# H-dibaryon & Hyperon interaction from recent Lattice QCD Simulations


Takashi Inoue, Nihon Univ.

for HAL QCD Collaboration

| | |
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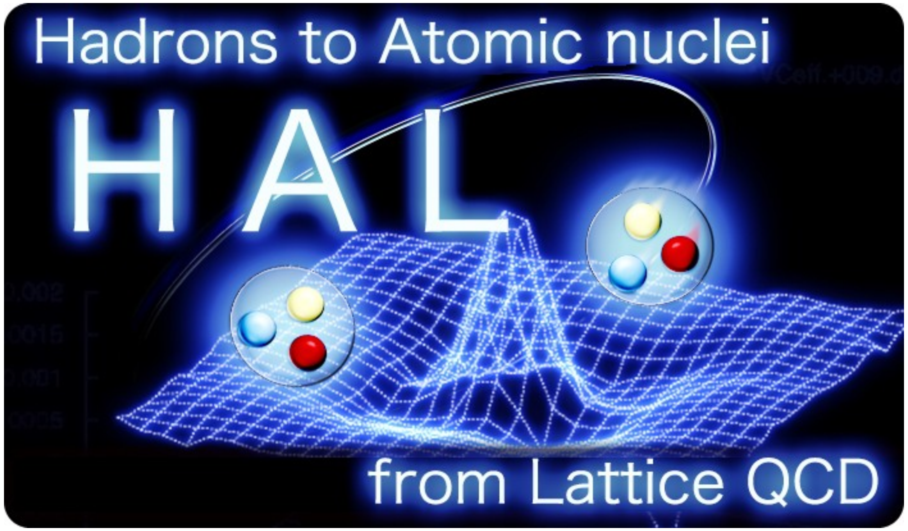



JAEA, Sep 9, 2011

# Introduction

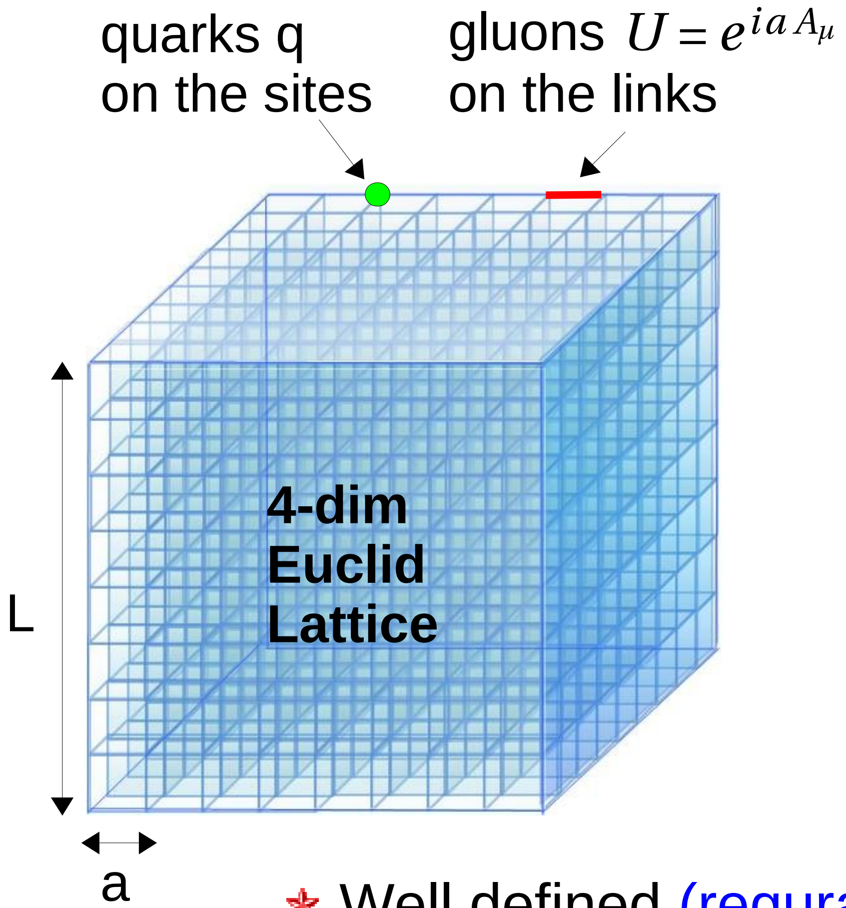
- H-dibaryon: predicted compact 6-quark state
  - R. L. Jaffe, Phys. Rev. Lett. 38 (1977) [MIT Bag model]
  - One of most famous candidates of exotic-hadron.
  - No Pauli exclusion, Flavor singlet, Attraction from OGE
  - Does H exists in nature? $B_H > 7$ MeV is ruled out by $_{\Lambda\Lambda}$He.
  - Possibility of a shallow bound state or a resonance remain.
- Hyperon interaction (YN, YY int.)
  - are important for neutron-star, super-nova phys. and so on.
  - however, are not well known due to lack of experimental data.
- Our purpose and goal
  1. We reveal BB int., including existence of the H-dibaryon, directly from QCD by using lattice simulation.
  2. We get deeper(or intuitive) understanding of BB int.
  3. People can apply them to many physics (super-nova etc).

# Plan of this talk

- Introduction
  - Brief background,  Our purpose and goal
  - Lattice QCD Simulation
  - Hadron system by LQCD – Our approach –
- Formulas & Setup
  - NBS w.f. and Potential
  - Lattice, Action and Facility,
  - Five ensembles
- Results
  - BB int. at the $SU(3)_F$ limit
  - H-dibaryon
  - Hyperon interactions
- Summary & Outlook

# Lattice QCD Simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu \left( i\partial_\mu - g\, t^a A^a_\mu \right) q - m\,\bar{q}\, q$$




Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ $U_i$ } generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturbative
- Highly predictable

# Hadron system by LQCD

- Conventional：use energy eigenstate (eigenvalue)
  - Lüscher's finite volume method for phase-shift
  - Infinite volume extrapolation to get bound state energy
- HAL：use the potential $V(r)+...$ from the NBS w.f.
  - Solve effective theory which reproduce T matrix from QCD
  - Advantages
    - No need to separate E eigenstate.
    - Just need to measure NBS w.f.
      Then extract the potential.
    - Demand a minimal lattice volume.
      No need to extrapolate to V=∞.
    - Can output many observables.

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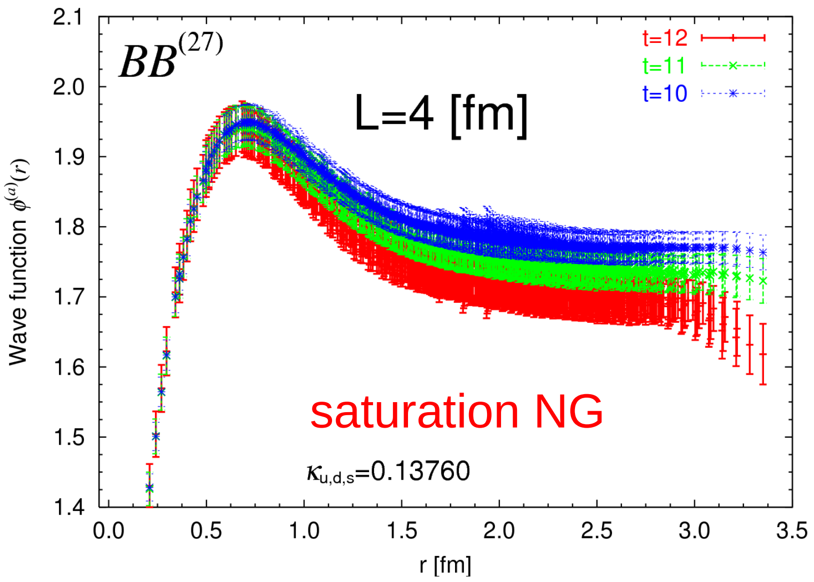

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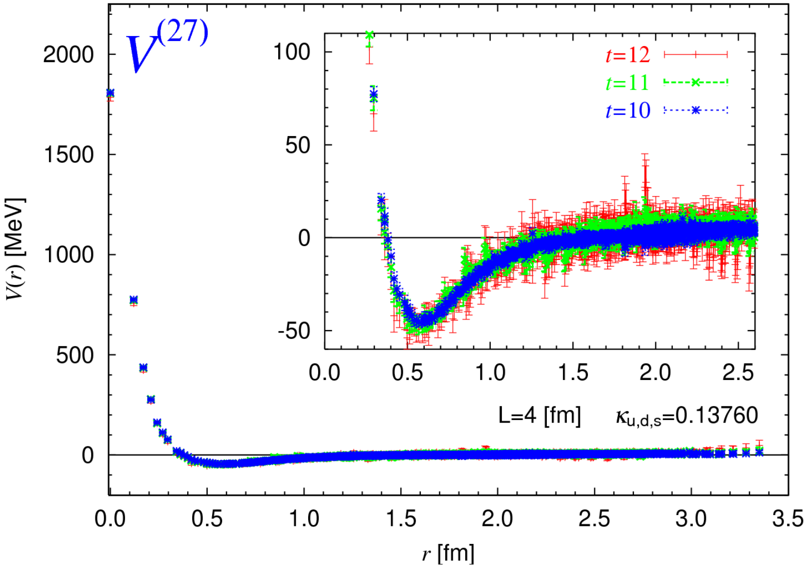

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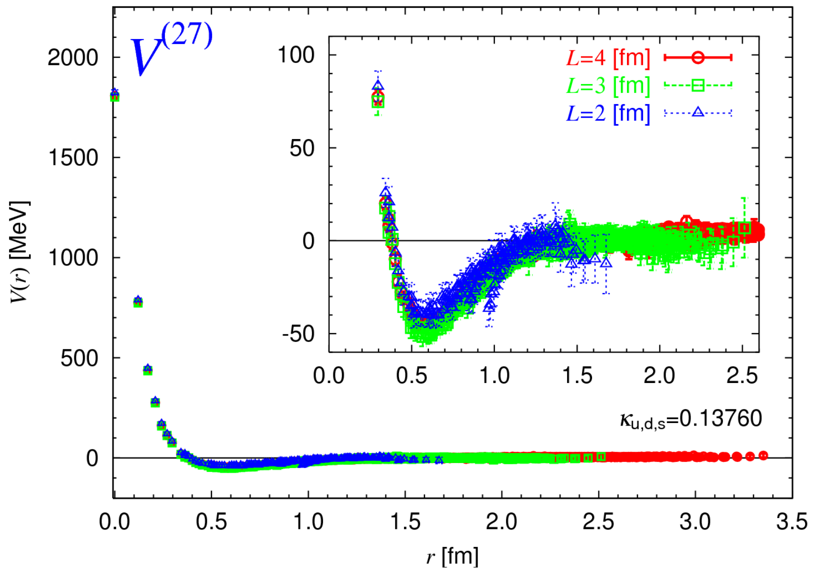

