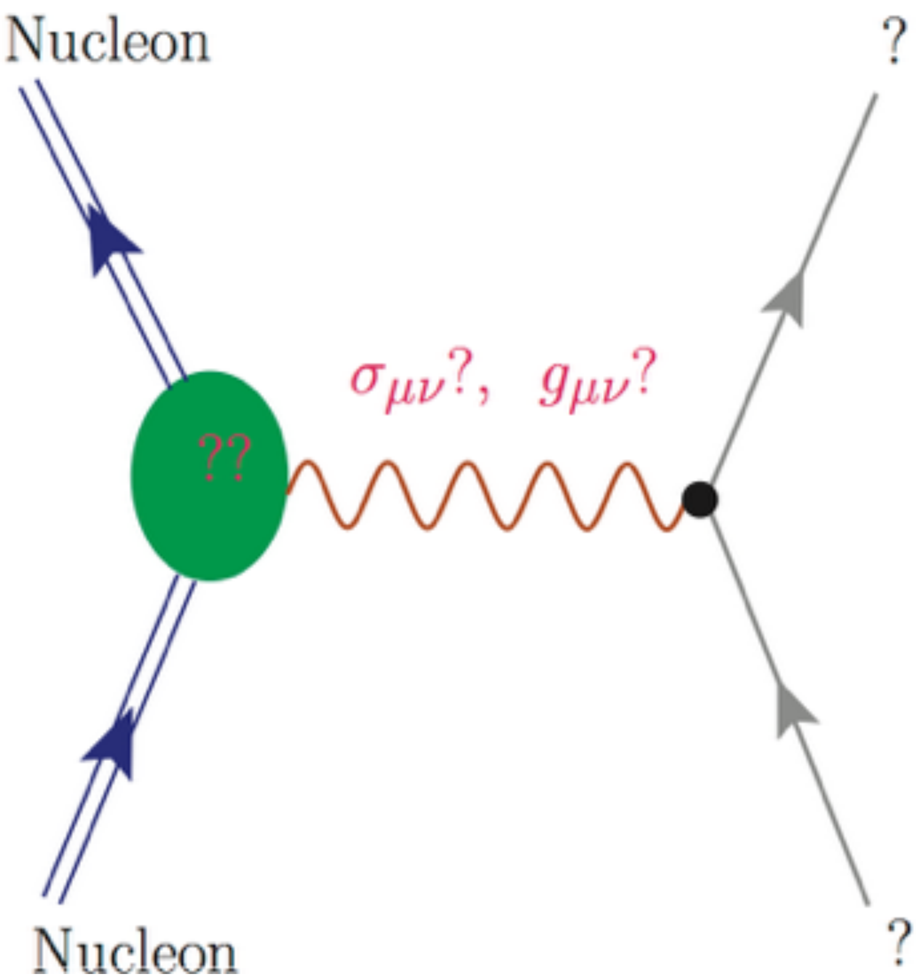
GPDs
Nucleon Tomography

Structure functions
Parton distributions

D. Brömmel, Dissertation (Regensburg U.)

Generalised Parton Distributions

Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors!**



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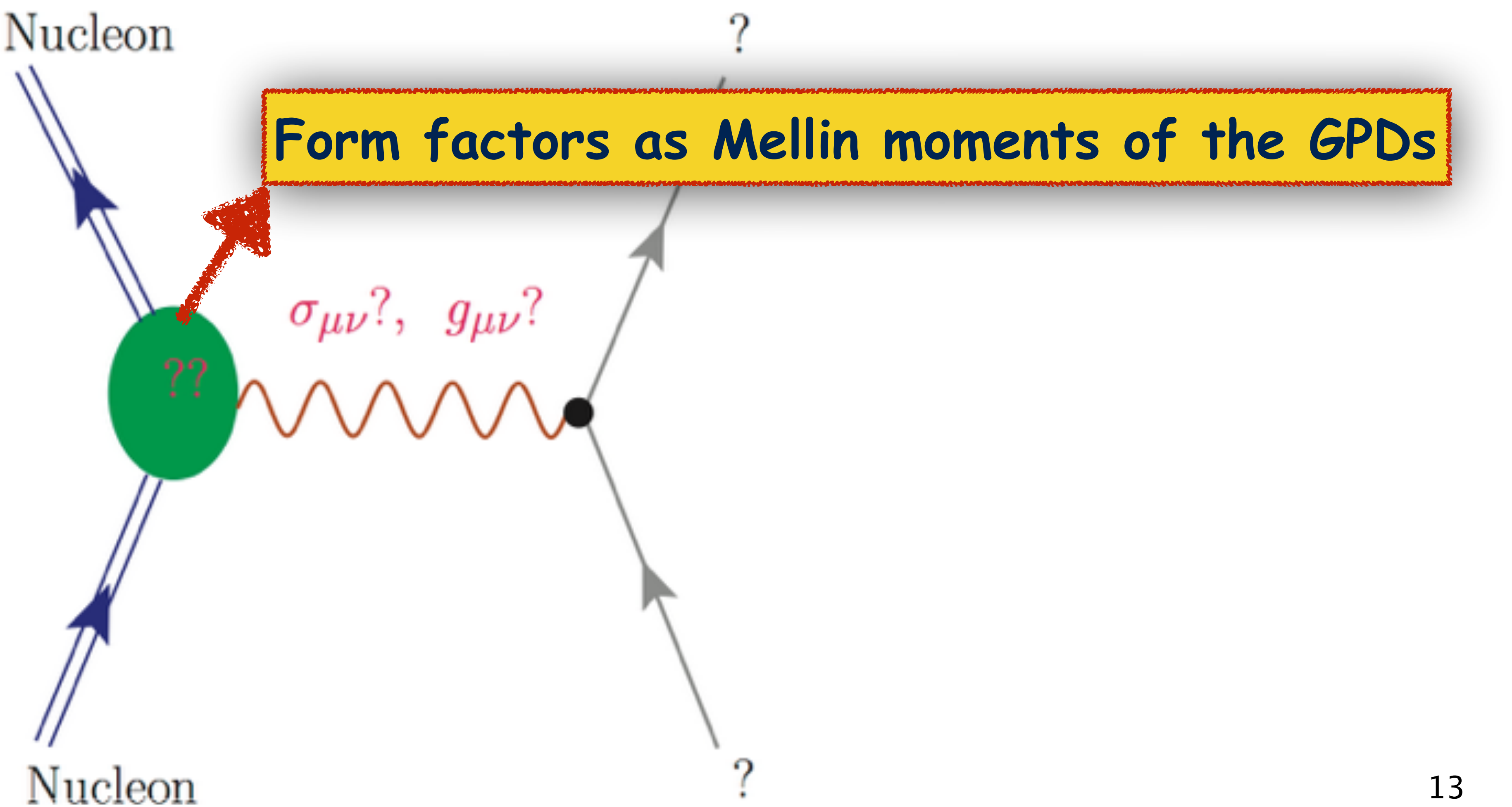

Model

Chiral quark-soliton model

Merits of the chiral quark-soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalisation scale is naturally given.
  $1/\rho \approx 600\,\mathrm{MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\mathrm{QSM}} = \int \mathcal{D}U \exp(-S_{\mathrm{eff}})$$

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \ln D(U)$$

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

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$$\boxed{D(U) = \partial_4 + H(U) + \hat{m}}$$

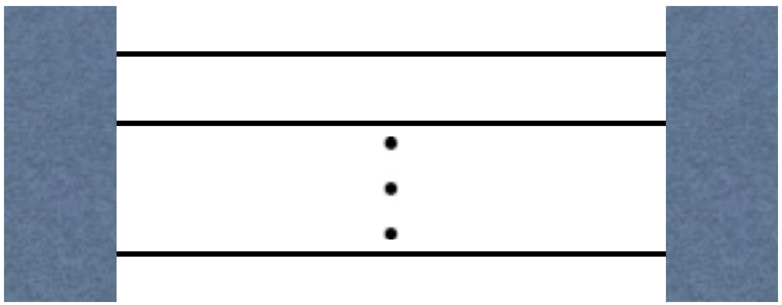
$$\boxed{\hat{m} = \mathrm{diag}(m_u, m_d, m_s)\gamma_4}$$

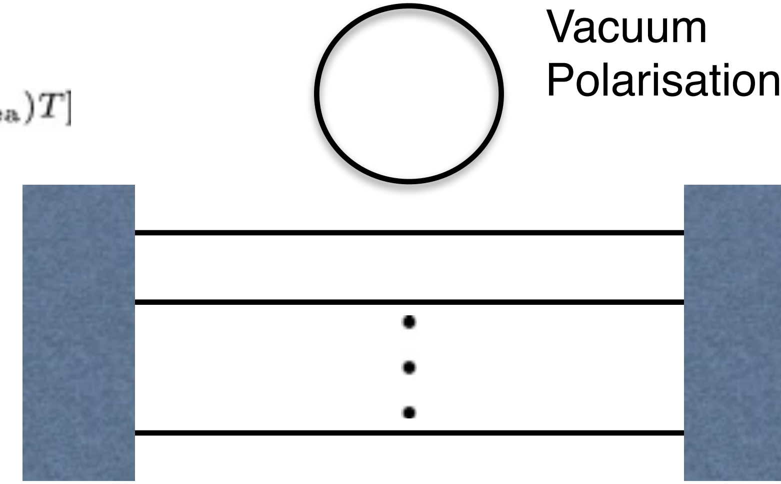
HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Chiral quark-soliton model

Classical solitons

$$\langle J_N(\vec{x},T)J_N^{\dagger}(\vec{y},-T)\rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\rm val}+E_{\rm sea})T]}$$

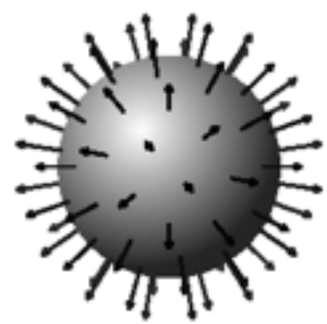




$$\frac{\delta}{\delta U}(N_c E_{\rm val} + E_{\rm sea}) = 0 \rightarrow M_{\rm cl} = N_c E_{\rm val}(U_c) + E_{\rm sea}(U_c)$$

Hedgehog Ansatz:

$$U_{\rm SU(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$


hedgehog

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton model

Collective (Zero-mode) quantisation

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

Zero-mode quantisation

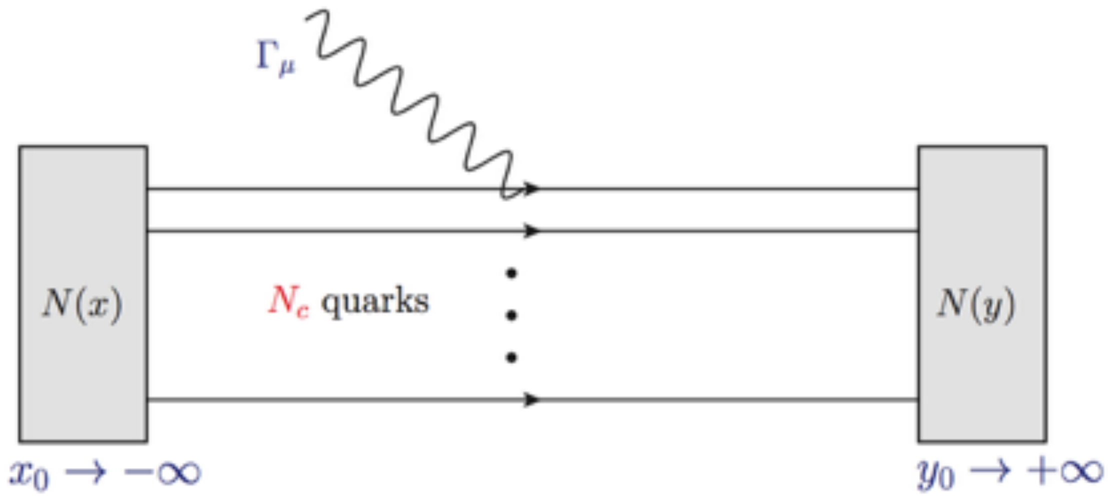
$$\boxed{\begin{aligned} U(\boldsymbol{x},t) &= R(t)U_c(\boldsymbol{x}-\boldsymbol{Z}(t))R^\dagger(t) \\ \int DU[\cdots] &\to \int DADZ[\cdots] \end{aligned}}$$

$$\Rightarrow \quad \mathcal{L} = -M_{sol} + \frac{I_1}{2}\sum_{i=1}^{3}\Omega_i{}^2 + \frac{I_2}{2}\sum_{i=4}^{7}\Omega_i{}^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

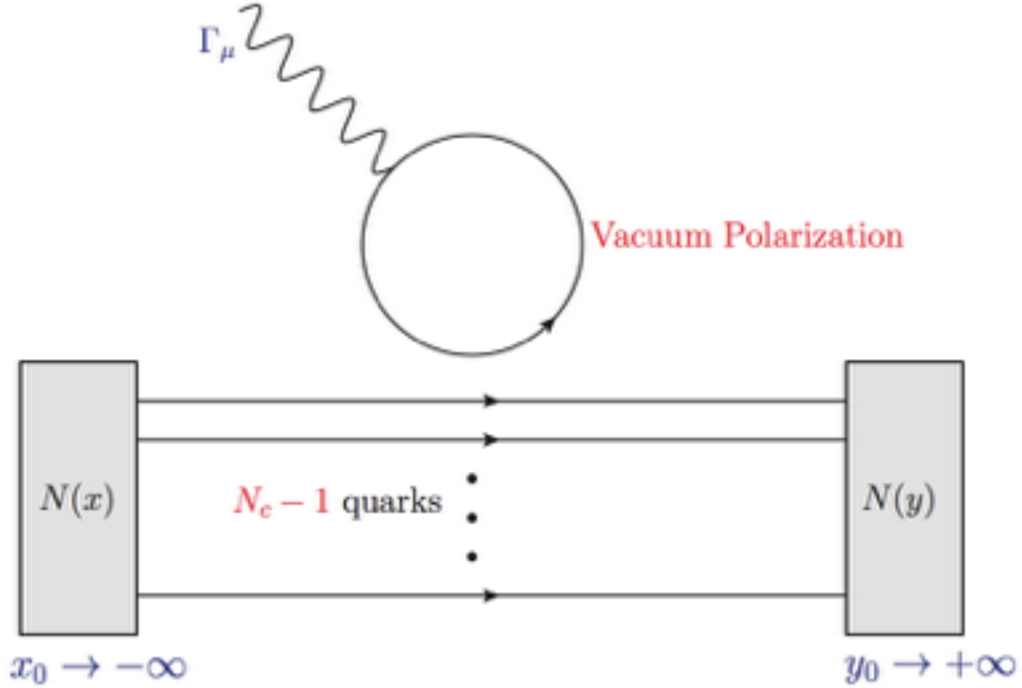
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Observables

Valence part



Sea part



Transverse Charge Densities (EM Form factors)

Transverse charge densities

Why transverse charge densities?

$$\langle P', S' | \bar{\psi}(\mathbf{0}) \gamma_\mu \hat{Q} \psi(\mathbf{0}) | P, S \rangle$$
$$= \bar{u}(p', s') \left(\gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s)$$

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GPDs

$$\int \frac{dx^-}{4\pi}\langle P', S'|\bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp)|P, S\rangle$$
$$= \frac{1}{2\bar{p}^+}\bar{u}(p', s')\left(\gamma^+ H_q(x, \xi, t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N} E_q(x, \xi, t)\right) u(p, s)$$

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$$\boxed{\begin{aligned} F_1(t) &= \sum_q e_q \int dx H_q(x, 0, t), \\ F_2(t) &= \sum_q e_q \int dx E_q(x, 0, t) \end{aligned}}$$

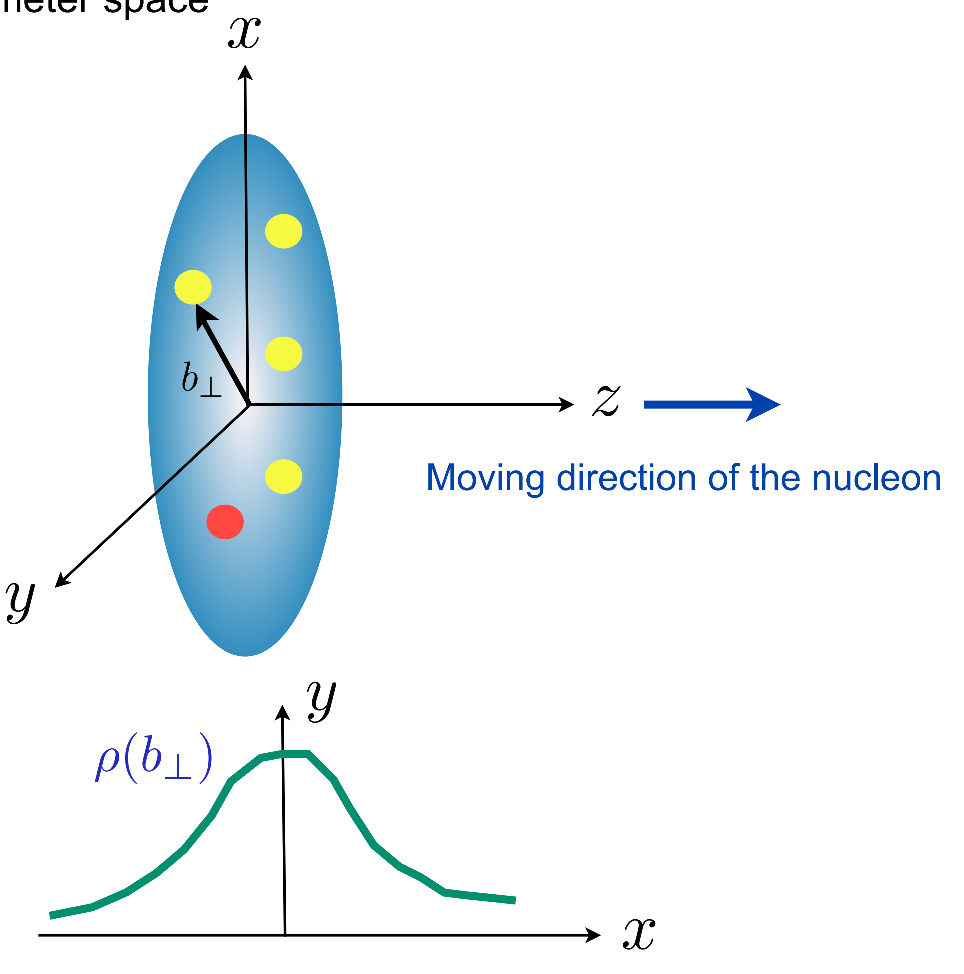
The EM form factors as the first moments of the vector GPDs

Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$\boxed{q(x,\,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x, -\mathbf{q}^2)}$$



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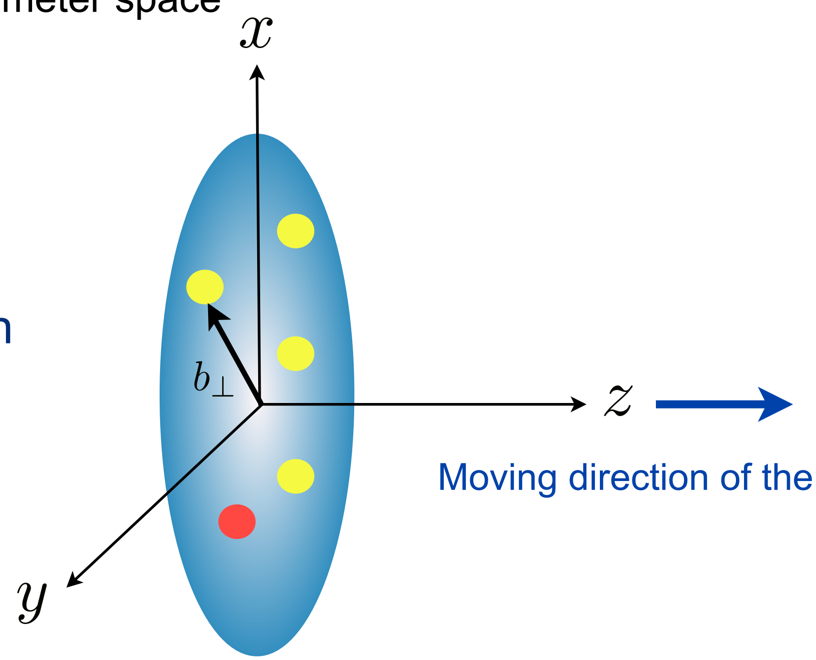
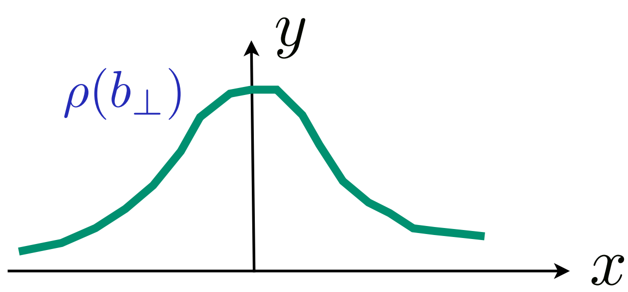
⇨ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).



$$\boxed{\begin{aligned}\rho(\mathbf{b}) &:= \sum_q e_q \int dx q(x,\,\mathbf{b}) \\ &= \int \frac{d^2q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q}\cdot\mathbf{b}}\end{aligned}}$$



Transverse charge densities

Inside an unpolarized nucleon

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

G.A. Miller, PRL **99,** 112001 (2007)

$$\rho_{\rm ch}^{\chi}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) F_1^{\chi}(Q^2)$$

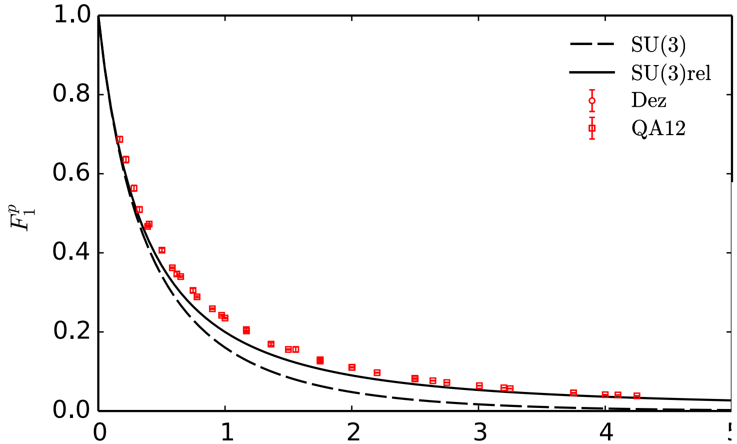
Inside a polarized nucleon

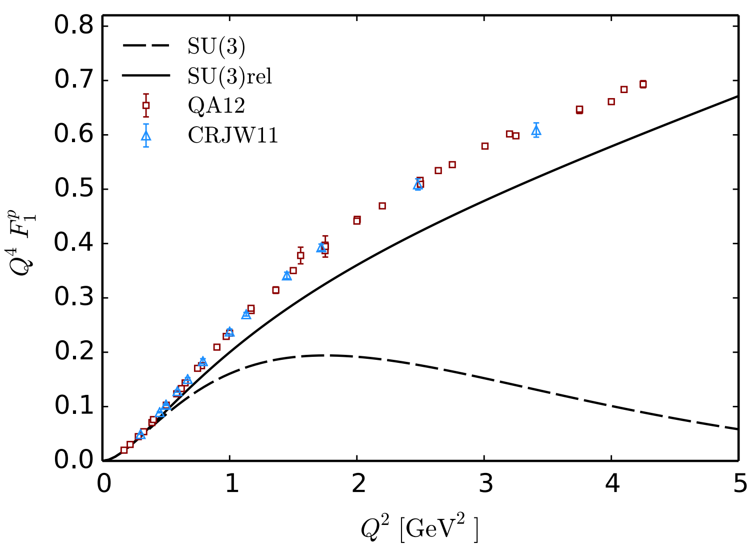
Carlson and Vanderhaeghen, PRL **100**, 032004

$$\rho_T^{\chi}(b) = \rho_{\rm ch}^{\chi}(b) - \sin(\phi_b - \phi_S) \frac{1}{2M_N} \int_0^\infty \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^{\chi}(Q^2)$$

