Exploring QCD Phase Diagram through Baryon-Multiplicity at Heavy Ion Collisions

Zn Collaboration
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Sept. 18, 2014
J-PARC

What is Z_N ?

Canonical Partition Function.

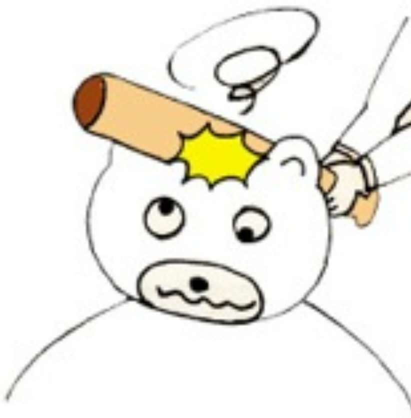
We will see it later.

Plan of the Talk

- Who is Nakamura ?
- Statistical Description of Fire-balls in A+A
  - Grand Canonical $Z_{GC}(T,\mu)$ vs Canonical $Z_C(T,n)$
- How to extract $Z_C(T,n)$ and how to use them.
  - Freese-out analysis and Net-Baryon Multiplicity
  - Lattice QCD
  - Moments
  - Lee-Yang zeros
- Summary with AA Collisions in J-PARC

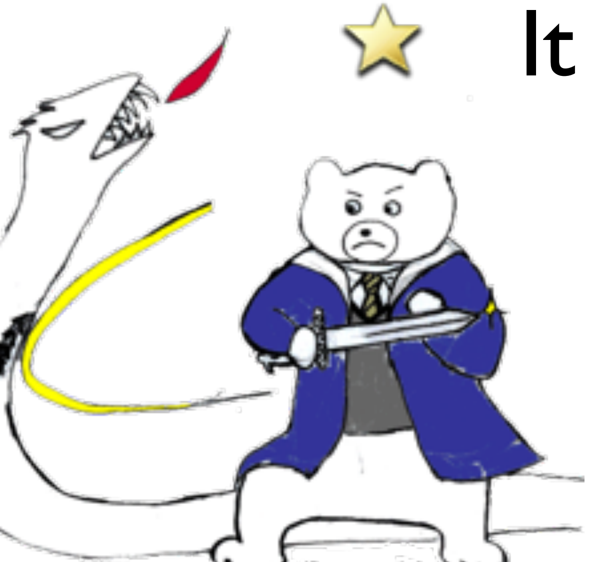
Self-Introduction (1)

- Born in 1949
- Theorist, but likes Experiment
  - Doctor thesis: High-Energy Hadron-Nucleus Interaction (Waseda Univ., 1979)
  - one experiment paper (gamma-deuteron at INS, Tokyo for Dibaryon search), PRL, 54, 1985



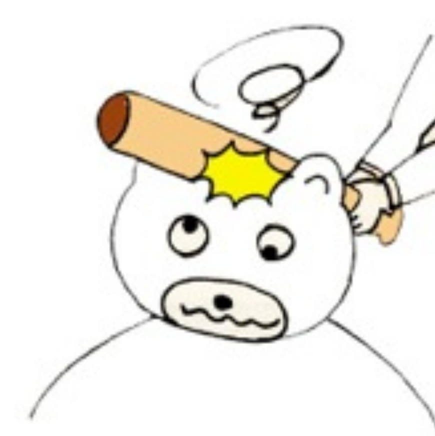
Self-Introduction (2)
Nakamura like to be an Early Trier
(in most case, too early,,,)

- Nakamura is the first person
  - to visit Bevatron as Japanese theoretical student, 1978, and to have strong impression from Nagamiya
  - to include dynamical fermions in lattice QCD, 1983
    - But, people said it is too expensive, and would have no future.
  - to calculate finite density QCD on lattice, 1984
    - It was color SU(2), and people said no physical interest.



Challenge !

Lose,,,



Self-Introduction (3)

- Nakamura is the first person
  - to win the Gordon-Bell prize as Japanese, 2000
    - ☆ But no one kew the Gordon-Bell prize in Japan at that time
      .
  - to calculate transport coefficients on lattice QCD, 2005
    - ☆ I compared it with AdS/CFT which has become famous.
  - to study Pan-flute in Europe as a Japanese
  - to play Pan-Flute at Yakushi-ji, 2012

薬師寺天武忌奉納演奏

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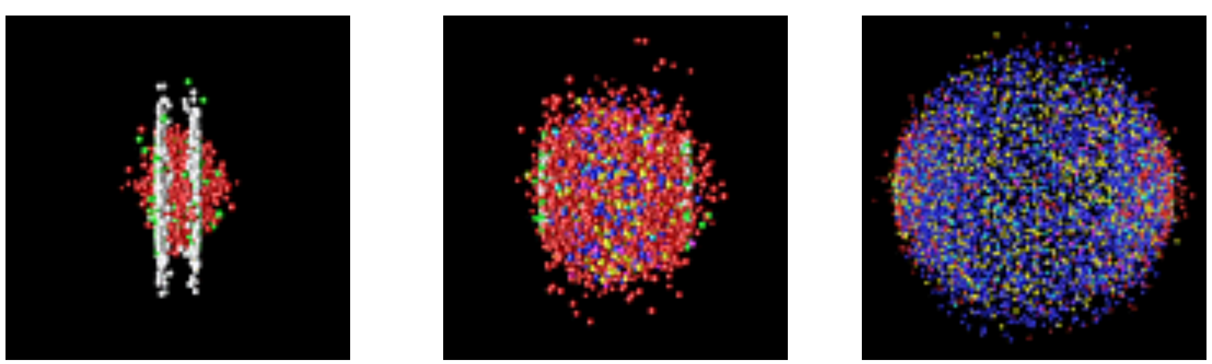
Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

with Two Parameters:

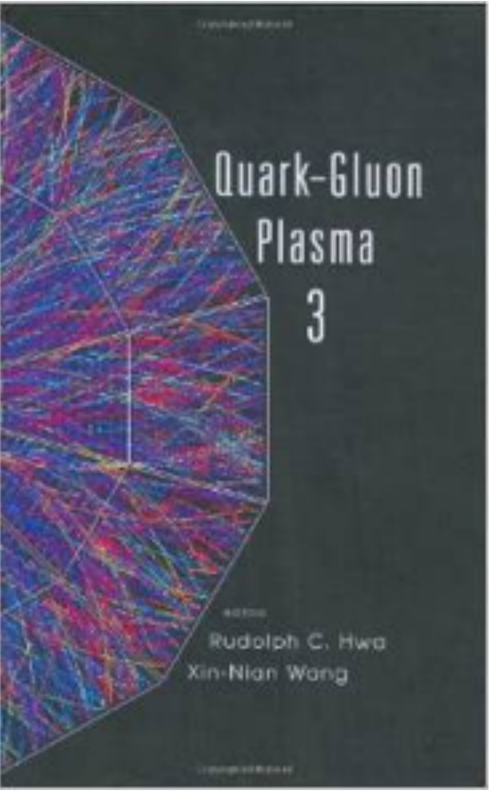
Chemical Potential, μ

and Temperature, T



$$Z(\mu, T)$$

Grand Canonical Partition Function



P. Braun-Munzinger , K. Redlich and J. Stachel
Quark Gluon Plasma 3, 491
arXiv:nucl-th/0304013

$$\ln Z(T,V,\vec{\mu}) = \sum_i \ln Z_i(T,V,\vec{\mu}),$$

$$\ln Z_i(T,V,\vec{\mu}) = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta\epsilon_i)],$$

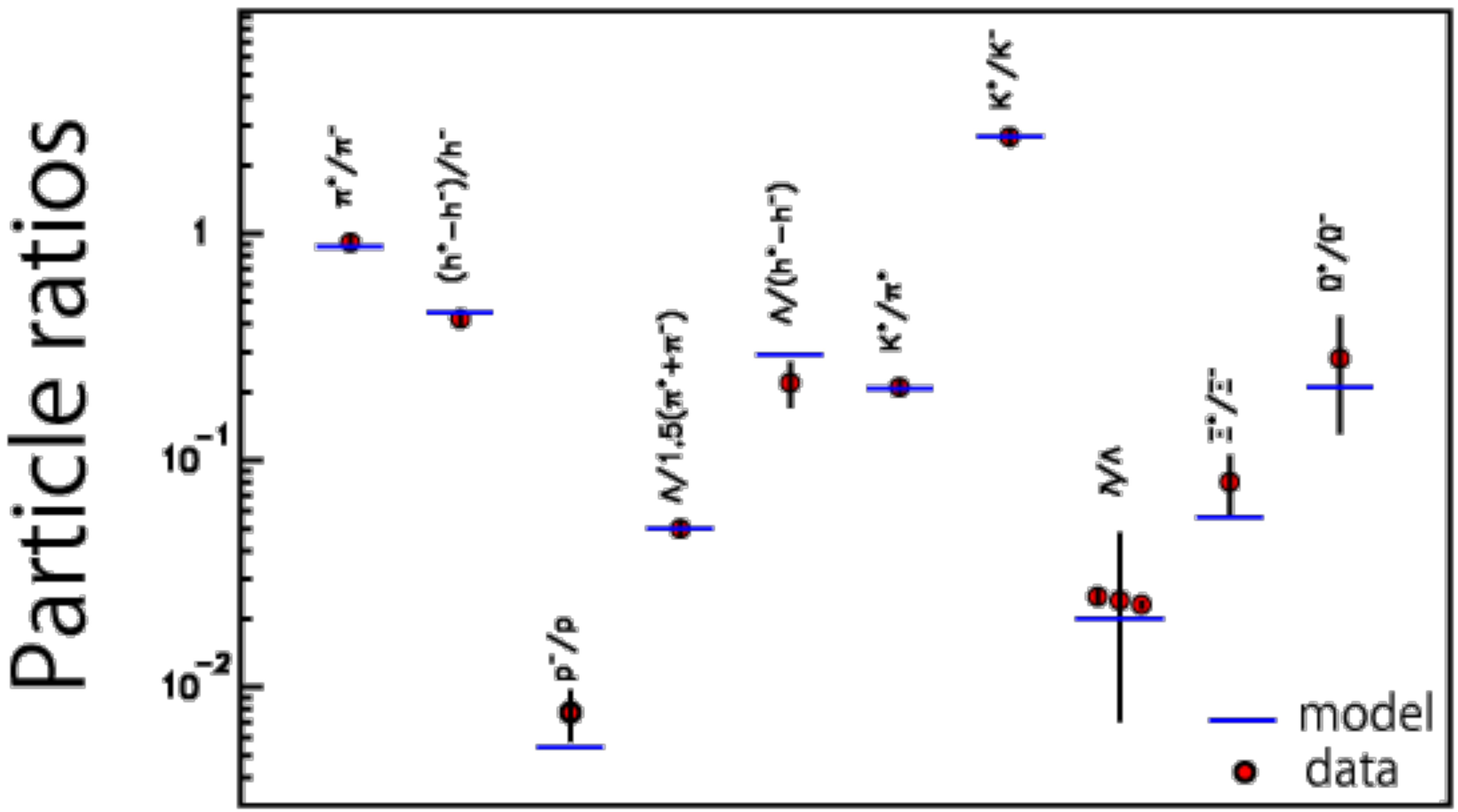
g_i spin--isospin degeneracy factor

(+) for fermions, (-) for bosons

$$\epsilon_i = \sqrt{p^2 + m_i^2}$$

$$\lambda_i(T,\vec{\mu}) = \exp(\frac{B_i\mu_B + S_i\mu_S + Q_i\mu_Q}{T})$$

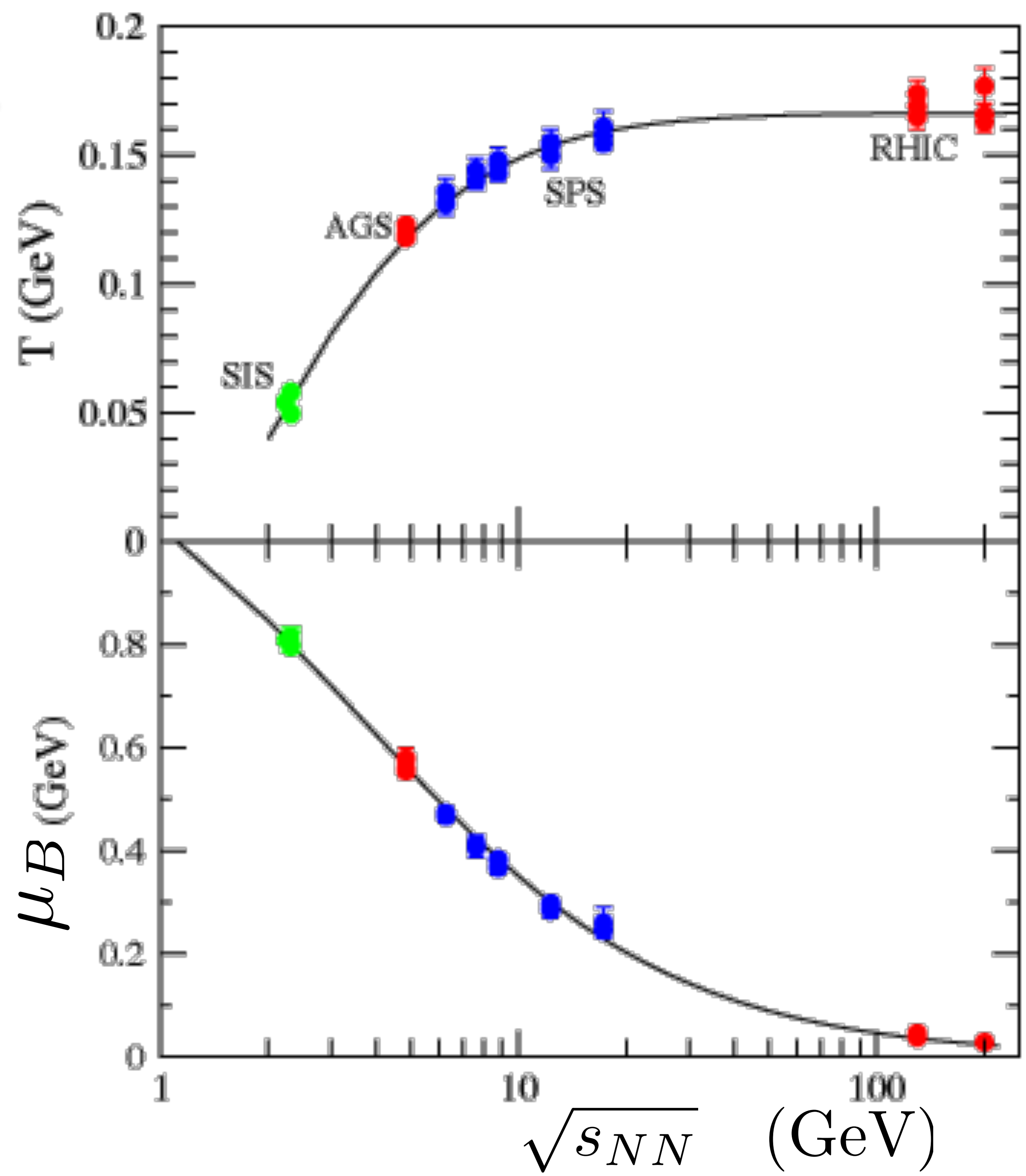
Parameters: T and μ

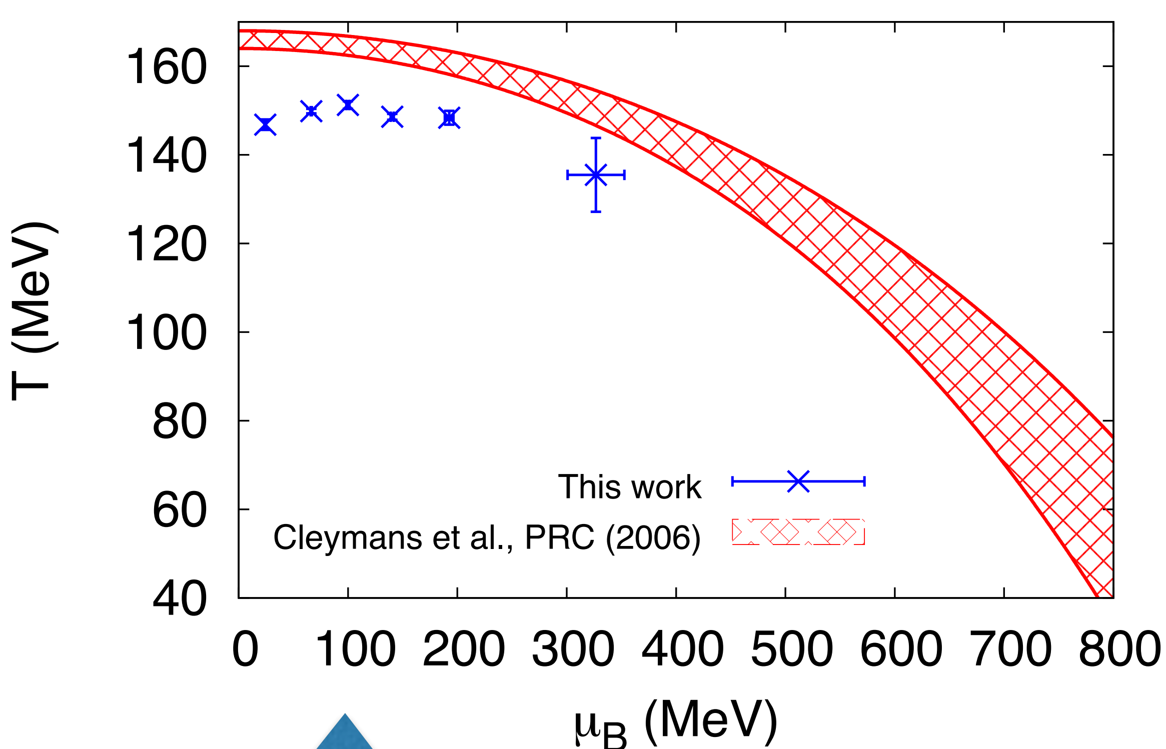


Pb–Pb collisions at 40 GeV/nucleon.
The thermal model calculations are obtained with T = 148 MeV and µB = 400 MeV