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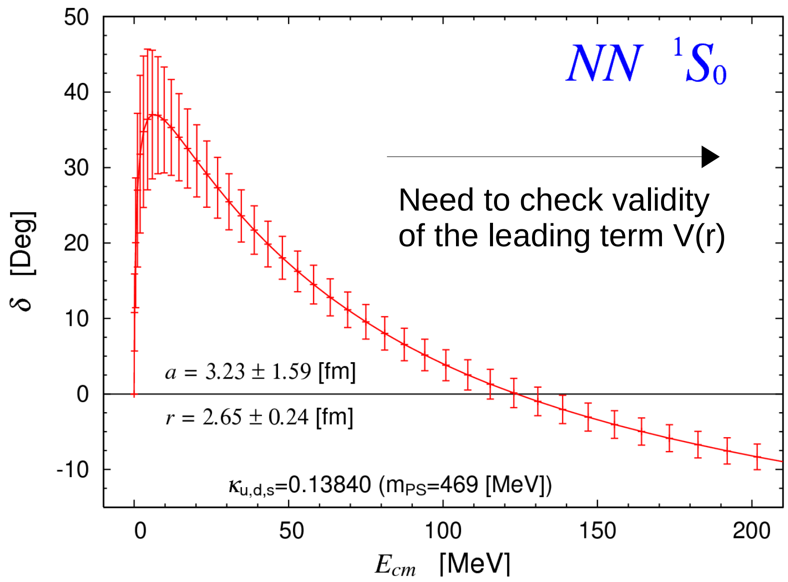

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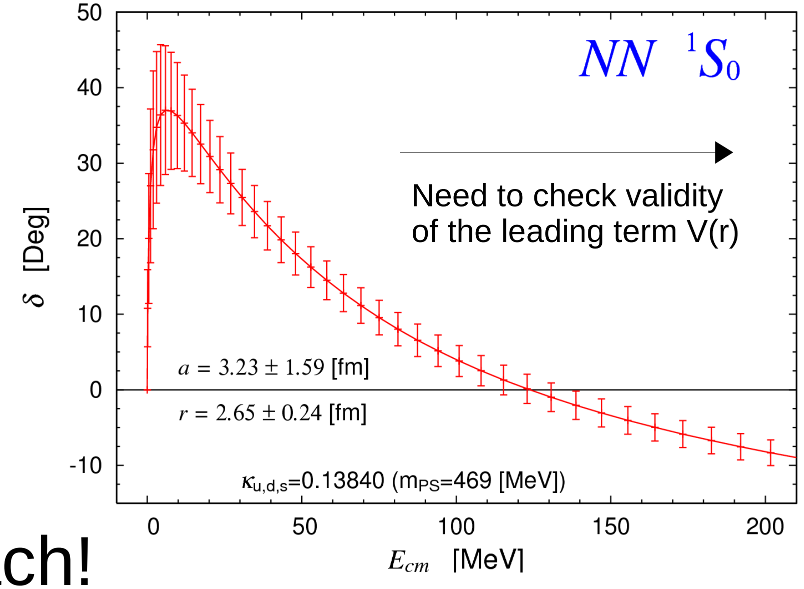



☆ We can probe the H in this approach!
(as far as H has a baryonic component)

# Formulas and Setup

# Nambu-Bethe-Salpeter w.f.

- NBS wave function

$$\psi^{(a)}(\vec{r},\, t) \overset{\text{def}}{=} \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, \overbrace{t) B_j(\vec{x}, t}^{\text{the same}}) | B{=}2, \text{a-plet} \rangle$$

QCD eigenstate

$$\propto \sum_{\vec{x}} G^{(a)}(\vec{x}+\vec{r}, \vec{x},\, t)$$

4-point function $G^{(a)}(\vec{x}, \vec{y},\, t-t_0) = \langle 0 | \underbrace{B_i(\vec{x}, t) B_j(\vec{y}, t)}_{\text{sink}} \, \underbrace{\overline{BB}^{(a)}(t_0)}_{\text{source}} | 0 \rangle$

- Point type octet baryon field operator at sink

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha}\, u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with} \quad \xi_i = \{c_i,\, \beta_i,\, \underline{x}\}$$

$$\Lambda_\alpha(x) = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

- Quark wall type BB source in the flavor irreducible rep.

e.g for flavor-singlet $\overline{BB}^{(1)} = -\sqrt{\frac{1}{8}}\, \overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\, \overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\, \overline{\mathrm{N}}\,\overline{\Xi}$

# Potential

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89(2010)
N. Ishii etal. [HAL QCD coll.] in preparation

NBS wave function $\psi(\vec{r},t) = \phi_{Gr}(\vec{r})e^{-E_{Gr}t} + \phi_{1st}(\vec{r})e^{-E_{1st}t} \cdots$

Euclidian space-time Schrödinger eq. of energy-eigen-compnent

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{Gr}(\vec{r}\,')e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{1st}(\vec{r})e^{-E_{1st}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{1st}(\vec{r}\,')e^{-E_{1st}t} = E_{1st}\phi_{1st}(\vec{r})e^{-E_{1st}t}$$

Non-local but energy independent

By adding equations

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

$\nabla$ expansion & truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')[V(\vec{r}) + \cancel{\nabla + \nabla^2}\ldots]$$

Therefor, in the leading

$$\boxed{V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \underline{\frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B}}$$

$t$-derivative at each $r$

different from previous method

must be $t$-indep. (non-trivial)

# Lattice, Action and Facility

| $\beta$ | a [fm] | Lattice | L [fm] |
|---|---|---|---|
| 1.83 | 0.121(2) | $32^3$ x 32 | 3.87 |

- Renormalization group improved Iwasaki gauge and Non-perturbatively O(a) improved Wilson quark
- We thank K.-I. Ishikawa and the PACS-CS group for providing their DDHMC/PHMC code to generate gauge configuration, and the Columbia Physics System for their lattice QCD simulation code.
- We enhance S/N of data by averaging on 4x4=16 source, and forward/backward propagation in time.
- All numerical computation carried at the supercomputer system T2K-Tsukuba.

# Five ensembles

T.I. et.al. Phys. Rev. Lett. 106, 162002(2011)

| K_uds | N_cfg | M_P.S. [MeV] | M_Vec [MeV] | M_Bar [MeV] | relative to |
|---|---|---|---|---|---|
| 0.13660 | 420 | 1170.9(7) | 1510.4(0.9) | 2274(2) | New |
| 0.13710 | 360 | 1015.2(6) | 1360.6(1.1) | 2031(2) | add cfg |
| 0.13760 | 480 | 836.8(5) | 1188.9(0.9) | 1749(1) | |
| 0.13800 | 360 | 672.3(6) | 1027.6(1.0) | 1484(2) | |
| 0.13840 | 600 | 468.9(8) | 830.6(1.5) | 1163(2) | New |

- We've made five ensembles with different hopping parameter, (=quark mass) corresponding to $M_{PS}$ = 1.17 [GeV] to 470 [MeV].
- With lightest quark (bottom row of the table),
  - p.s. meson is a little lighter than the physical kaon.
  - baryon is a little lighter than the physical sigma baryon.
- Now, the simulated hadron world is not so far from the real world, although the $SU(3)_F$ breaking is not taken into account.

# Results

# BB int. at SU(3)$_F$ limit

- In the limit, convenient basis exist to describe the int.

$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

flavor irreducible rep.

- In S-wave, no off-diagonal interaction exists.