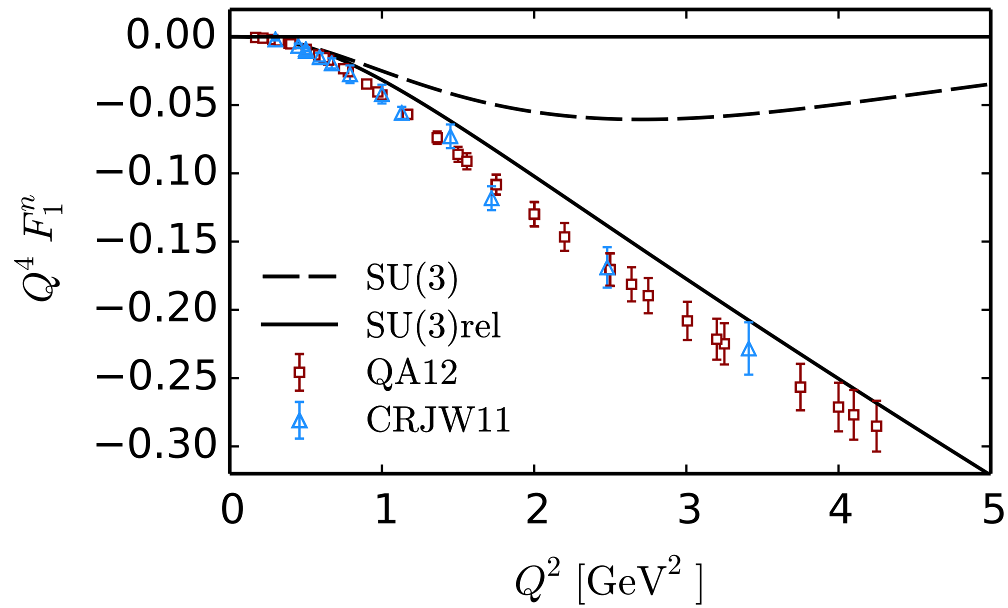
Dirac & Pauli Form factors

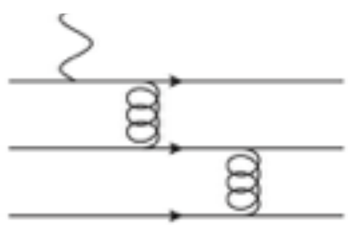
Proton





Neutron



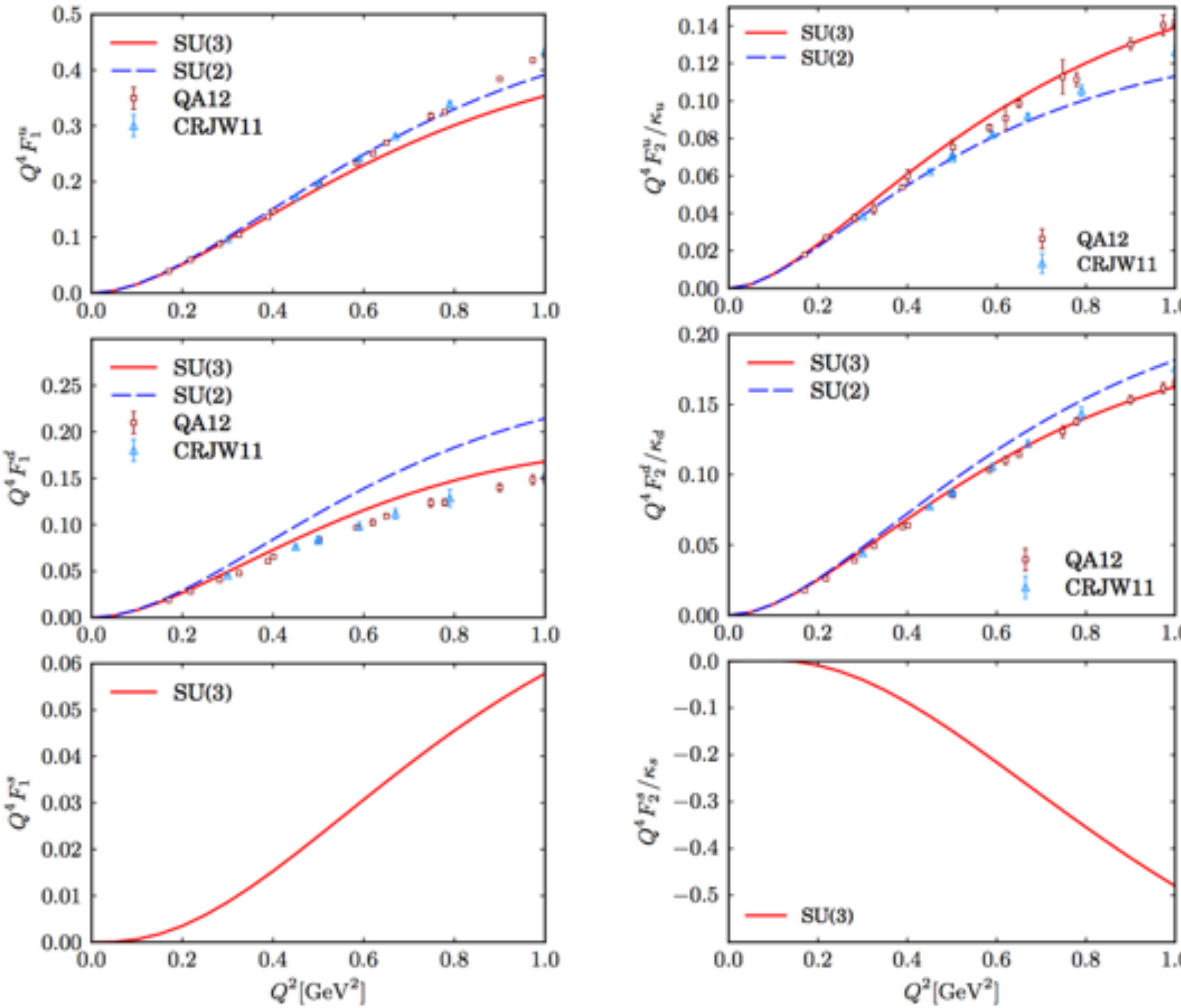


$F_1(Q^2) \sim \frac{1}{Q^4},\; Q^2 \to \infty$ $\qquad$ $F_2(Q^2) \sim \frac{1}{Q^6},\;\; Q^2 \to \infty$

S. J. Brodsky and G. R. Farrar, PRD 11, 1309 (1975).

Silva, Urbano, HChK, hep-ph/1305.6373

Dirac & Pauli Form factors



Up quark FFs

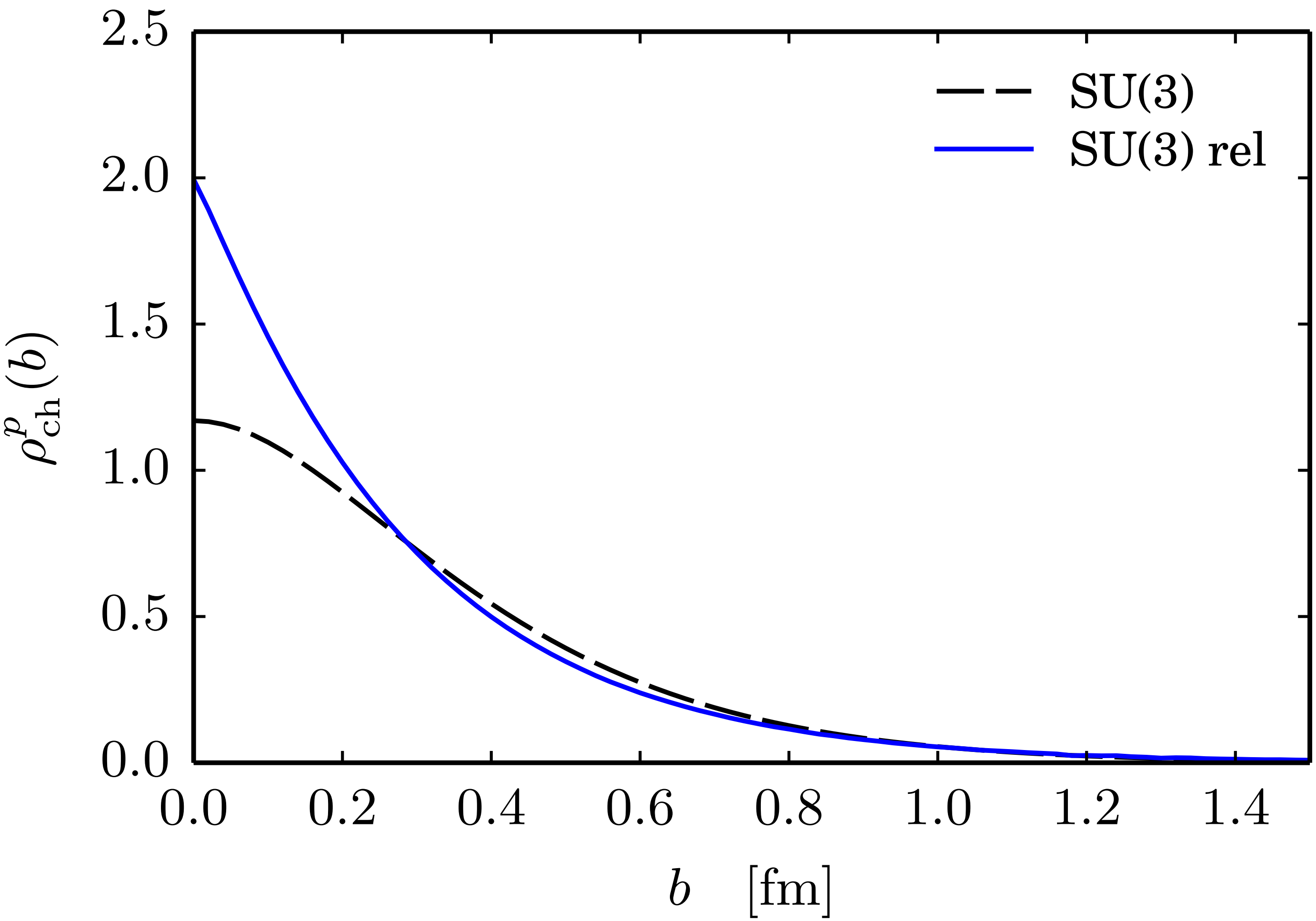
Down quark FFs

Strange quark FFs

Silva, Urbano, HChK, hep-ph/1305.6373

Results

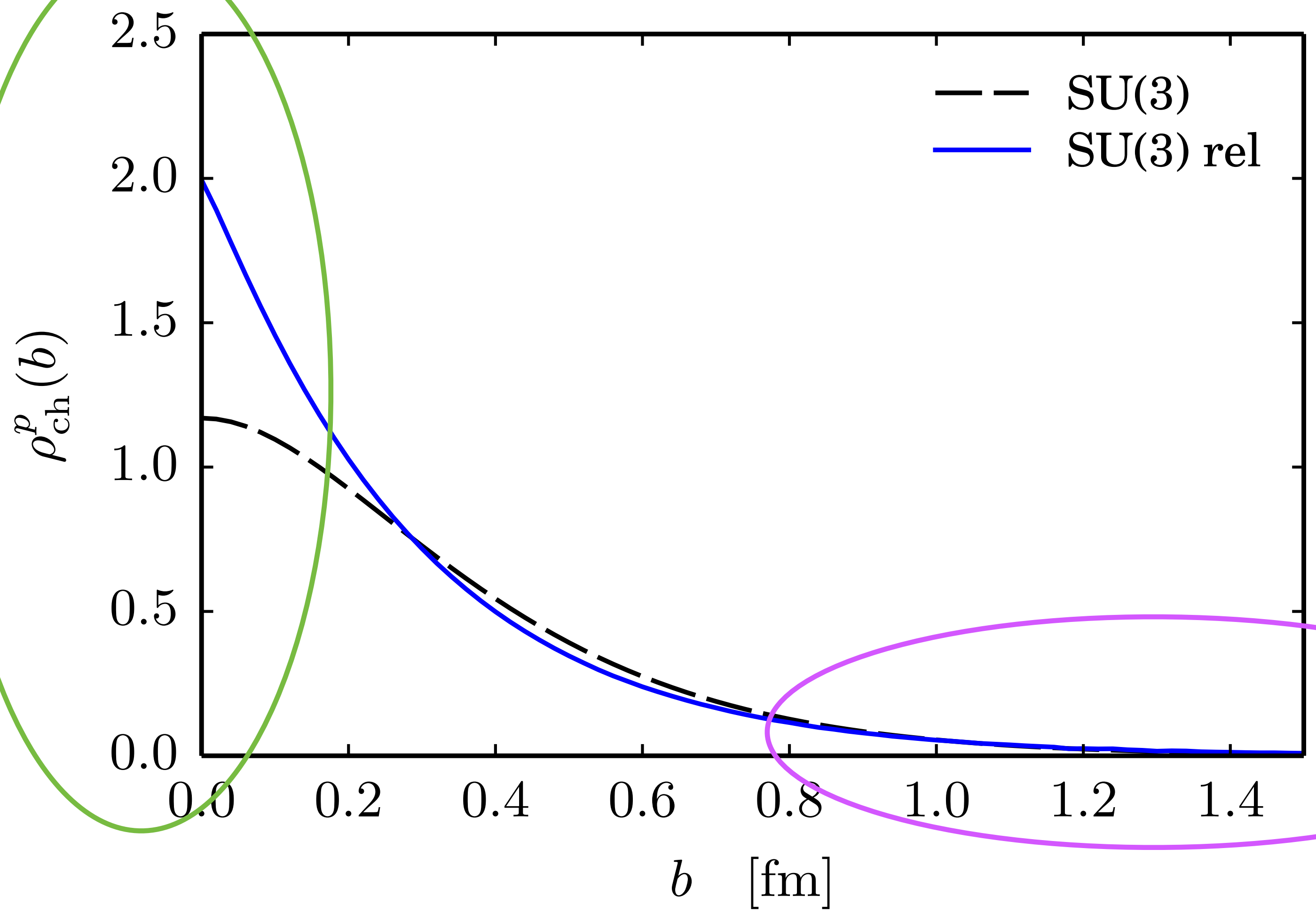
Transverse charge densities inside an **unpolarized** proton



Silva, Urbano, Hohn, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** proton



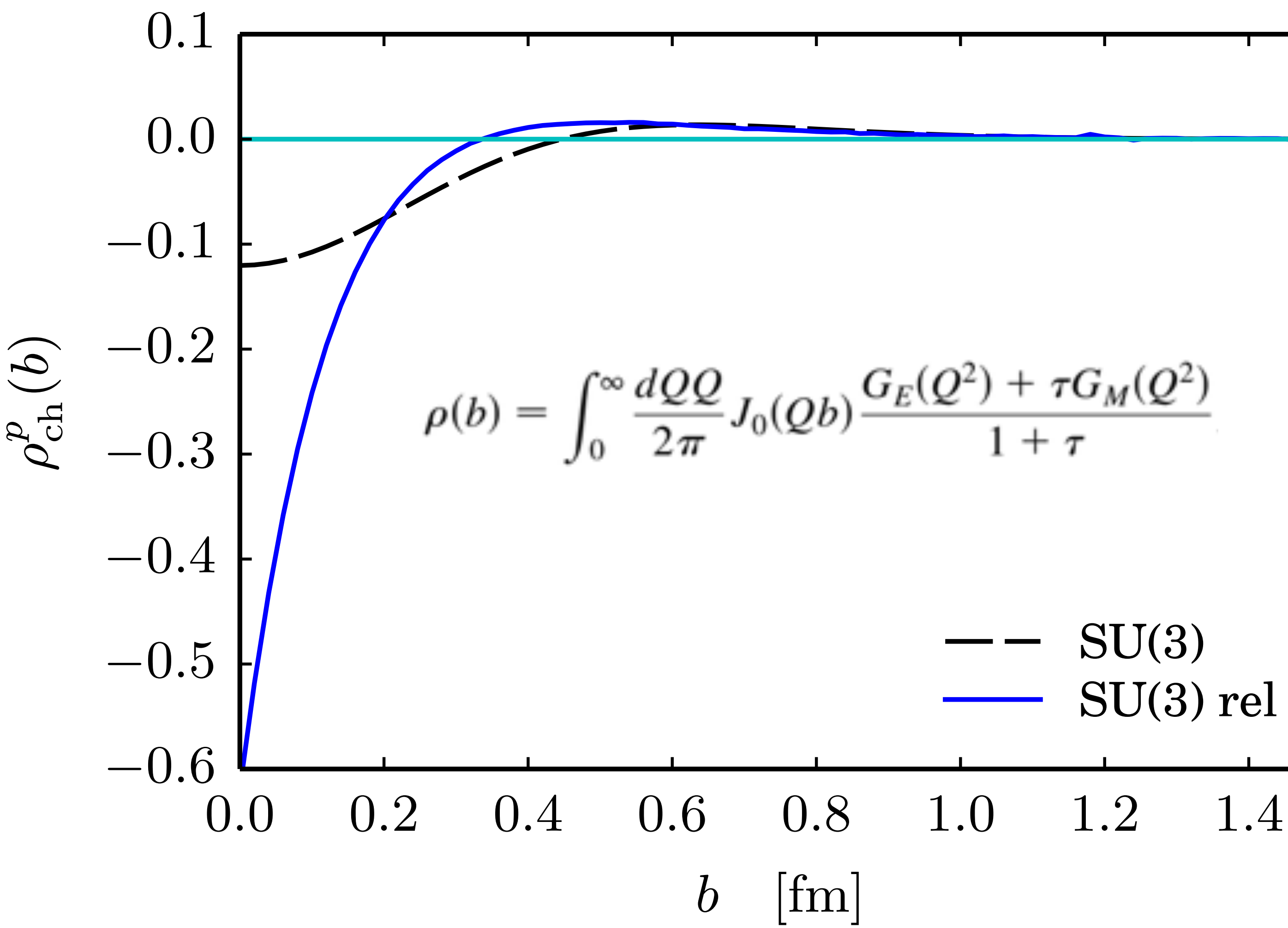
Long positive tail:
Possible positive pion cloud

Centered positive charge distribution

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** neutron

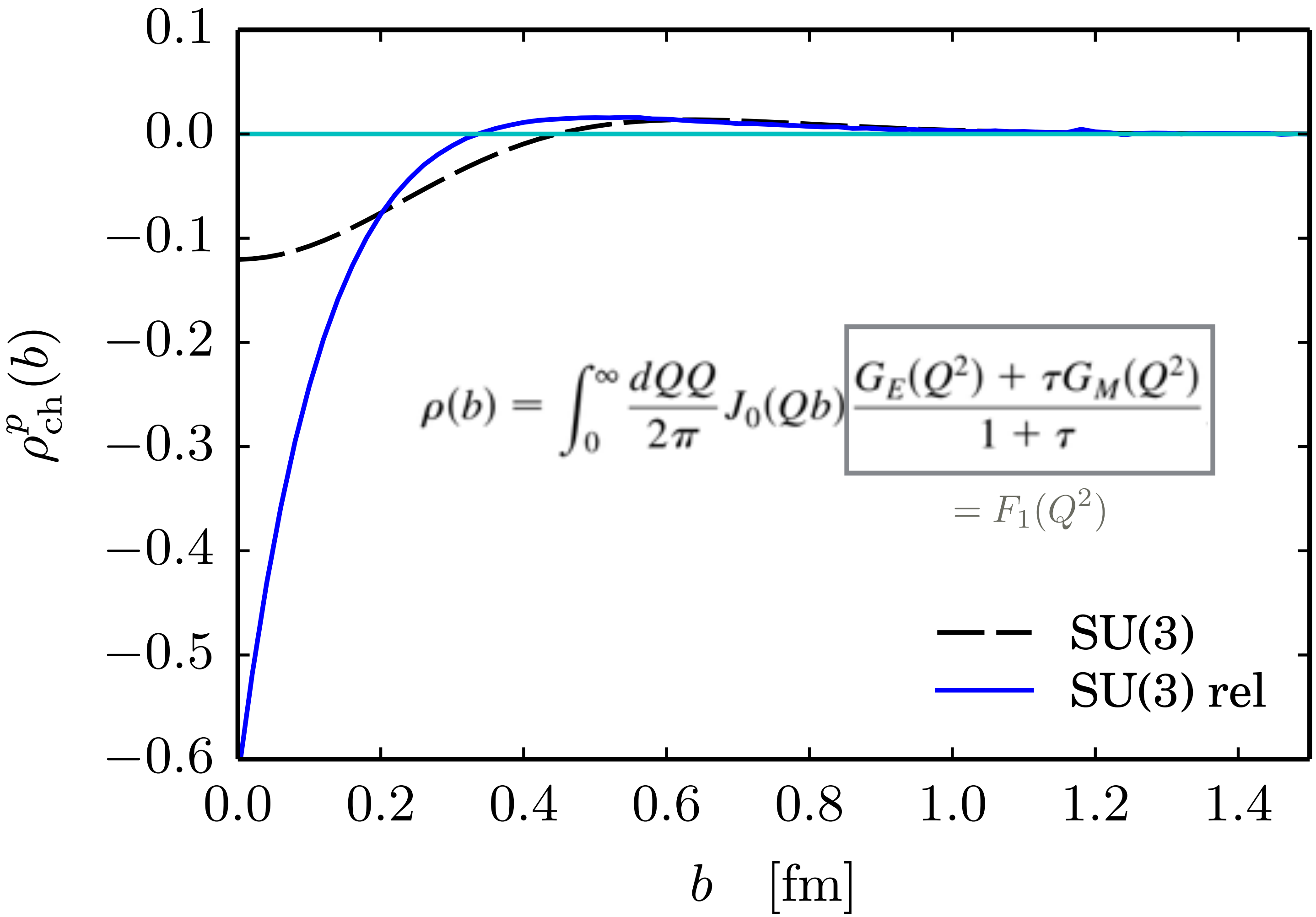


$\tau = Q^2/4M_N^2$

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** neutron



$$\rho(b) = \int_0^\infty \frac{dQQ}{2\pi} J_0(Qb) \boxed{\frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}}$$

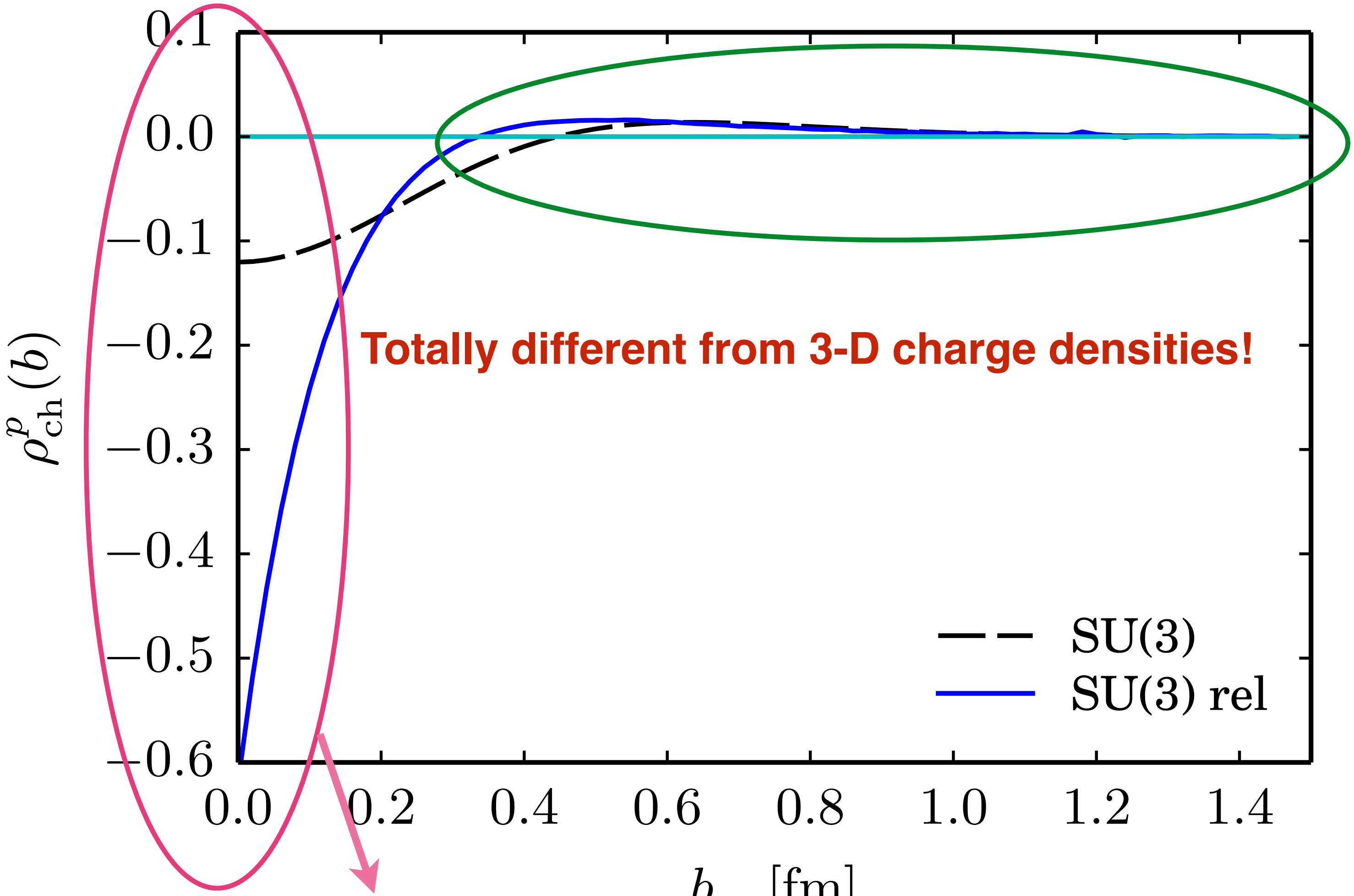
$$\tau = Q^2/4M_N^2$$

$$= F_1(Q^2)$$

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** neutron



Positive pion tail

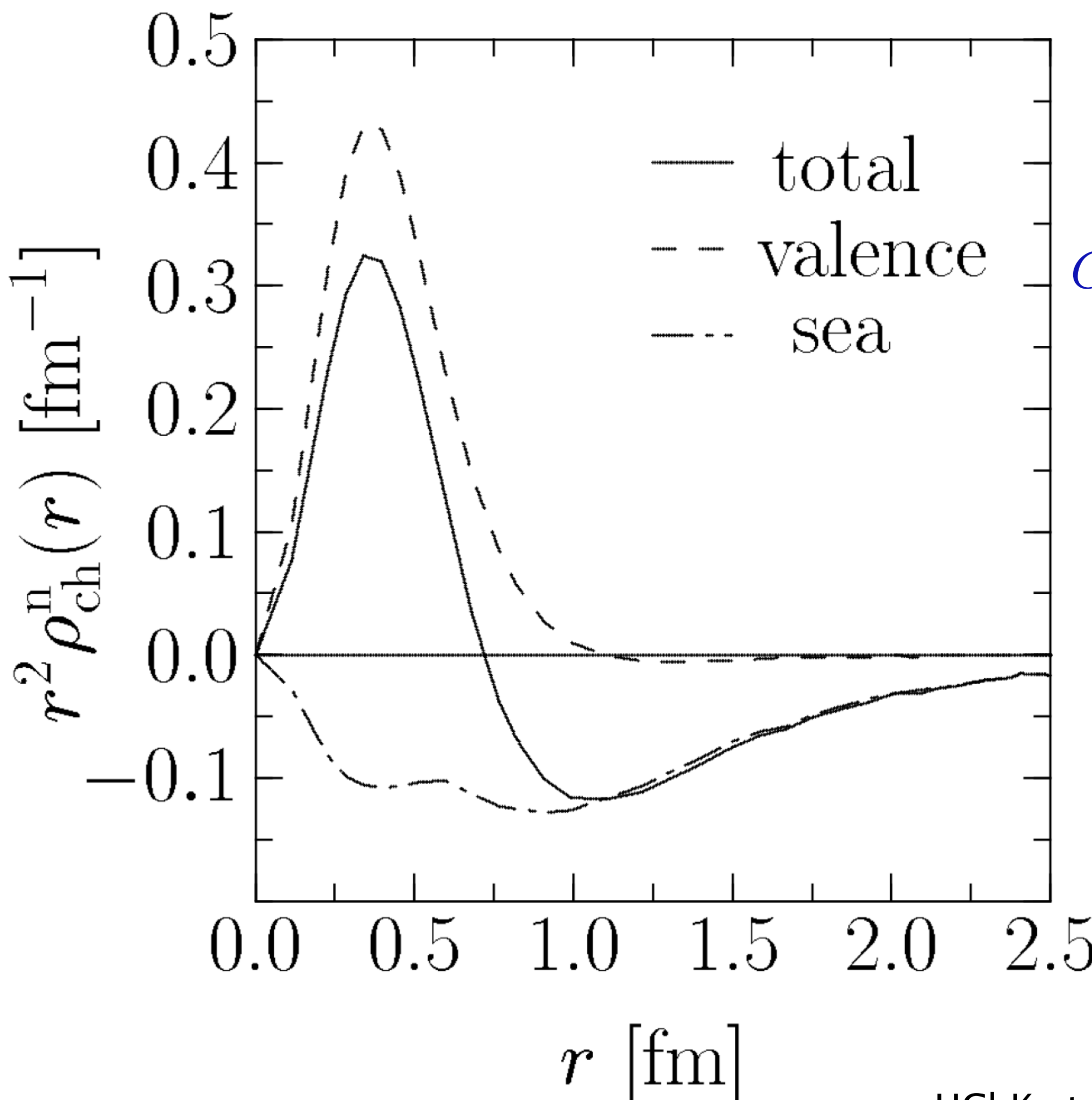
Surprisingly, negative charge distribution in the center of the neutron!