Freeze-out Analysis

J.Cleymans et al.,
Phys. Rev. C73, (2006) 034905.





Alba et al., arXiv:1403.4903

including also higher moments of multiplicities

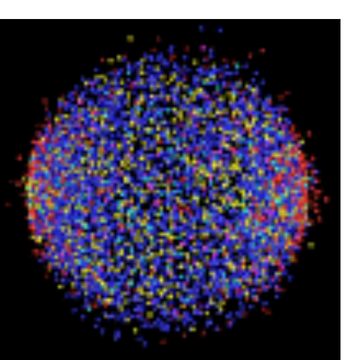
Statistical Description is good at least as a first approximation

with Two Parameters Chemical Potential, μ
and Temperature, T

$Z_{GC}(\mu, T)$ Grand Canonical Partition Function

Alternative: Number, n and Temperature, T

$Z_C(n, T)$ Canonical Partition Function



or Z_N

They are equivalent and related as

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

$$\xi \equiv e^{\mu/T}$$ Fugacity

Let us prove it !

$$Z(\mu, T) \Leftrightarrow Z_n(T)$$

Grand Canonical — Canonical

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n | e^{-(H-\mu\hat{N})/T} | n \rangle$$

$$= \sum_n \langle n | e^{-H/T} | n \rangle \, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T} \right)$$

Fugacity

This is very useful relation.

The partition function stands for the Probability

$$Z_{GC}(\mu, T) = \sum_n Z_n(T)\xi^n$$

System with μ and T

Probability to find (net-)baryon number= n

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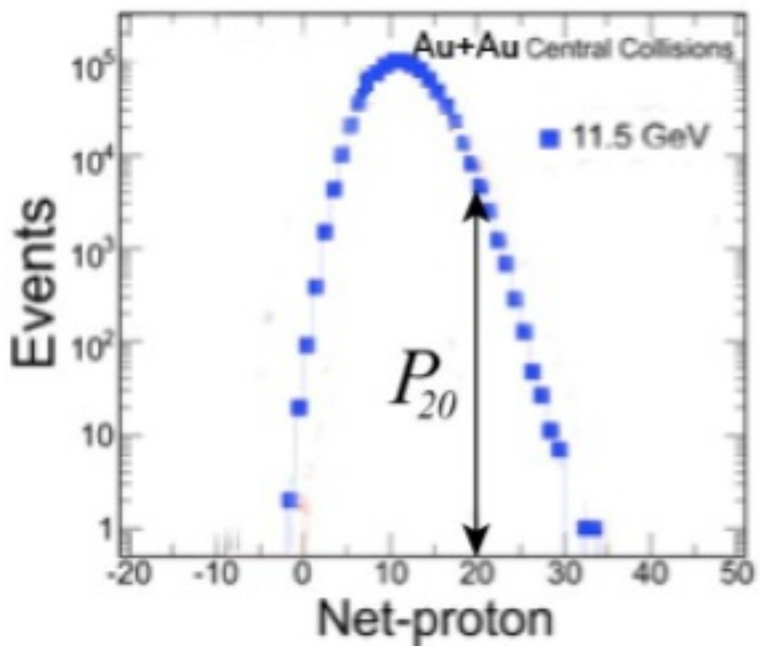
We extract Z_n from experimental multiplicity



$$P_n = Z_n \xi^n$$

$$\left(\xi \equiv e^{\mu/T} \right)$$

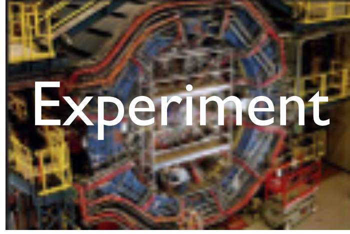
ξ unknown

$$Z_n = P_n / \xi^n$$

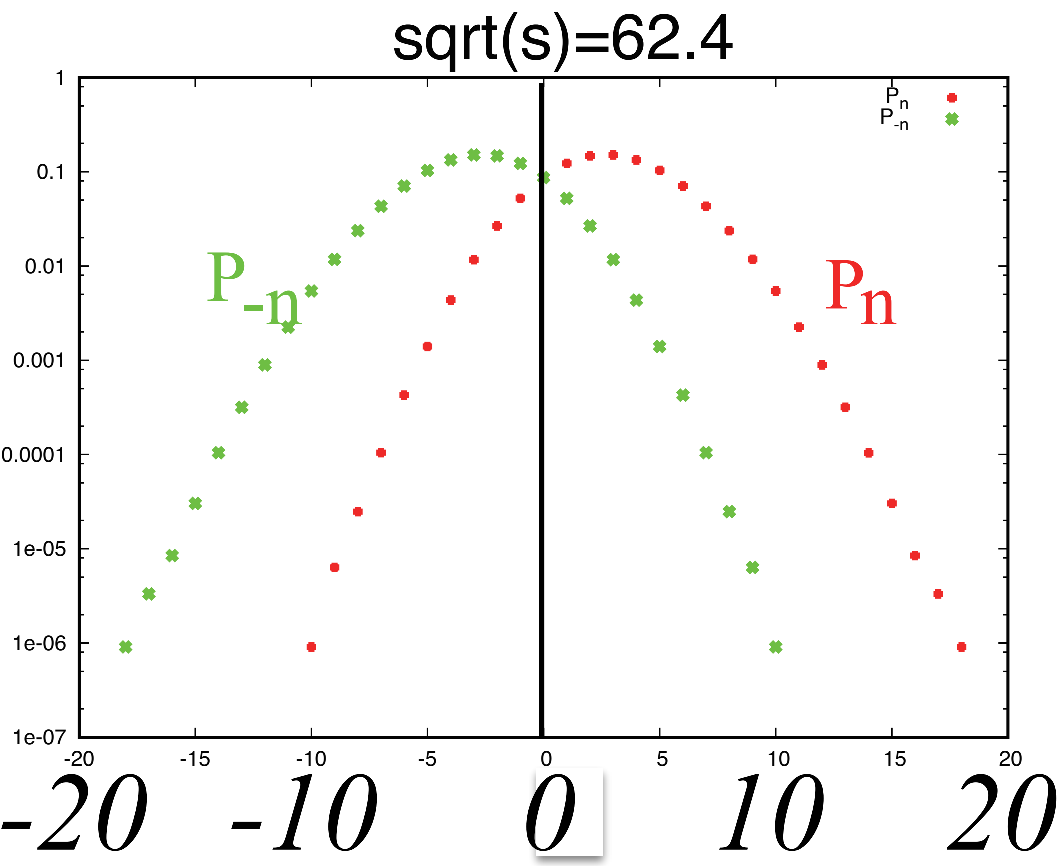
We require

$$Z_{+n} = Z_{-n}$$

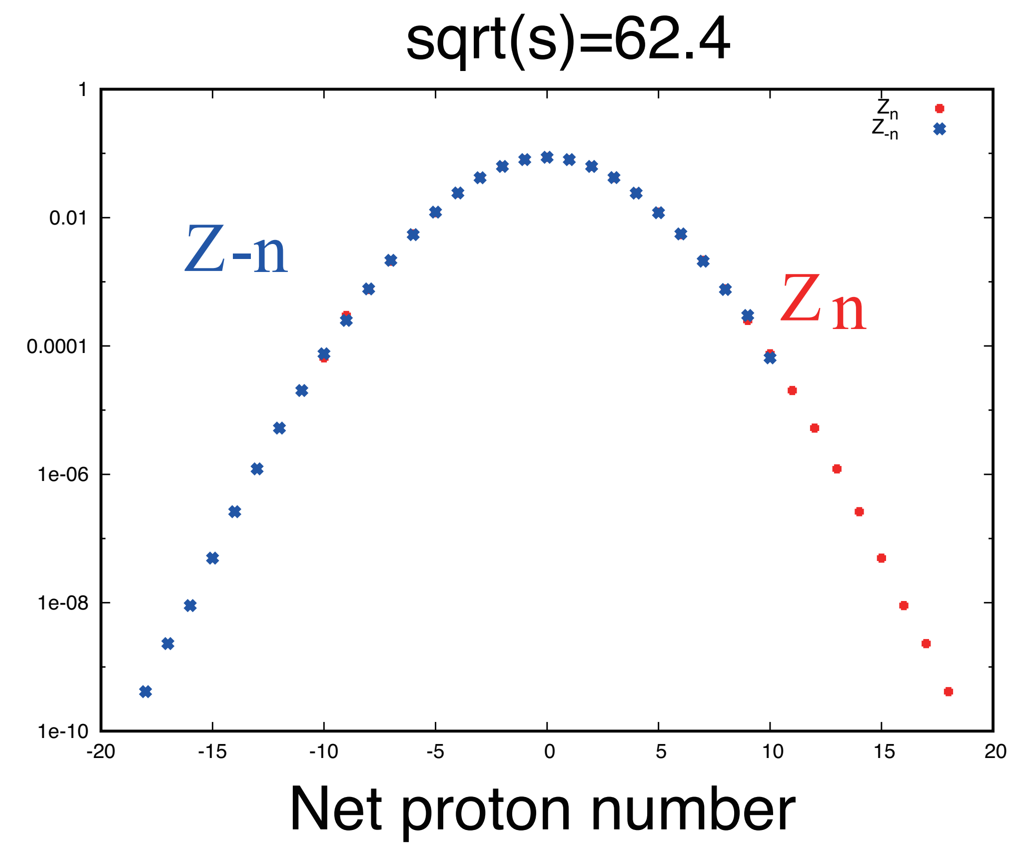
(Particle-AntiParticle Symmetry)



Demand $Z_{+n} = Z_{-n}$



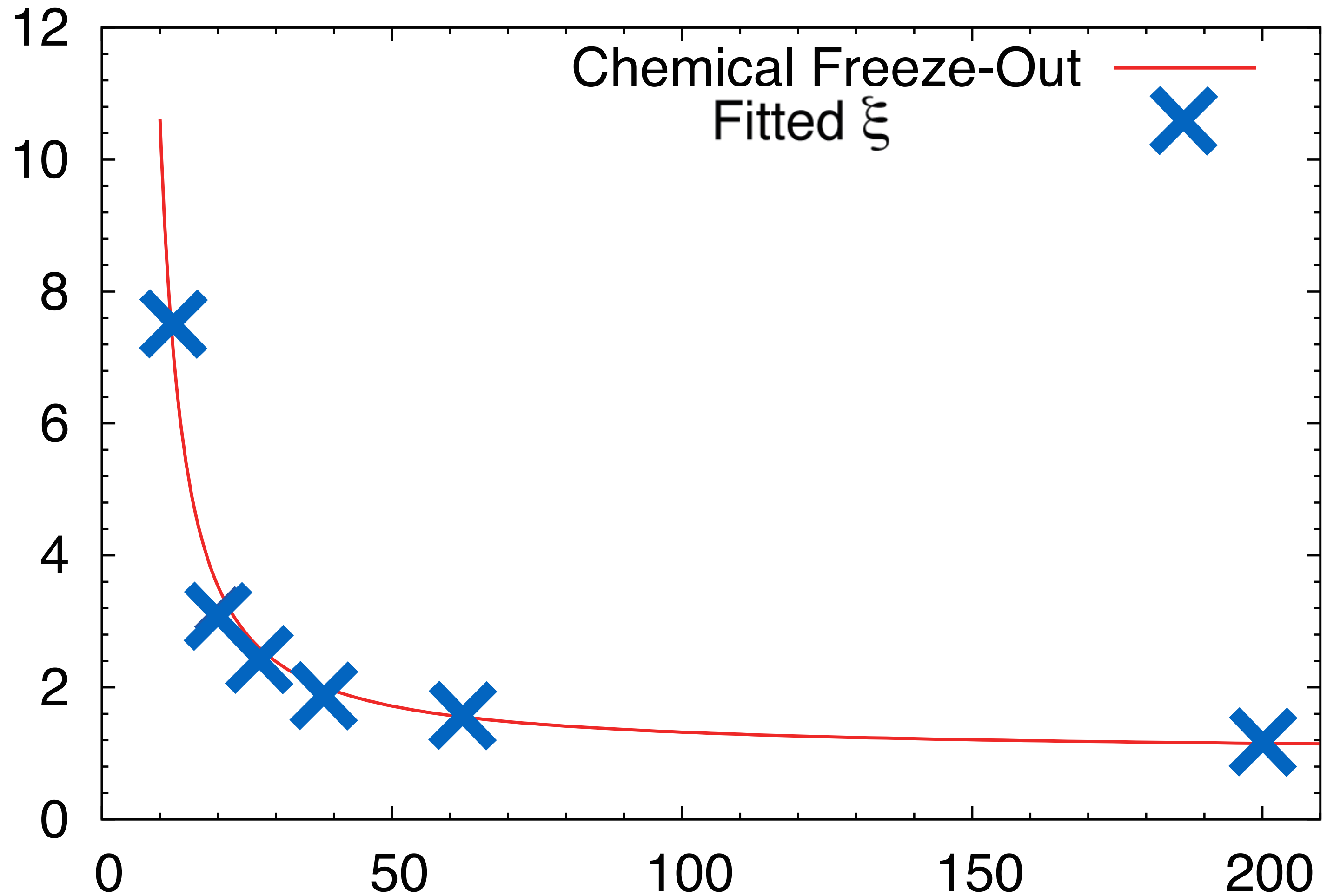
Tune ξ



n

$$Z_n = P_n/\xi^n$$

Fitted ξ are very consistent with those by Freeze-out Analysis.