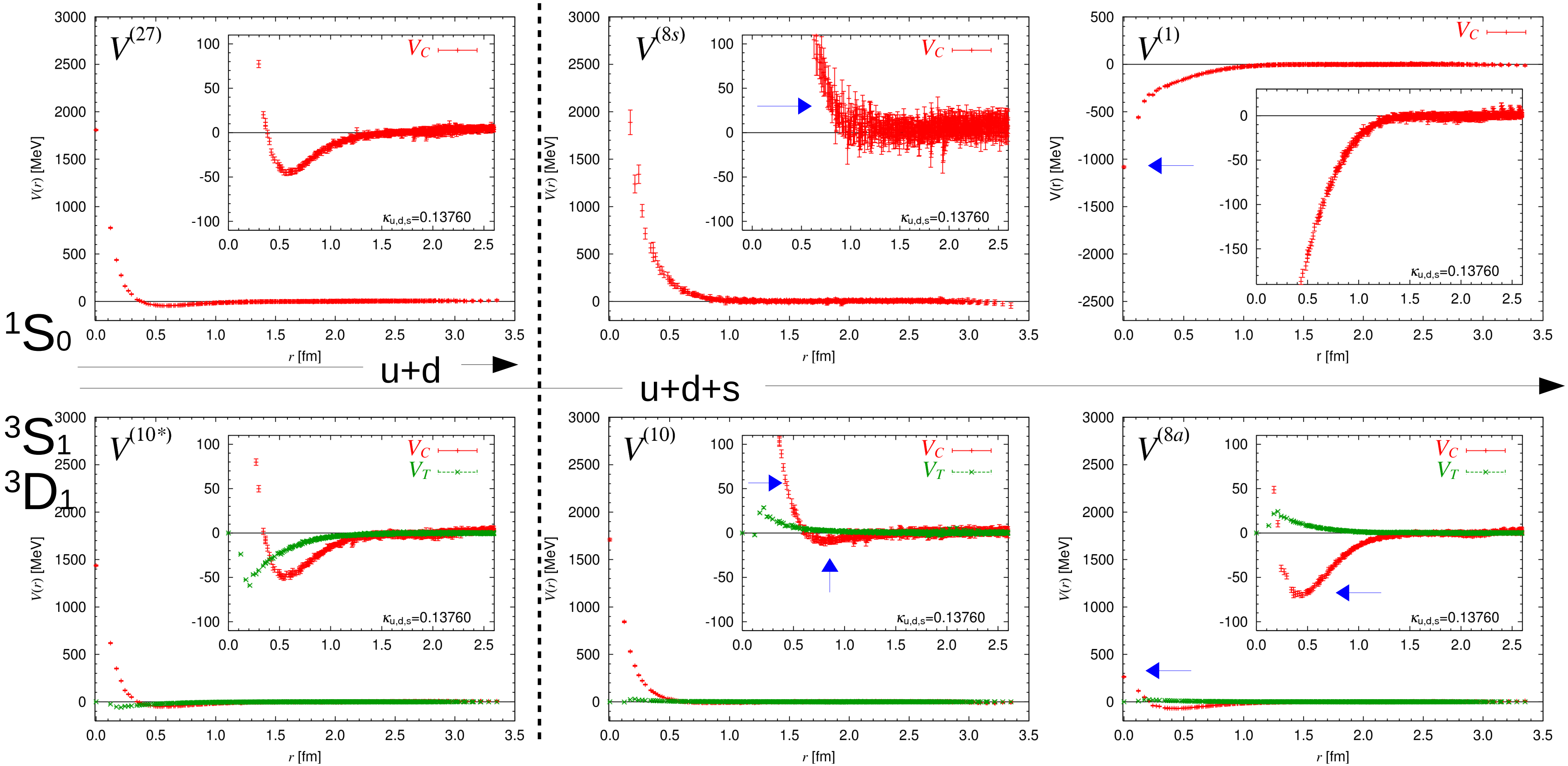
Fermi statistics leads to

$$^1S_0 \;:\; V^{(27)}(r),\quad V^{(8s)}(r),\quad V^{(1)}(r)$$

$$^3S_1 \;:\; V^{(10^*)}(r),\quad V^{(10)}(r),\quad V^{(8a)}(r)$$

- Six $V^{(a)}(r)$ contain essential flavor-spin structure of BB int.
- We can reconstruct all baryon-base interaction (eg. ΛN) by using these $V^{(a)}(r)$ with SU(3) C.G. coefficients.
- The $V^{(a)}$ is useful to pin down physical origin of particular feature, since effective models assume flavor symmetry.

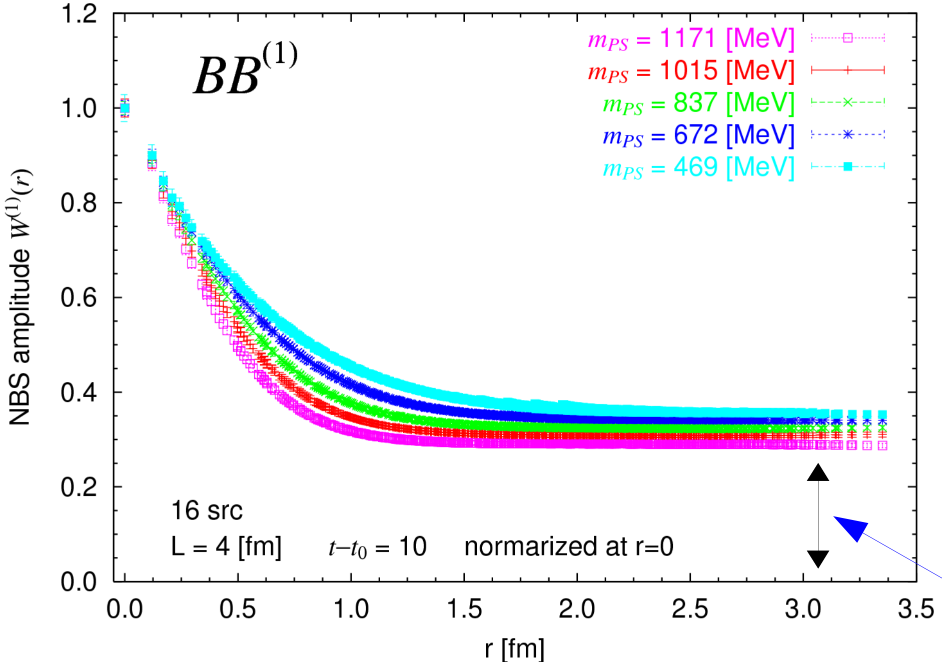
# BB int. at SU(3)$_F$ limit

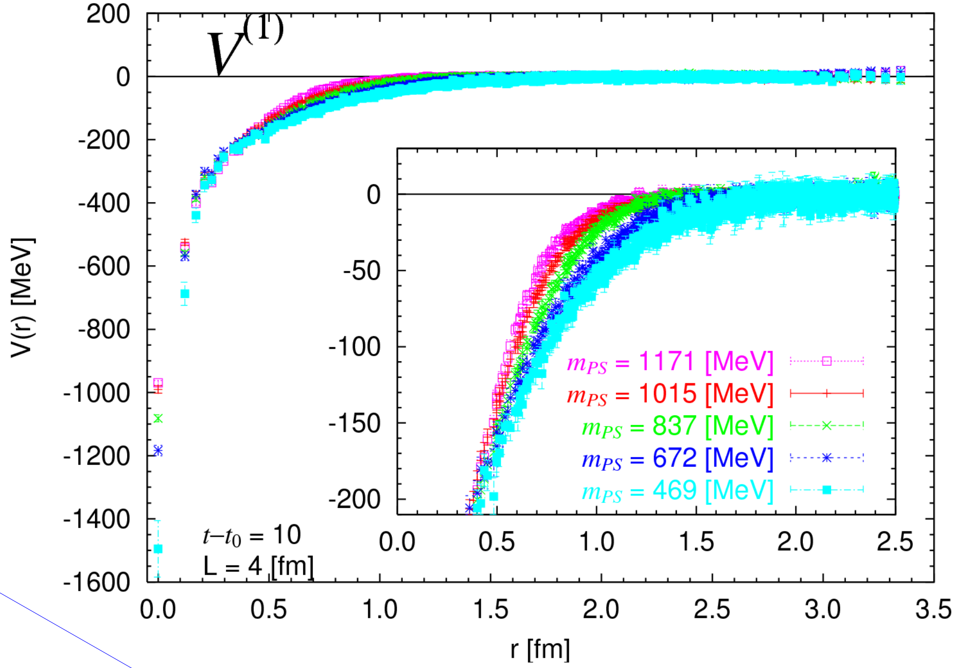



- At the SU(3)$_F$ limit corresponding to $M_\pi$ = $M_K$ = 837 [MeV].
- QM is true at small $r$. Especially, no repulsion in $1_F$ channel.
- This indicate possibility of a bound H-dibaryon in the limit.