G.A. Miller, PRL **99,** 112001 (2007)

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Old **3-D charge** densities inside an **unpolarized** neutron

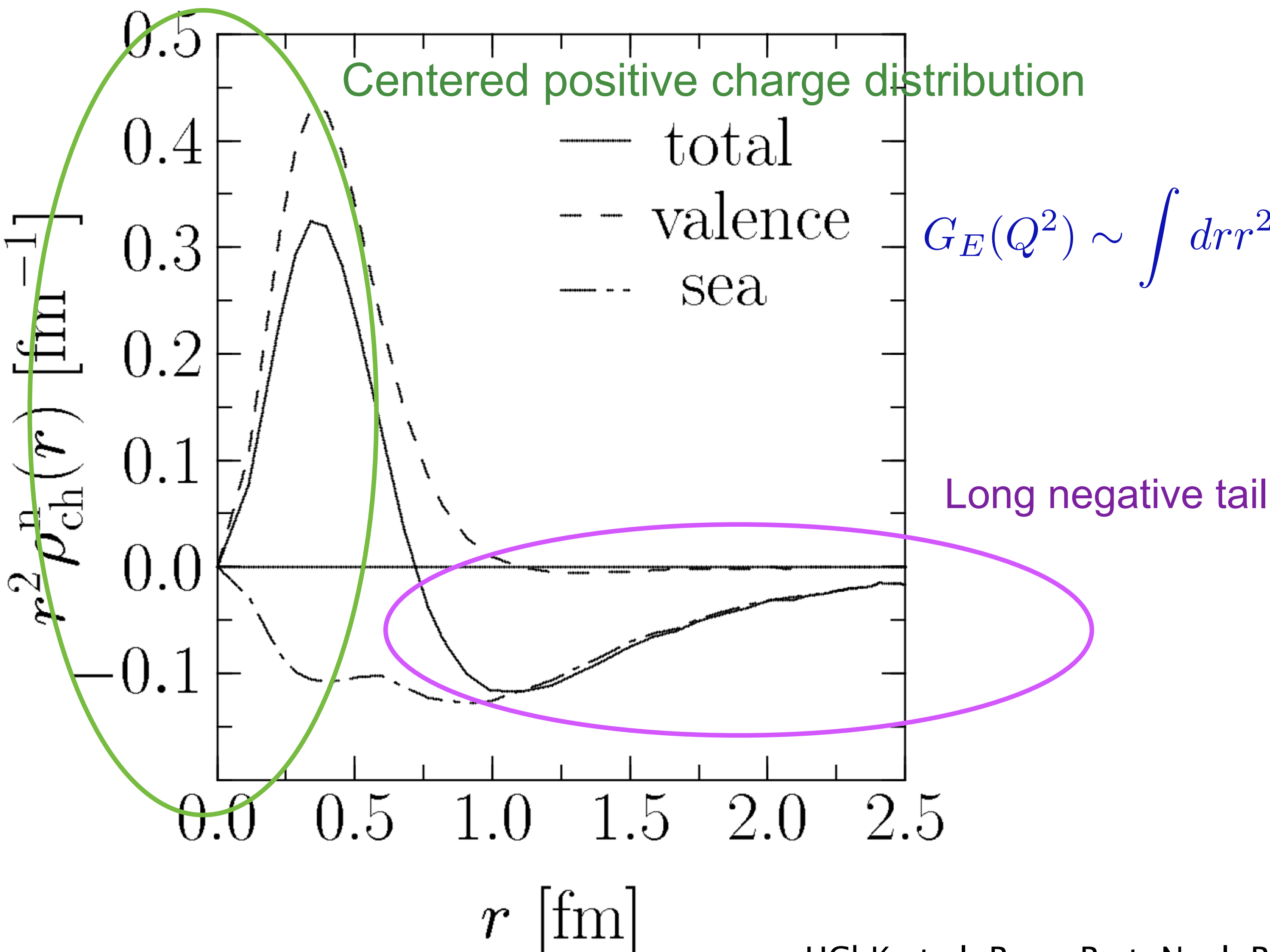


$$G_E(Q^2) \sim \int dr r^2 j_0(Qr) \rho_{\mathrm{ch}}(r)$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Results

Old **3-D charge** densities inside an **unpolarized** neutron

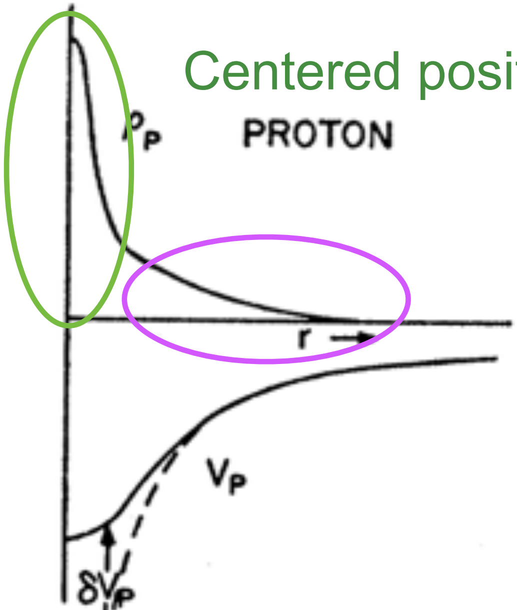


$$G_E(Q^2) \sim \int dr r^2 j_0(Qr)\rho_{\rm ch}(r)$$

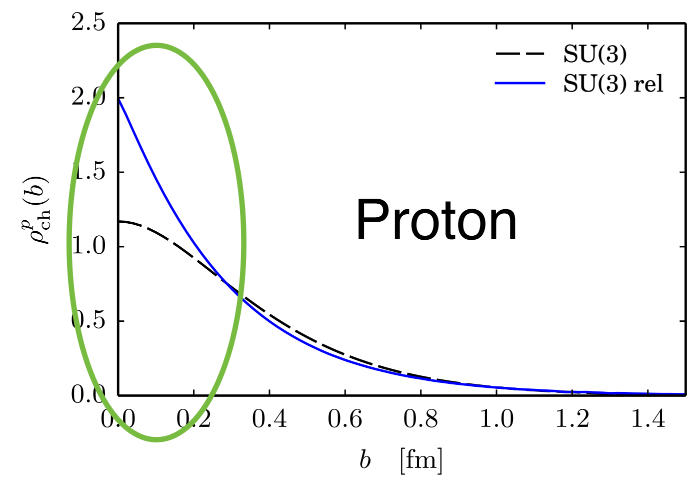
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Results

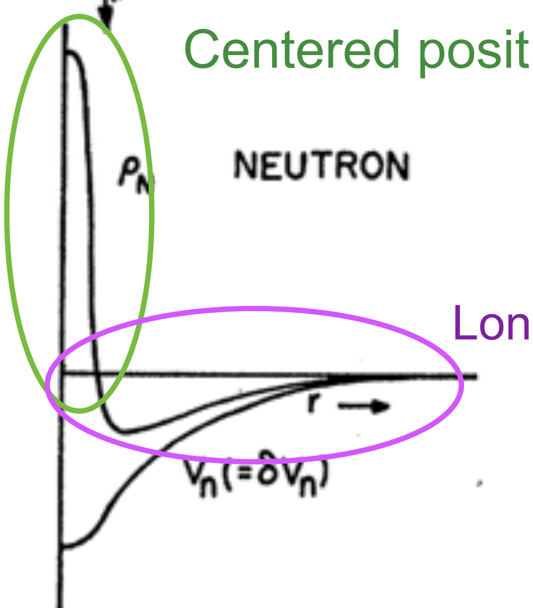
Centered positive charge distribution



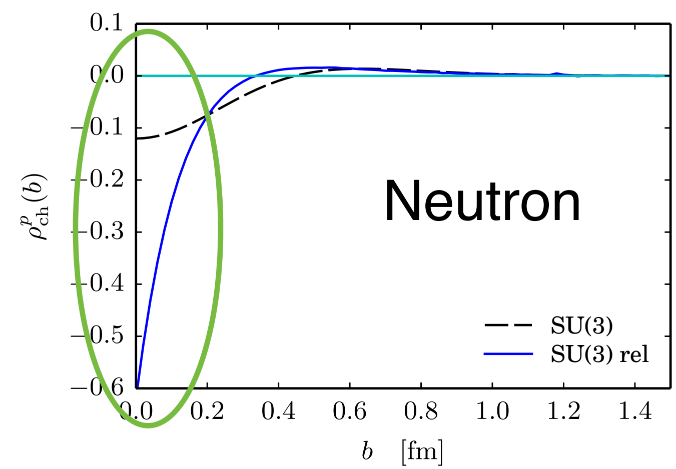
Long positive tail:
Positive pion cloud



Centered positive charge distribution



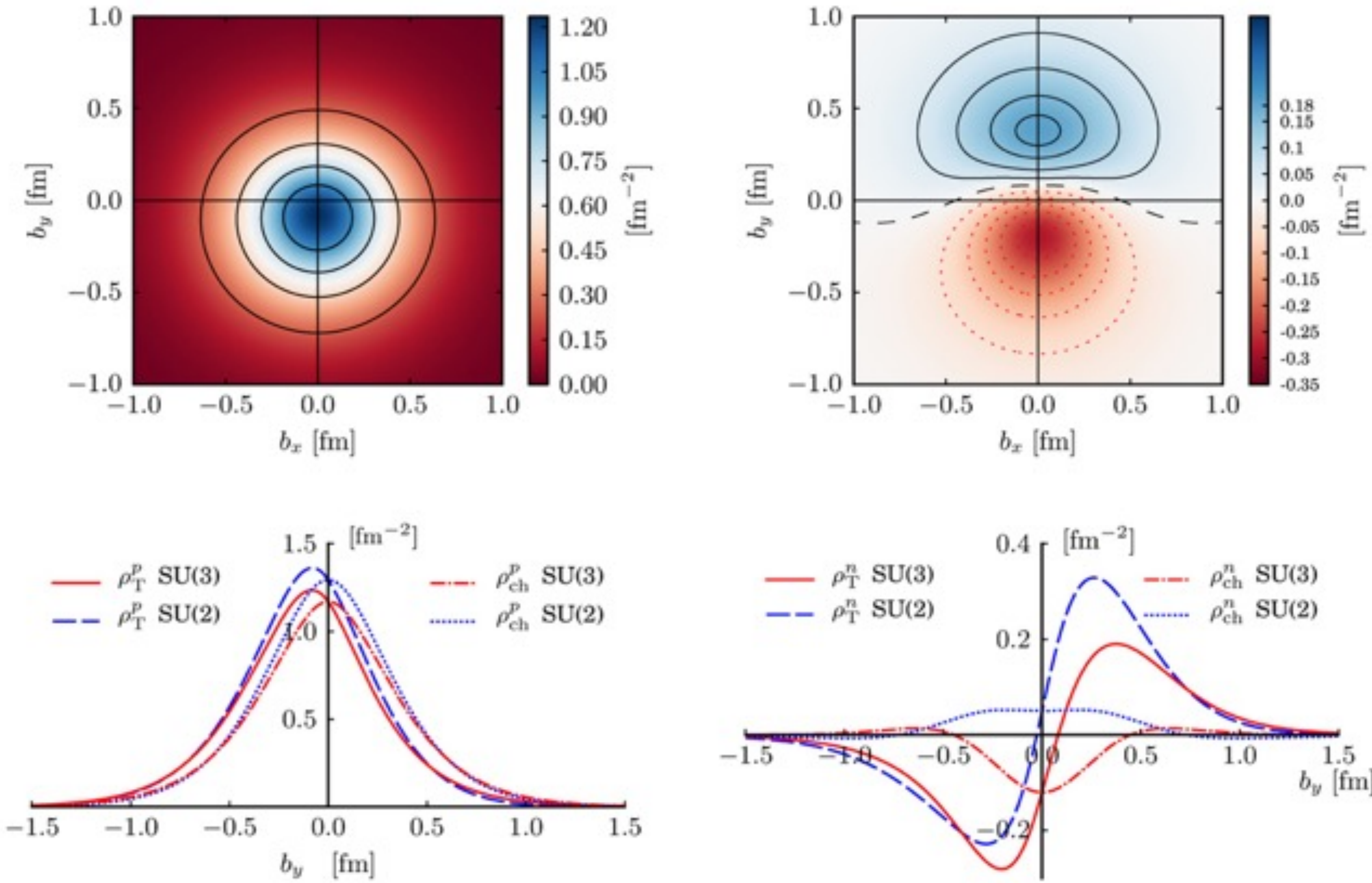
Long negative tail



Yennie, Levy, & Ravenhall, Rev. Mod. Phys. **29**, 144 (1957)

Results

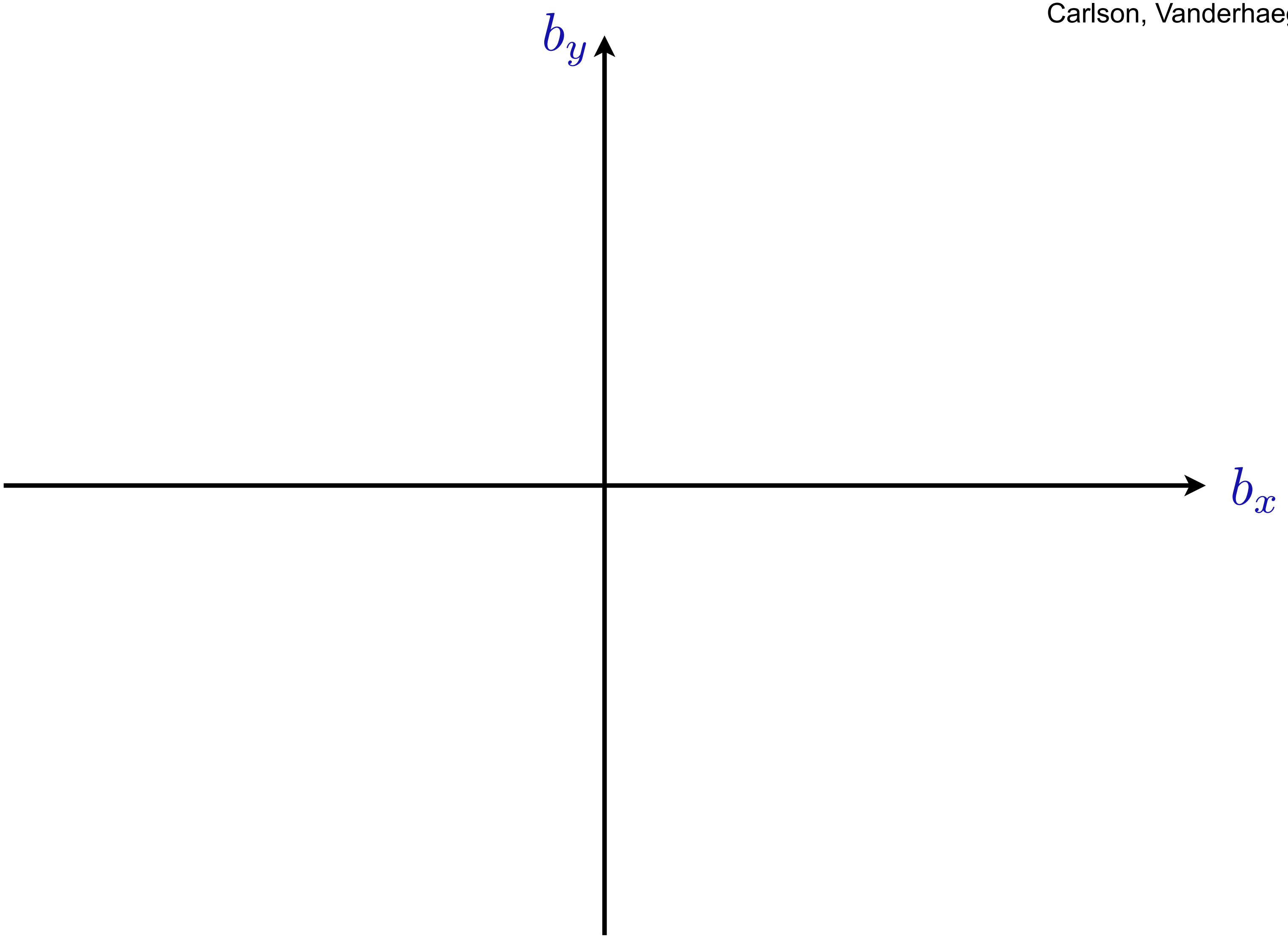
Transverse charge densities inside an **polarized** nucleon



Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

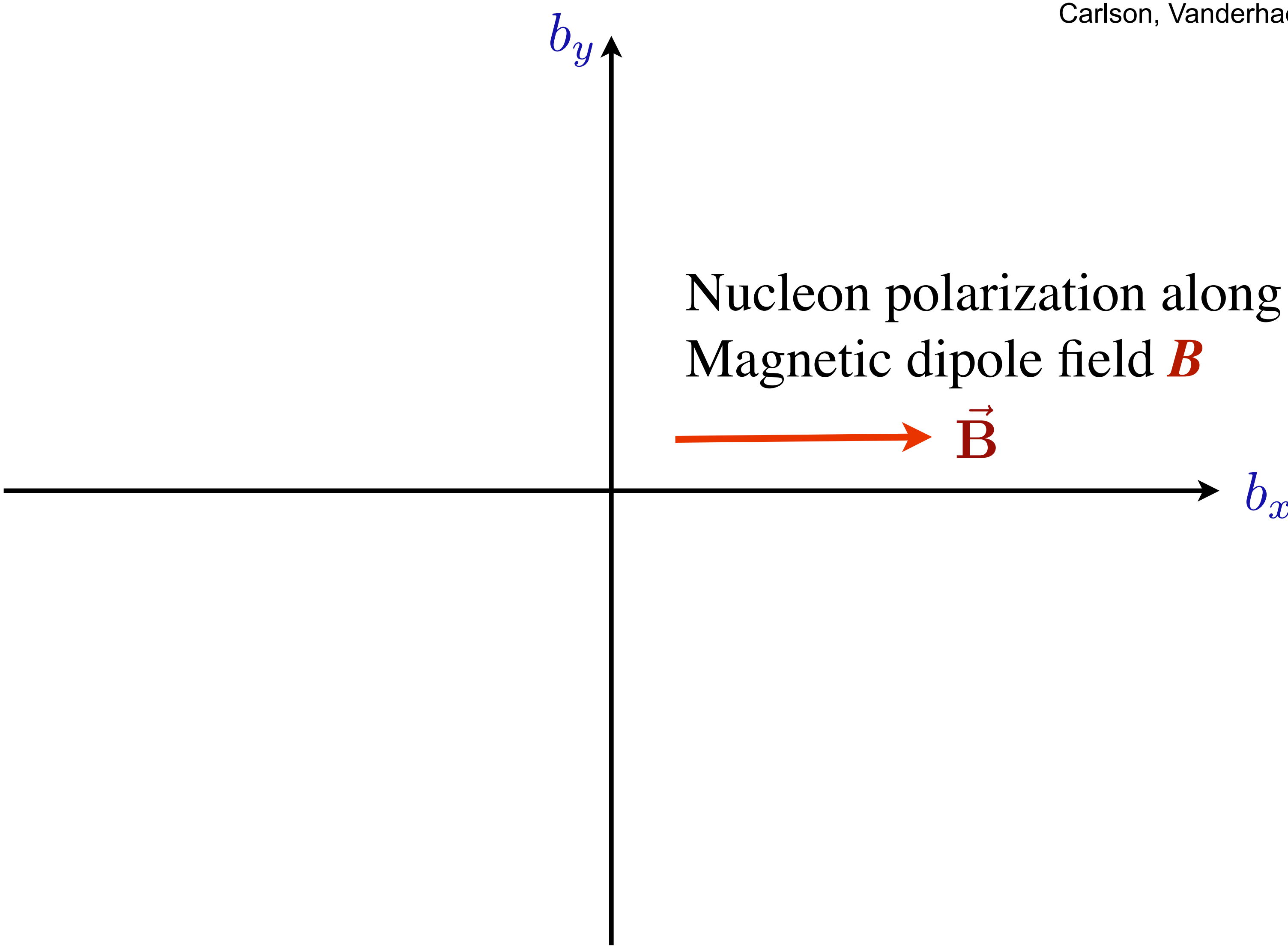
Carlson, Vanderhaeghen, PRL **100**, 032004



Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004

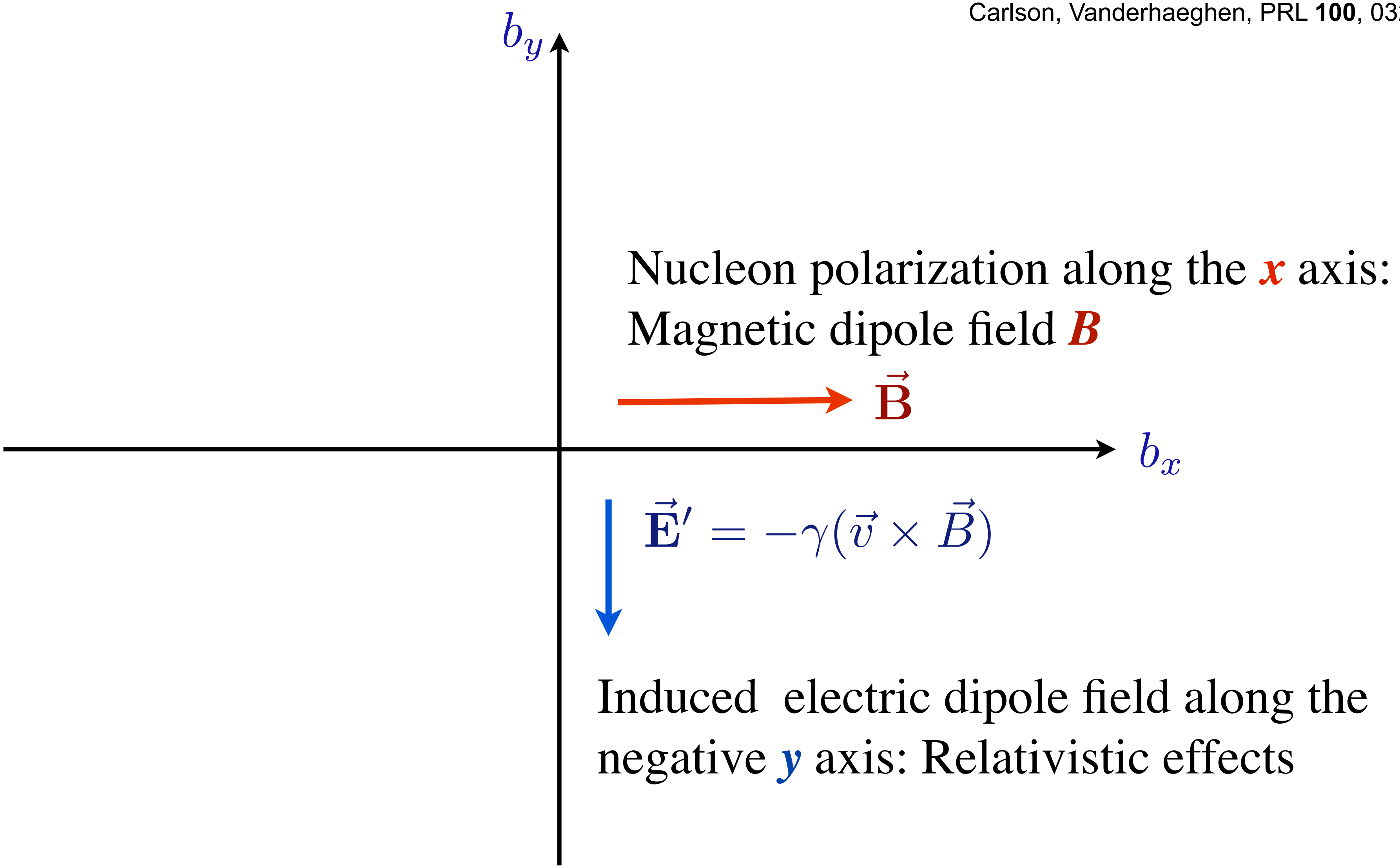


Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

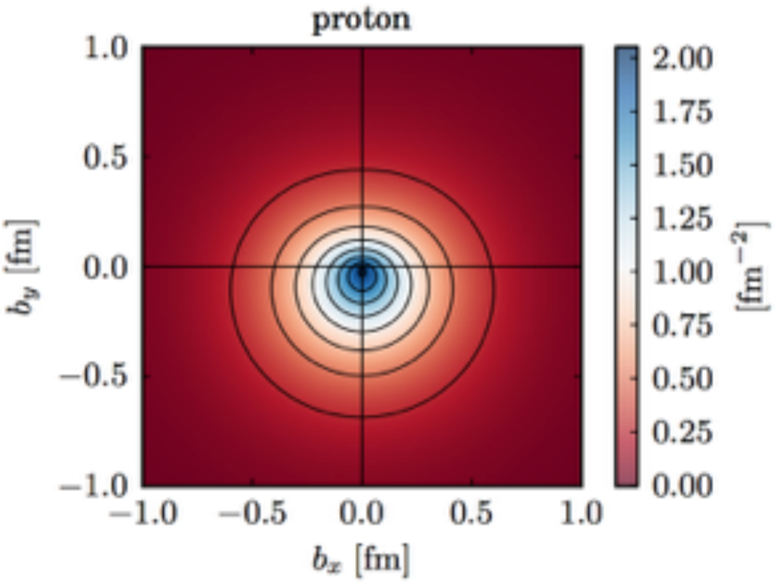
Carlson, Vanderhaeghen, PRL **100**, 032004



Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004



Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

$\vec{\mathrm{B}}$

b_x

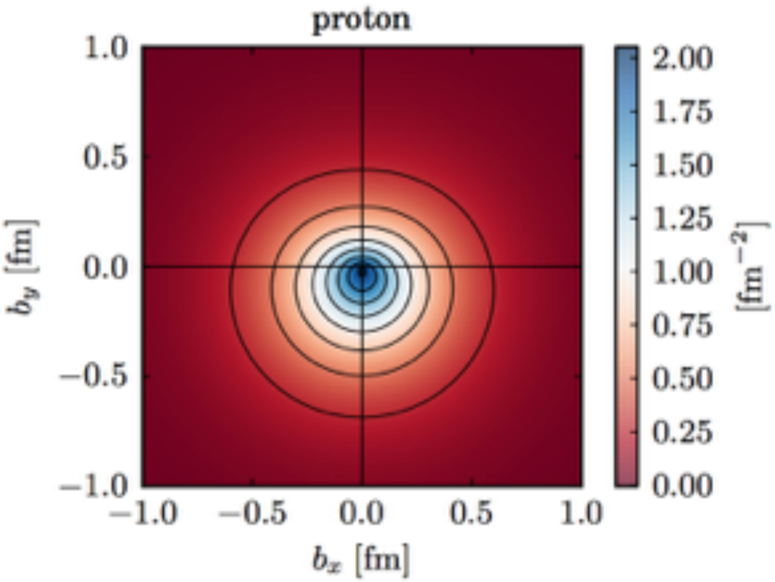
$$\vec{\mathbf{E}}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004

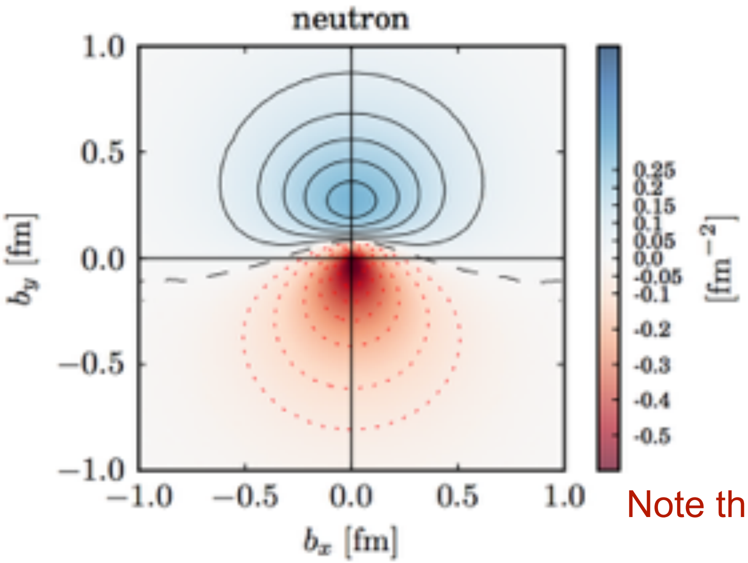


Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

$\vec{\mathrm{B}}$

b_x

$\vec{\mathbf{E}}' = -\gamma(\vec{v} \times \vec{B})$



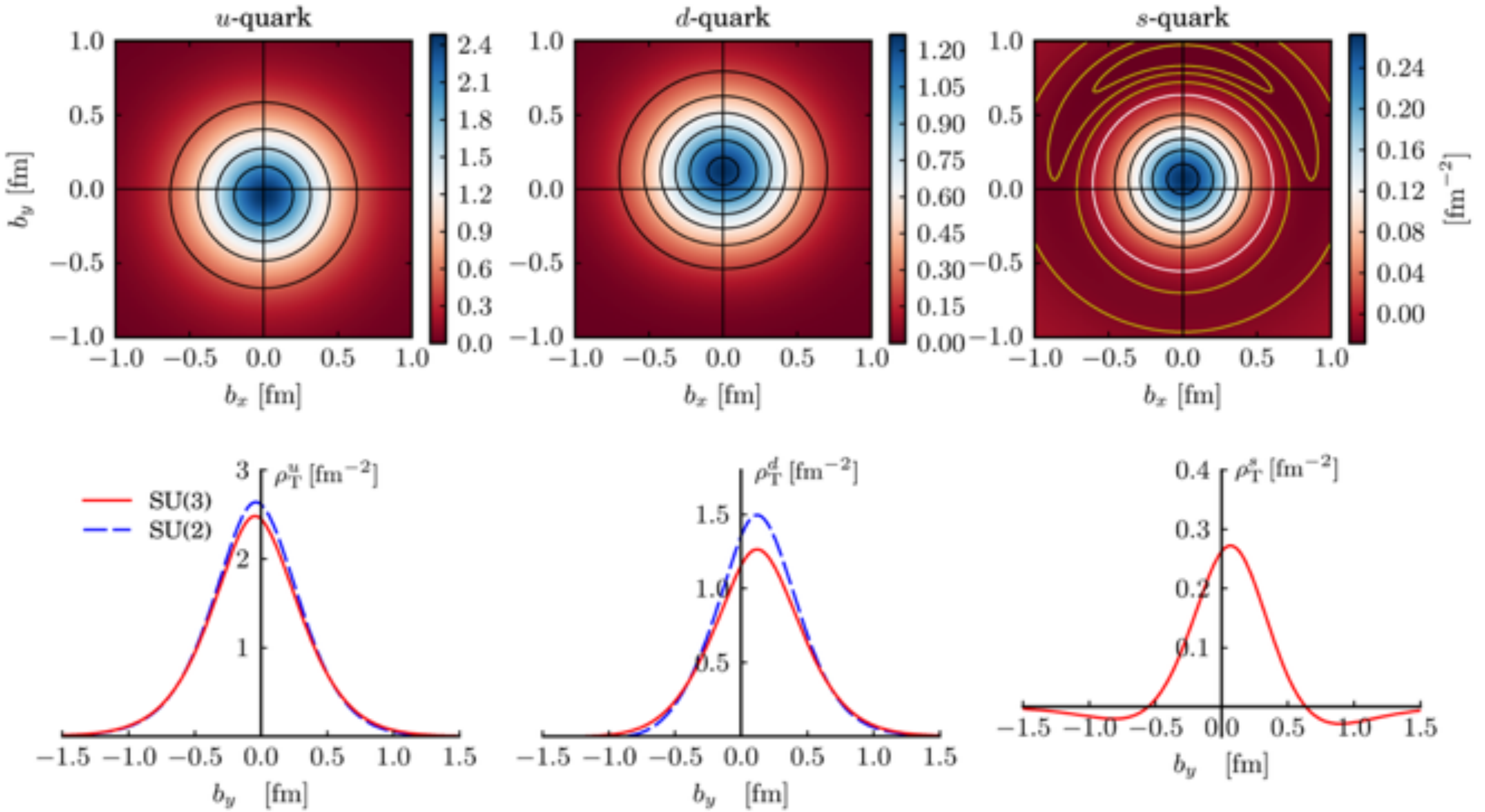
Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Note that the neutron anomalous magnetic moment is negative!

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Flavor-decomposed Transverse charge densities inside a polarized nucleon



Silva, Urbano, HChK, hep-ph/1305.6373

Transverse Spin Densities (Tensor form factors)

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^{\chi}\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[H_T^{\chi}(Q^2)i\sigma^{\mu\nu} + E_T^{\chi}(Q^2)\frac{\gamma^{\mu}q^{\nu} - q^{\mu}\gamma^{\nu}}{2M} + \tilde{H}_T^{\chi}(Q^2)\frac{(n^{\mu}q^{\nu} - q^{\mu}n^{\nu})}{2M^2}\right]u_s(p)$$

$$\int_{-1}^{1} dx H_T^{\chi}(x,\xi=0,t) = H_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx E_T^{\chi}(x,\xi=0,t) = E_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx \tilde{H}_T^{\chi}(x,\xi=0,t) = \tilde{H}_T^{\chi}(q^2)$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^{\chi}\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\Big[H_T^{\chi}(Q^2)i\sigma^{\mu\nu} + E_T^{\chi}(Q^2)\frac{\gamma^{\mu}q^{\nu} - q^{\mu}\gamma^{\nu}}{2M} + \tilde{H}_T^{\chi}(Q^2)\frac{(n^{\mu}q^{\nu} - q^{\mu}n^{\nu})}{2M^2}\Big]u_s(p)$$

$$\boxed{\int_{-1}^{1} dx H_T^{\chi}(x,\xi=0,t) = H_T^{\chi}(q^2),}$$

$$\int_{-1}^{1} dx E_T^{\chi}(x,\xi=0,t) = E_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx \tilde{H}_T^{\chi}(x,\xi=0,t) = \tilde{H}_T^{\chi}(q^2)$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^{\chi}\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[\boxed{H_T^{\chi}(Q^2)}i\sigma^{\mu\nu} + E_T^{\chi}(Q^2)\frac{\gamma^{\mu}q^{\nu} - q^{\mu}\gamma^{\nu}}{2M} + \tilde{H}_T^{\chi}(Q^2)\frac{(n^{\mu}q^{\nu} - q^{\mu}n^{\nu})}{2M^2}\right]u_s(p)$$

$$\boxed{\int_{-1}^{1} dx H_T^{\chi}(x,\xi=0,t) = H_T^{\chi}(q^2),}$$

$$\int_{-1}^{1} dx E_T^{\chi}(x,\xi=0,t) = E_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx \tilde{H}_T^{\chi}(x,\xi=0,t) = \tilde{H}_T^{\chi}(q^2)$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2}\int\frac{d\Omega}{4\pi}\langle N_{\frac{1}{2}}(p')|\psi^{\dagger}\gamma^k q^k\lambda^{\chi}\psi|N_{\frac{1}{2}}(p)\rangle$$

$$\kappa_T^{\chi} = -H_T^{\chi}(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the **transverse spin quark densities** inside the nucleon.

Tensor form factors

Tensor charges and anomalous tensor magnetic moments are scale-dependent.

$$\delta q(\mu^2) = \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)}\right)^{4/27}\left[1 - \frac{337}{486\pi}\left(\alpha_S(\mu_i^2) - \alpha_S(\mu^2)\right)\right]\delta q(\mu_i^2)\,,$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\left[1 - \frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}{\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.248\,\mathrm{GeV}$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$\lvert \delta d / \delta u \rvert$	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$\lvert\delta d/\delta u\rvert$	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

$$
\begin{array}{lll}
\text{SIDIS [16] } (0.80\,\text{GeV}^2): & \delta u = 0.54^{+0.09}_{-0.22}\,, & \delta d = -0.231^{+0.09}_{-0.16}, \\
\text{SIDIS [16] } (0.36\,\text{GeV}^2): & \delta u = 0.60^{+0.10}_{-0.24}\,, & \delta d = -0.26^{+0.1}_{-0.18}, \\
\text{Lattice [21] } (4.00\,\text{GeV}^2): & \delta u = 0.86 \pm 0.13\,, & \delta d = -0.21 \pm 0.005\,, \\
\text{Lattice [21] } (0.36\,\text{GeV}^2): & \delta u = 1.05 \pm 0.16\,, & \delta d = -0.26 \pm 0.01\,, \\
\chi\text{QSM } (0.36\,\text{GeV}^2): & \delta u = 1.08\,, & \delta d = -0.32\,,
\end{array}
$$

[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

$\mu^2 = 0.36\,\mathrm{GeV}^2$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 $\boxed{(3.70)}$
κ_T^d	1.83	1.83	1.90 $\boxed{(2.35)}$
κ_T^s	$0.2 \sim -0.2$		
κ_T^u/κ_T^d	1.95	2.02	1.58

. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

$\mu^2 = 0.36\,\mathrm{GeV}^2$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 (3.70)
κ_T^d	1.83	1.83	1.90 (2.35)
κ_T^s	$0.2 \sim -0.2$		
κ_T^u/κ_T^d	1.95	2.02	1.58

The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Transverse spin density

$$\rho(\mathbf{b},\,\mathbf{S},\,\mathbf{s}) = \frac{1}{2}\Bigg[\, H(b^2) - S^i\epsilon^{ij}b^j\,\frac{1}{M_N}\,\frac{\partial E(b^2)}{\partial b^2} - s^i\epsilon^{ij}b^j\,\frac{1}{M_N}\,\frac{\partial \kappa_T(b^2)}{\partial b^2}$$
$$+\, s^i S^i \left\{ H_T(b^2) - \frac{1}{4M_N^2}\nabla^2 \tilde{H}_T(b^2) \right\}$$
$$+\, s^i \left(2b^i b^j - b^2\delta^{ij}\right) S^j\,\frac{1}{M_N^2}\left(\frac{\partial}{\partial b^2}\right)^2 \tilde{H}_T(b^2) \Bigg]\,,$$

M. Diehl & Ph. Haegler, Euro.Phys. J., C 44 (2005) 87.

Transverse spin density

$$\rho(\mathbf{b},\, \mathbf{S},\, \mathbf{s}) = \boxed{\frac{1}{2}\left[\; H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \;\right]}$$

$$[\mathbf{S}, \mathbf{s}] = [(1,0), (0,0)], \;\; [\mathbf{S}, \mathbf{s}] = [(0,0), (1,0)]$$

M. Diehl & Ph. Haegler, Euro.Phys. J., C 44 (2005) 87.

Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \boxed{\frac{1}{2}\left[H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]}$$

$$[\mathbf{S}, \mathbf{s}] = [(1,0),(0,0)], \;\; [\mathbf{S}, \mathbf{s}] = [(0,0),(1,0)]$$

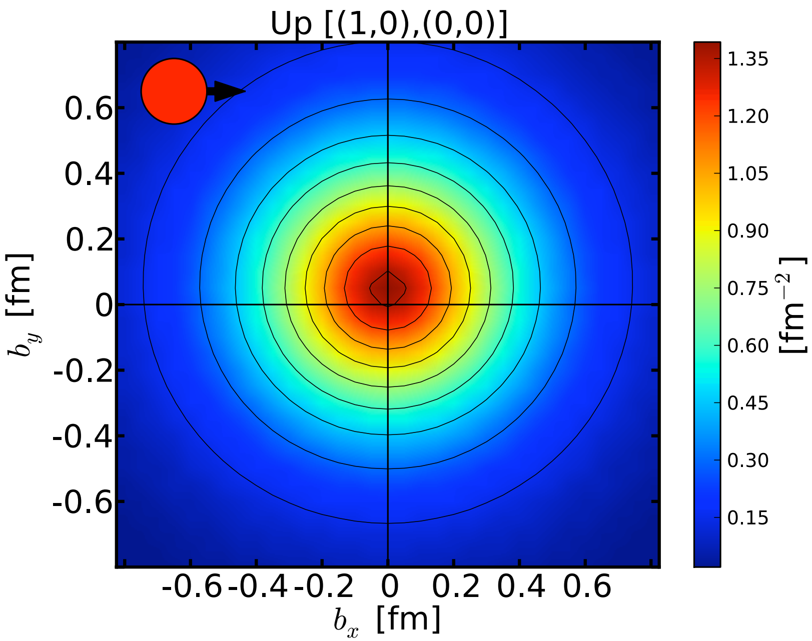
$$\mathcal{F}^\chi(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^\chi(Q^2)$$

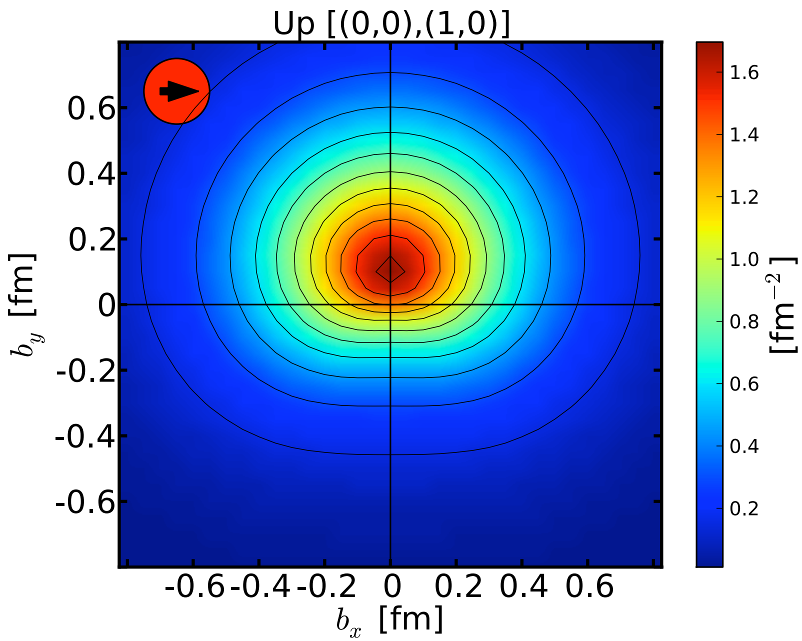
$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

M. Diehl & Ph. Haegler, Euro.Phys. J., C 44 (2005) 87.

Results

Up quark transverse spin density inside a nucleon

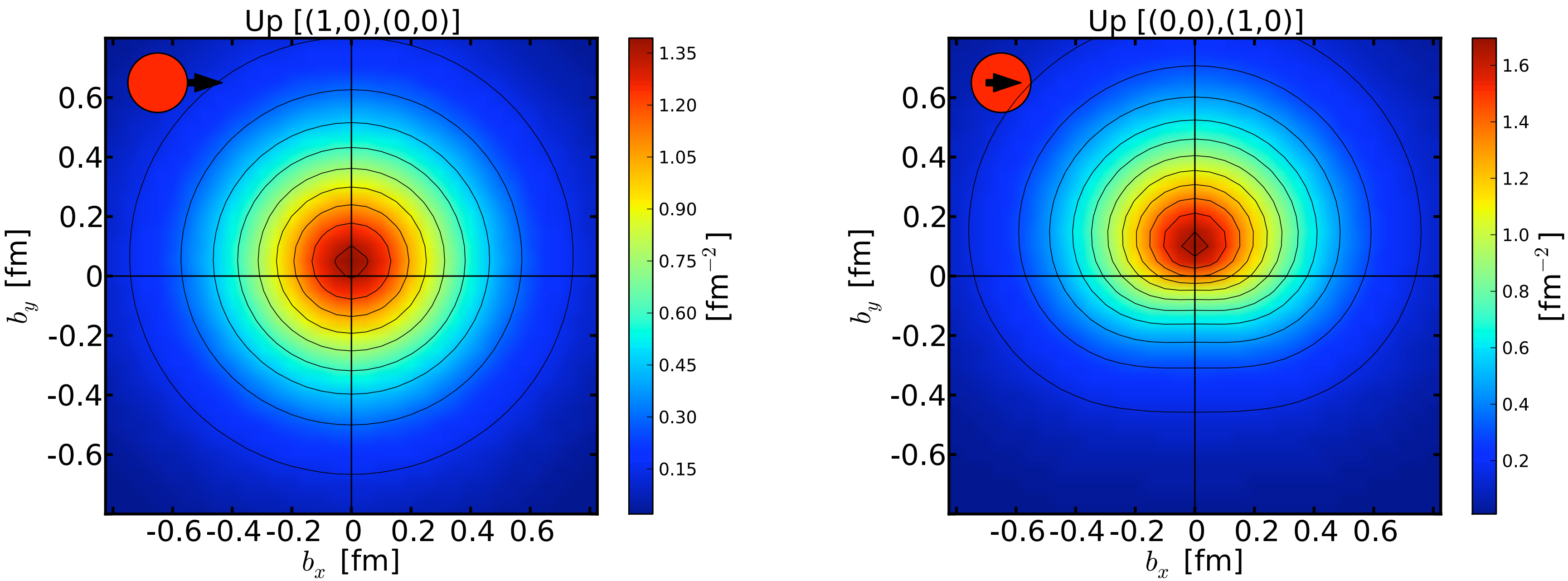




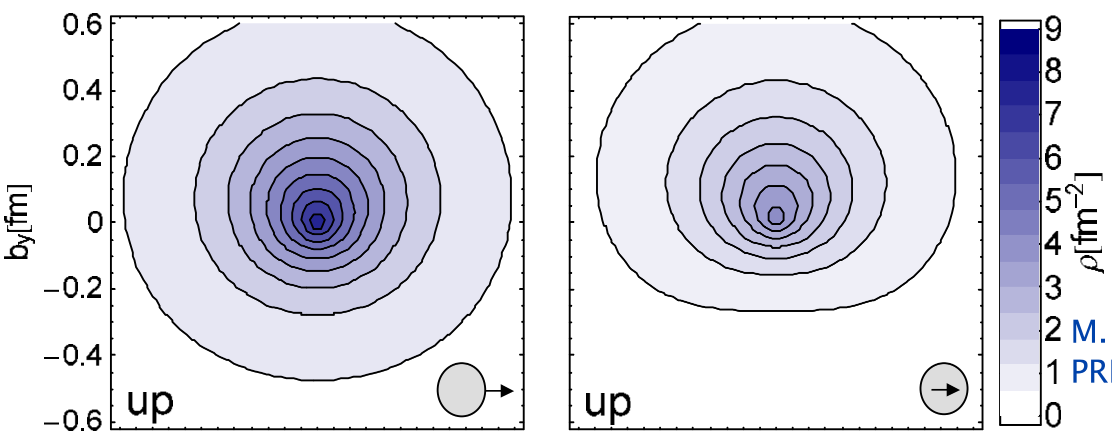
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Results