ξ

$\sqrt{s}\ GeV$

$\left(\xi \equiv e^{\mu/T}\right)$

× This work

— Freeze-out

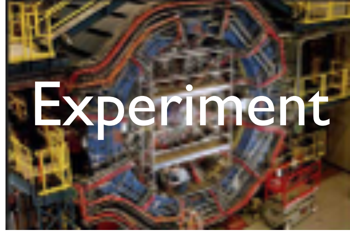
J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

Comparison of obtained ξ

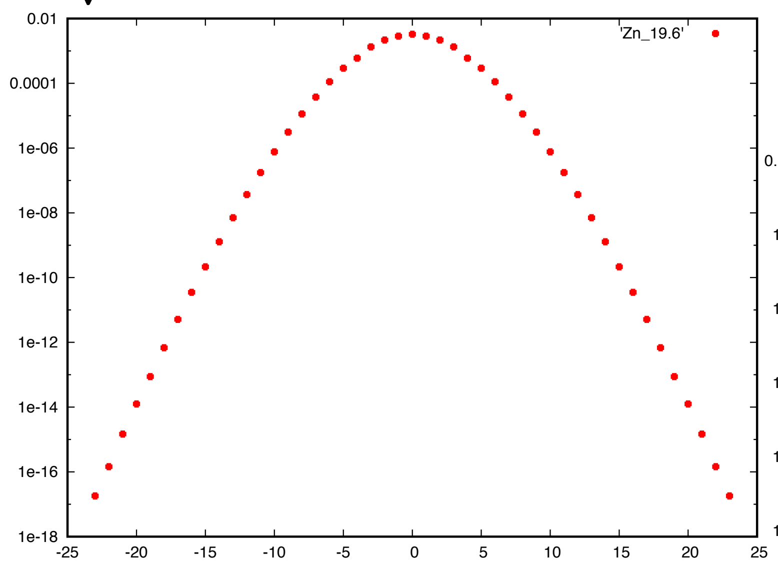
$$\xi \equiv e^{\mu/T}$$

$\sqrt{s_{NN}}$ GeV	Cleymans(06)	Aba(14)	Our
11.5	8.04	11.1	7.48
19.6	3.62	3.65	3.21
27	2.62	2.58	2.43
39	1.98	1.93	1.88
62.4	1.55	1.53	1.53
200	1.18	1.18	1.18

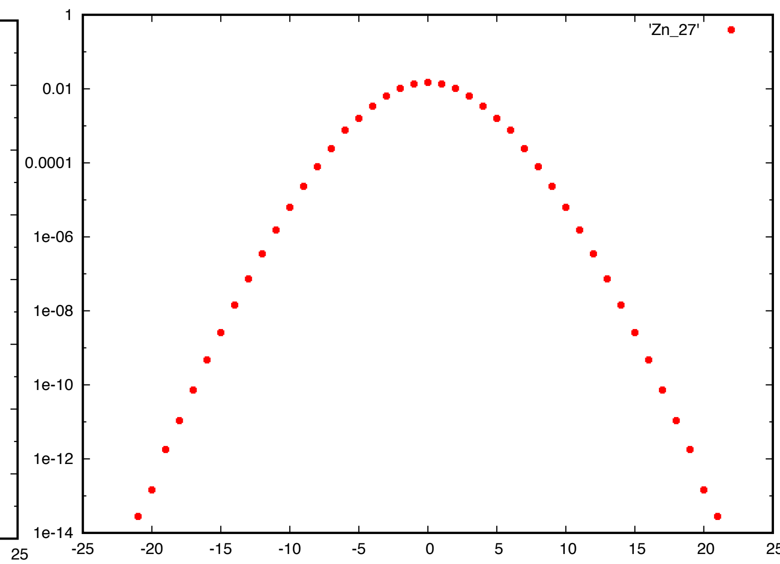
Z_n from RHIC data



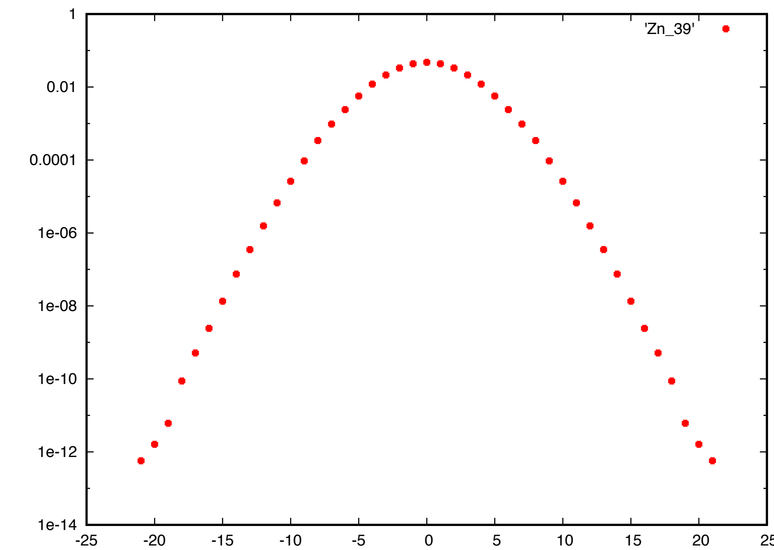
$\sqrt{s} = 19.6\text{GeV}$



$\sqrt{s} = 27\text{GeV}$

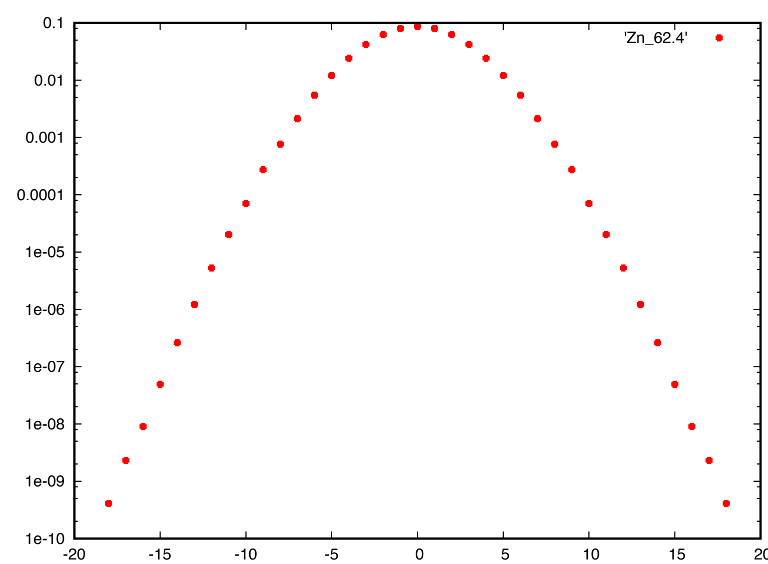


$\sqrt{s} = 39\text{GeV}$

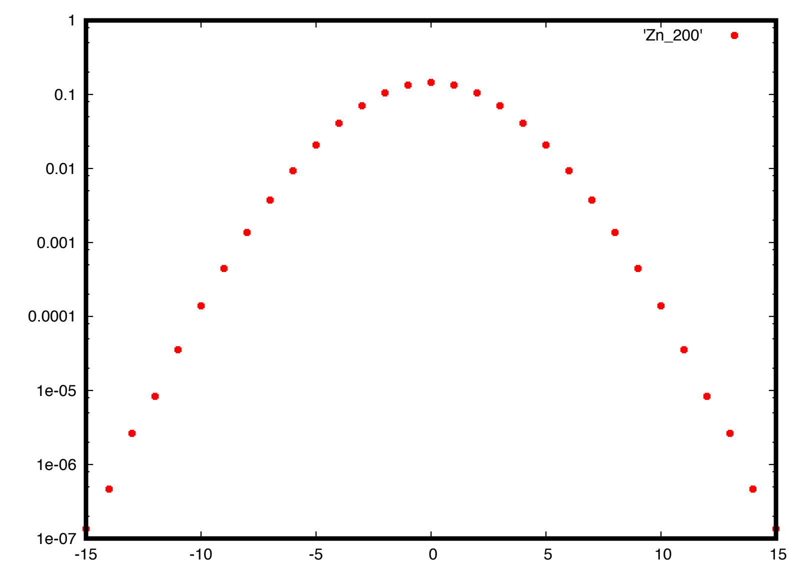


Can I see Difference?

$\sqrt{s} = 62.4\text{GeV}$



$\sqrt{s} = 200\text{GeV}$



Yes, You Can !
We will see it.

Yes, very useful, because

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

$(\xi \equiv e^{\mu/T}$: Fugacity$)$

$Z_n(T)$ ← $Z(\xi, T)$ at some ξ and T

$Z_n(T)$ → $Z(\xi, T)$ at ANY ξ

for both Experiments and Lattice

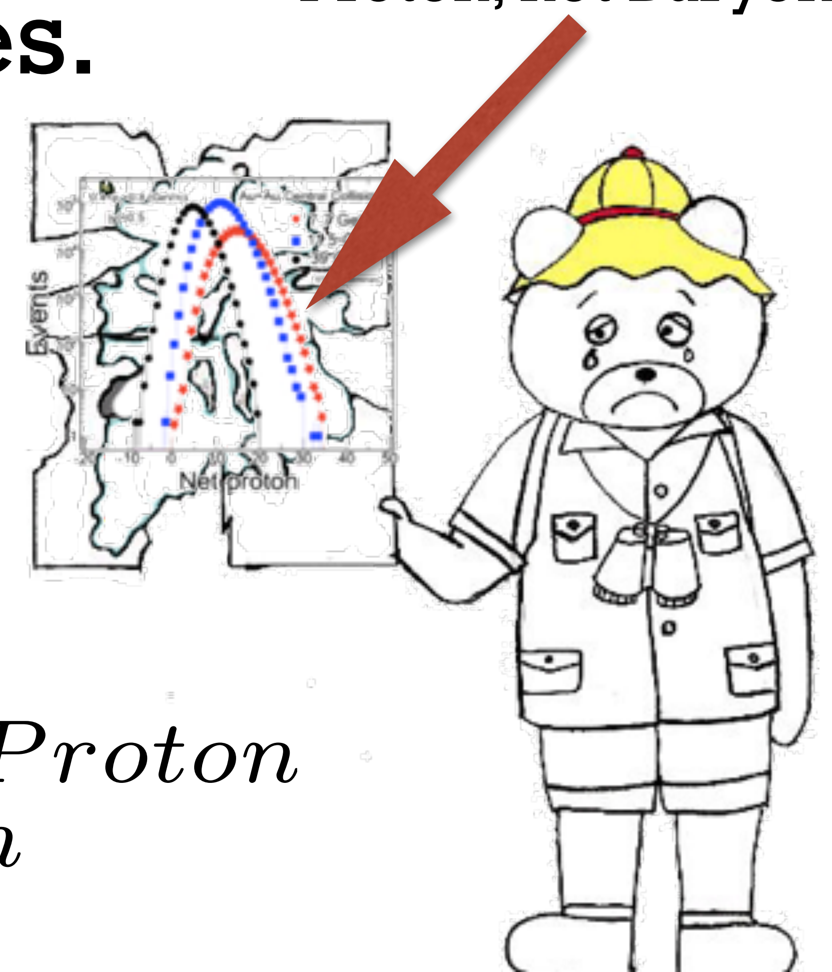
(Current) Weak Points

1) Experimental Multiplicity Data

Net-Proton and Not net-Baryon

One can prove $Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$ only for Conserved Quantities.

Proton, not Baryon



Possible approaches:

i) Wait for Net-Baryon data, or Net-Charge data.

ii) Study and analyze data assuming $Z_n^{Baryon} \sim Z_n^{Proton}$

2) N_{max} is not very large.

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lower estimation of larger density contribution.

We can calculate
also by Lattice QCD Z_n

But Sign Problem on Lattice ?

Sign Problem
One Slide Review

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields})\ \det D\ e^{-(\text{Gluon Action})}$$

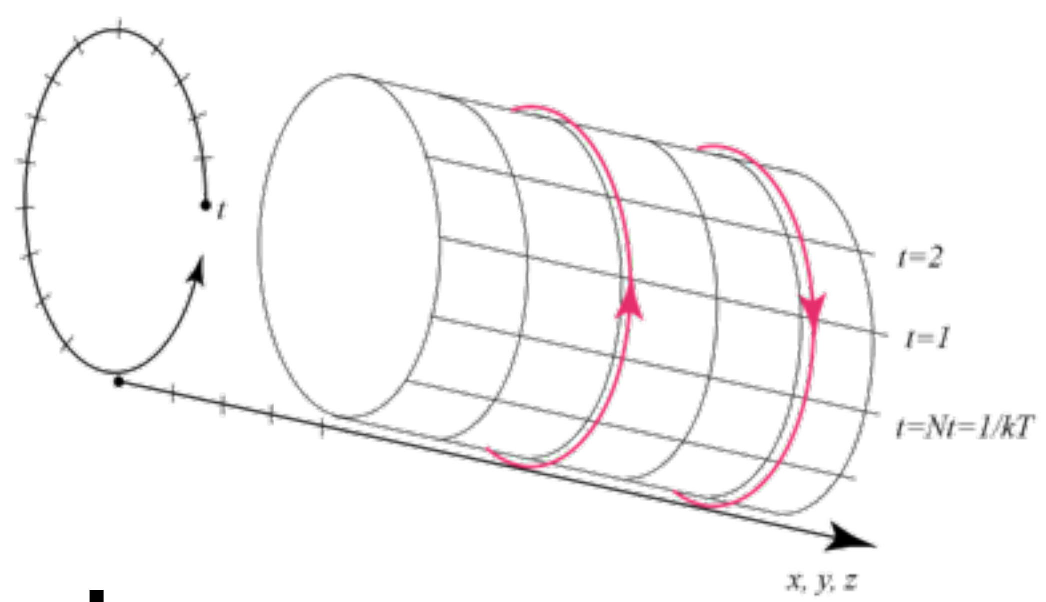
$$\begin{aligned}\det D &= \exp(\mathrm{Tr}\log D) \\ &= \exp\left(e^{+\mu/T}Q^{+} + e^{-\mu/T}Q^{-} + \cdots\right)\end{aligned}$$

$Q^{+} \Leftrightarrow Q^{-}$

Complex Conjugate

If $\mu = 0 \Rightarrow \det D$ real

$\mu \neq 0 \Rightarrow \det D$ complex



$$\det D \quad = \exp\left(e^{+\mu/T}Q^{+} + e^{-\mu/T}Q^{-} + \cdots\right)$$

Q^{+} ⟷ Q^{-} Complex Conjugate

If μ Pure Imaginary ➡ $\det D$ real

A.Hasenfratz and Toussant, 1992

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

Great Idea ! But practically it did not work.