# H-dibaryon

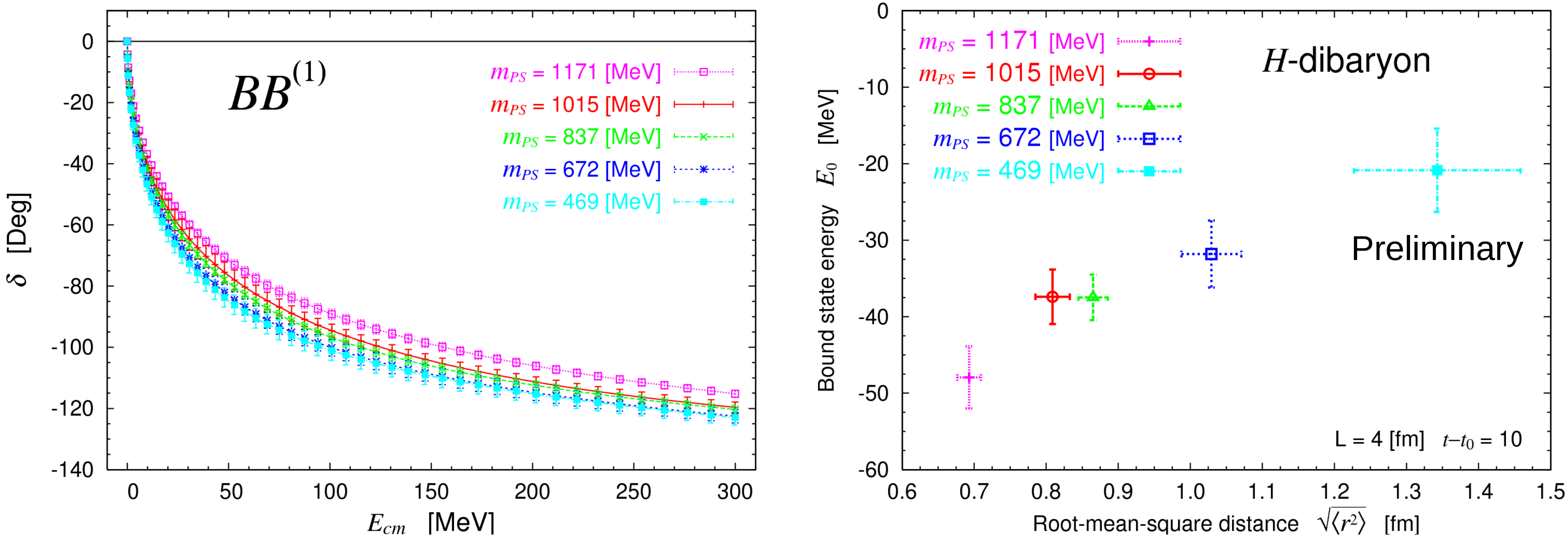
# NBS w.f. & potential



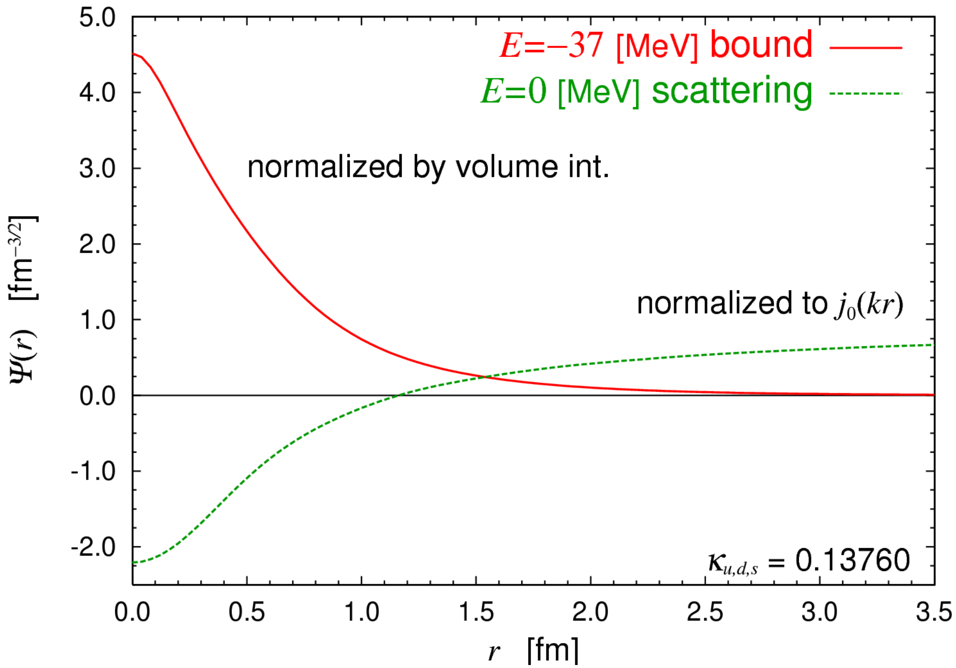

- Left: Measured NBS w.f. of the 1F channel
  - The finite value at large distance is excited states contribution as well as a finite volume effect. (demonstrated in later)
- Right: Extracted potential of the 1F channel
  - $V^{(1)}(r)$ become more attractive as quark mass decrease.

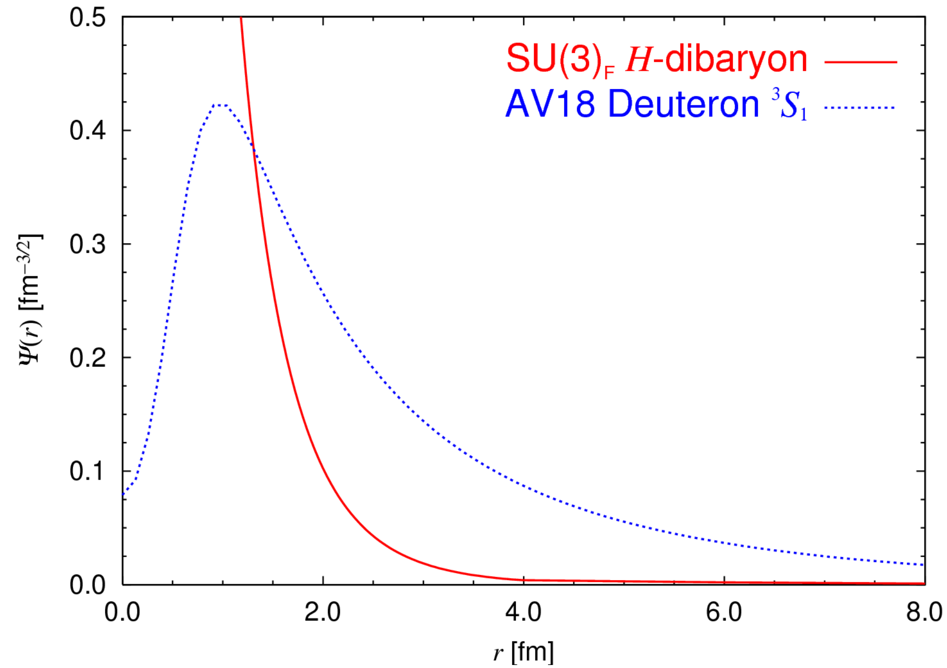
# Observables




- Left: scattering phase shift v.s. $E_{cm}$
  - shows existence of one discrete state below threshold.
- Right: obtained ground state
  - which is 20 - 50 MeV below from free BB ie. 3q-3q.
  - This means that there is a 6-quark bound state in the $1_F$ channel.
  - A stable(bound) H-dibaryon exists in these $SU(3)_F$ limit world!

# Size of H-dibaryon




- Left: Wave function of the lowest two states of V$^{(1)}$.
  - the measured NBS w.f. is a superposition of red and green with the finite volume effect.
- Right: Comparison between the H and the physical deuteron.
  - One can get feeling of the H-dibaryon: It is compact.

# Hyperon interaction

# Potential in baryon-base

- In flavor SU(3) broken world, e.g. the physical one, the baryon-basis are used instead of flavor-basis.
- In the SU(3) limit, the baryon-base potential $V_{ij}(r)$ can be obtained by a unitary rotation of the potential $V^{(a)}(r)$.

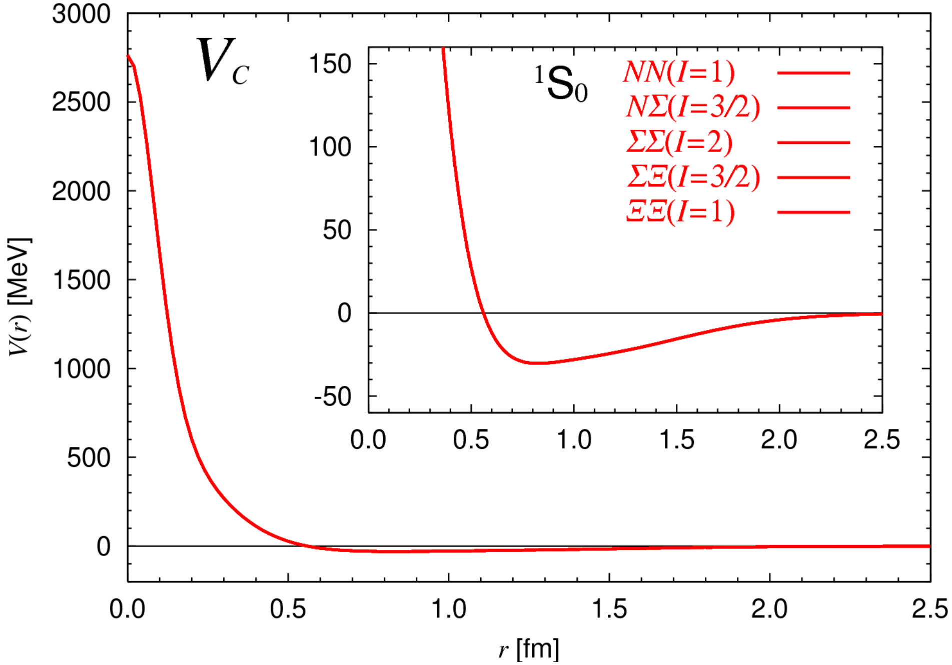
e.g. S=−2, I=0 sector

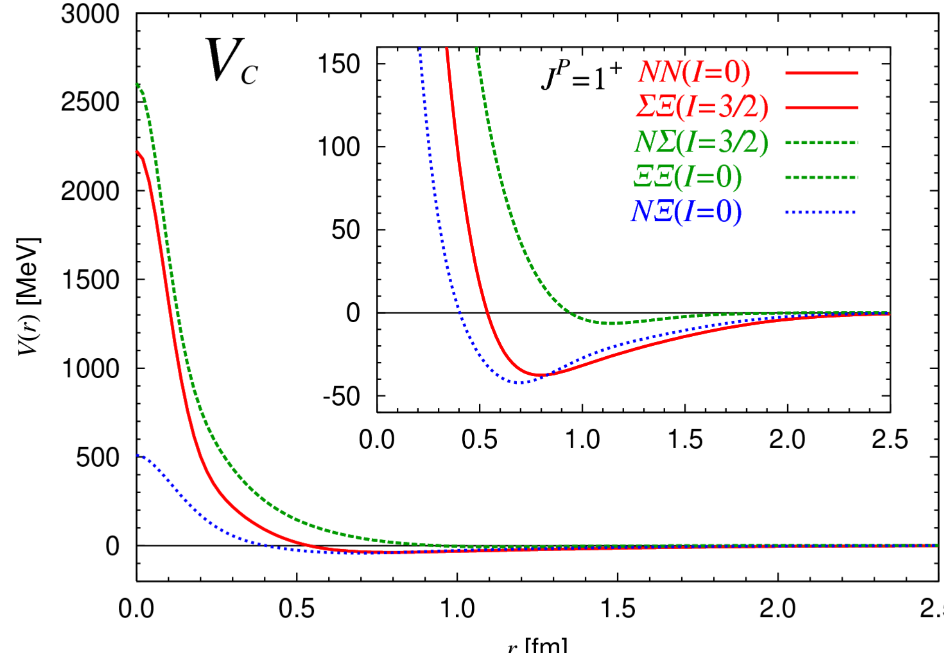
coupled channel

$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle \Xi N | \end{pmatrix} = U \begin{pmatrix} \langle 27 | \\ \langle 8 | \\ \langle 1 | \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t = \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

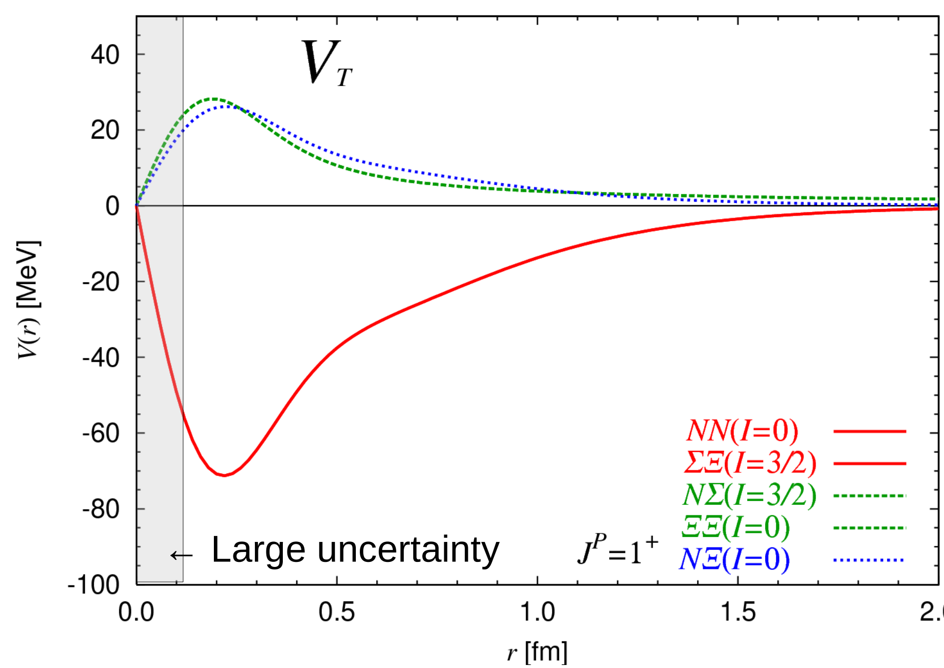
- I show you potentials $V_{ij}(r)$ at the lightest quark mass ($K_{uds}$ = 0.13840, $M_{ps}$=469 MeV) obtained with the $V^{(a)}$ in an analytic function fitted to data.