Up quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

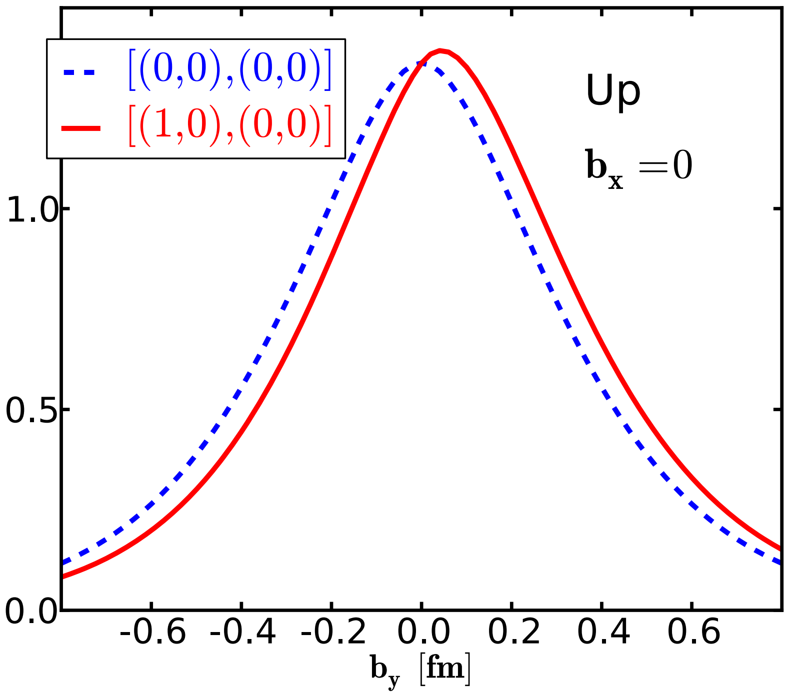


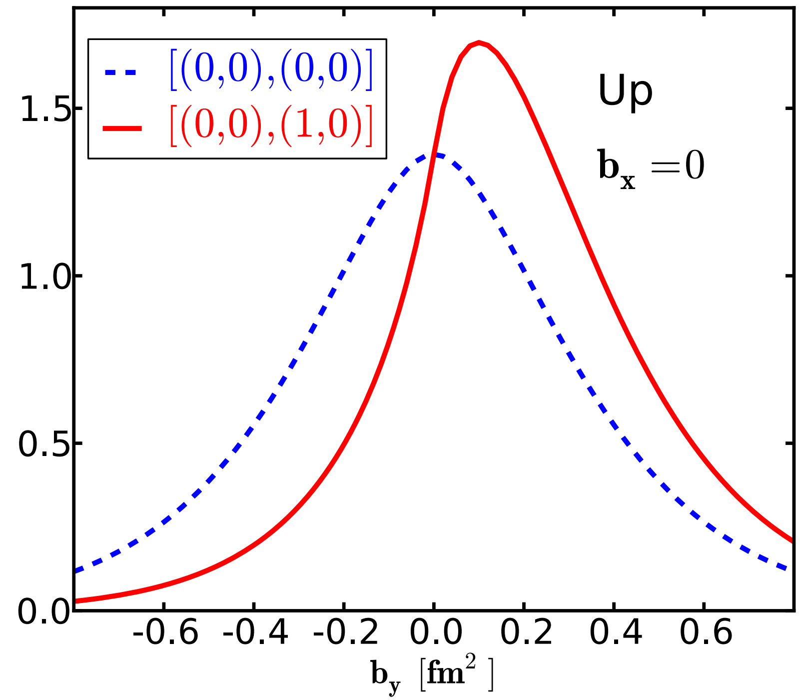
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

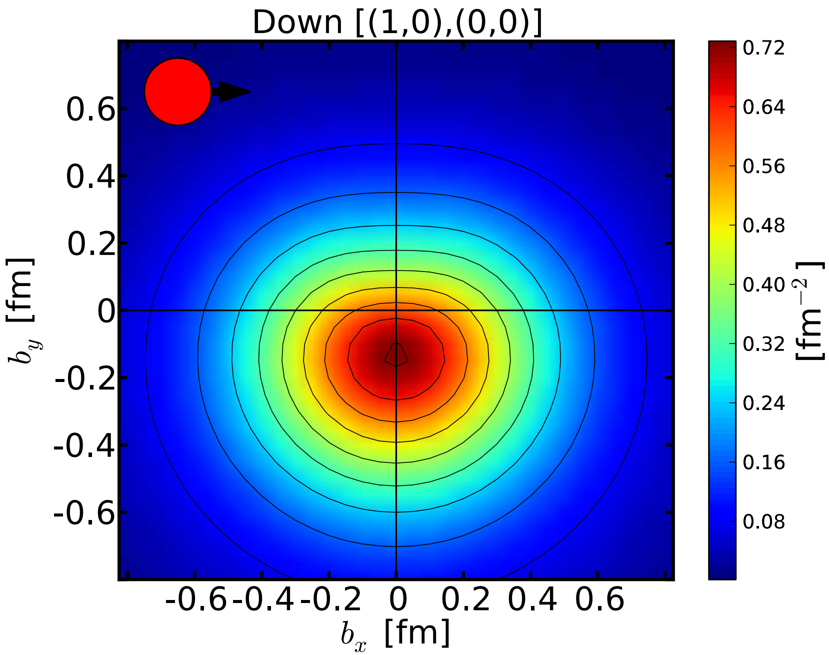
Up quark transverse spin density inside a nucleon

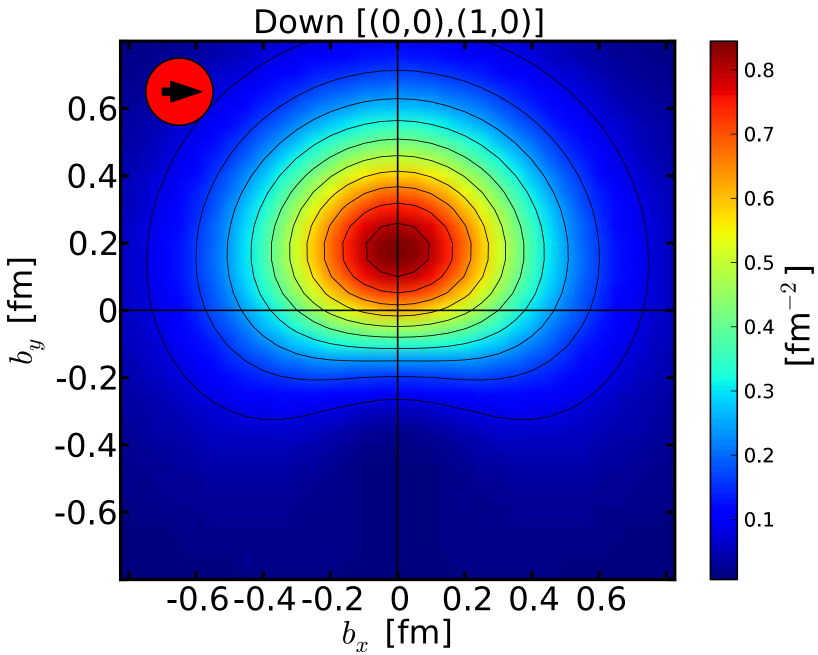




Results

Down quark transverse spin density inside a nucleon

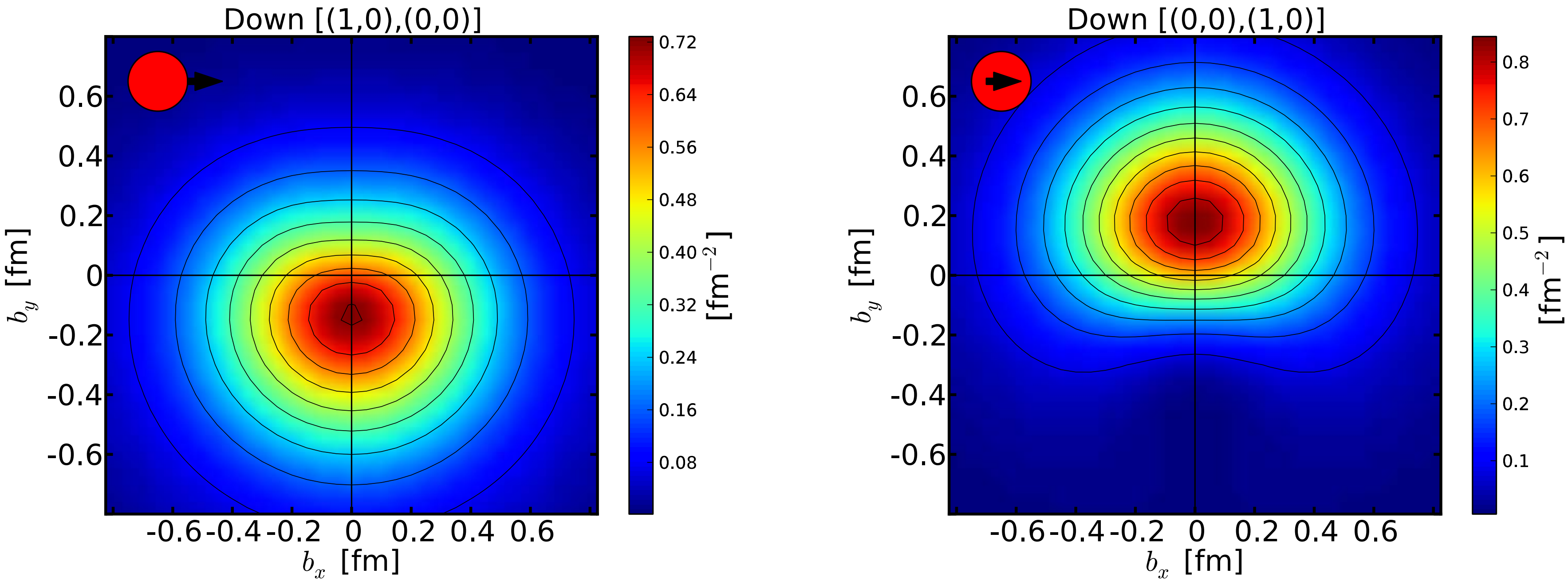




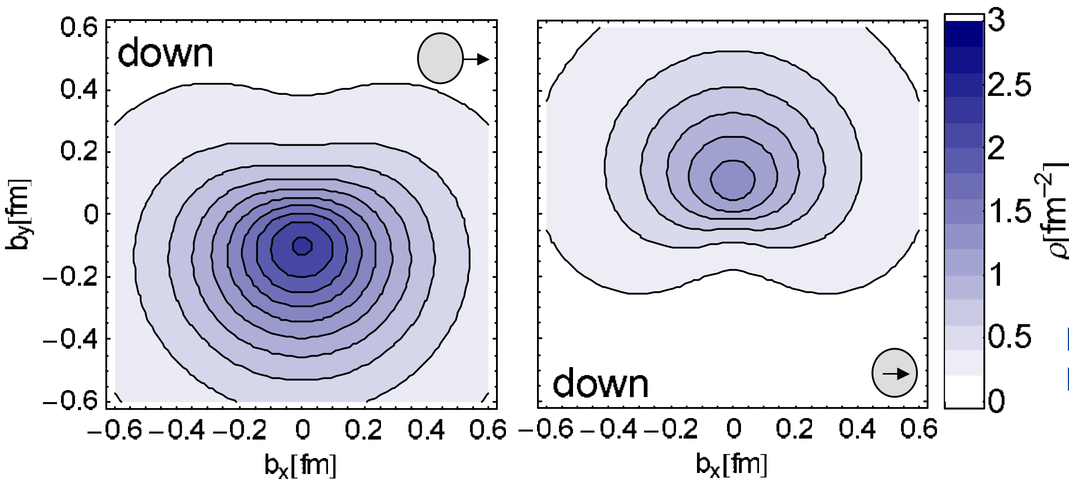
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Results

Down quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

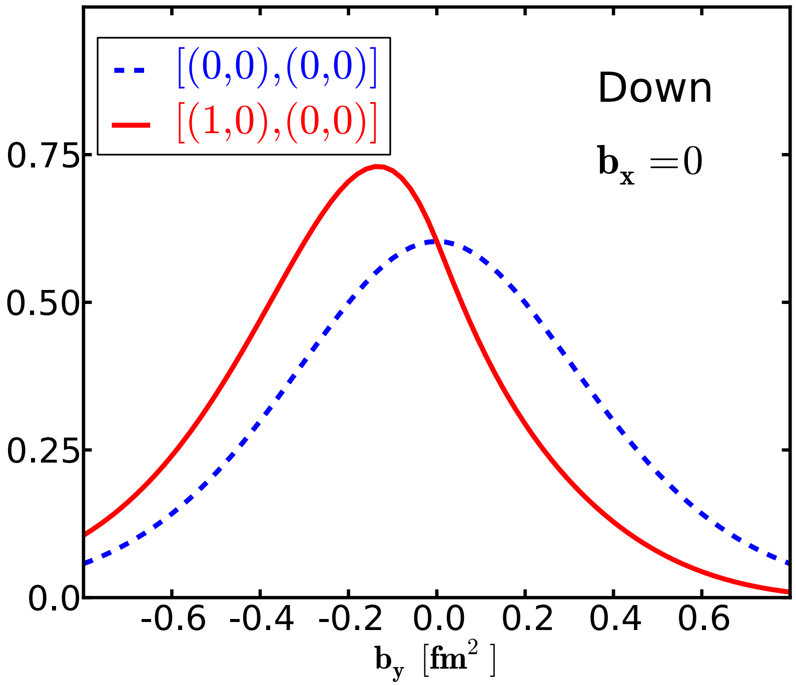


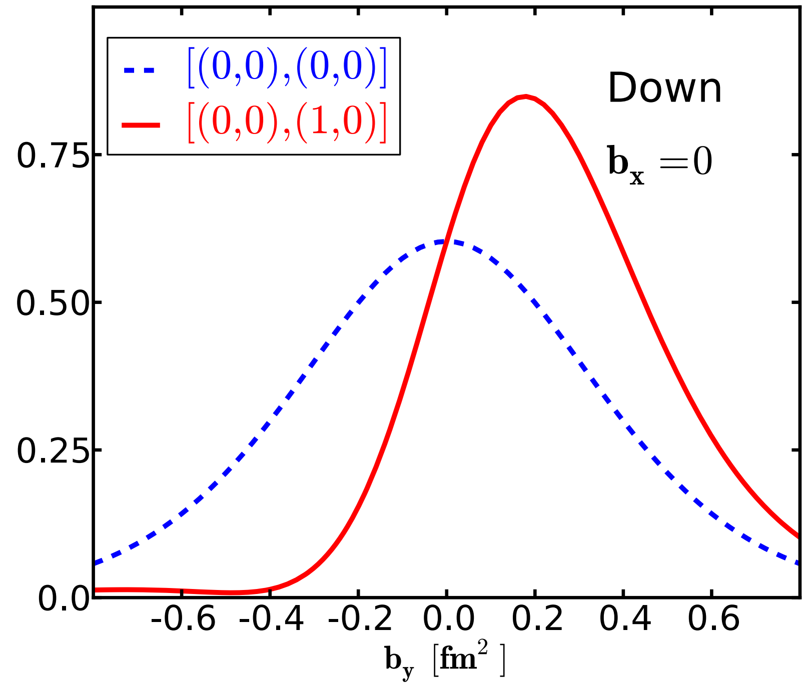
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

Results

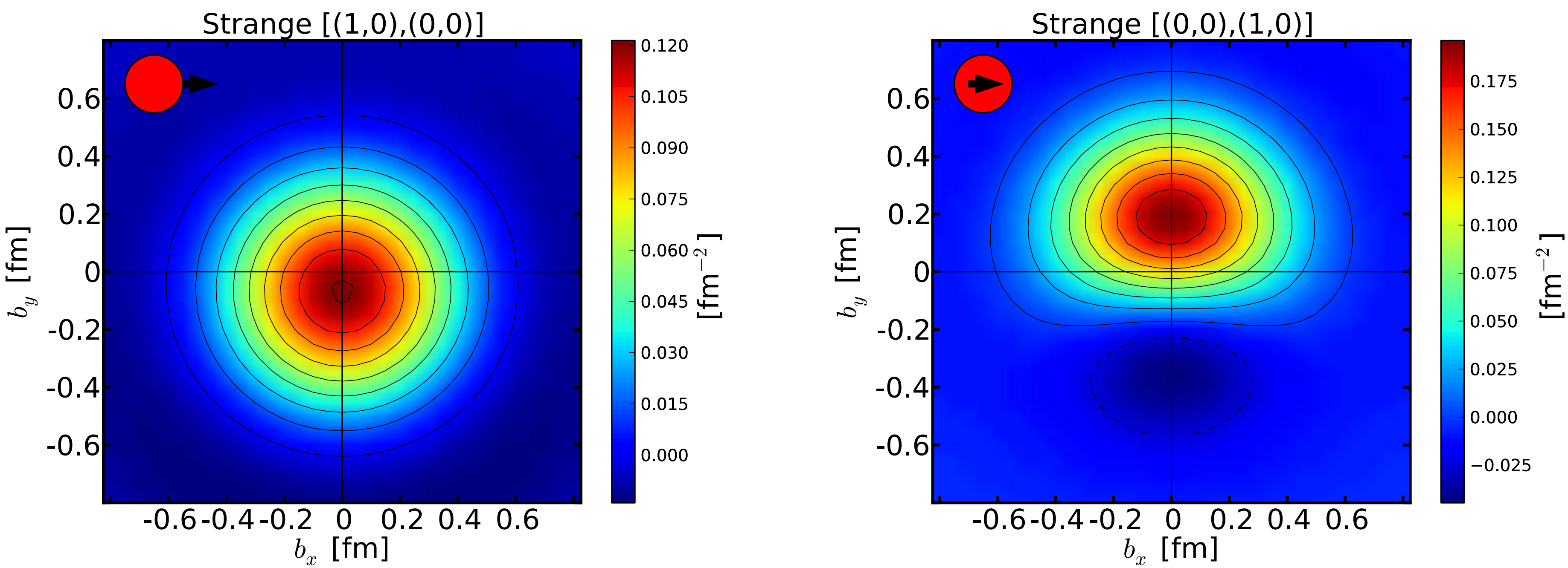
Down quark transverse spin density inside a nucleon





Results

Strange quark transverse spin density inside a nucleon

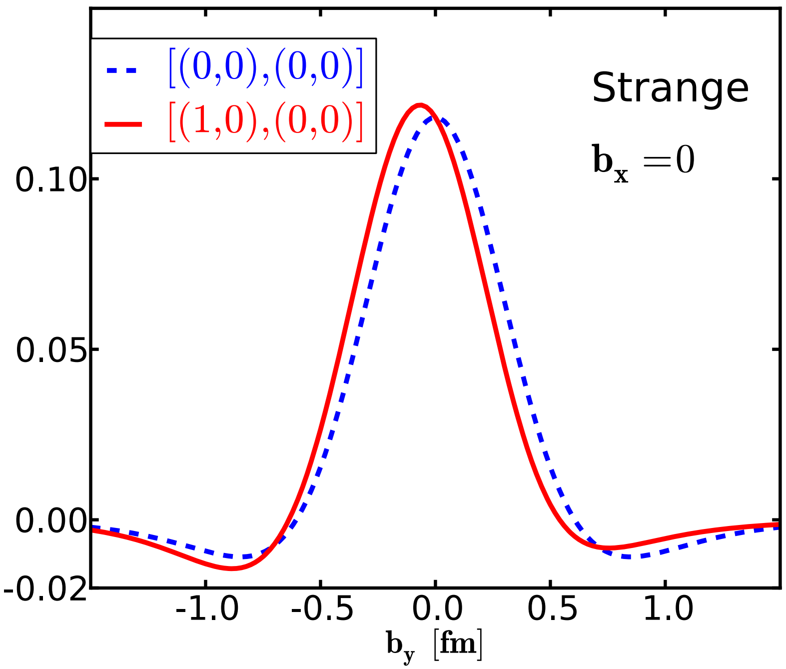


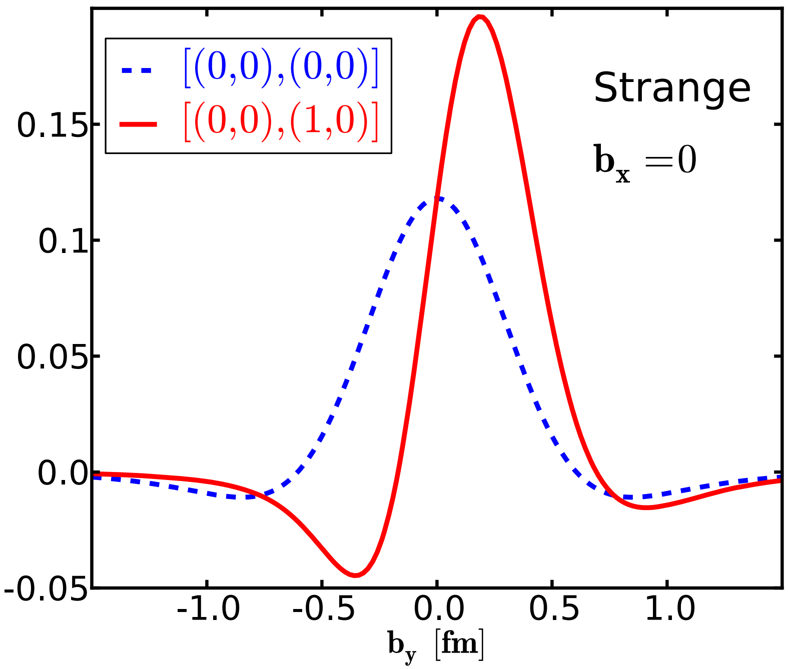
T. Ledwig & H–Ch.K, PRD 85, 034041 (2012)

This is the **first** result of the strange quark transverse spin density inside a nucleon

Results

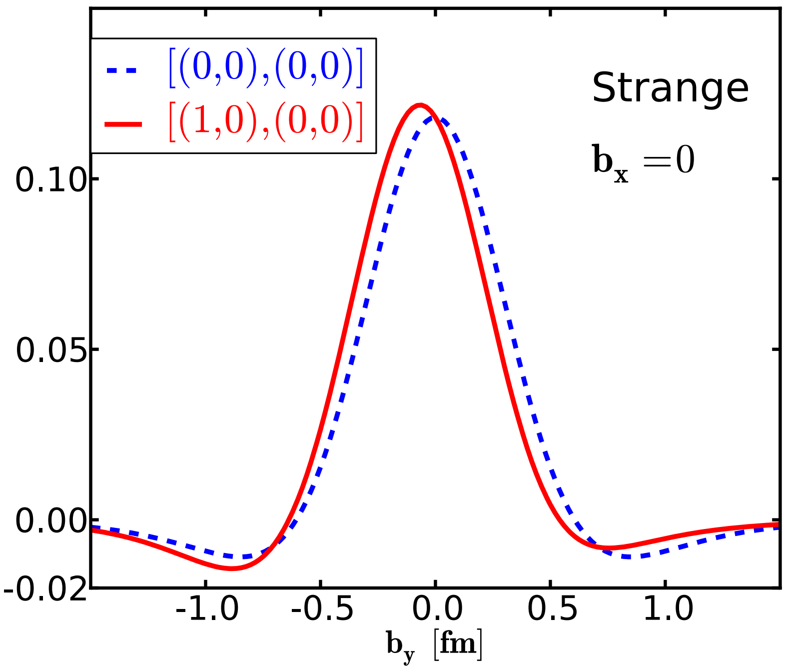
Strange quark transverse spin density inside a nucleon

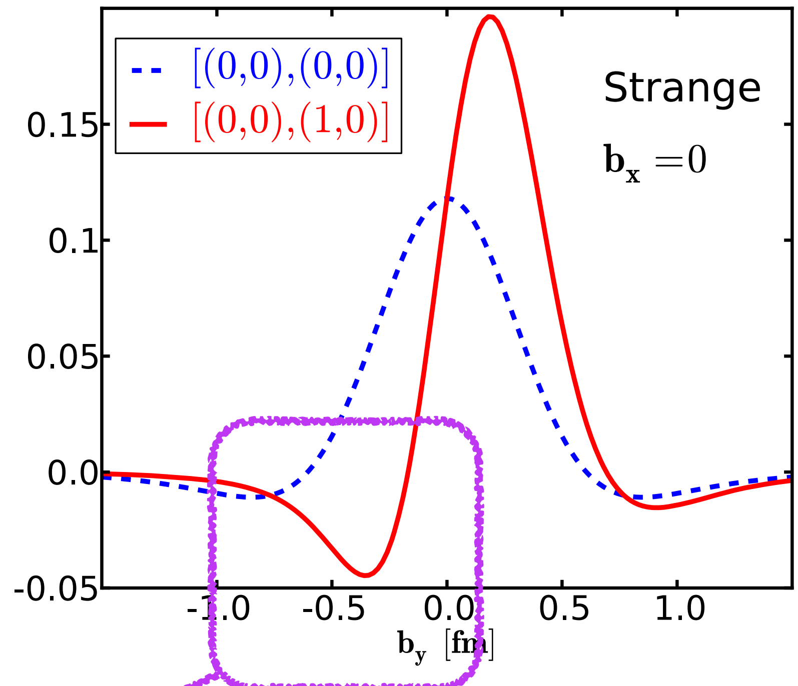




Results

Strange quark transverse spin density inside a nucleon





Polarized to the negative direction in the b plane.