Zn Collaboartion Method:

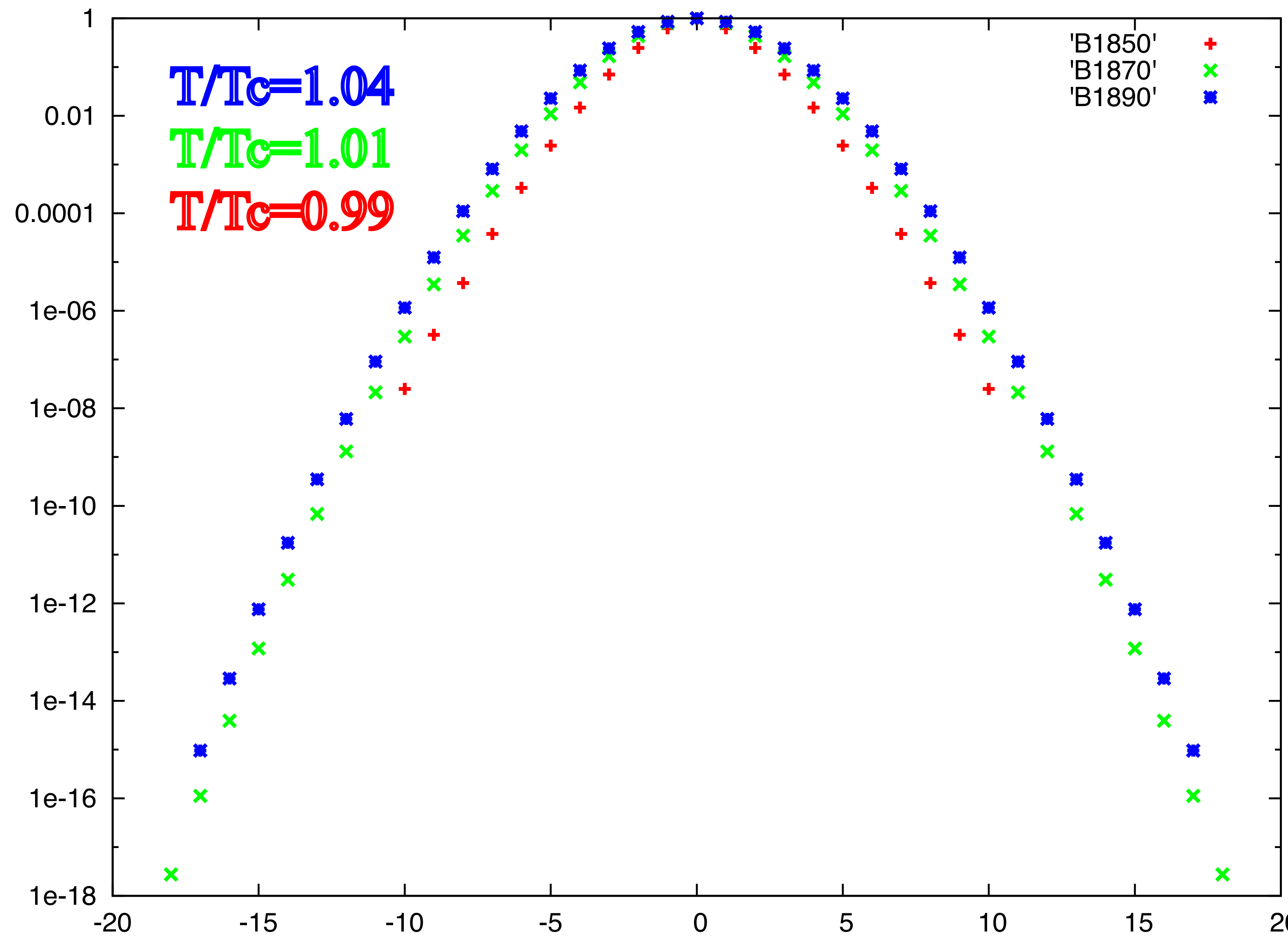
$$Z_C(n,T) = \int \frac{d\theta}{2\pi} \int \frac{\det(\theta)}{\det(\theta_0)} \det(\theta_0) e^{-(\text{Gluon Action})}$$

θ integration ➡ Multi-Precision (50 - 100)

Lattice Data

Can I see Difference?

Zn



Yes, You Can !
Wait a moment.

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$

Is this useful ?

Yes, because

1) We can calculate Z at any ξ (i.e., μ)

2) We can calculate Z even at complex ξ

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  - **Moments**
  - Lee-Yang zeros
- Summary

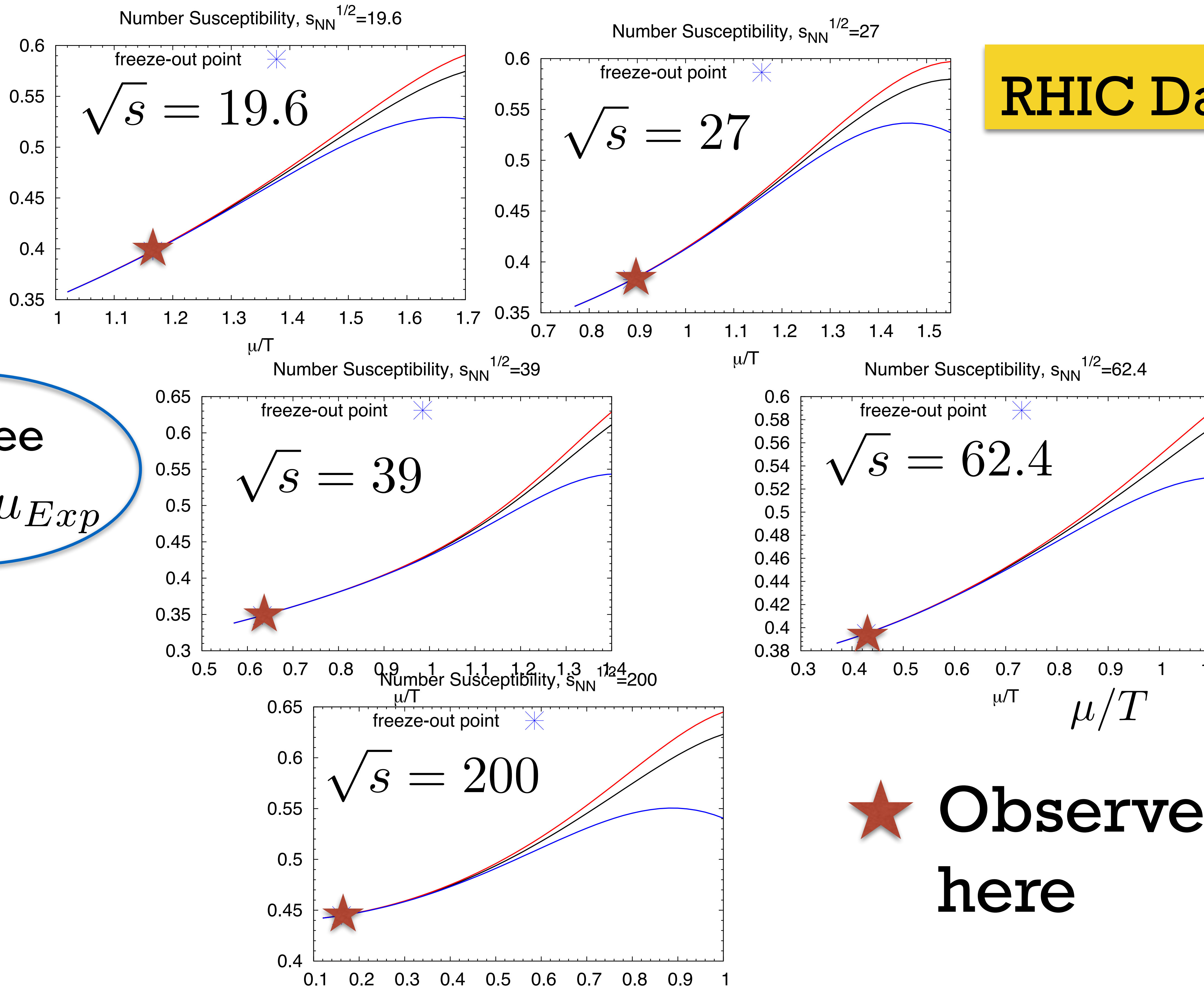
Moments λ_k

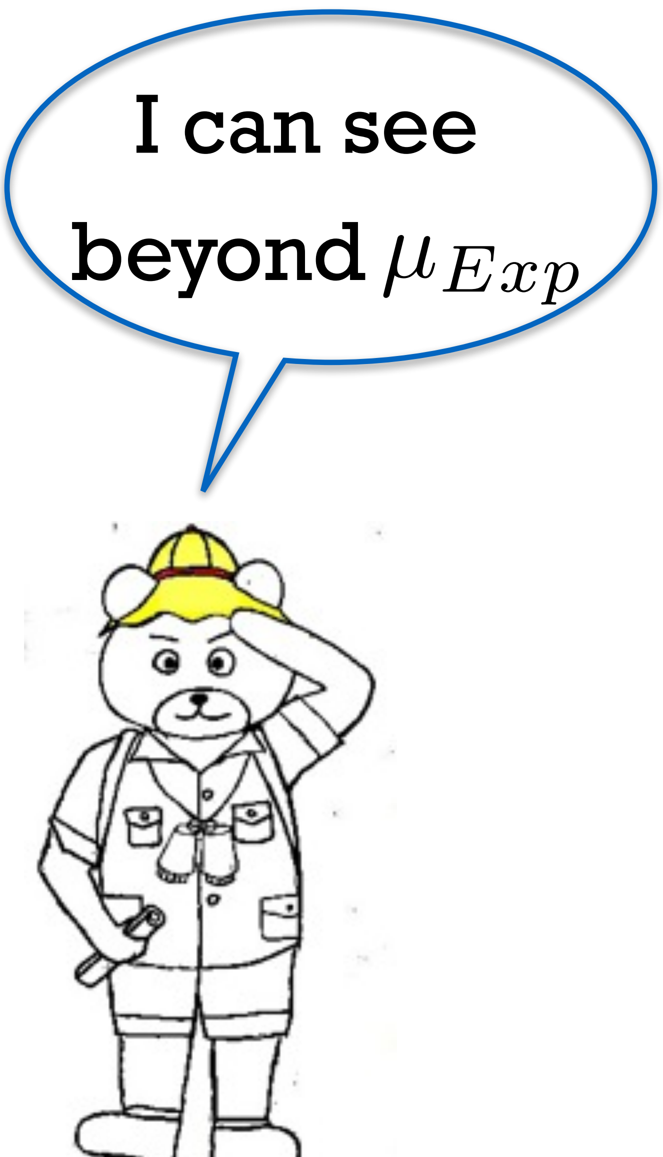
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