# Uncoupled (exclusive) channels

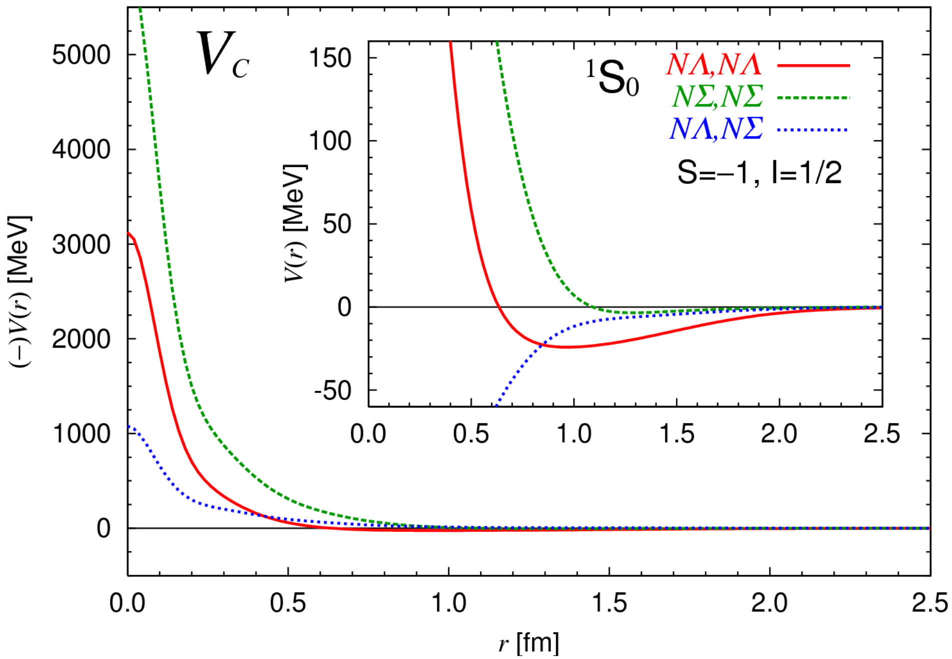


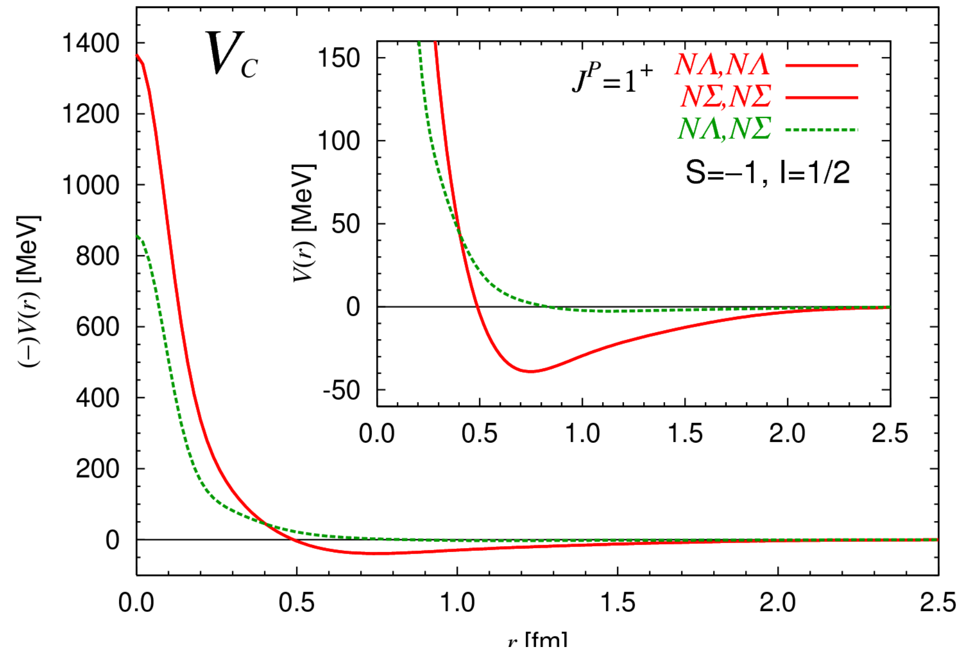



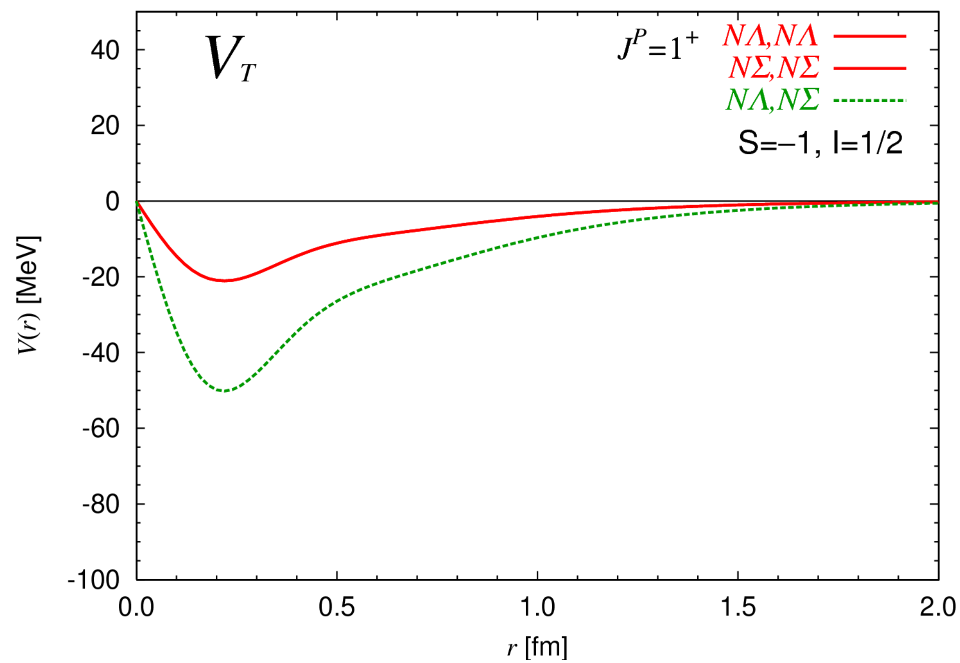

- Some BB channels belong to a flavor irr-rep. exclusively.
- Potential of such *BB* channel is nothing but $V^{(a)}(r)$.
- S=0 and S=−4 *BB* channels are completely exclusive.



# S=−1, I=1/2 sector

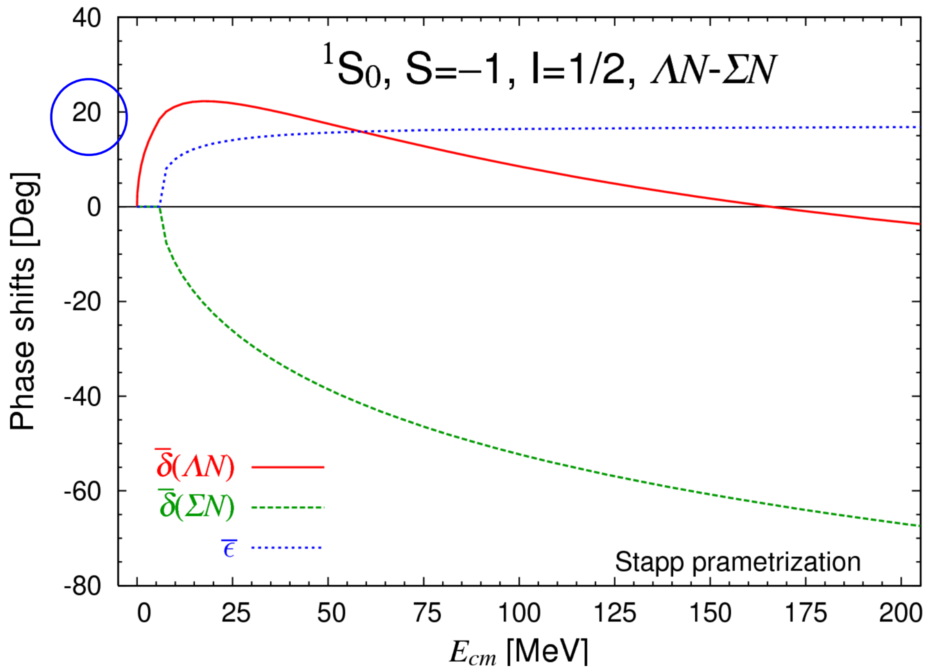


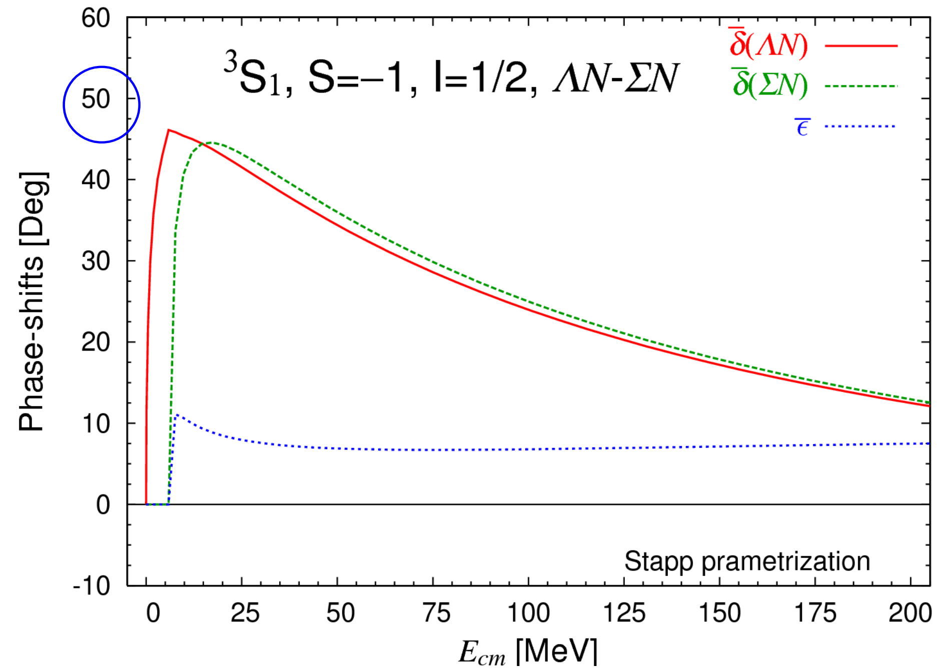




- NΛ - NΣ(I=1/2) coupled.
- $^1S_0$
  - $V_{N\Lambda}$ has attractive well.
  - $V_{N\Sigma}$ is strongly repulsive.
- $J^P = 1^+$
  - Both have an attractive well.
  - NΛ - NΣ strongly coupled.
  - Off-diagonal $V_T$ is stronger.

