Extended Brueckner-Hartree-Fock theory in many body system - Importance of pion in nuclei -

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In collaboration with

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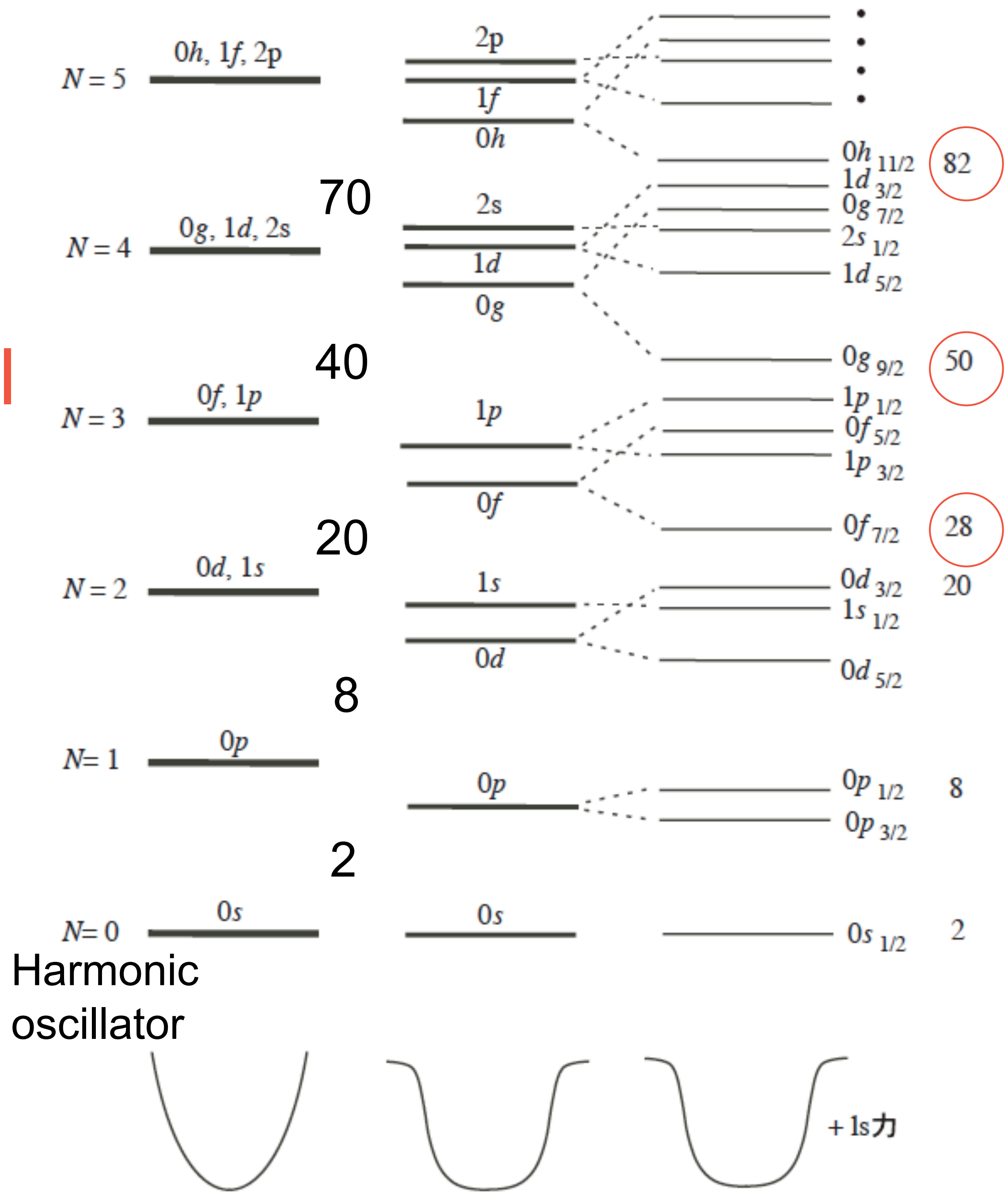
Pion is important in Nuclear Physics！