$$\lambda_k \equiv \left(T \frac{\partial}{\partial \mu} \right)^k \log Z$$

Susceptivility as a function of μ/T

RHIC Data

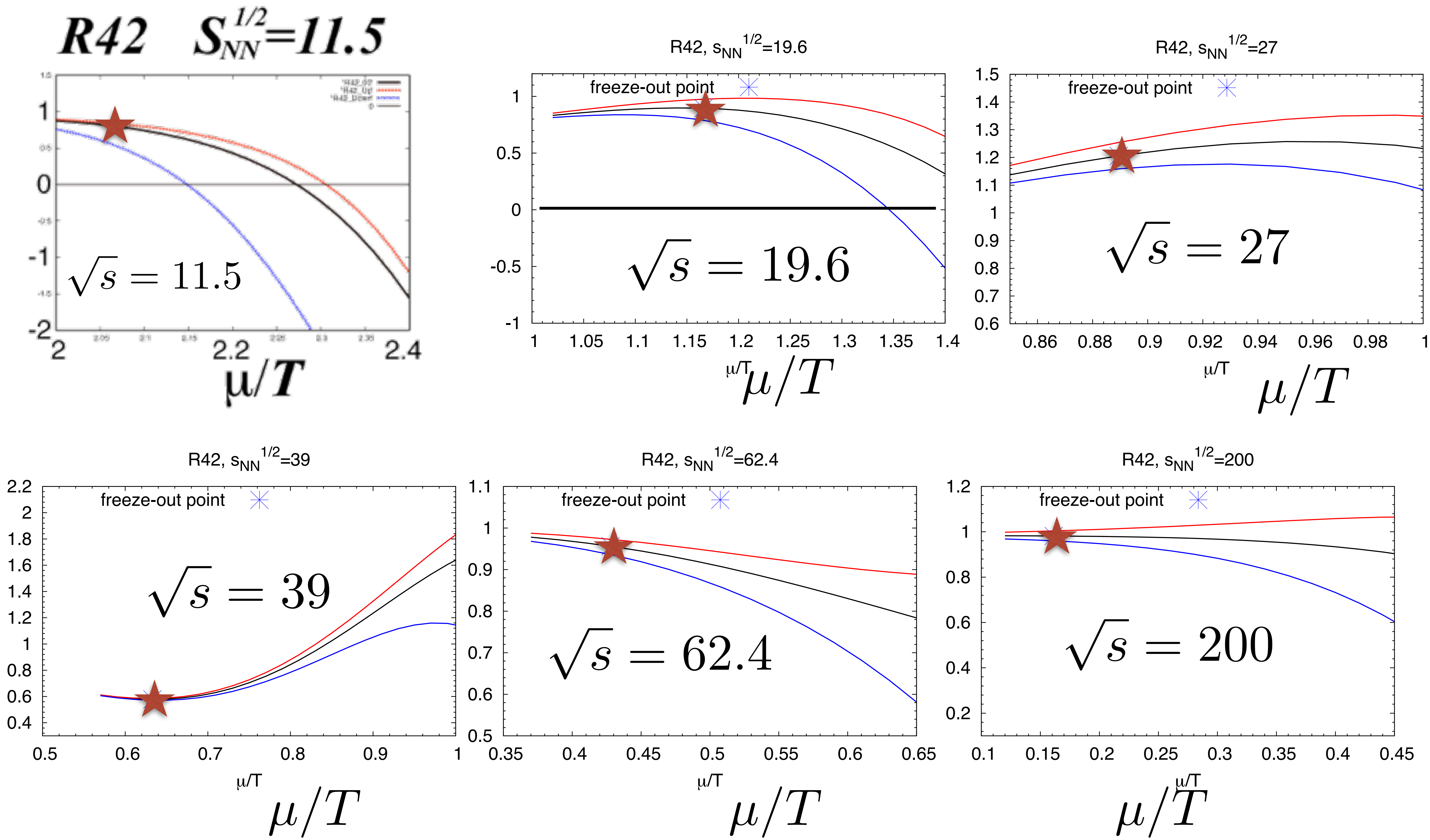




★ Observed here

RHIC Data

Kurtosis $\frac{\lambda_4}{\lambda_2}$ as a function of $\frac{\mu}{T}$

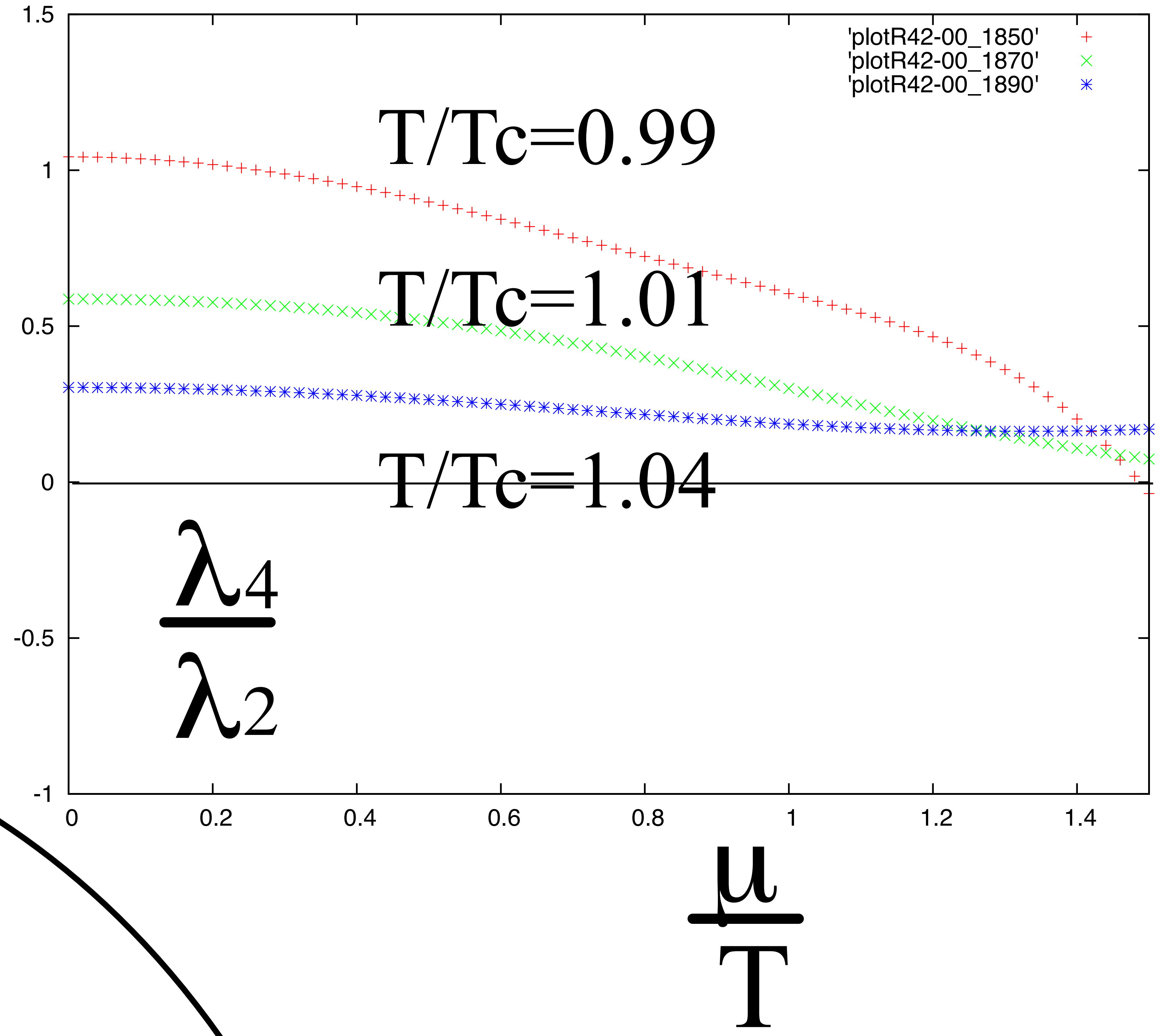


Data are taken at μ_0
You calculate the moments at $\mu > \mu_0$

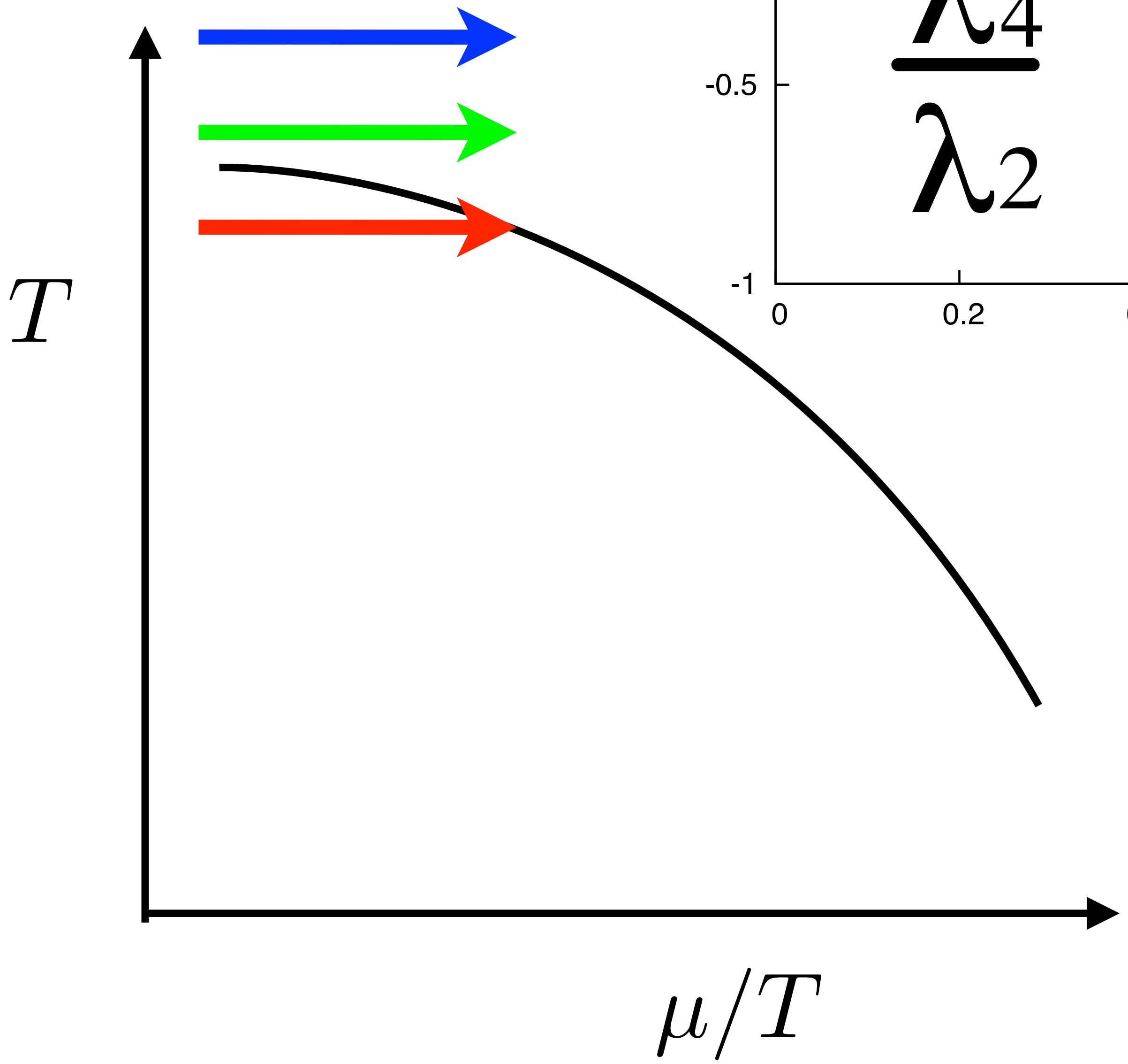
Magic ?
or Cheating ?

No Magic !
We use all Z_n data,
$(-N_{max} \leq n \leq +N_{max})$
that are usually not employed.

Lattice



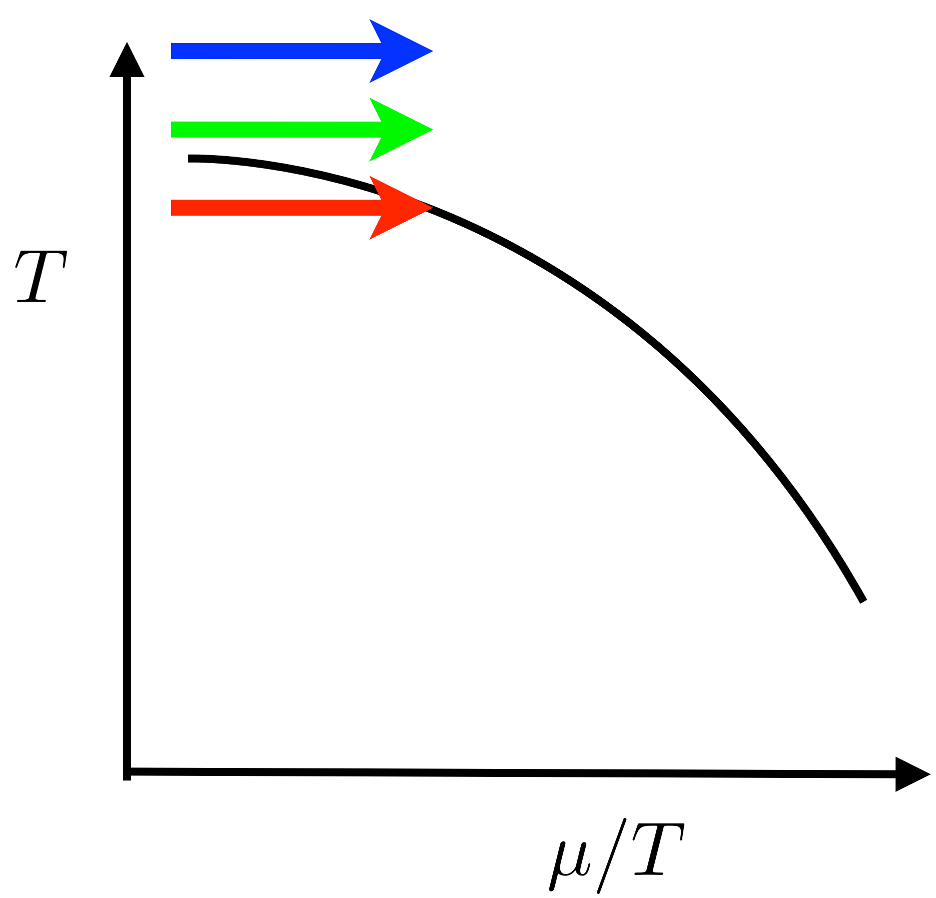
1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
'plotR42-00_1850'
'plotR42-00_1870'
'plotR42-00_1890'
T/Tc=0.99
T/Tc=1.01
T/Tc=1.04
λ4
λ2
μ
T

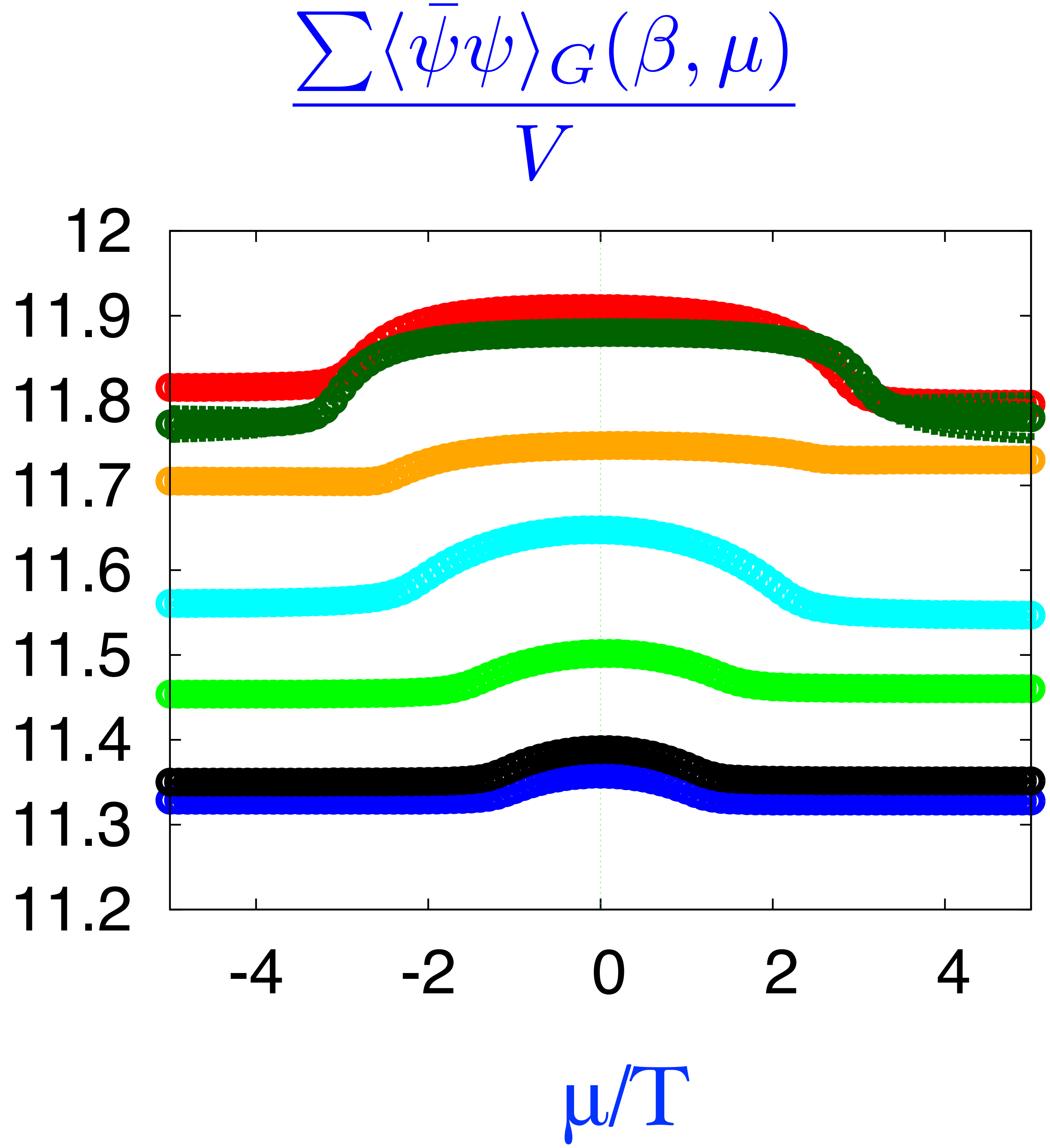


T
μ/T

Chiral Condensate

Lattice





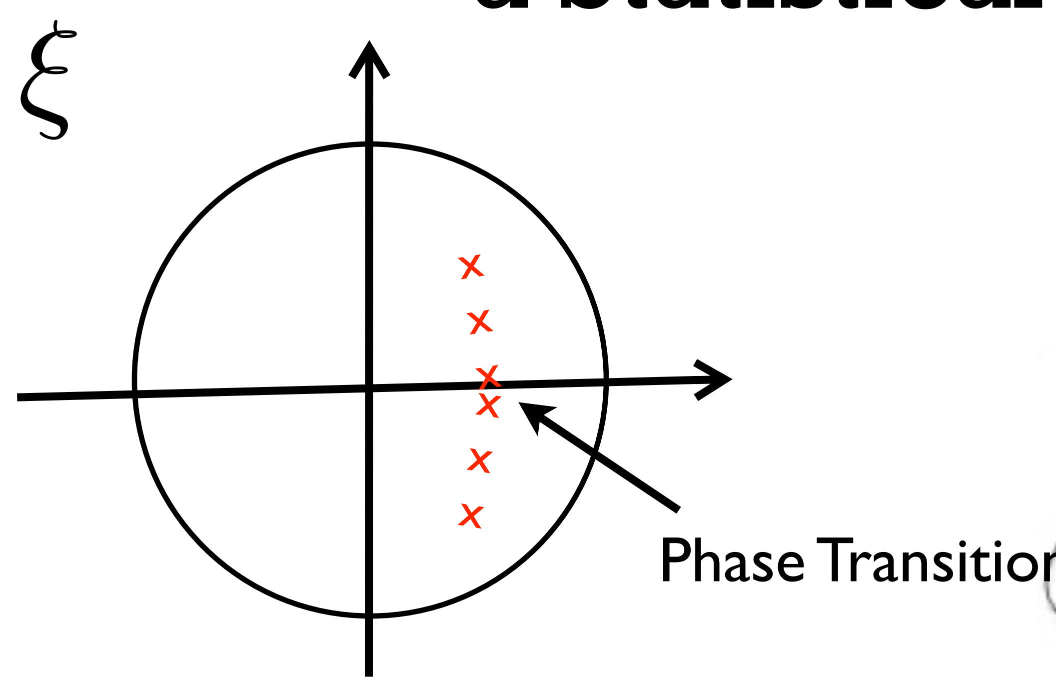
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lee-Yang Zeros (1952)

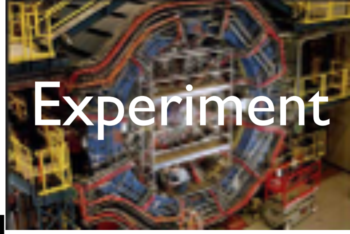
Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