# YN scattering in S=−1, I=1/2



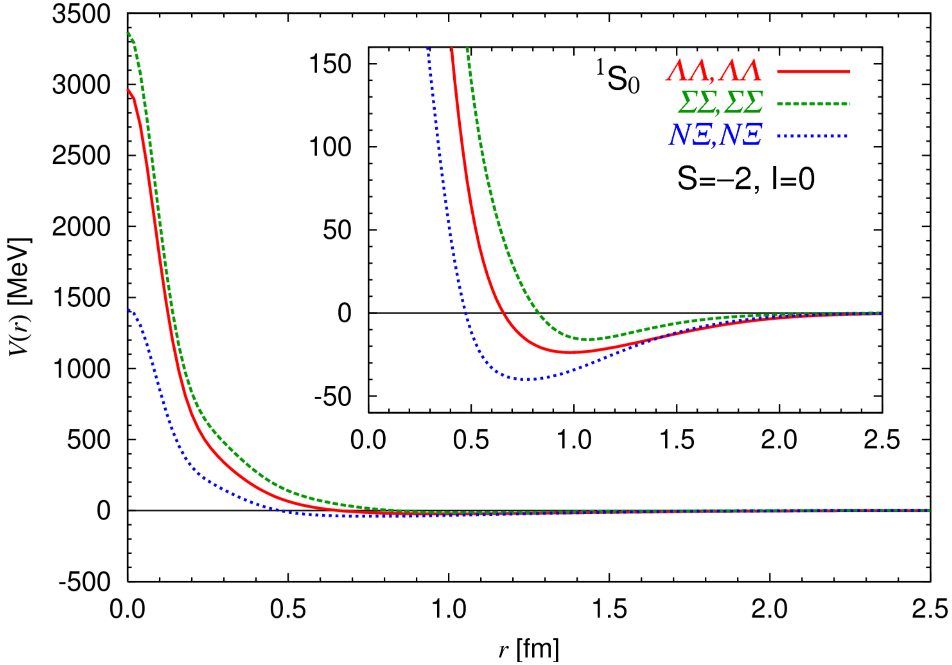

- Phase-shift and mixing-angle as a function of energy

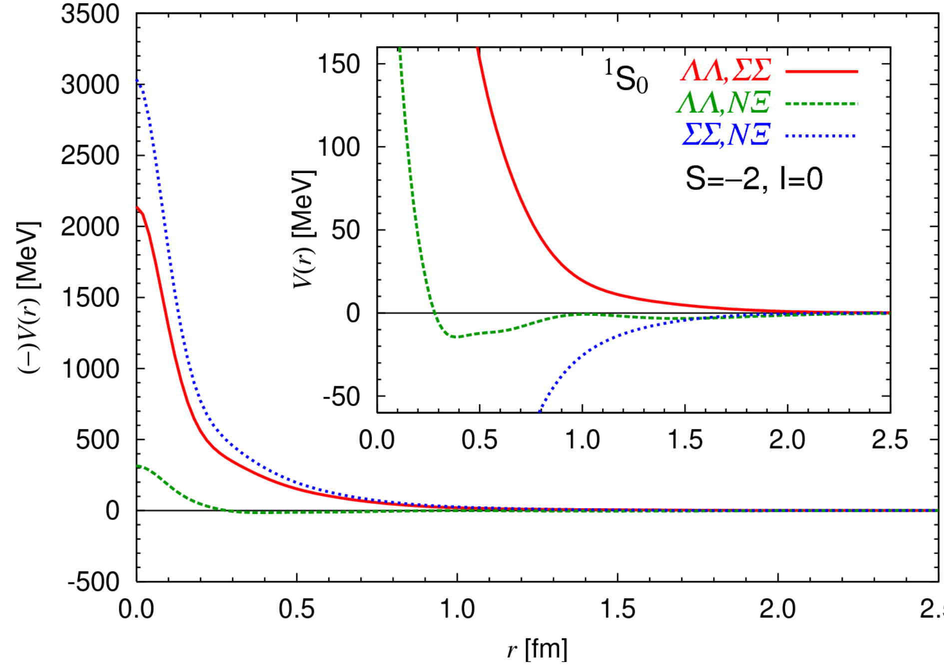
flavor SU(3) limit small SU(3) breaking

- from $V$(r) at (K$_{ud}$, K$_s$)=(0.13760, 0.13760) and $M_B$ at (0.13760, 0.13710)
- where $M_N$ = 1791.1, $M_\Lambda$ = 1827.0, $M_\Sigma$ = 1833.8, $M_\Xi$ = 1860.0 [MeV]
- Effective central potential is used for $^3S_1$ channel.

- Essential features must remain at the physical point.
- Future experiment will confirm them. Exciting!