- Yukawa（1934）predicted pion as a mediator of nuclear interaction to form nucleus
- Meyer-Jansen（1949）introduced shell model—beginning of Nuclear Physics
- Nambu（1960）introduced the chiral symmetry and its breaking produced mass and the pion as pseudo-scalar particle

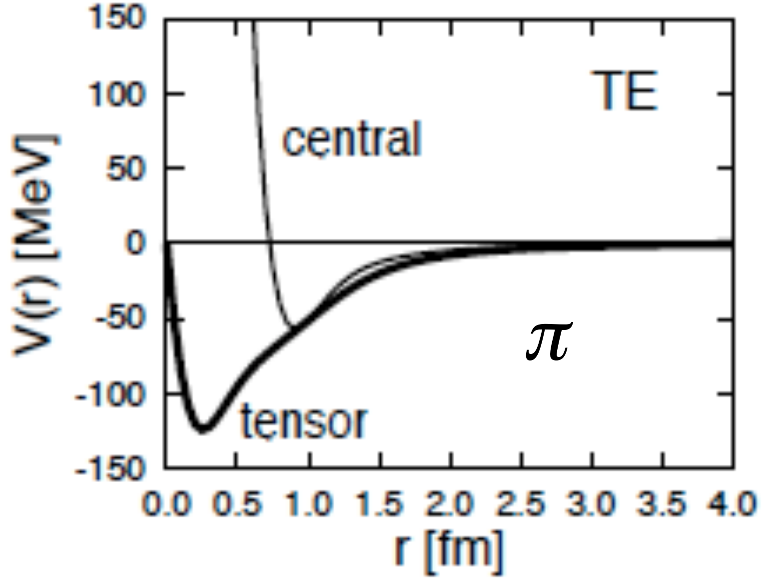
Shell model
(Meyer-Jensen)

- Phenomenological
- Strong spin-orbit interaction added by hand
- Magic number
- 2,8,20,28,50,82



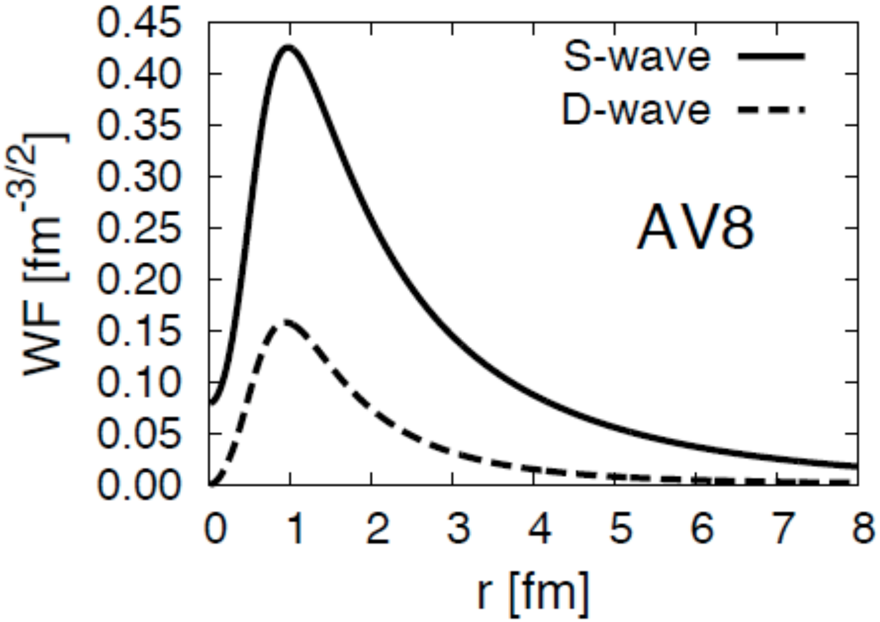
The importance of pion is clear in deuteron

NN interaction



S=1 and L=0 or 2

$$\Psi_d = u(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M} + w(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M}$$