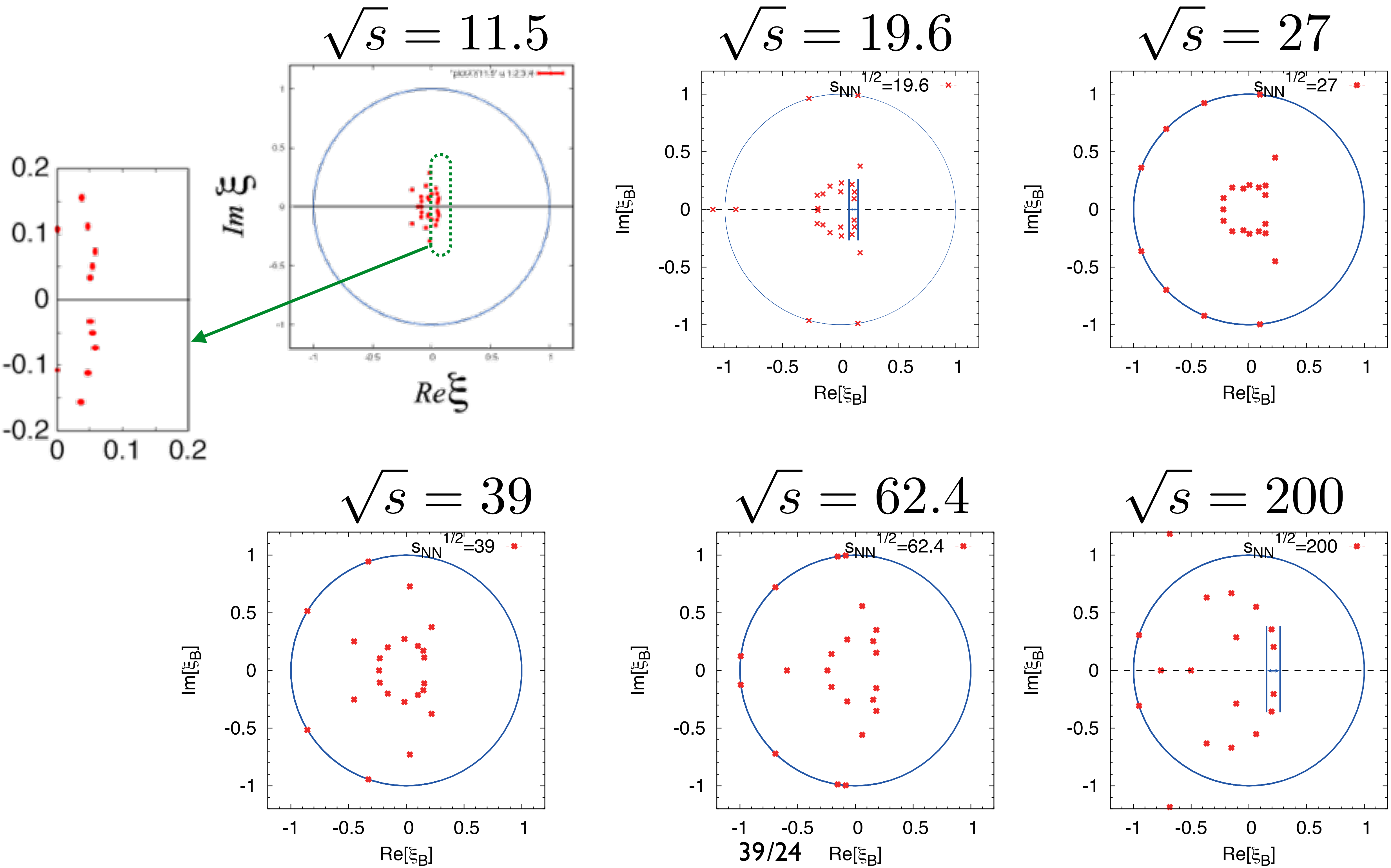
Great Idea to investigate a Statistical System



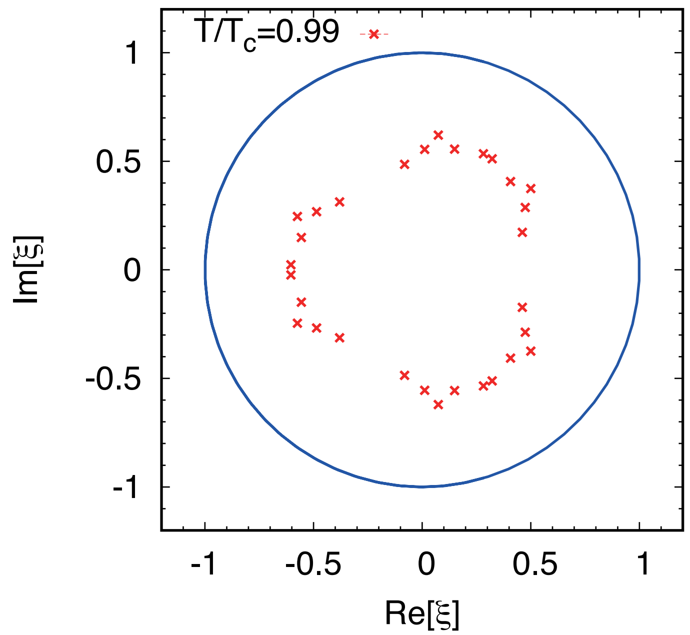
Lee-Yang Zeros Experimental Data (RHIC)



Lee-Yang Zeros: RHIC Experiments

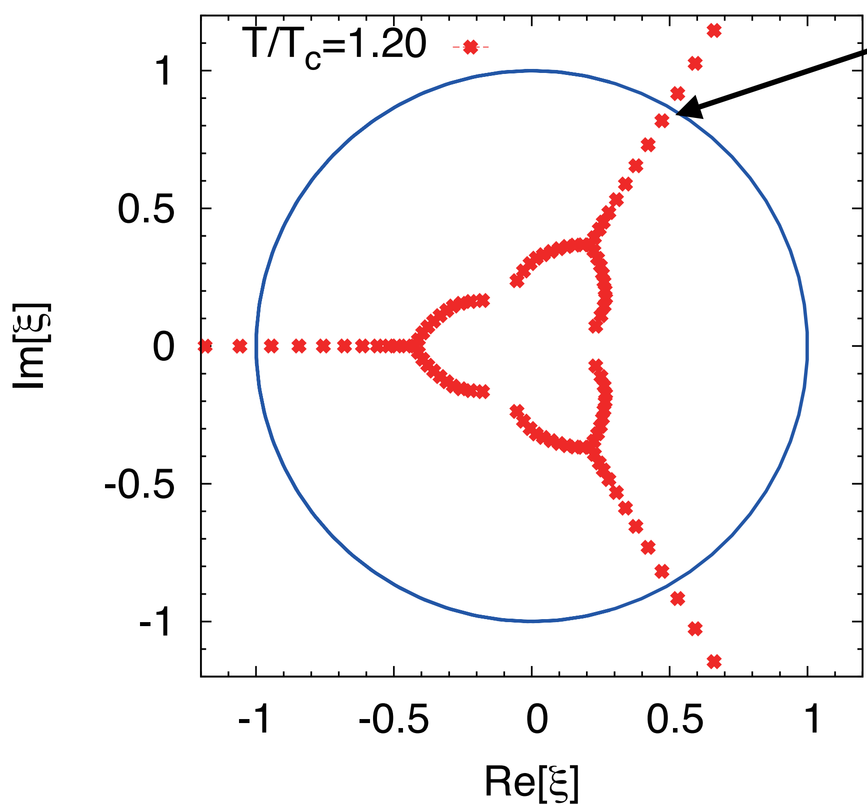


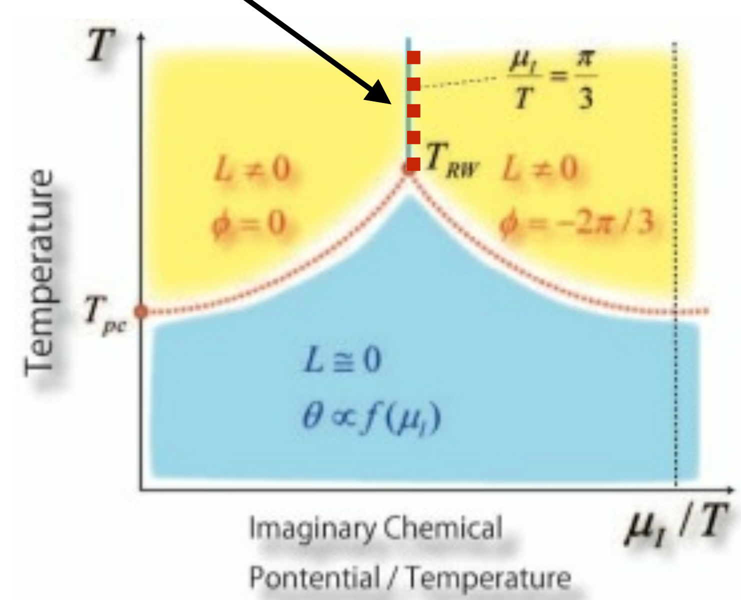
Lattice Data



$\beta = 1.85$ $T/T_c \sim 0.99$

Roberge-Weise Transition !

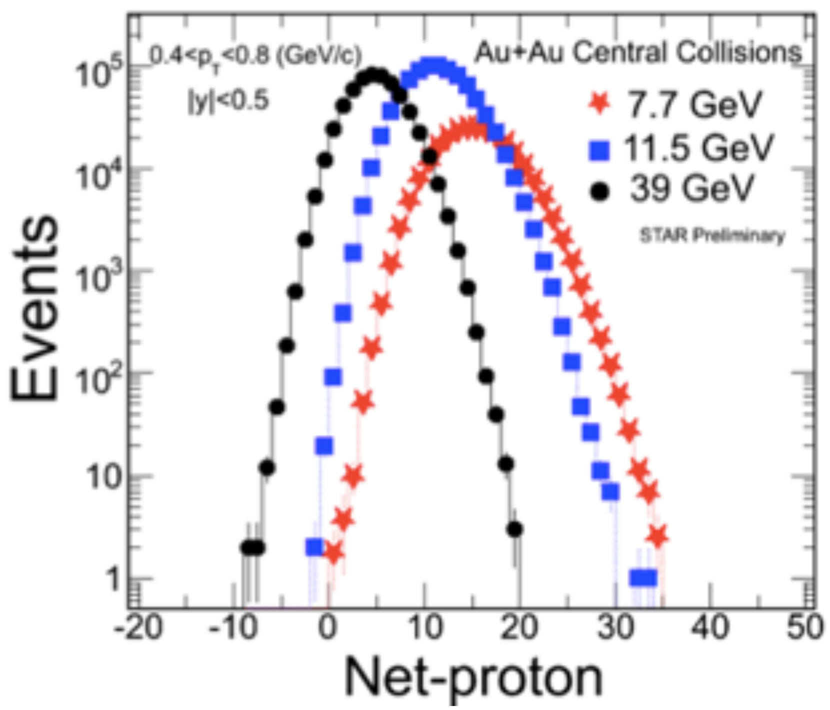




$T/T_c \sim 1.20$

and How

What are Multiplicity Distributions telling us on QCD Phase Diagram ?



Experimental Data

Extract $Z_n(T)$

Construct

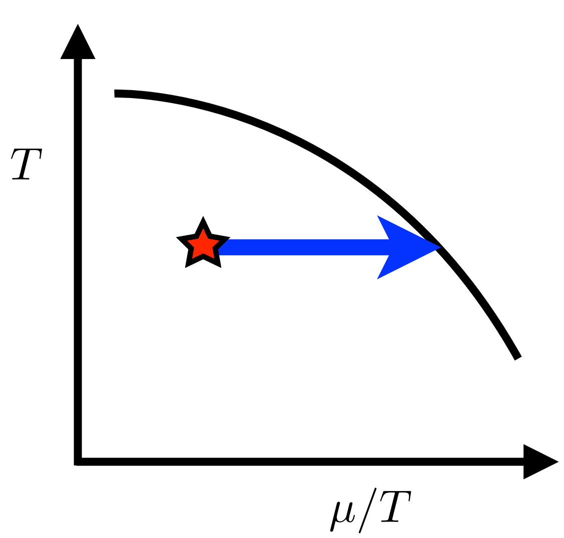
$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Construct

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Calculate Moments at $\mu \geq \mu_{\text{Experiment}}$

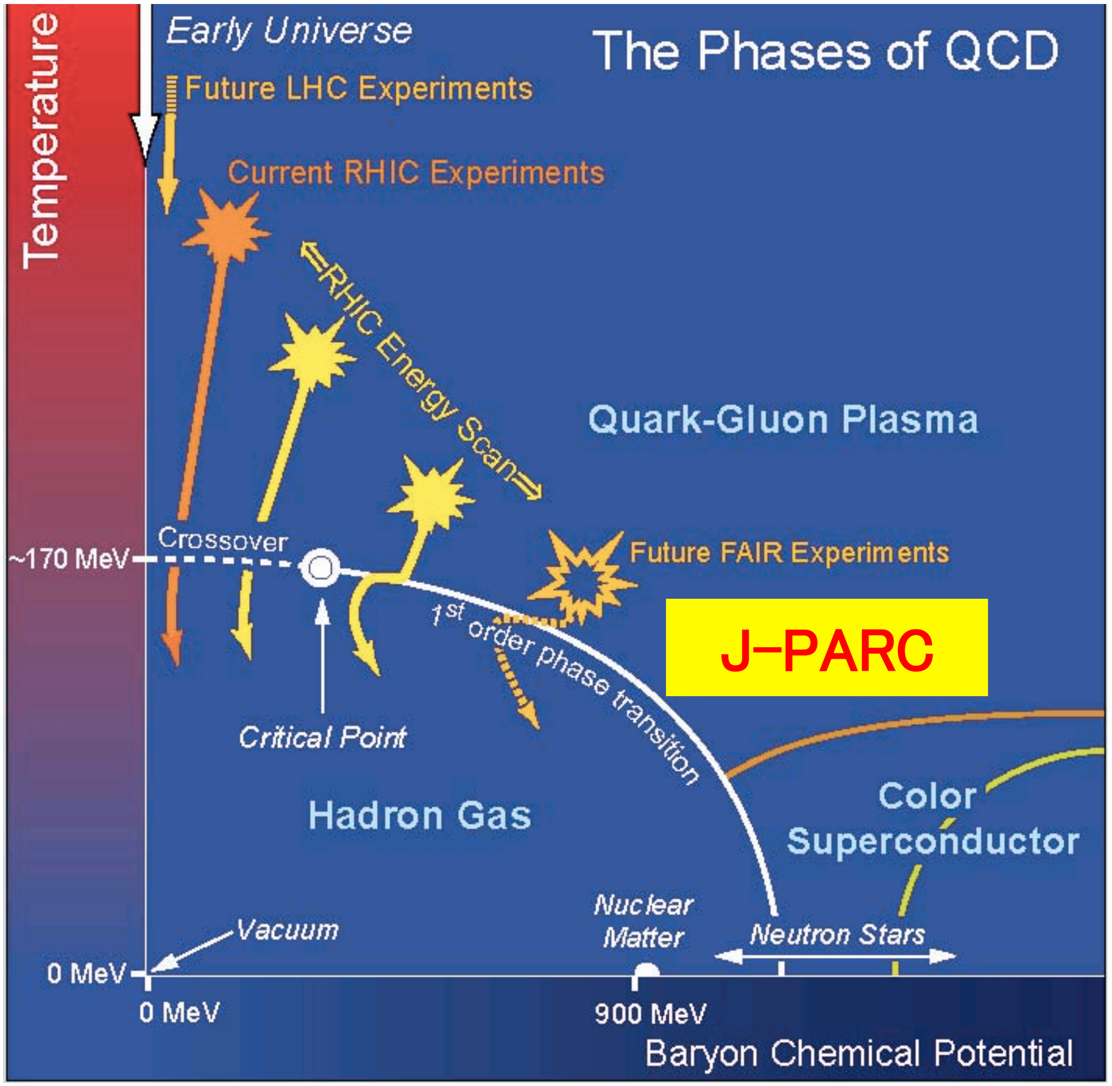
Construct Lee-Yang Zeros



The current Net-Proton data is a Test-Bed.
But even they suggest the phase boundary.