EMT form factors:
Stability of the nucleon

EMT form factors

Energy-momentum tensor form factors

$$\langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N}\right.$$

$$\left. + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\right]u(p)$$

EMT form factors

Energy-momentum tensor form factors

$$\langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p')\Big[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\Big]u(p)$$

GPDs

$$\int\frac{dx^-}{4\pi}\langle P',S'|\bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp)|P,S\rangle = \frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N}E_q(x,\xi,t)\right)u(p,s)$$

EMT form factors

Energy-momentum tensor form factors

$$\langle N(p')|T^{Q,G}_{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N}\right.$$
$$\left. + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\right]u(p)$$

GPDs

$$\int \frac{dx^-}{4\pi}\langle P',S'|\bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp)|P,S\rangle$$
$$= \frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N}E_q(x,\xi,t)\right)u(p,s)$$

The EMT form factors as the second moments of the isoscalar vector GPDs

$$\int_{-1}^{1} dx\, x \sum_f H_q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}d_1^Q(t)\xi^2,$$
$$\int_{-1}^{1} dx\, x \sum_f E_q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}d_1^Q(t)\xi^2,$$

EMT form factors

In the Breit frame,

$$T^Q_{\mu\nu}(\mathbf{r},\mathbf{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\exp(i\Delta\cdot\mathbf{r})\langle p', S'|T^Q_{\mu\nu}(0)|p,S\rangle$$

$$M_2(t) - \frac{t}{4M_N^2}\left(M_2(t) - 2J(t) + \frac{4}{5}d_1(t)\right) = \frac{1}{M_N}\int d^3r e^{-i\mathbf{r}\cdot\Delta}T_{00}(\mathbf{r},\mathbf{s})$$

Momentum fractions carried by quarks and gluons

$$M_2^Q(0) = \int_0^1 \mathrm{d}x \sum_q x(f_1^q + f_1^{\bar{q}})(x),$$

$$M_2^G(0) = \int_0^1 \mathrm{d}x x f_1^g(x),$$

EMT form factors

In the Breit frame,

$$T^Q_{\mu\nu}(\mathbf{r},\mathbf{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\exp(i\Delta\cdot\mathbf{r})\langle p',S'|T^Q_{\mu\nu}(0)|p,S\rangle$$

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Momentum fractions carried by quarks and gluons

$$M_2^Q(0) = \int_0^1 \mathrm{d}x \sum_q x\boxed{(f_1^q + f_1^{\bar{q}})}(x),$$

Unpolarized parton distributions

$$M_2^G(0) = \int_0^1 \mathrm{d}x x f_1^g(x),$$

EMT form factors

$$J^Q(t) + \frac{2t}{3} J^{Q'}(t) = \int \mathrm{d}^3\mathbf{r} e^{-i\mathbf{r}\boldsymbol{\Delta}} \varepsilon^{ijk} s_i r_j T^Q_{0k}(\mathbf{r}, \mathbf{s}),$$

$$d_1^Q(t) + \frac{4t}{3} d_1^{Q'}(t) + \frac{4t^2}{15} d_1^{Q''}(t)$$
$$= -\frac{M_N}{2} \int \mathrm{d}^3\mathbf{r} e^{-i\mathbf{r}\boldsymbol{\Delta}} T^Q_{ij}(\mathbf{r}) \left(r^i r^j - \frac{\mathbf{r}^2}{3} \delta^{ij} \right),$$

Constraints

$$M_2(0) = \frac{1}{M_N} \int \mathrm{d}^3\mathbf{r} T_{00}(\mathbf{r}, \mathbf{s}) = 1,$$ Nucleon Mass

$$J(0) = \int \mathrm{d}^3\mathbf{r} \varepsilon^{ijk} s_i r_j T_{0k}(\mathbf{r}, \mathbf{s}) = \frac{1}{2},$$ Nucleon Spin

$$d_1(0) = -\frac{M_N}{2} \int \mathrm{d}^3\mathbf{r} T_{ij}(\mathbf{r}) \left(r^i r^j - \frac{\mathbf{r}^2}{3} \delta^{ij} \right) \equiv d_1$$ D-term

Stability of the nucleon

$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\delta_{ij}$$

Stability of the nucleon

$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + \boxed{p(r)}\delta_{ij}$$

$$\boxed{\int_0^\infty dr\, r^2 p(r) = 0}$$

: Stability condition of the nucleon

Stability of the nucleon

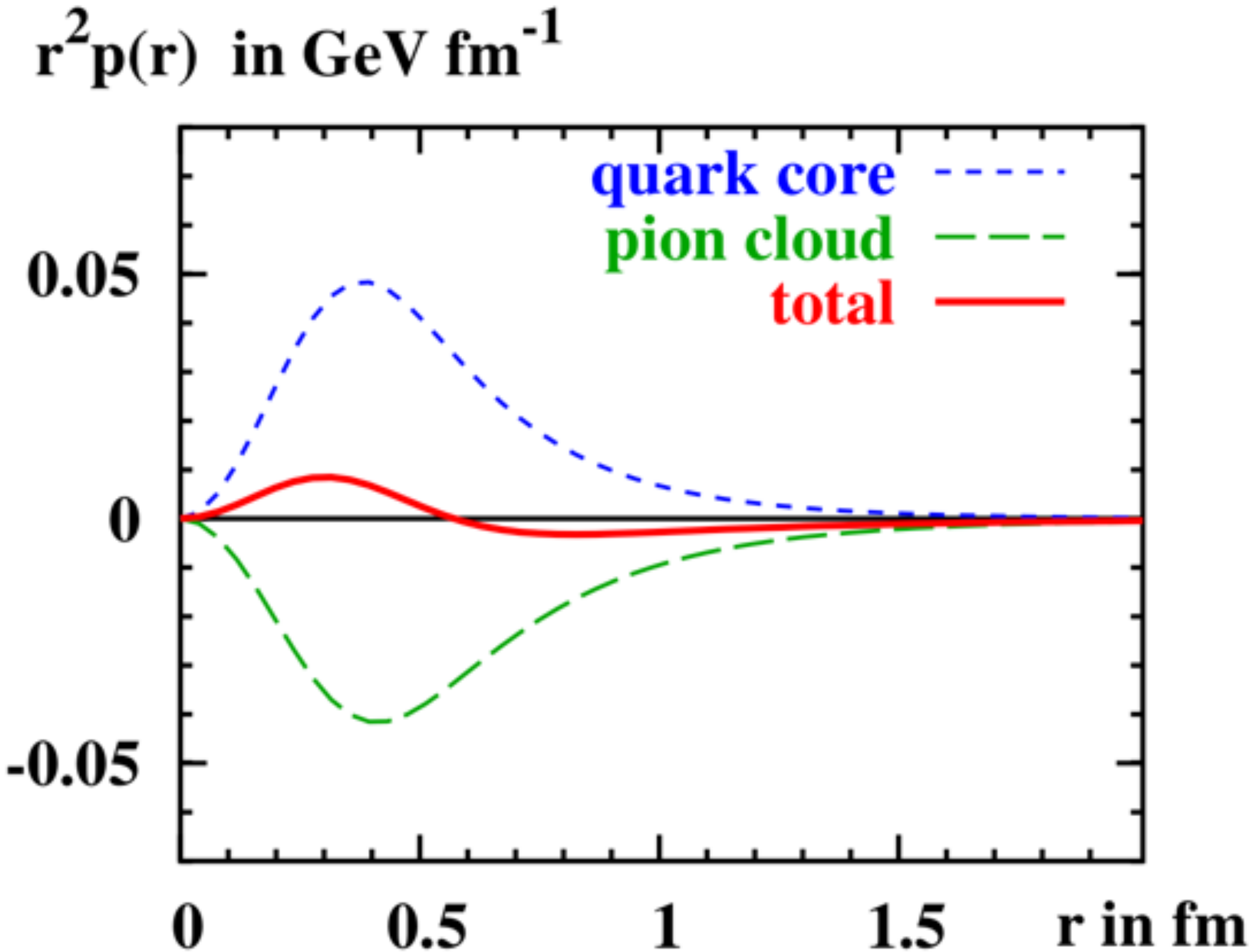
$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + \boxed{p(r)}\,\delta_{ij}$$

$$\boxed{\int_0^\infty dr\, r^2 p(r) = 0}$$

: Stability condition of the nucleon

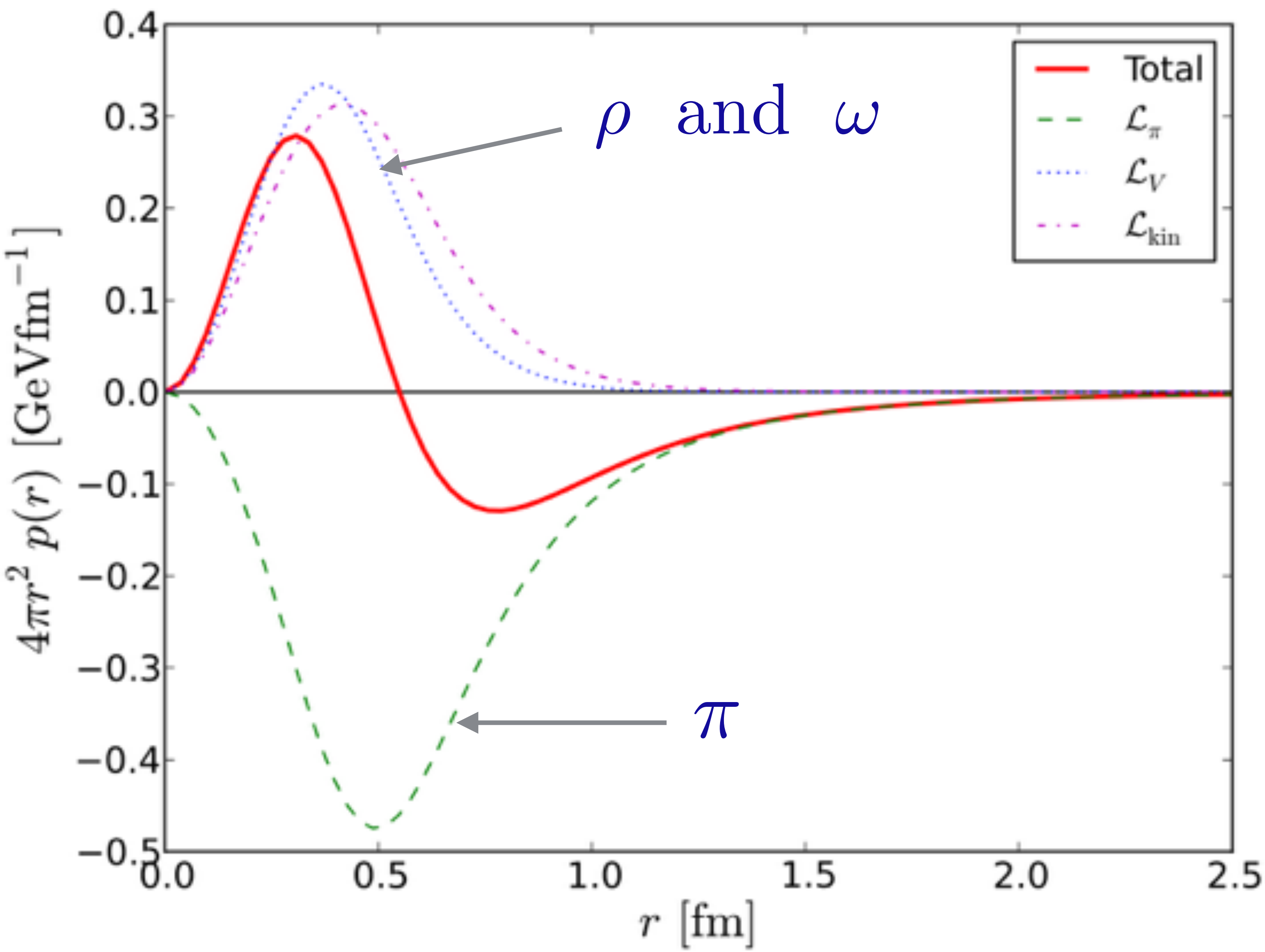
Any model for the nucleon should satisfy this condition!

Stability of the nucleon: XQSM



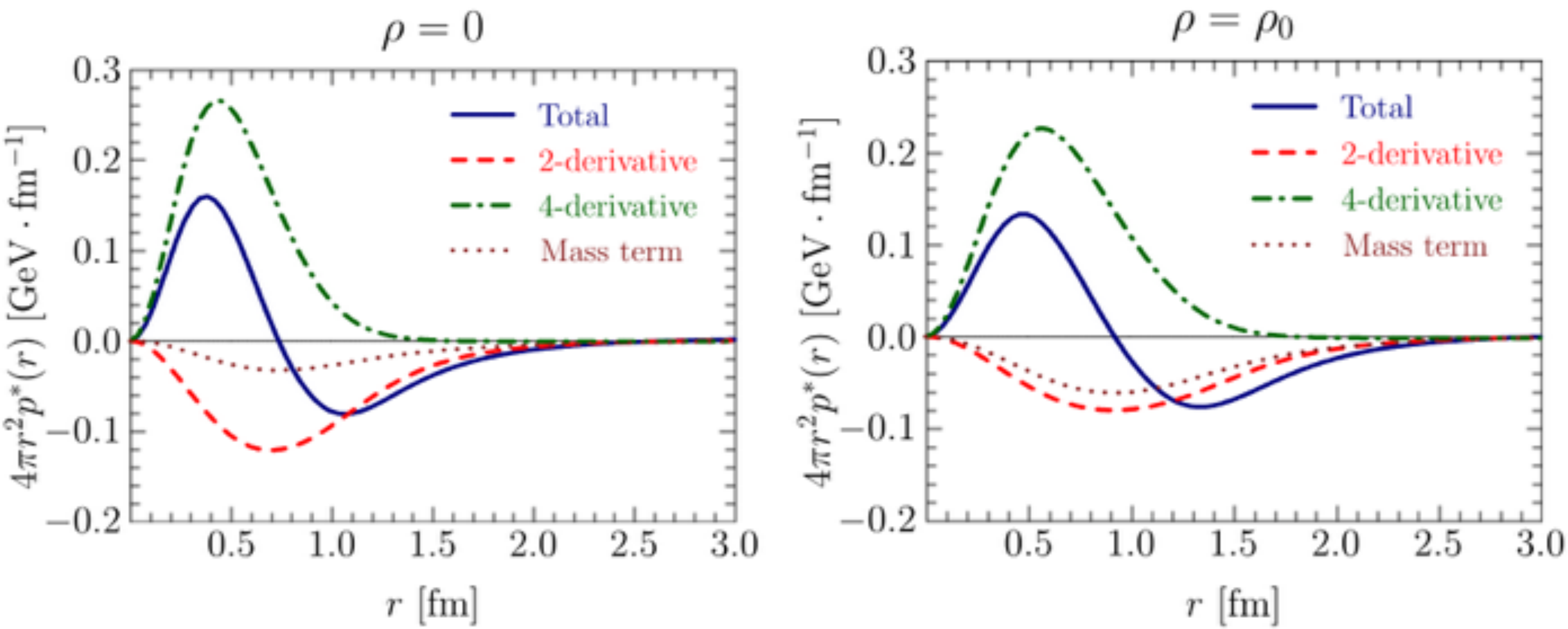
Goeke et al, PRD 75, 094021

Stability of the nucleon: pi-rho-omega model



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

Stability of the nucleon: Skyrme



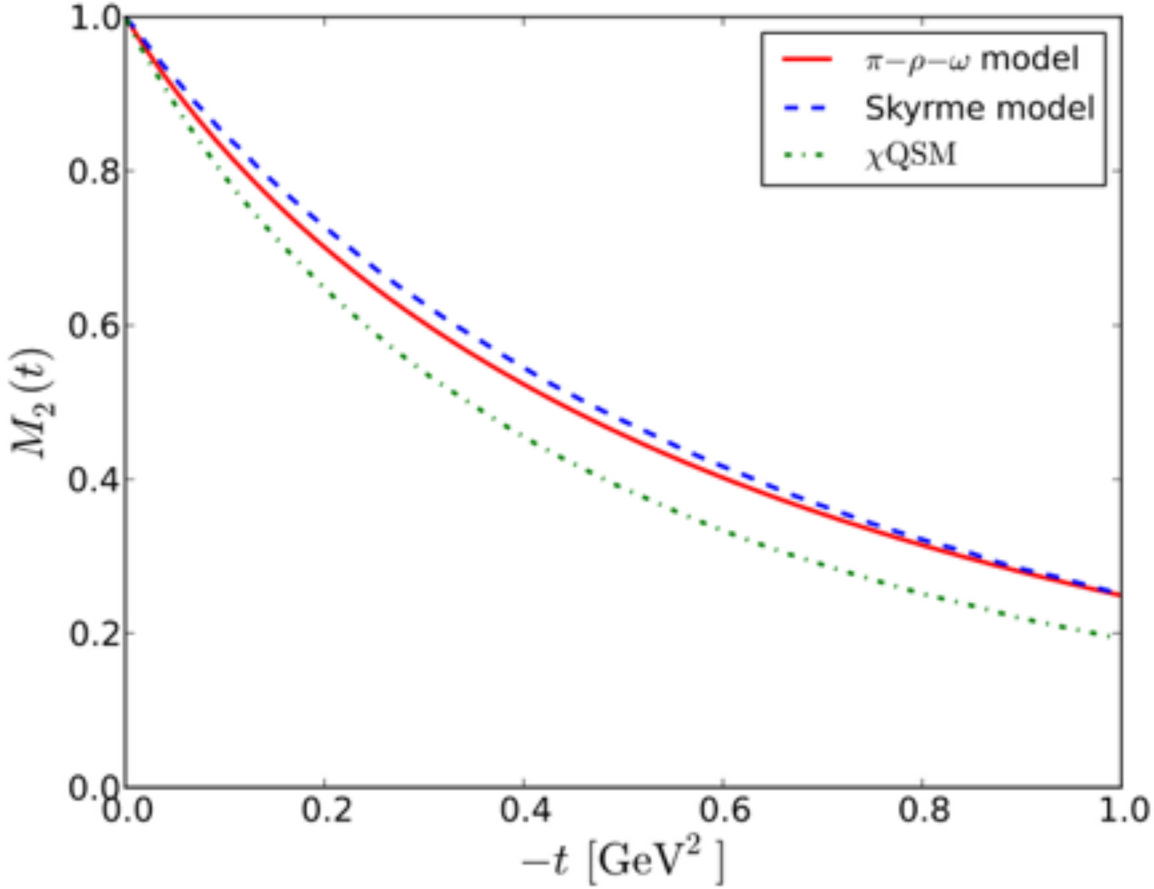
It is quite nontrivial to satisfy the stability of the nucleon!

HChK et al, PLB 718, 625 (2012)

EMT form factors: Results

Mass form factors

$$M_2(t) = \frac{1}{M_{\rm sol}} \int d^3r\, T_{00}(r)\, j_0(r\sqrt{-t}) - \frac{t}{5M_{\rm sol}^2} d_1(t)$$

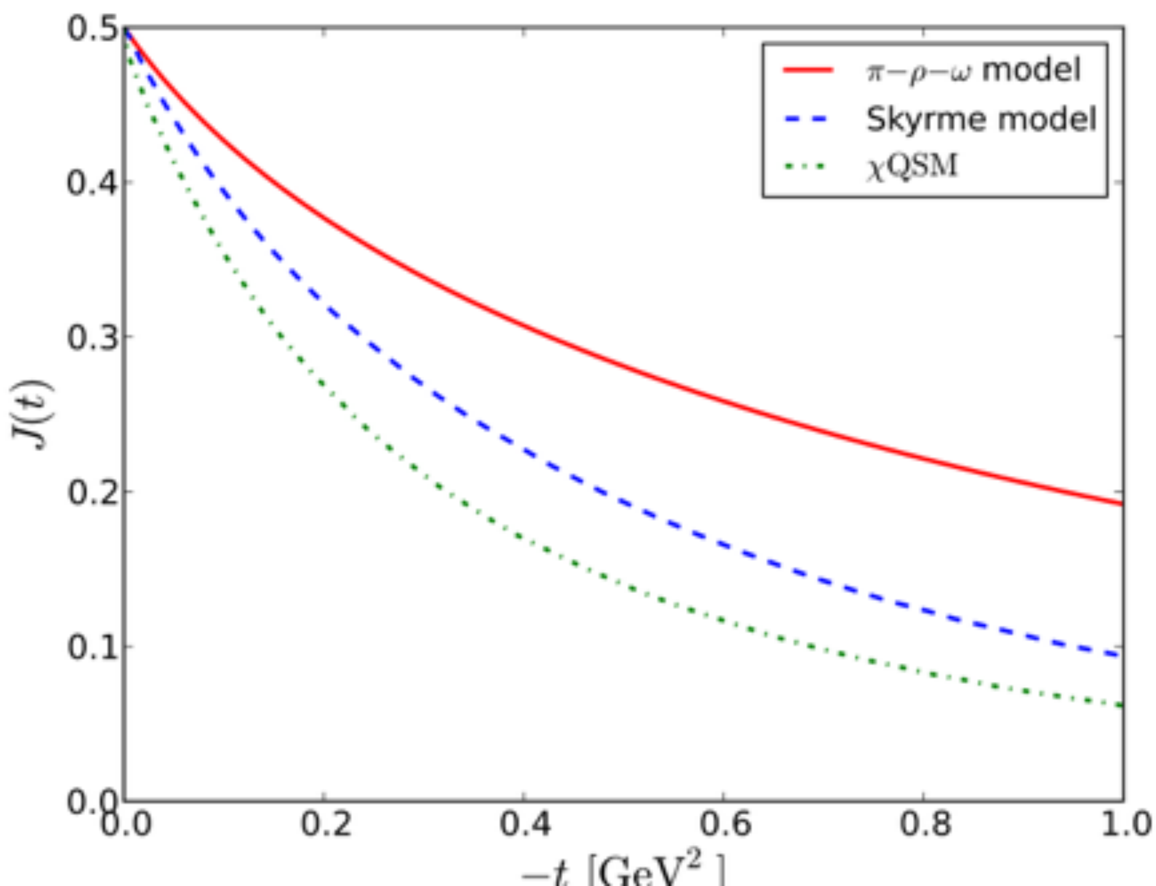


J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

EMT form factors: Results

Spin form factors

$$J(t) = 3\int \mathrm{d}^3r\,\rho_J(r)\,\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \qquad T^{0i}(\vec{r},\vec{s}) = \frac{\mathrm{e}^{ilm}r^l s^m}{(\vec{s}\times\vec{r})^2}\,\rho_J(r)$$



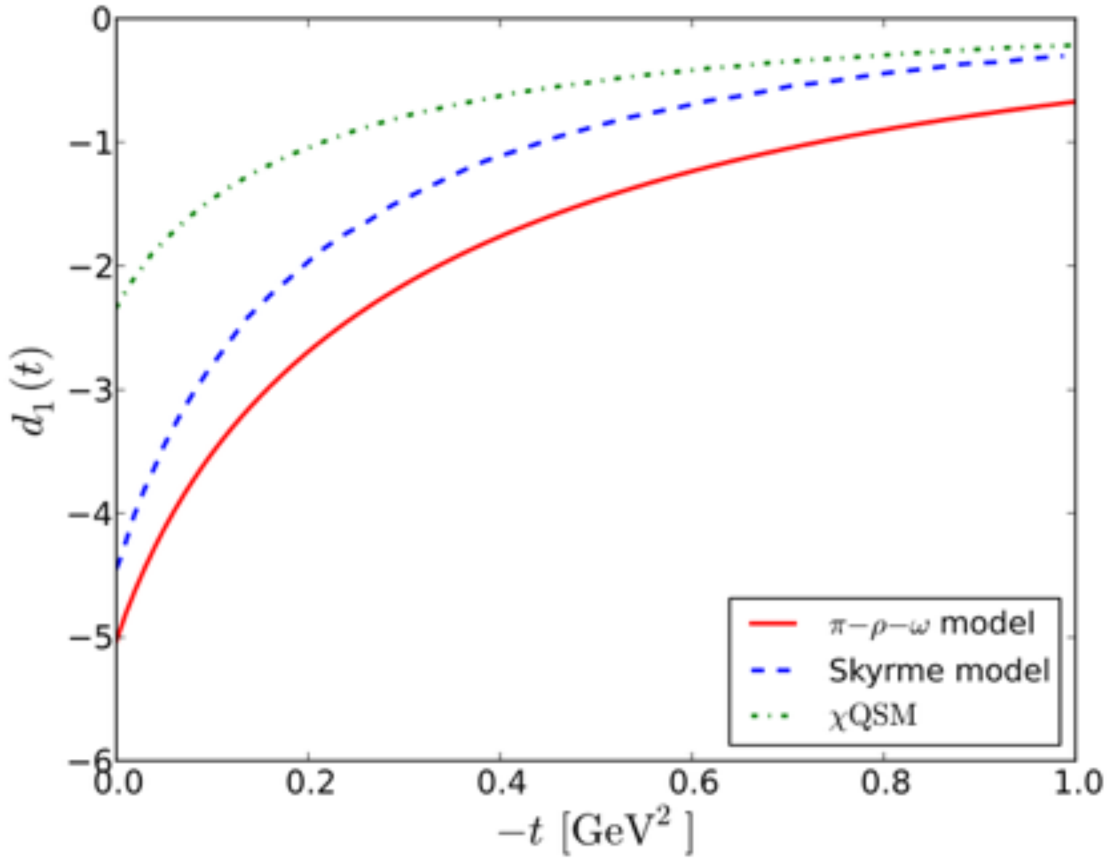
J. Jung, U. Yakhshiev, HChK

EMT form factors: Results

d1 form factors

$$d_1(t) = \frac{15M_{\rm sol}}{2}\int \mathrm{d}^3r\; p(r)\;\frac{j_0(r\sqrt{-t})}{t}$$

$$T^{ij}(r) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

EMT form factors: Results

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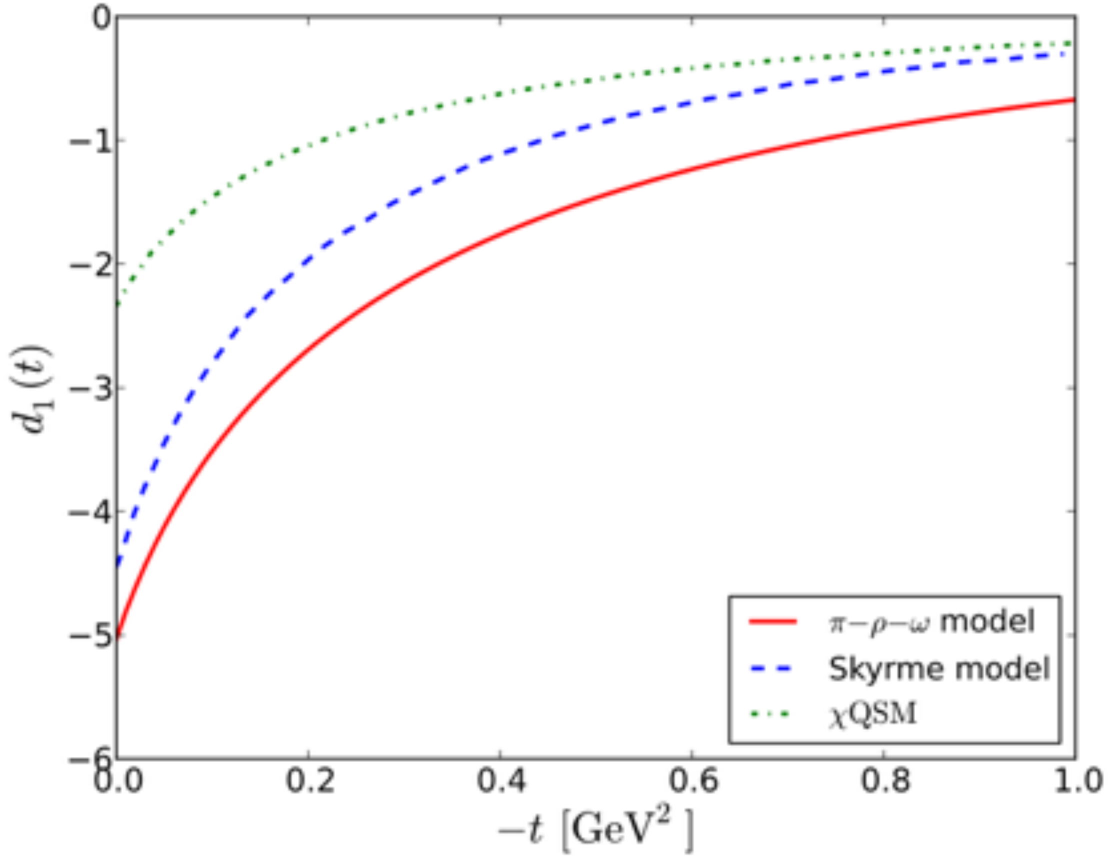
$$T^{ij}(r) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

$$\boxed{d_1 < 0}$$

To secure the stability of a particle

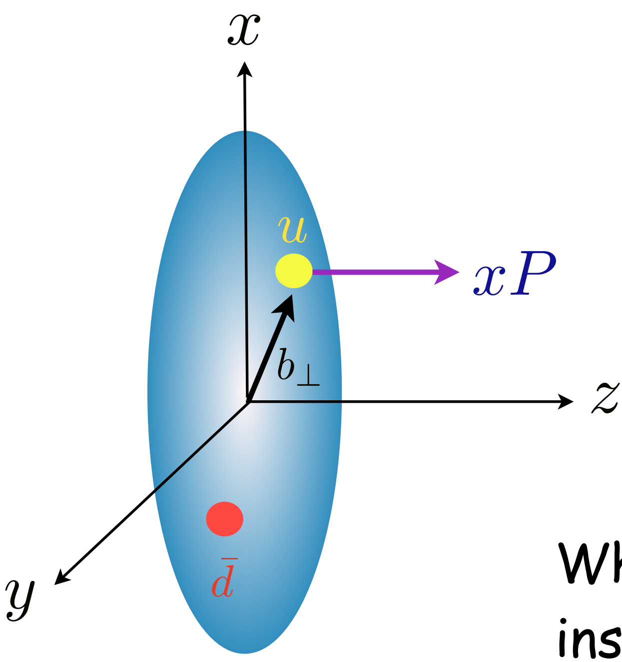


J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

Transverse spin structure of the pion

The spin structure of the Pion

Vector & Tensor Form factors of the pion



Pion: Spin S=0

Longitudinal spin structure is trivial.

$$\langle \pi(p')|\bar{\psi}\gamma_3\gamma^5\psi|\pi(p)\rangle = 0$$

What about the transversely polarized quarks inside a pion?