@genkenpion

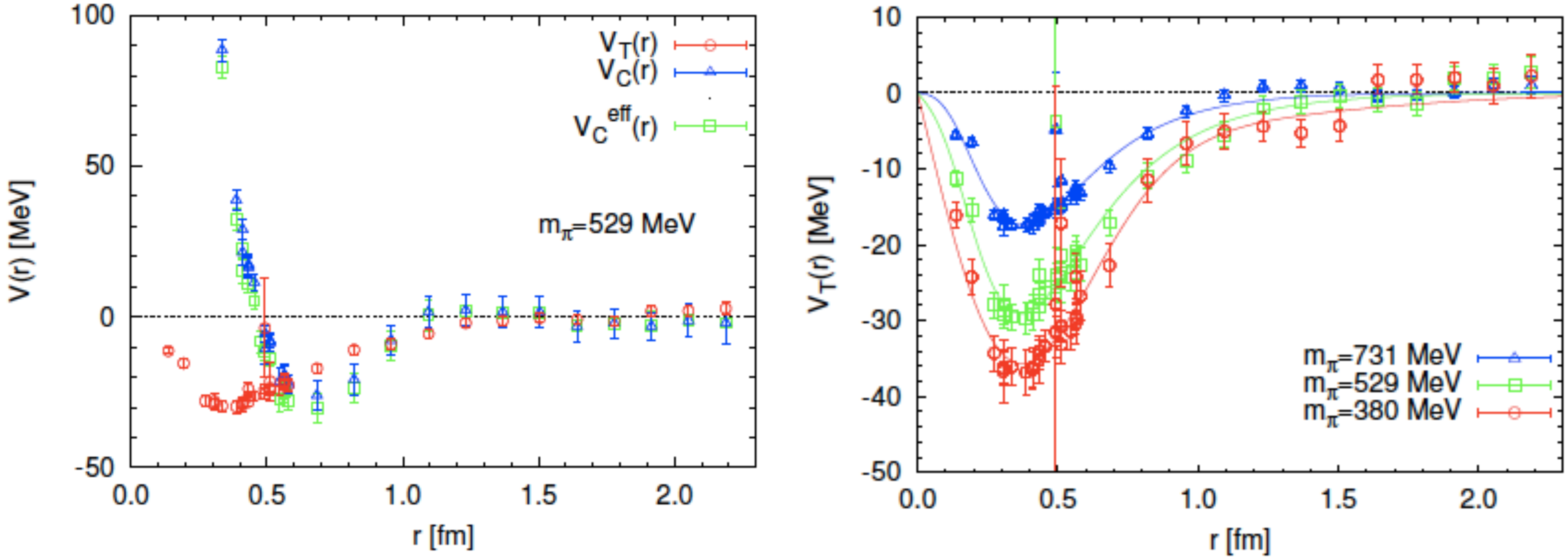
Deuteron (1^+)

Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensorc	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(*D*)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