Sako@QM2014

"Towards the Heavy-Ion Program at J-PARC"



Hadron Seminar @J-Parc Takao Sakaguchi

"High Enegy" Program (50 GeV MR)

- Ion species
  - p, Si, Cu, Au, U

    Au→U
  - Baryon density
    - $7.5\rho_0 \rightarrow 8.6\rho_0$ (JAM)
  - Duration at $\rho>5\rho_0$
    - 4 → 7 fm/c
- Beam energy
  - 1 - 11.6 AGeV (U) ($\sqrt{s}\downarrow NN =4.9GeV$
  - Possibly 19 AGeV($\sqrt{s}\downarrow NN =6.2GeV$)
- Rate
  - 10^{10}-10^{11} ions per cycle (~a few sec)

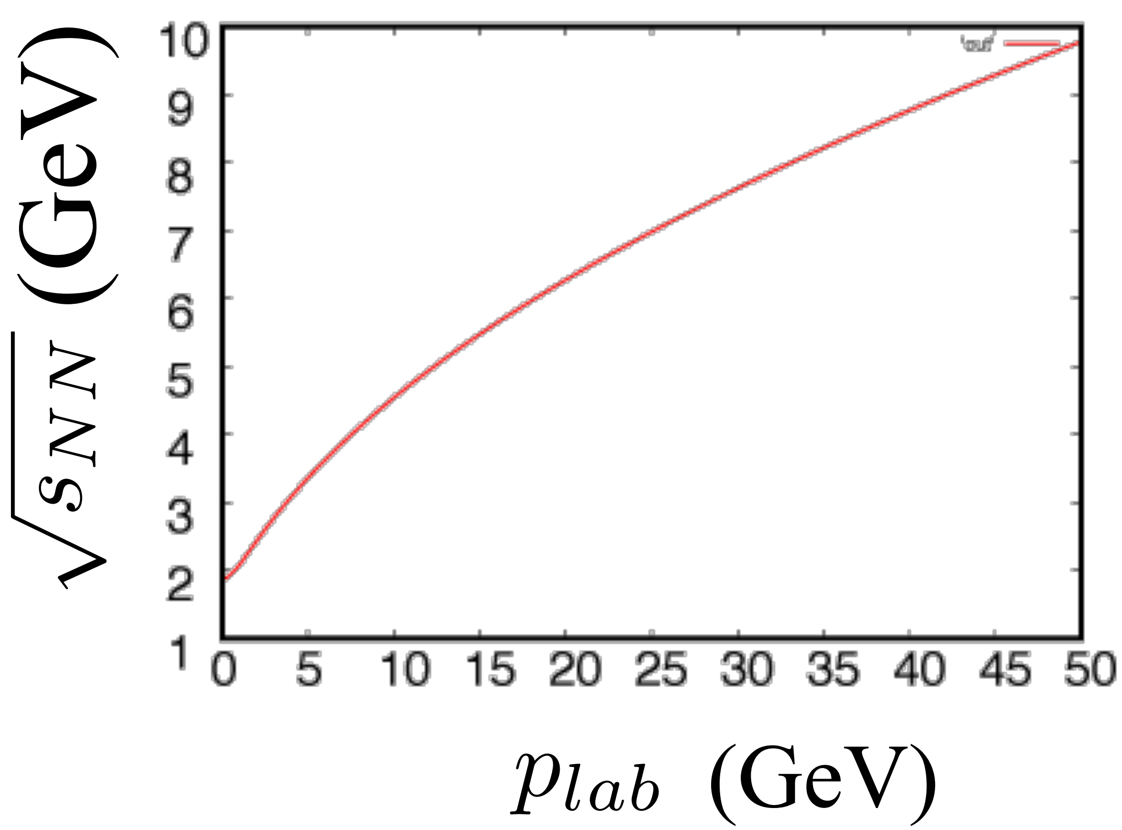
$A + B$ in a lab. frame (fixed target)

$$s = (p_a + p_b)^2 = M_A^2 + M_B^2 + 2E_a M_b$$

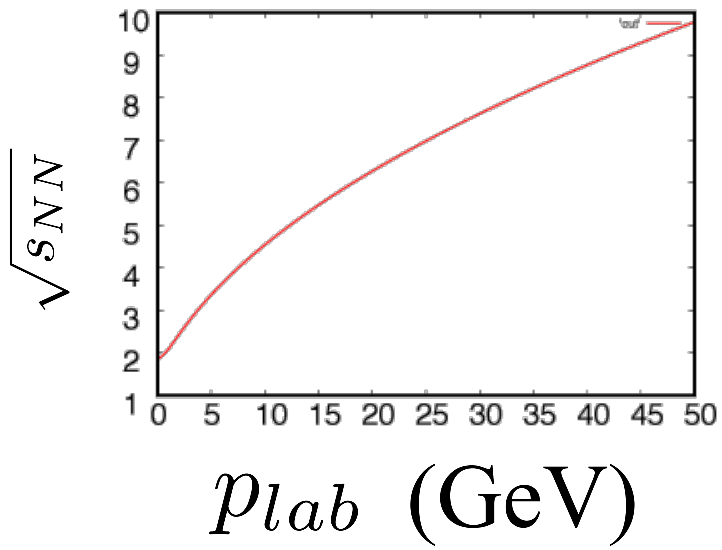
For simplicity, $A = B$

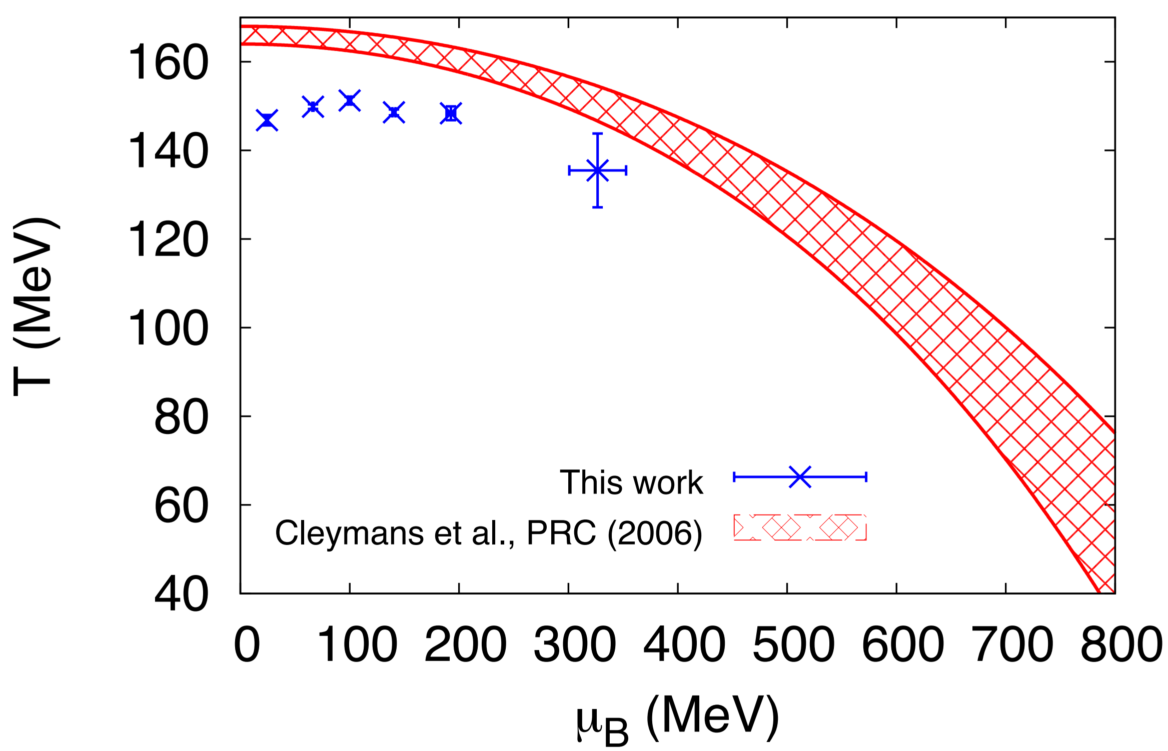
$$M_A = M_B = A m_N \quad E_a = A\sqrt{\vec{p}_{lab}^2 + m_N^2}$$

$$\sqrt{s_{NN}} = \frac{\sqrt{s}}{A} = \sqrt{2\left(m_N + \sqrt{\vec{p}_{lab}^2 + m_N^2}\right) m_N}$$

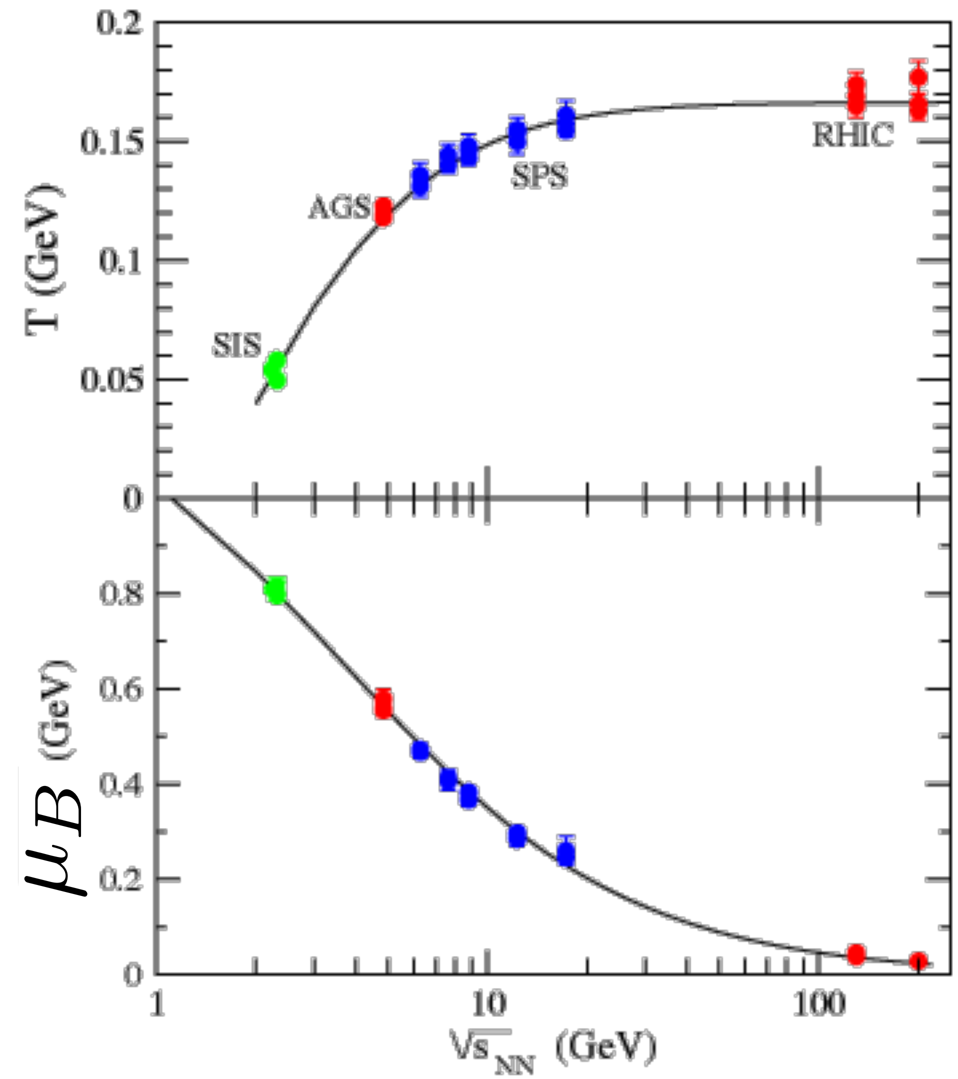


J-PARC search regions ?

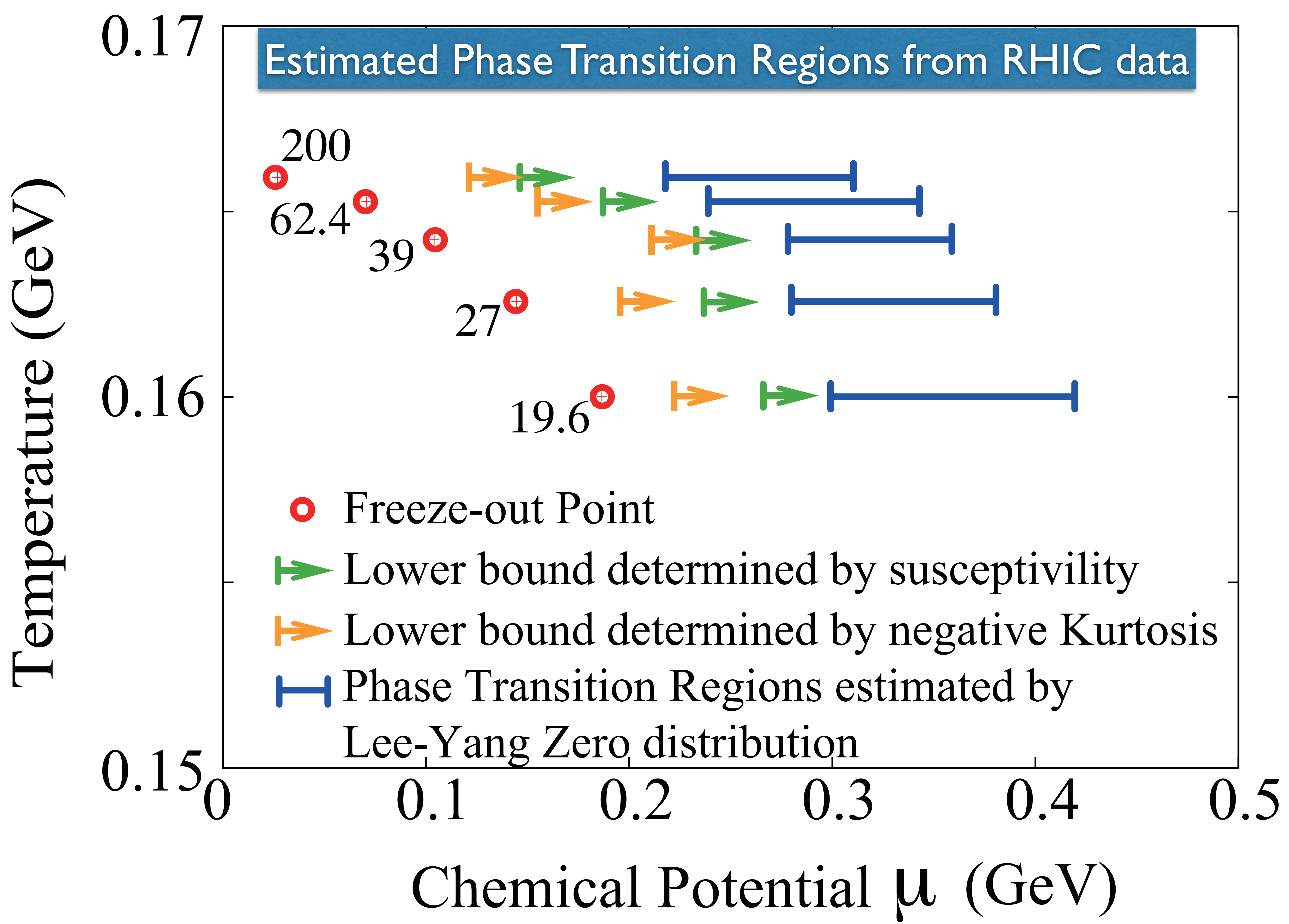




Alba et al., arXiv:1403.4903



J.Cleymans et al.,
Phys. Rev. C73, (2006) 034905.



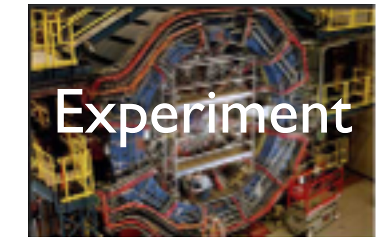
Estimated Phase Transition Regions from RHIC data
0.17
0.16
0.15
0
0.1
0.2
0.3
0.4
0.5
200
62.4
39
27
19.6
Temperature (GeV)
Chemical Potential μ (GeV)
Freeze-out Point
Lower bound determined by susceptivility
Lower bound determined by negative Kurtosis
Phase Transition Regions estimated by Lee-Yang Zero distribution

Summary

- A+A collision data at RHIC around indicate we are near the QCD phase transition line.
  If J-PARC may join this challenge, it will contribute a lot.
- Zn analysis give us a power to predict higher density.
  - ⭐ Large statistic at large n is important
- Lattice QCD has now power to calculate high density.