→ Internal spin structure of the pion

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Spin probability densities in the transverse plane

A_{n0}: Vector densities of the pion, B_{n0}: Tensor densities of the pion

$$\int_{-1}^{1} dx\, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2)$$

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

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Vector and Tensor form factors of the pion

$$\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)$$

$$\langle \pi^+(p_f) | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(p_i) \rangle = \mathcal{AS}\left[\frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\text{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} B_{ni}(Q^2)\right]$$

Nonlocal chiral quark model

Gauged Effective Nonlocal Chiral Action

$$S_{\rm eff} = -N_c \mathrm{Tr}\ln\left[i\not{D} + im + i\sqrt{M(iD,m)}U^{\gamma_5}\sqrt{M(iD,m)}\right]$$

$$D_\mu = \partial_\mu - i\gamma_\mu V_\mu$$

The nonlocal chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- "Derived" from QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter!

$\rho \approx 0.3\,\mathrm{fm},\ R \approx 1\,\mathrm{fm}$

Dilute instanton liquid ensemble

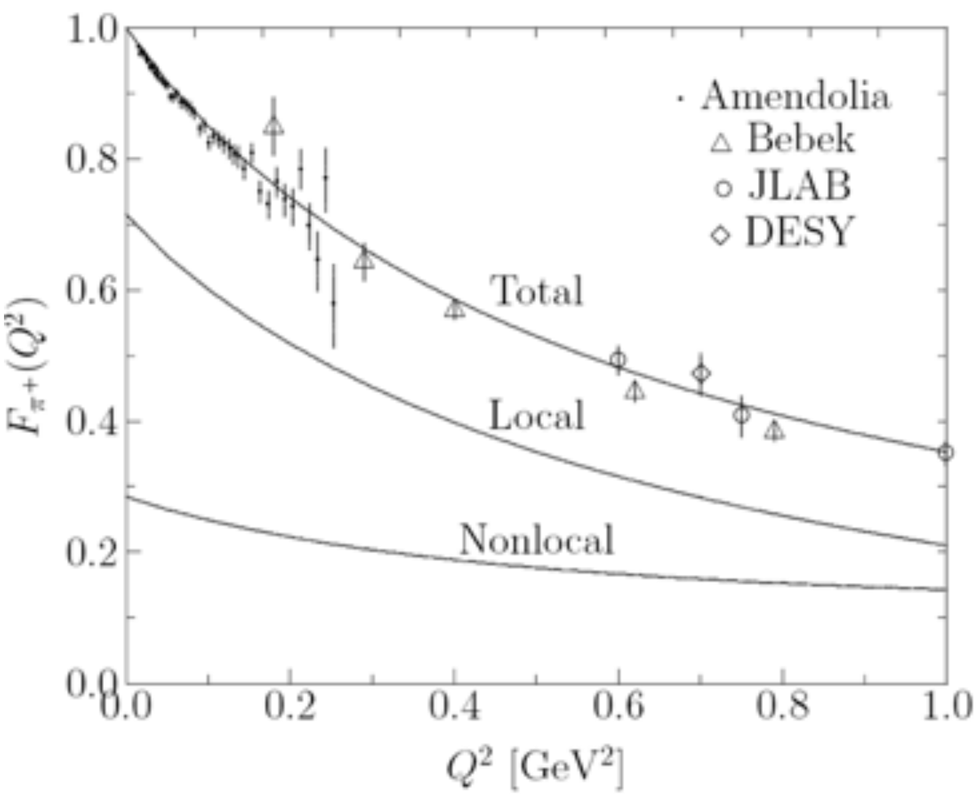
$\mu \approx 600\,\mathrm{MeV}$

D. Diakonov & V. Petrov Nucl.Phys. B272 (1986) 457
H.-Ch.K, M. Musakhanov, M. Siddikov Phys. Lett. B **608**, 95 (2005).
Musakhanov & H.-Ch. K, Phys. Lett. B **572**, 181-188 (2003)

EM Form factor of the pion

EM form factor (A_{10})

$$\langle\pi(p_f)|\psi^\dagger\gamma_\mu\hat{Q}\psi|\pi(p_i)\rangle = (p_i+p_f)A_{10}(q^2)$$



$$\sqrt{\langle r^2\rangle} = 0.675\,\mathrm{fm}$$

$$\sqrt{\langle r^2\rangle} = 0.672 \pm 0.008\,\mathrm{fm}\ (\mathrm{Exp})$$

$$F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1+Q^2/M^2}$$

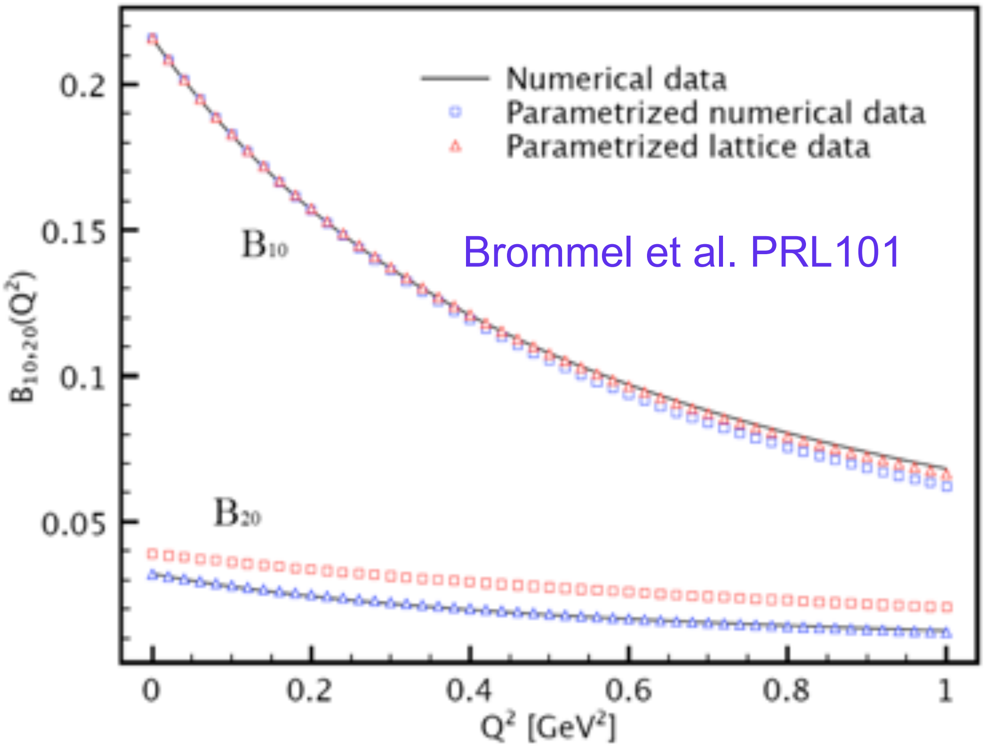
M(Phen.): 0.714 GeV

M(Lattice): 0.727 GeV

M(XQM): 0.738 GeV

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

Tensor Form factor of the pion



RG equation for the tensor form factor

$$B_{10}(Q^2,\,\mu) = B_{10}(Q^2,\,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma/2\beta_0}$$

$$\gamma_1 = 8/3,\ \gamma_2 = 8,\ \beta_0 = 11N_c/3 - 2N_f/3$$

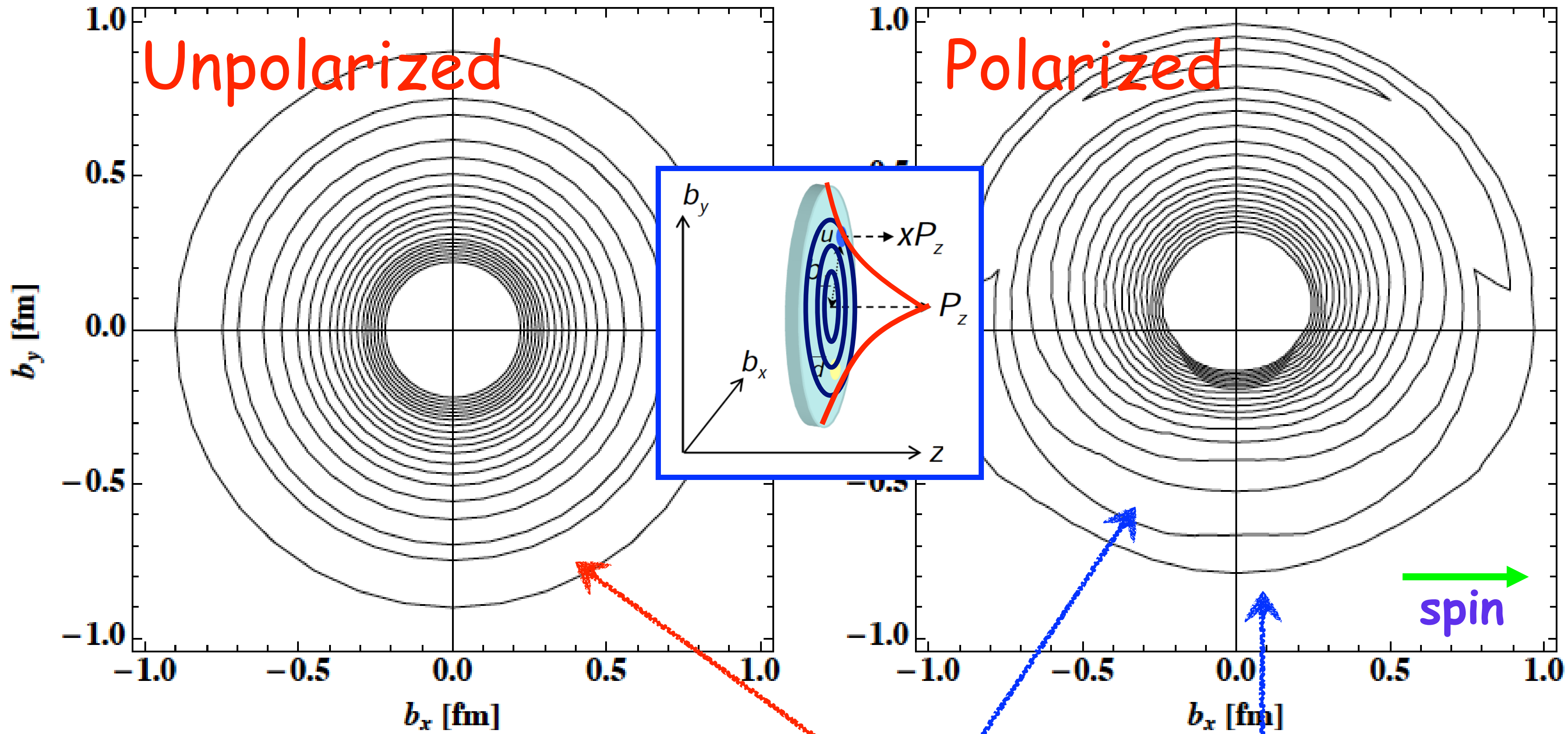
p-pole parametrization for the form factor

$$B_{10}(Q^2) \;=\; B_{10}(0)\left[1+\frac{Q^2}{pm_p^2}\right]^{-p}$$

S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

For the kaon, S.i. Nam & HChK, Phys. Lett. **B**707, 546 (2012)

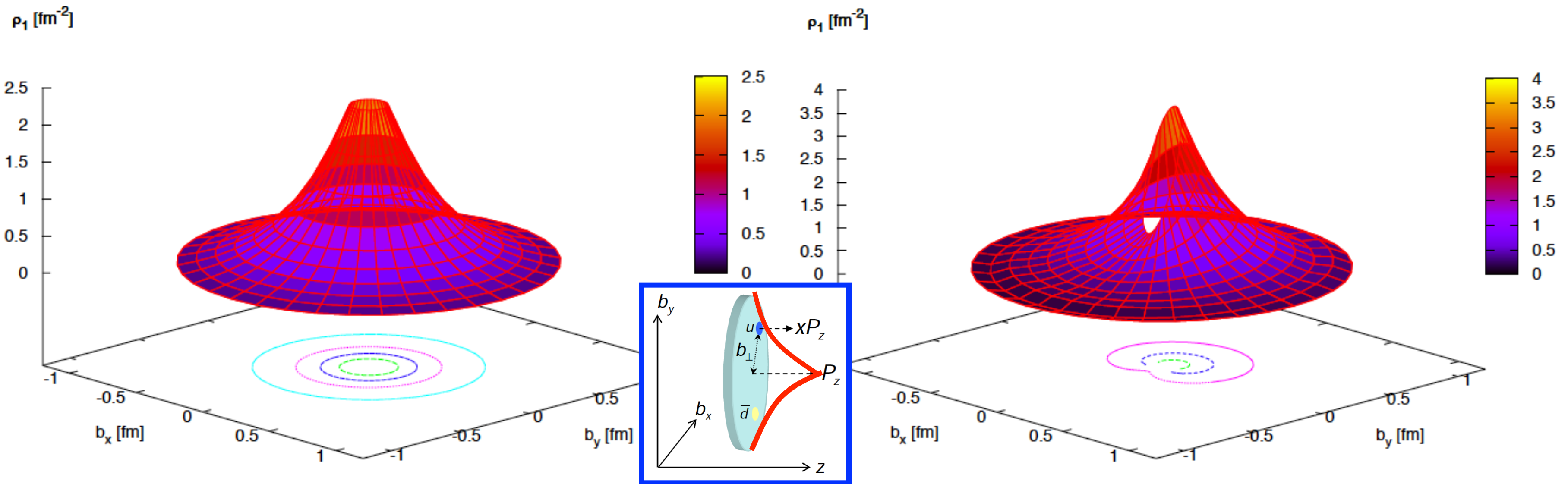
Spin density of the quark



$$\rho_1\left(b_\perp, s_x = \pm\frac{1}{2}\right) = \frac{1}{2}\left[A_{10}(b^2) \mp \frac{b\sin\theta}{m_\pi} B'_{10}(b^2)\right]$$

Polarization

Spin density of the quark



Significant distortion appears for the polarized quark!

$m_\pi = 140$ MeV	$B_{10}(0)$	m_{p_1} [GeV]	$\langle b_y \rangle$ [fm]	$B_{20}(0)$	m_{p_2} [GeV]
Present work	0.216	0.762	0.152	0.032	0.864
Lattice QCD [7]	0.216 ± 0.034	0.756 ± 0.095	0.151	0.039 ± 0.099	1.130 ± 0.265

Results are in a good agreement with the lattice calculation!

EMT form factors
stability of the pion

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab} H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, x H_\pi^{I=0}(x,\xi,t) = A_{20}(t) + 4\xi^2 A_{22}(t)$$

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_{\pi}^{I=0}(x,\xi,t) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle\pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, xH_{\pi}^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2\boxed{A_{22}(t)}$$: Generalized form factors of the pion

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t)=\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle\pi^a(p')|\bar\psi(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\,xH_\pi^{I=0}(x,\xi,t)=\boxed{A_{20}(t)}+4\xi^2\boxed{A_{22}(t)}$$: Generalized form factors of the pion

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle\pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle=\frac{\delta^{ab}}{2}\left[(tg_{\mu\nu}-q_\mu q_n u)\Theta_1(t)+2P_\mu P_\nu\Theta_2(t)\right]$$

$$T_{\mu\nu}(x)=\frac{1}{2}\bar\psi(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$: QCD EMT operator for the quark part

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle\pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, xH_\pi^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2\boxed{A_{22}(t)}$$
: Generalized form factors of the pion

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle\pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_n u)\boxed{\Theta_1(t)} + 2P_\mu P_\nu\boxed{\Theta_2(t)}\right]$$

EMTFFs (Gravitational FFs)

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$
: QCD EMT operator for the quark part

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)}\langle \pi^a(p')|\bar\psi(-\lambda n/2)\not n[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, x H_\pi^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2 \boxed{A_{22}(t)}$$: Generalized form factors of the pion

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}[(tg_{\mu\nu} - q_\mu q_n u)\boxed{\Theta_1(t)} + 2P_\mu P_\nu \boxed{\Theta_2(t)}]$$

EMTFFs (Gravitational FFs)

$$T_{\mu\nu}(x) = \frac{1}{2}\bar\psi(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$: QCD EMT operator for the quark part

$$\boxed{\Theta_1 = -4A_{22}^{I=0},\ \Theta_2 = A_{20}^{I=0}}$$

Stability of the pion

Time component of the EMT matrix element gives the pion mass.

$$\langle \pi^a(p)|T_{44}(0)|\pi^b(p)\rangle\big|_{t=0} = -2m_\pi^2\Theta_2(0)\delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\big|_{t=0} = \frac{3}{2}\delta^{ab}t\,\Theta_1(t)\Big|_{t=0}$$

Zero in the chiral limit

$$\mathcal{P} = \langle \pi^a(p)|T_{ii}(0)|\pi^a(p)\rangle$$
$$= \boxed{\frac{12N_c mM}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2}} + \boxed{\frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}}$$

(Based on the local model)

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

$$\mathcal{P} = \frac{12N_c mM}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2} + \frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2 l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}$$

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\,\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

Quark condensate

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\,\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

Quark condensate

Pion decay constant

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

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Quark condensate

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

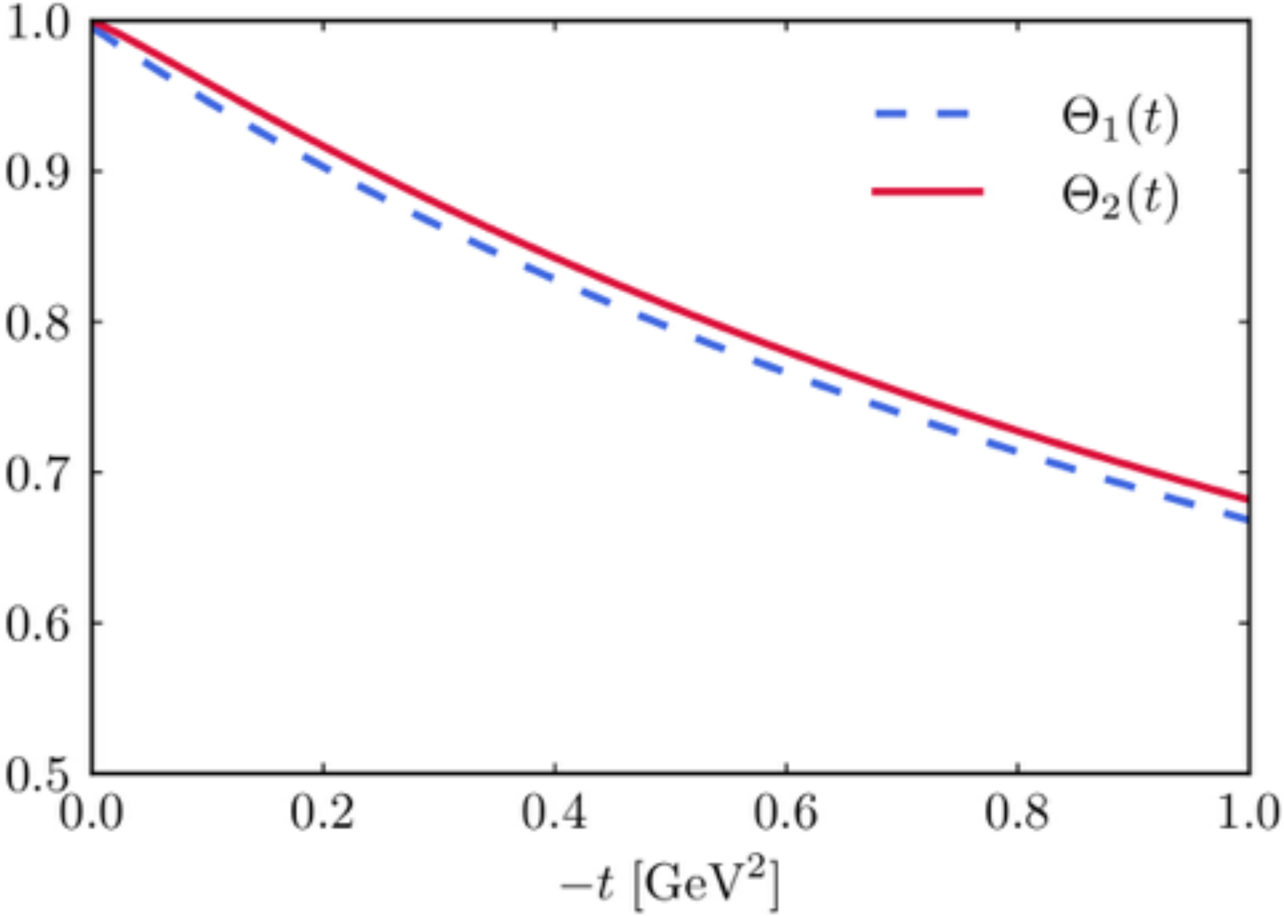
Pion decay constant

$$\mathcal{P} = \frac{3M}{f_\pi^2\overline{M}}\left(m\langle\bar{\psi}\psi\rangle + m_\pi^2 f_\pi^2\right) = 0\ !$$

by the Gell-Mann-Oakes-Renner relation to linear m order

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Energy-momentum Tensor FFs

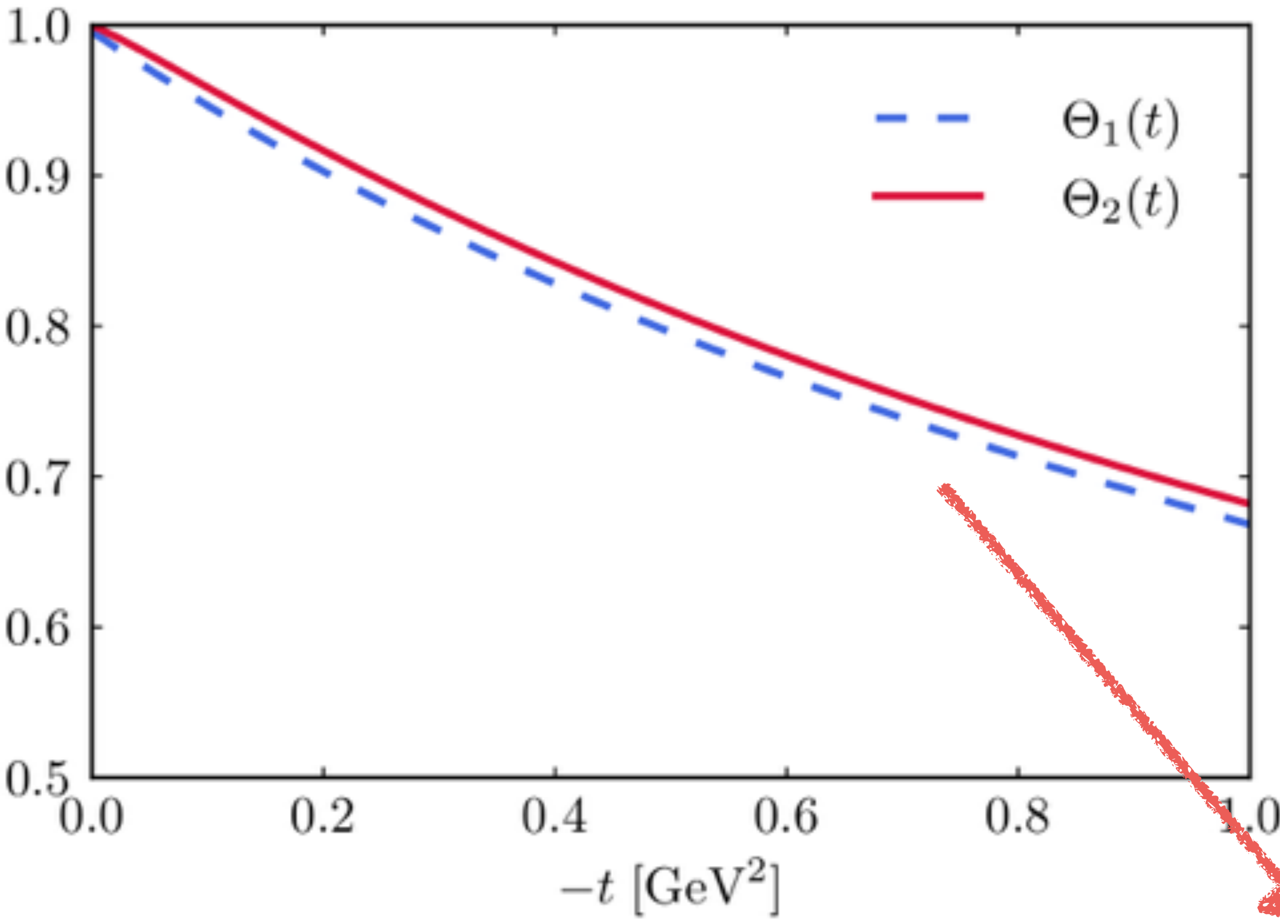


$$\Theta_1 = \Theta_2$$

in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Energy-momentum Tensor FFs