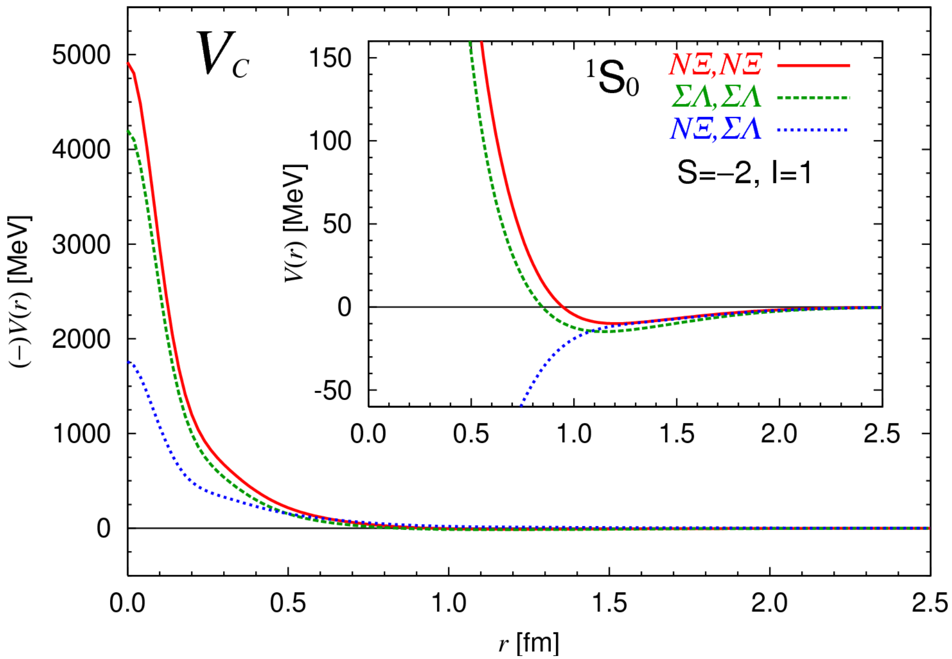
# S=−2, I=0 sector

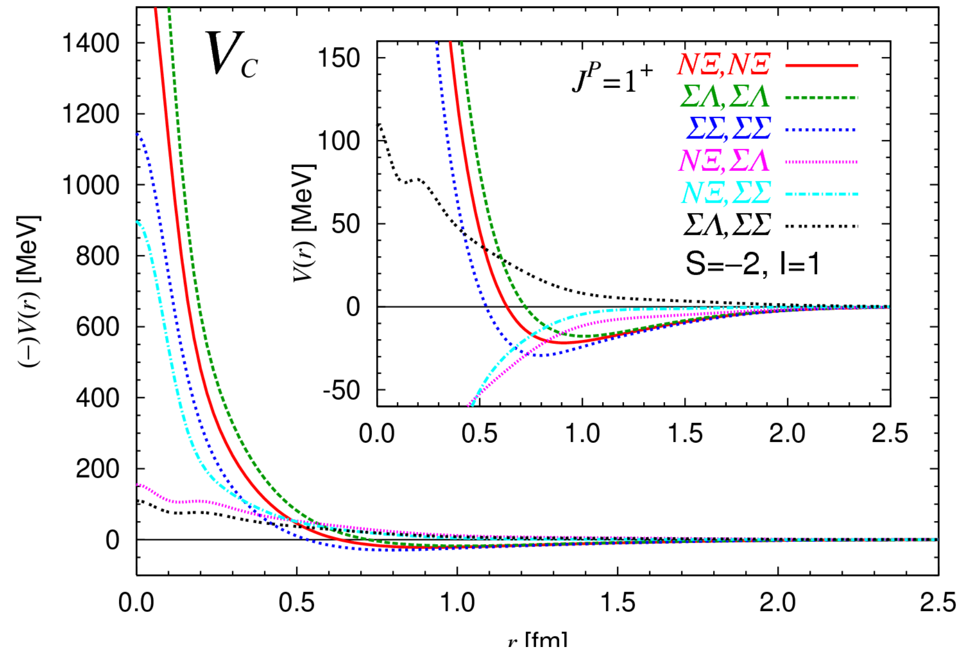


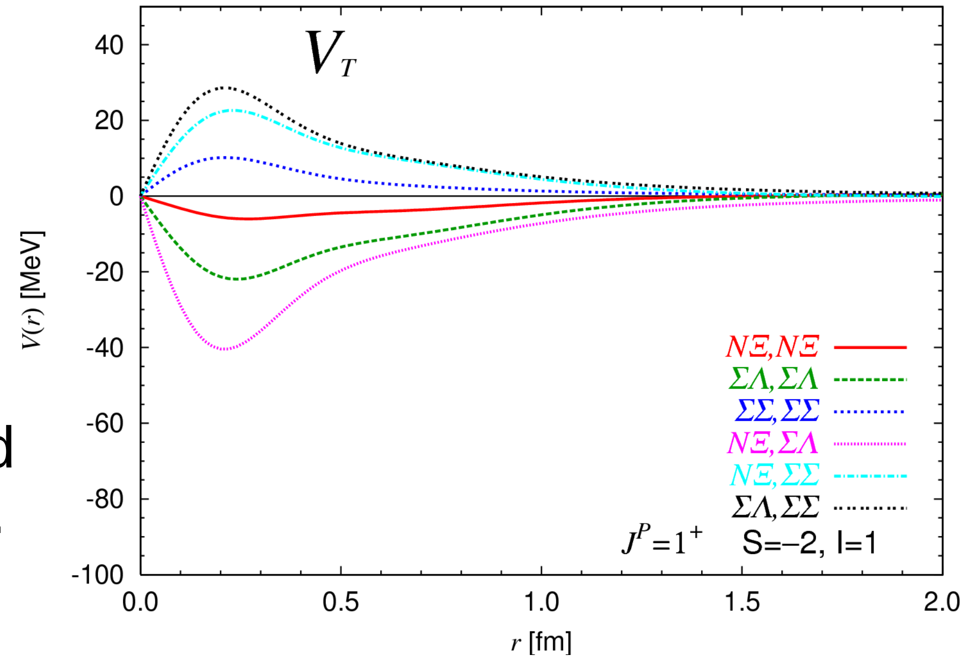




- ΛΛ – NΞ – ΣΣ coupled. Left: diagonal. Right: Off-diagonal.
- It is flavor symmetric(spin singlet), and involve flavor singlet ch.
- Channel coupling int. are comparable to diagonal ones, except for the ΛΛ - NΞ transition (small sign change in $V_{ΛΛ\text{-}NΞ}$ must be artifact).
- Interaction is most attractive in NΞ channel, although it has not much real meaning because channel coupling is strong.

# S=−2, I=1 sector

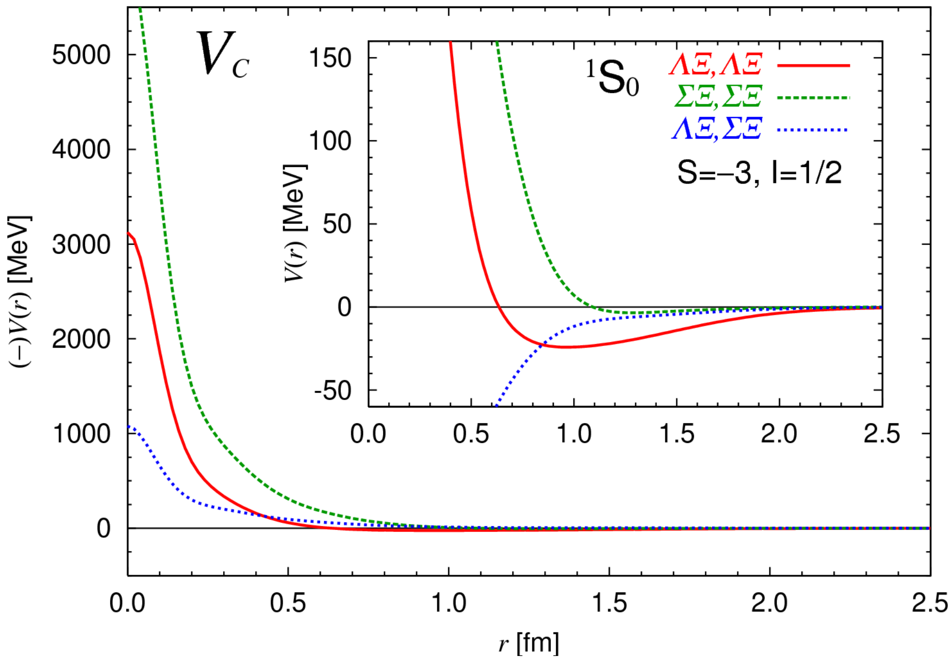


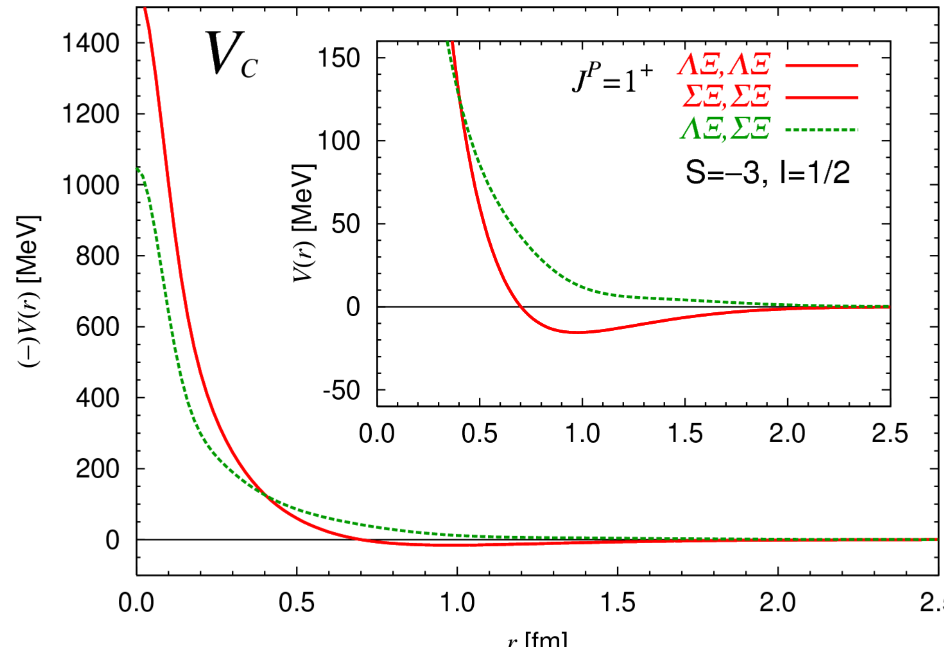



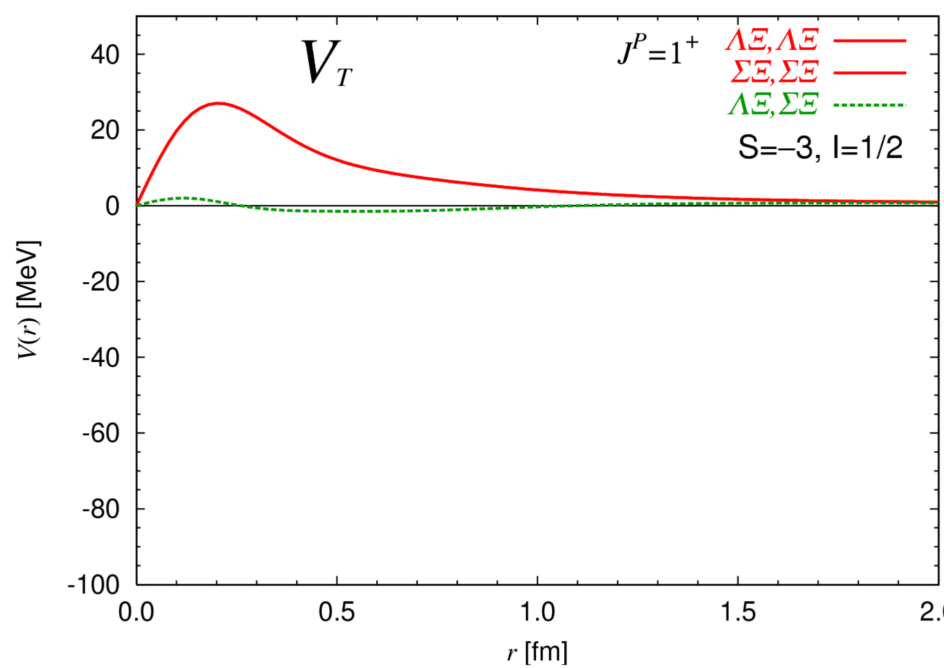




- NΞ – ΣΛ (−ΣΣ) coupled.
- $^1S_0$
  - $V_{N\Xi} \approx V_{\Sigma\Lambda}$, moderate coupling
- $J^P = 1^+$
  - Need to solve 6-channel-coupled Schrödinger eq. for observables.

# S=−3, I=1/2 sector



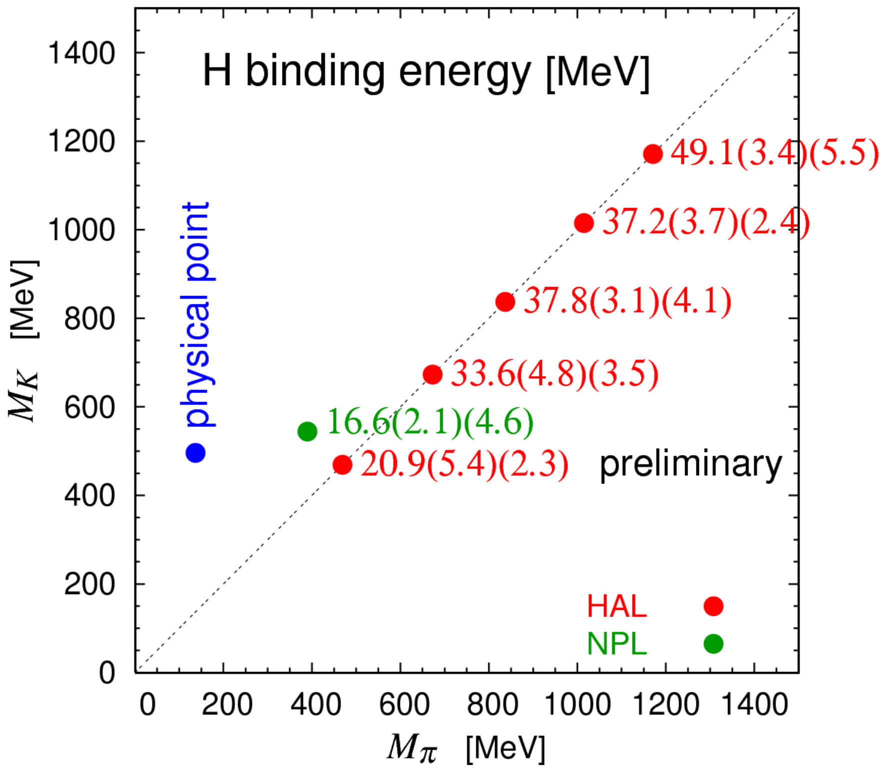

- ΛΞ − ΣΞ coupled.
- $^1S_0$
  - $V_{\Lambda\Xi} = V_{\Lambda N}$, $V_{\Sigma\Xi} = V_{\Sigma N}$
- $J^P=1^+$
  - strong channel coupling by $V_C$.
  - hyperon-changing $V_T$ vanishes.