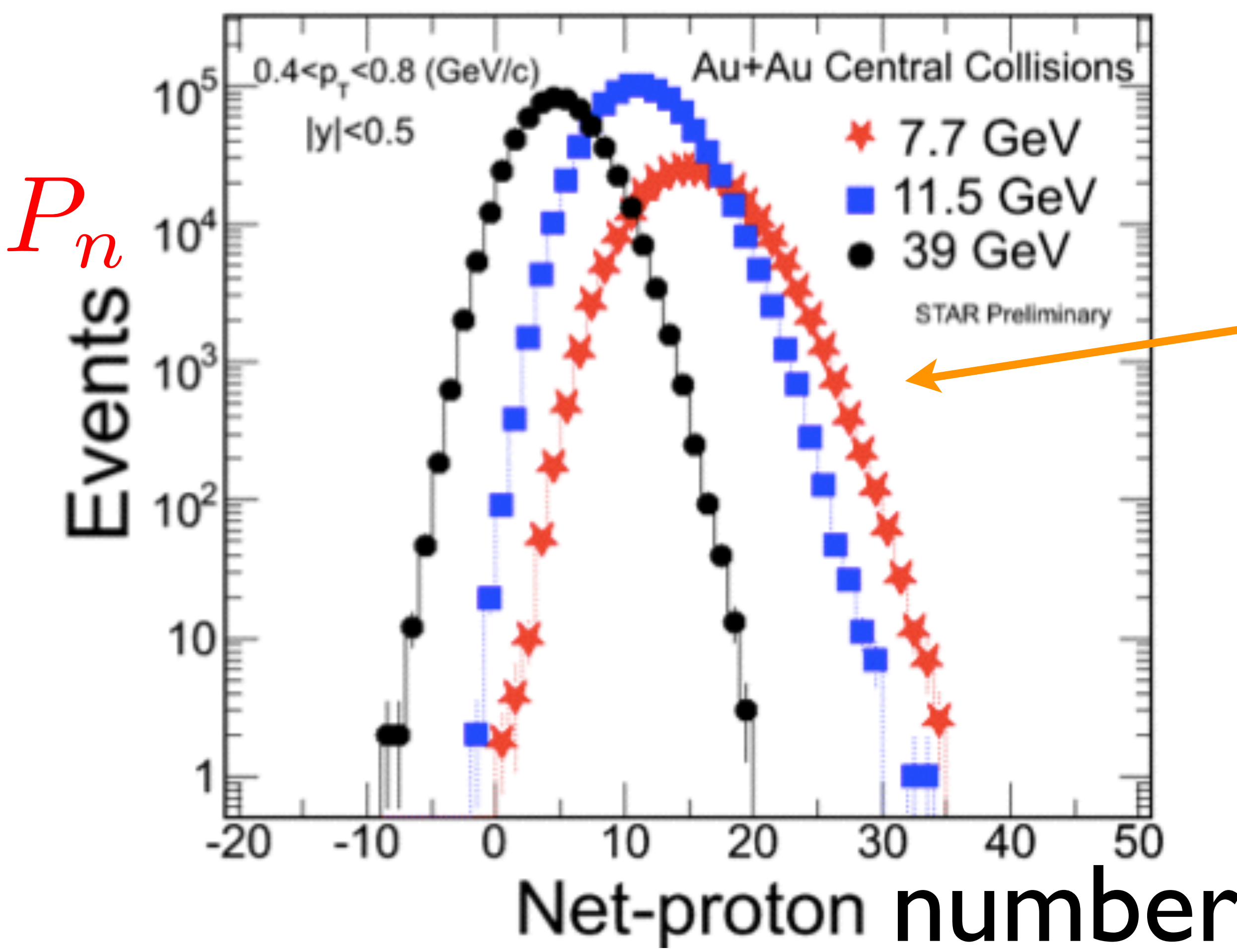
Backup Slide

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$



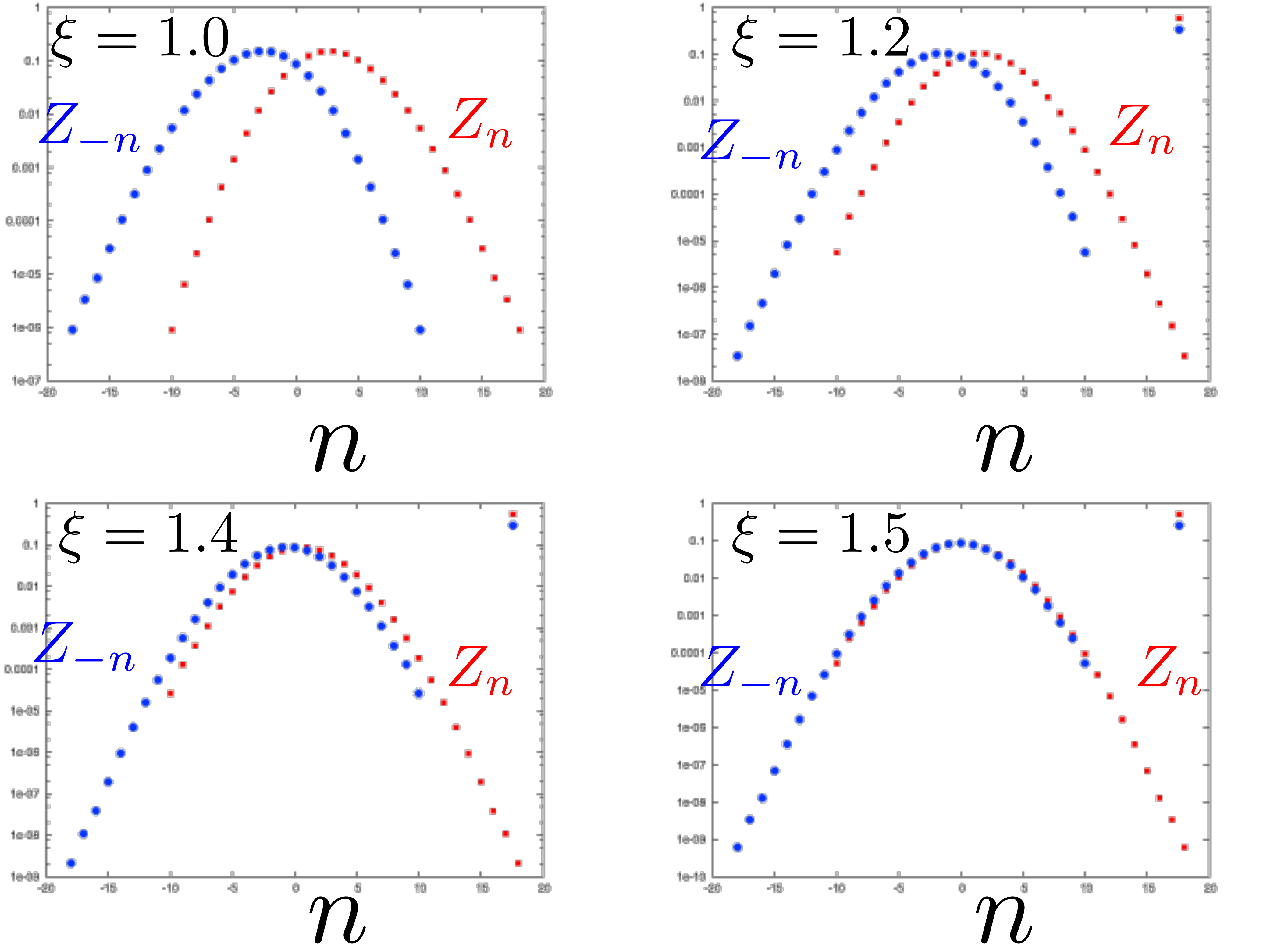
Partition Function is Sum of the Probabilities P_n with n ...

If I know ξ, then I have Z_n.



$$Z_n = P_n / \xi^n$$

From Experiment

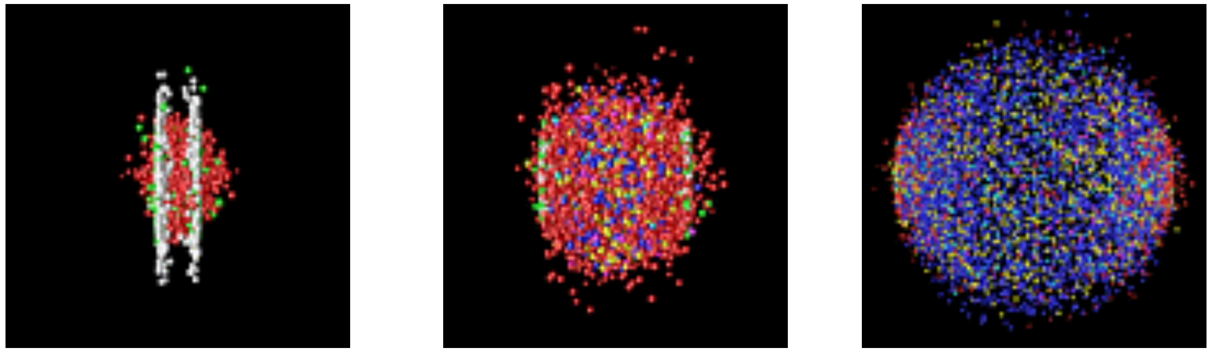


Final Value $\xi = 1.534$

We assume

the Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

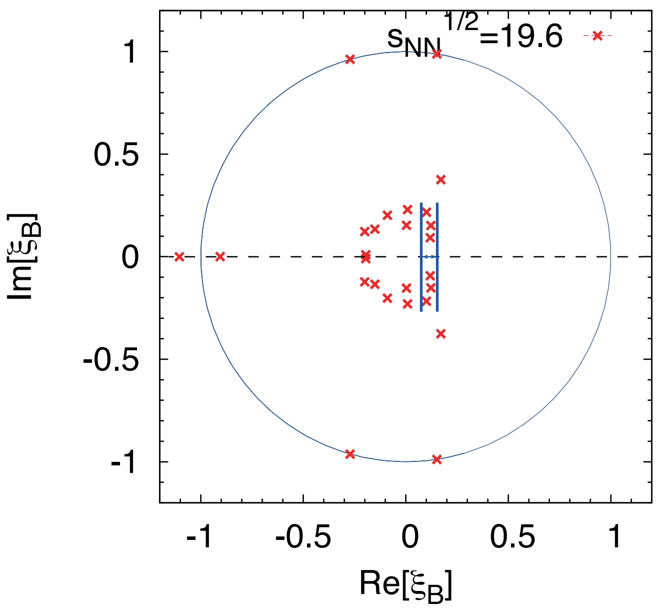
with μ (chemical Potential)
and T (Temperature)

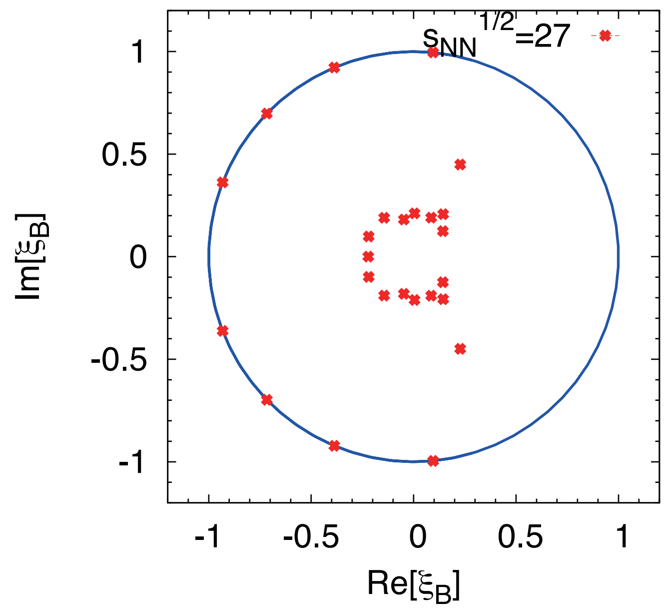


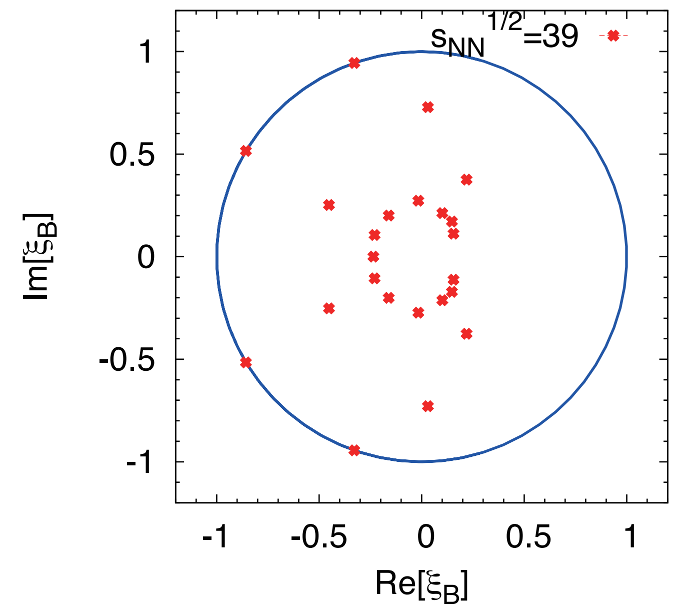
$$Z(\mu, T)$$

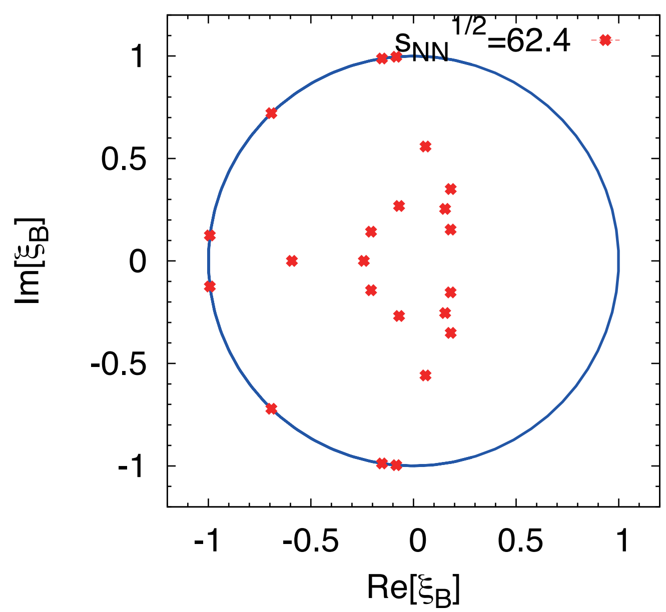
Grand Canonical Partition Function

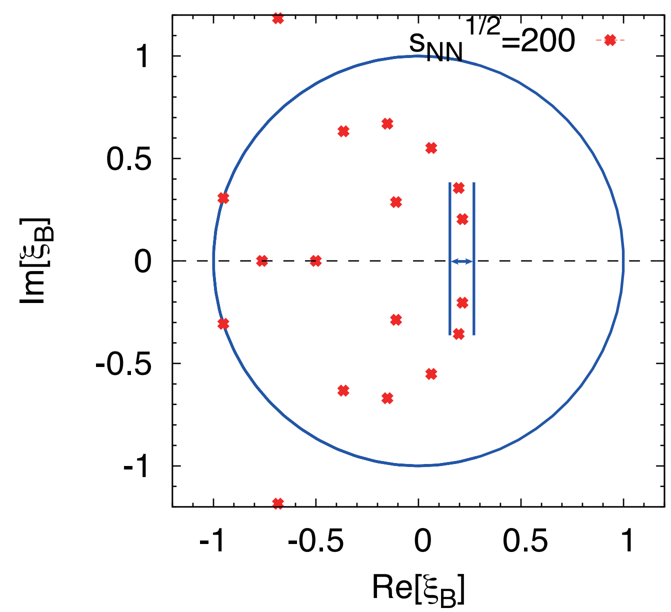
Lee-Yang Zeros: RHIC Experiments











Hunting the QCD Phase Transition Regions

Find Rooms where No Criminal.

⇒ The Target is in other Room.