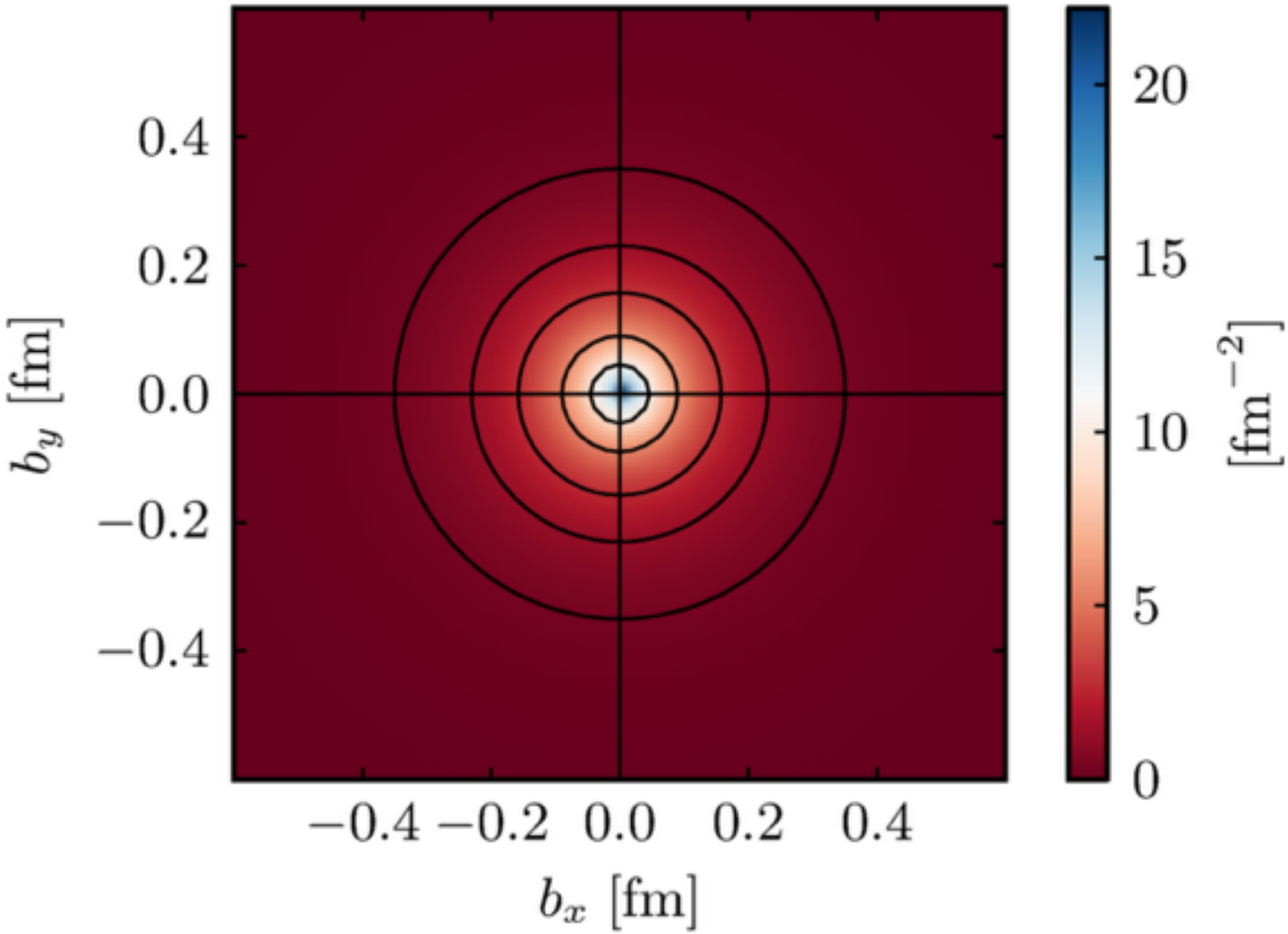
$$\Theta_1 = \Theta_2$$

in the chiral limit

The difference arises from the explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

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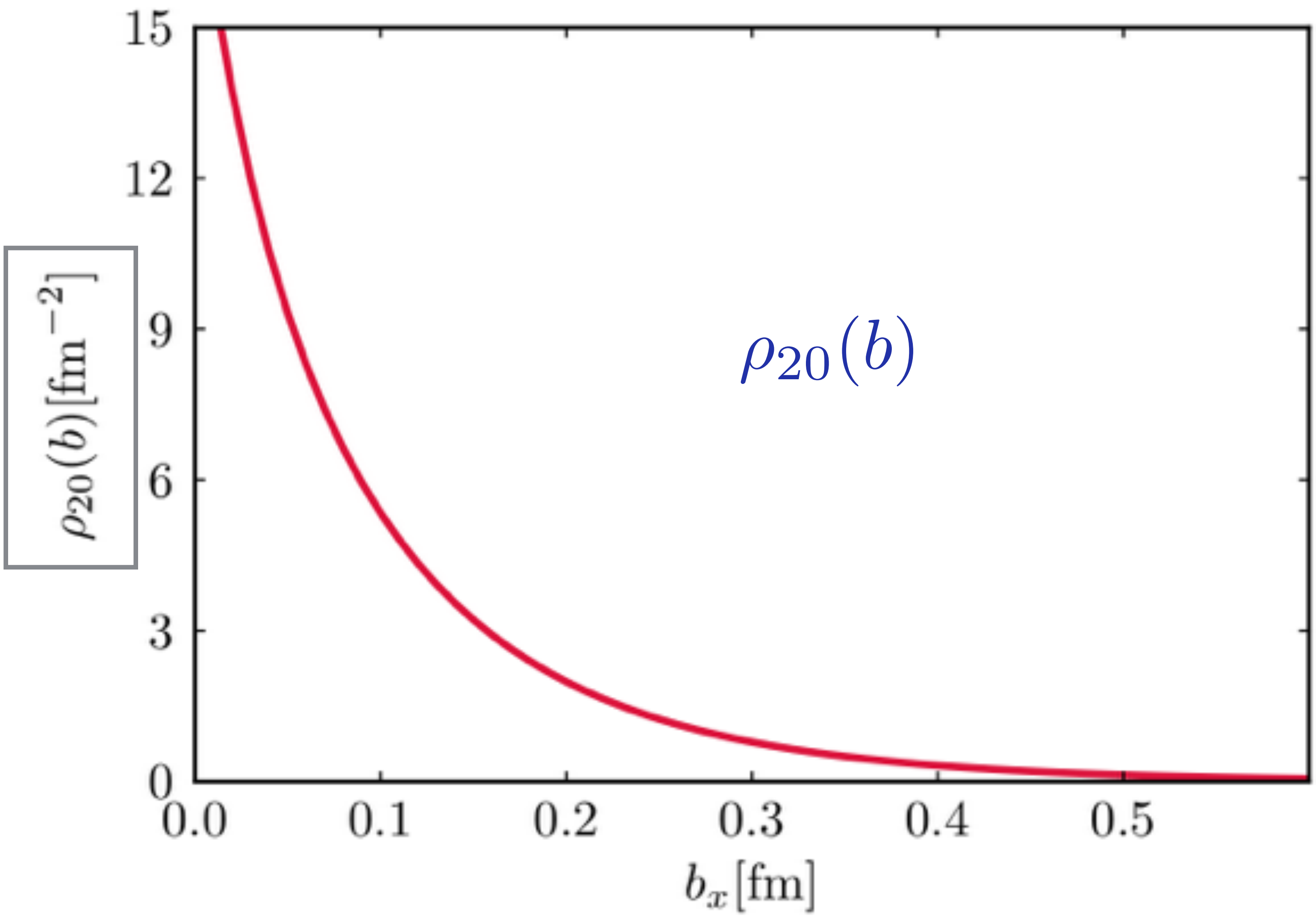
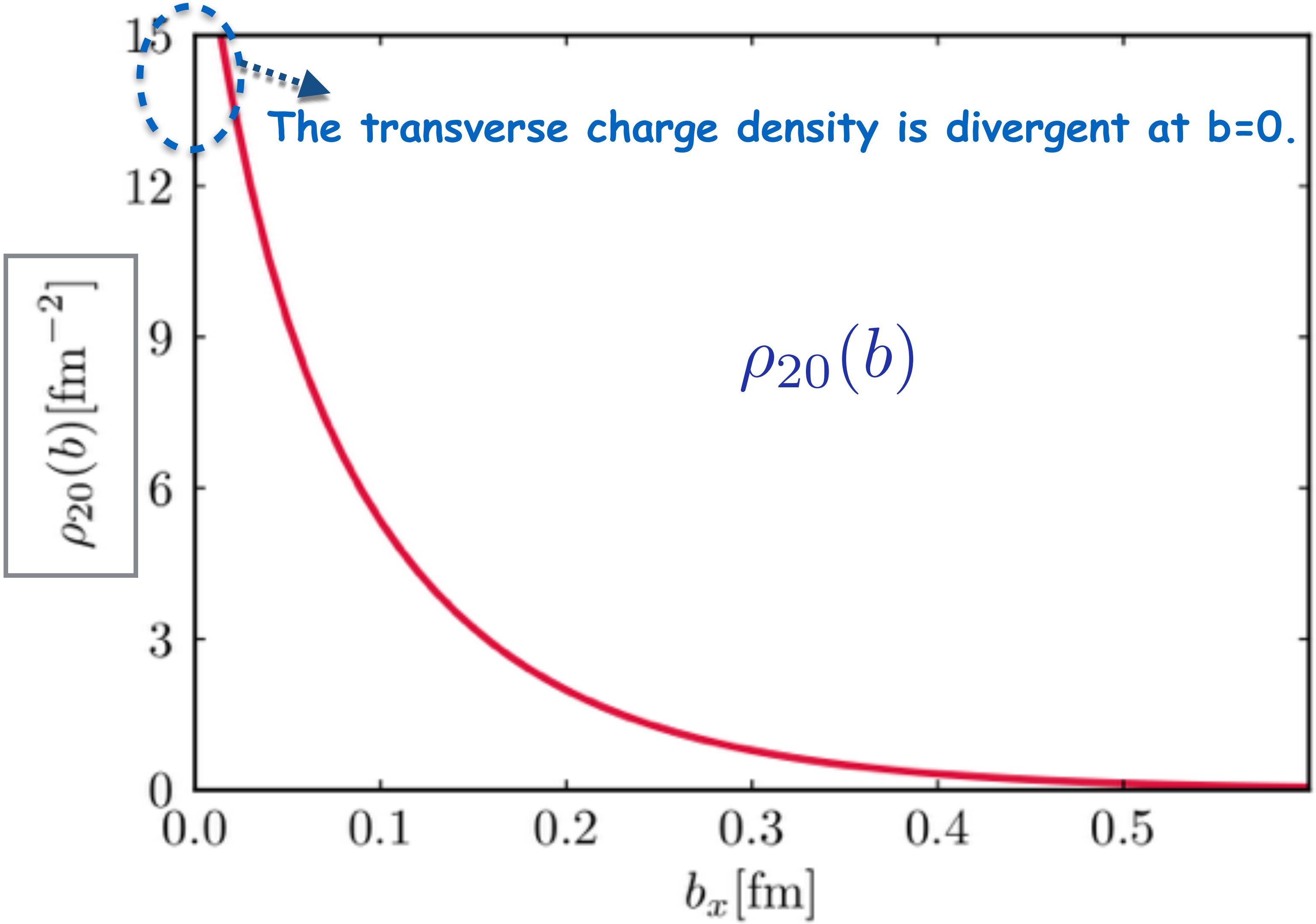


H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

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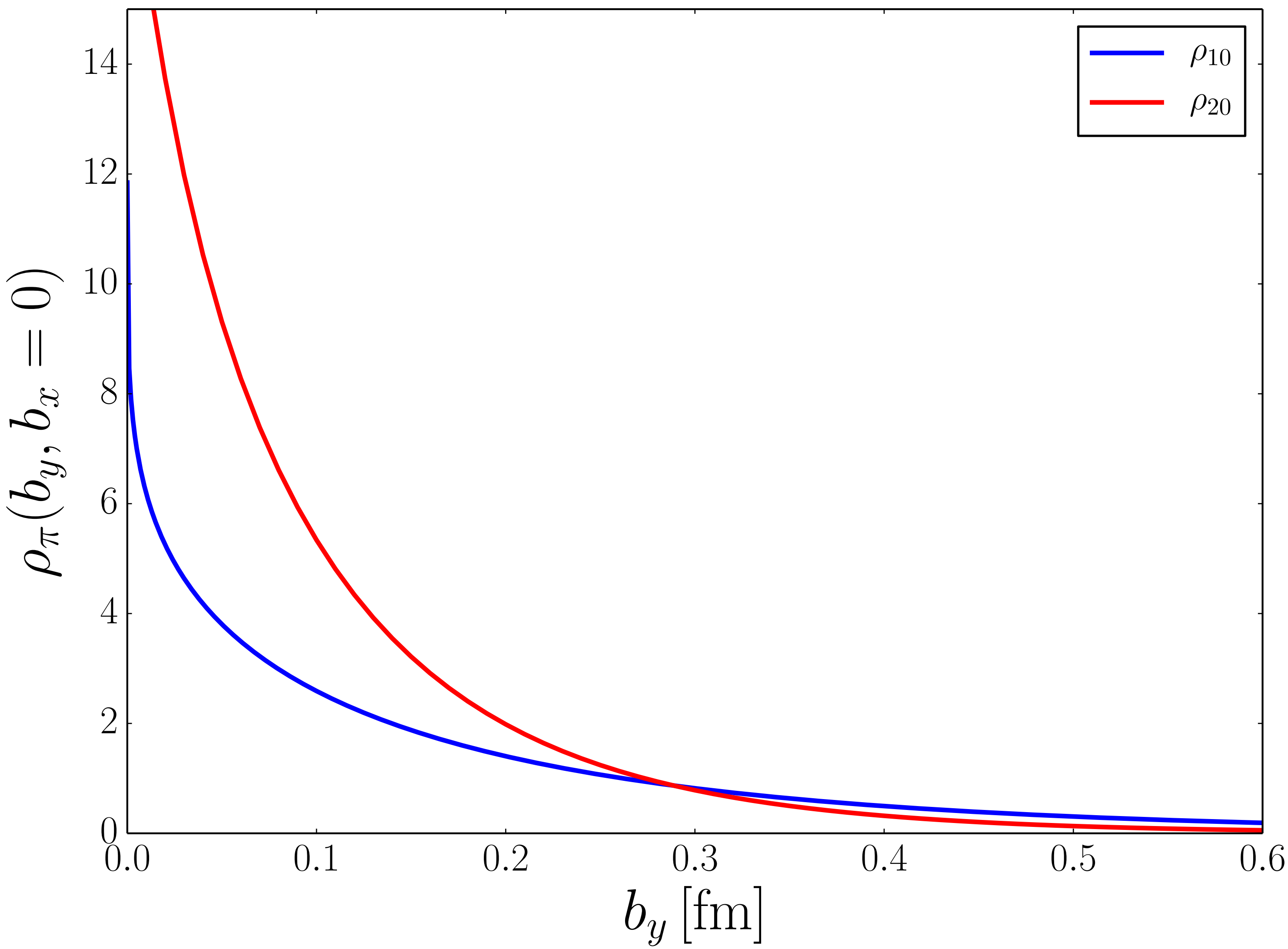

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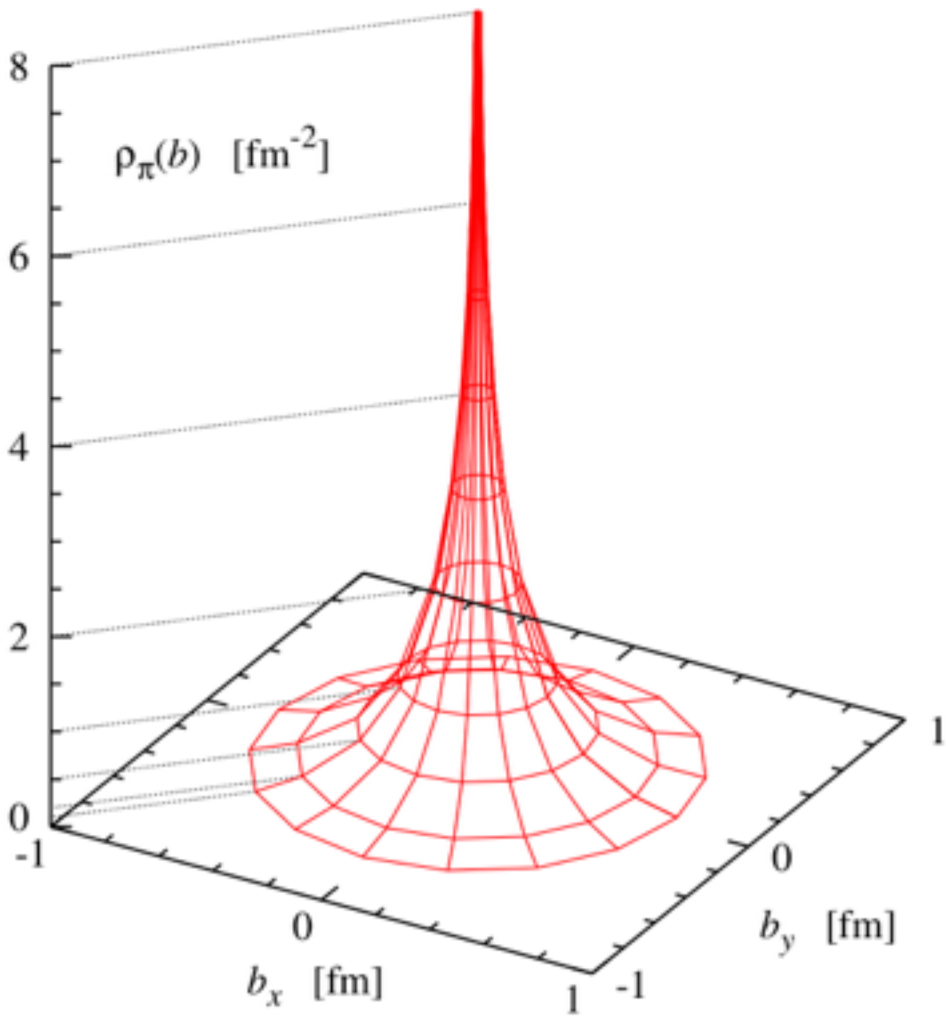
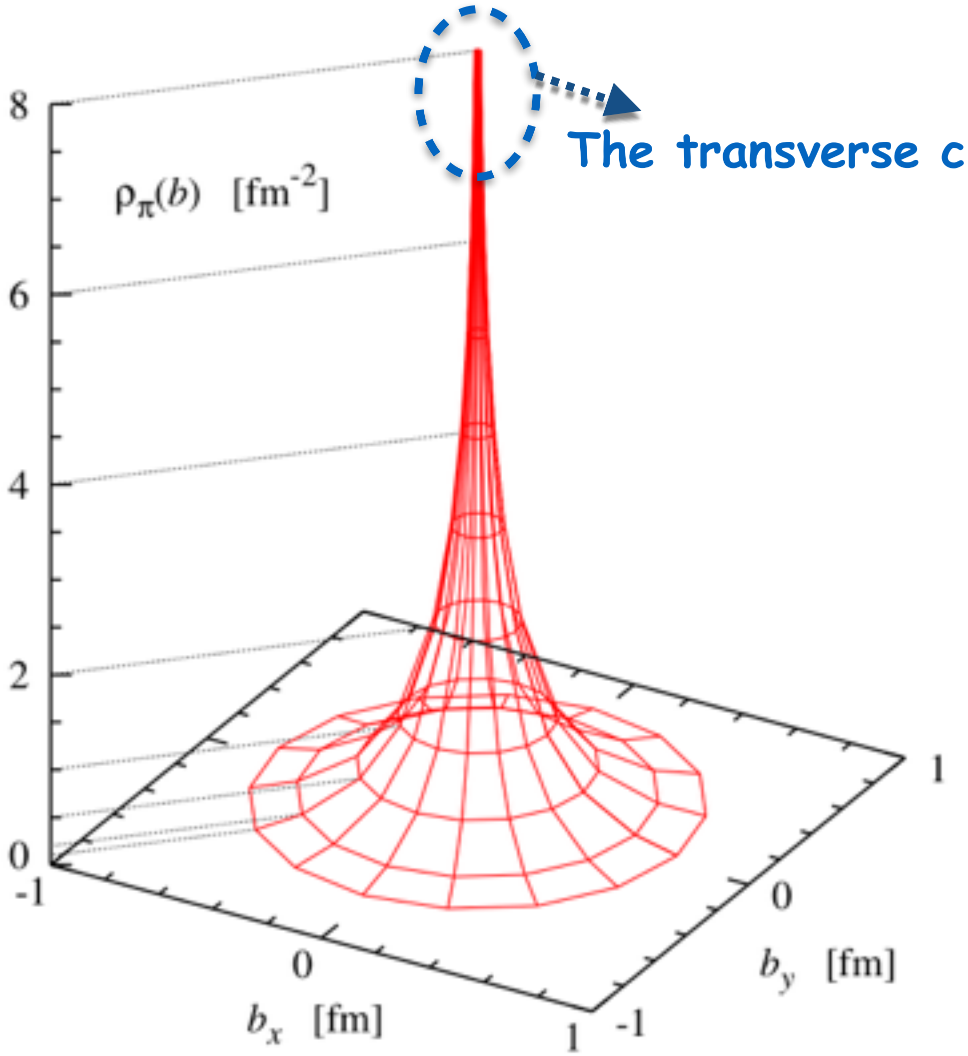


$$\rho_{10}(b)$$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

Transverse charge density of the pion



The transverse charge density is divergent at b=0.

$$\rho_{10}(b)$$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

Summary & Conclusion

Summary

- We have reviewed recent investigations on the charge and spin structures of the nucleon and the pion, based on the chiral quark(-soliton) model.

- We have derived the EM and tensor form factors of the nucleon, from which we have obtained its transverse charge & spin densities. The results are compared with the lattice and "experimental" data.

- We also showed the EMT form factors of the nucleon and the pion. The stabilities of the nucleon and the pion, which are quite nontrivial, were also discussed.

Outlook

- Weak GPDs and generalized form factors are under investigation (A relevant work will be published soon).

- The excited states for the nucleon and the hyperon can be investigated (Extension of the XQSM is under way).

- Internal structure of Heavy-light quark systems (Derivation of the Partition function is close to the final result.)

- New perspective on hadron tomography

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!