# Summary

- We've introduced motivation and purpose.
  - We want to explore for the H-dibaryon in QCD by using Lattice.
  - We start with flavor SU(3) limit to avoid complication.
- We've explained our approach.
  - We'll extract observables exclusively through the interaction potential.
  - By using both time and spacial derivative of the NBS w.f. at each point, we can extract the potential even without the ground-state-saturation.
  - We've tested and confirmed our new technique. ($t$-indep. and $L$-indep.)
- We've done full QCD lattice simulation to probe the H.
  - $32^3$ x 32 lattice, L=3.87 [fm], Iwasaki gauge, clover quark
- We've found a bound=stable H-dibaryon
  - in flavor SU(3) symmetric world at $M_{PS}$ = 470 [MeV] – 1.17 [GeV],
  - its binding energy is 20 – 50 [MeV] depending on the quark mass,
  - and size is compact compared to usual B=2 system.

# Summary & Outlook

- Plot of H binding energy from recent full QCD simulations.
  - SR. Beane etal [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011)
  - Obtained binding energy from the two groups looks consistent.
  - But, all data points are still away from the physical point.



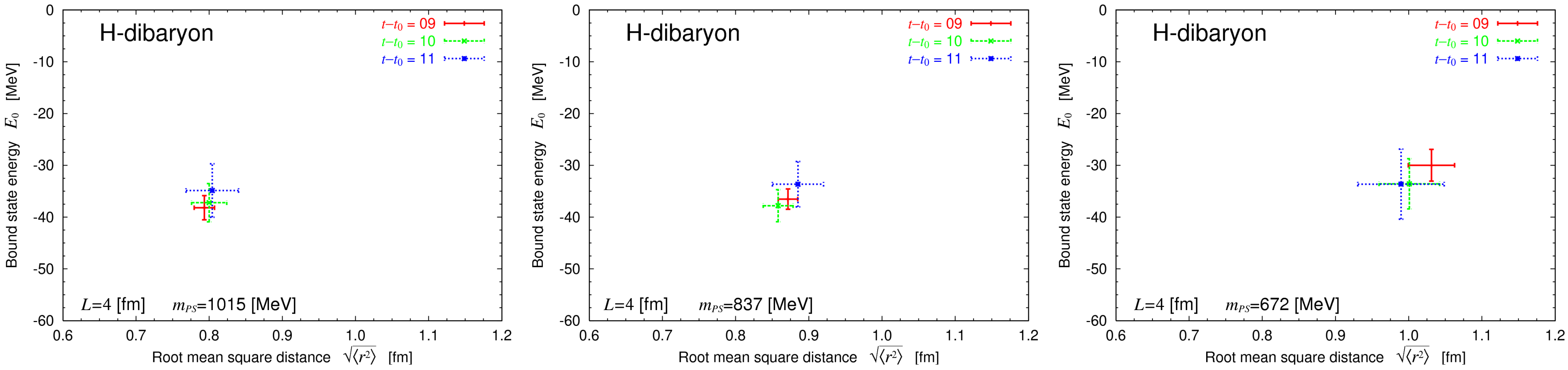

- We'll continue this study and put more data on this plot.
- We'll take flavor SU(3) breaking into account soon.
- We'll get information of H-dibaryon in the physical world in near future.

Thank You!

# Backup slides

# Binding energy of H

- Choice of time slice t = 9, 10 , 11 [a]




- All results agree within statistical error.
- Error from choice of fit function is less than few % ie. negligible.
- Final results for biding energy of H-dibaryon:

$$m_{P.S.} = 1015 \text{ MeV}: \ B_H = 37.2\,(3.7)\,(2.4) \ \text{MeV}$$
$$m_{P.S.} = 837 \text{ MeV}: \ B_H = 37.8\,(3.1)\,(4.2) \ \text{MeV}$$
$$m_{P.S.} = 672 \text{ MeV}: \ B_H = 33.6\,(4.8)\,(3.5) \ \text{MeV}$$

# FAQ

1. Does the potential depend on the choice of source?

2. Does the potential depend on choice of field operator at sink?

3. Does the potential $U(x,y)$ or $V(r)$ depend on energy?

# FAQ

1. Does the potential depend on the choice of source?

➔ No. Some sources may enhance excited states in NBS w.f. However, the new method has no problem with excited states at all.

2. Does the potential depend on choice of field operator at sink?

➔ Yes. It can be regarded as the “scheme” to define potential. Note that the potential itself is not physical observable. We'll obtain unique result for observables irrespective to the choice, as long as the potential U(x,y) is deduced exactly.

3. Does the potential U(x,y) or V(r) depend on energy?

➔ By construction, U(x,y) is non-local but energy independent. While, its leading order truncation V(r) obtained here, is local but velocity dependent. However, we know that the dependence in NN case is very small and negligible at least at E = 0 – 45 MeV. If we get dependence, we''ll determine the next leading terms from it.

# FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

5. Is the H a compact six-quark object or a tight BB bound state?

# FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

➔ Yes. Because a large energy dependence means that

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{Gr}}(\vec{r})\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{1st}}(\vec{r})\right]\phi_{1st}(\vec{r})e^{-E_{1st}t} = E_{1st}\phi_{1st}(\vec{r})e^{-E_{1st}t}$$

then $V(\vec{r}) \equiv \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$ would have a large $t$-dep.

5. Is the H a compact six-quark object or a tight BB bound state?

➔ Both. There is no distinct separation between two, because baryon is nothing but a 3-quark in QCD. Imagine a compact 6-quark object in $(0S)^6$ configuration. This configuration can be rewritten in a form of $(0S)^3 \times (0S)^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r. This shows that a compact six-quark object can have a baryonic component, which we measure in the NBS w.f. We've established existence of a stable QCD eigenstate which couples to BB state. We do NOT insist that another "H" doesn't exist which cannot couple to BB.

# FAQ

6. Do you insist such a deeply bound H exists in the real world?

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

# FAQ

6. Do you insist such a deeply bound H exists in the real world?

➔ No. With SU(3)F breaking, three BB thresholds in S=−2,I=0 sector split as $E^{\mathrm{Th}}_{\Lambda\Lambda} < E^{\mathrm{Th}}_{N\Xi} < E^{\mathrm{Th}}_{\Sigma\Sigma}$ . Therefore, we expect that the binding energy of H measured from $E^{\mathrm{Th}}_{\Lambda\Lambda}$ is much smaller than the present value, or even H is above $E^{\mathrm{Th}}_{\Lambda\Lambda}$ in the real world.

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

➔ It is a measure of spacial distribution of baryonic matter in H. It corresponds to the point matter root mean square distance of deuteron (= 2 x 1.9 [fm]).

# Problem

- Free 2-body energy spectrum in finite volume w/ periodic B.C.

$$p_{x,y,z} = \frac{2 n_{x,y,z} \pi}{L} \quad \text{then} \quad K_{nx,ny,nz} = \left(n_x^2 + n_y^2 + n_z^2\right) \underline{\left(\frac{2\pi}{L}\right)^2 \frac{1}{2\mu}}$$

e.g. in $L = 2$ [fm], $2\mu = M = 1750$ [MeV] case, 220 [MeV]

therefor $E_{Gr} = 2M + 0,\ E_{1\text{st}} = 2M + 220,\ E_{2\text{nd}} = 2M + 440$

- Even with interaction, spectrum is essentially the same.
- State realized in lattice at time $t$ $\quad |t\rangle = |Gr\rangle + |1\text{st}\rangle \cdots$

NBS w.f. $\quad \psi(\vec{r}, t) = \phi_{Gr}(\vec{r}) e^{-E_{Gr} t} + \underline{\phi_{1\text{st}}(\vec{r}) e^{-E_{1\text{st}} t} \cdots}$

Depending on $\Delta E \equiv E_{1\text{st}} - E_{Gr}$, if $t$ is large enough

$\psi(\vec{r}, t) \simeq \phi_{Gr}(\vec{r}) e^{-E_{Gr} t}$ Ground State Saturation

Exponential tail

- On $L = 2$ [fm] lattice, G.S.S. is realized at $t \geq 10$ [a].

# SU(3) Octet Baryon operator

$$p_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) u(\xi_3)$$
$$n_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) d(\xi_3)$$

$$\Sigma^+_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, u(\xi_1) s(\xi_2) u(\xi_3)$$
$$\Sigma^0_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\sqrt{\frac{1}{2}}\left[d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3)\right]$$
$$\Sigma^-_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, d(\xi_1) s(\xi_2) d(\xi_3)$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\sqrt{\frac{1}{6}}\left[d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3)\right]$$

$$\Xi^0_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, s(\xi_1) u(\xi_2) s(\xi_3)$$
$$\Xi^-_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, s(\xi_1) d(\xi_2) s(\xi_3)$$

- With corrected phase $\bar{1} = -\epsilon^{123} = -(ds - sd) = sd - ds$

# Irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} + \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^+}\,\overline{\Sigma^-} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^0}\,\overline{\Sigma^0} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^-}\,\overline{\Sigma^+}$$

$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} + \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} - \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^+} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^+}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} - \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} + \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

# Various Theoretical Approaches to Nuclei