Progress of Theoretical Physics, Vol. 123, No. 1, January 2010

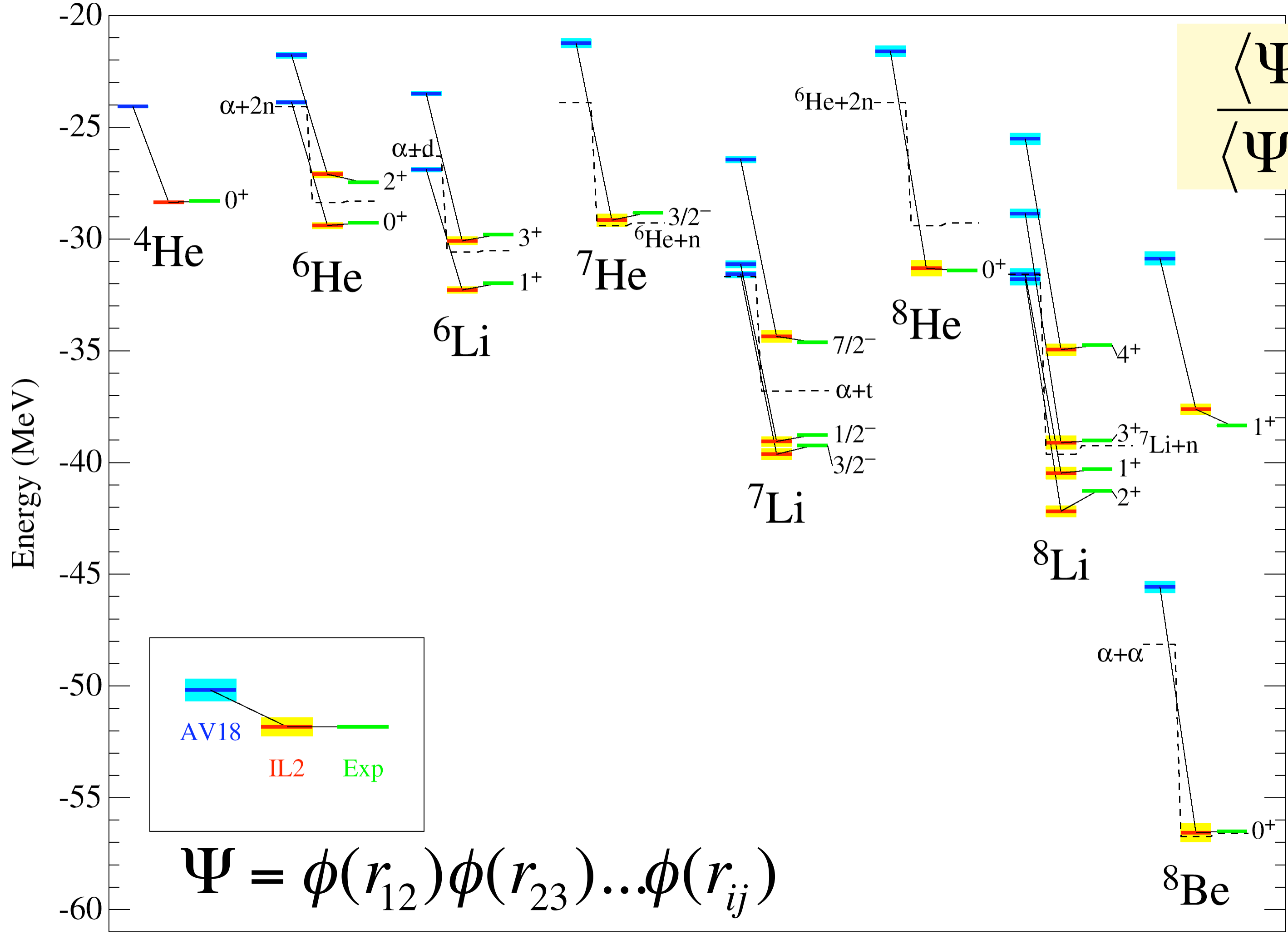
Theoretical Foundation of the Nuclear Force in QCD and Its Applications to Central and Tensor Forces in Quenched Lattice QCD Simulations

Sinya AOKI,[1] Tetsuo HATSUDA[2] and Noriyoshi ISHII[2]



Variational calculation of few body system with NN interaction

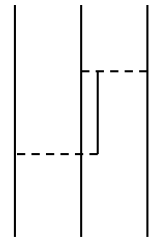
VMC+GFMC



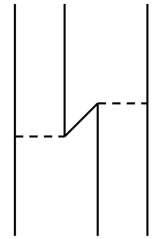
$$\Psi = \phi(r_{12})\phi(r_{23})\ldots\phi(r_{ij})$$

$$\frac{\langle\Psi|V_\pi|\Psi\rangle}{\langle\Psi|V_{NN}|\Psi\rangle} \sim 80\%$$

V_{NNN}



Fujita-Miyazawa



Relativistic

C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

Heavy nuclei (Super model)

Pion is key

Pion is important in nucleus

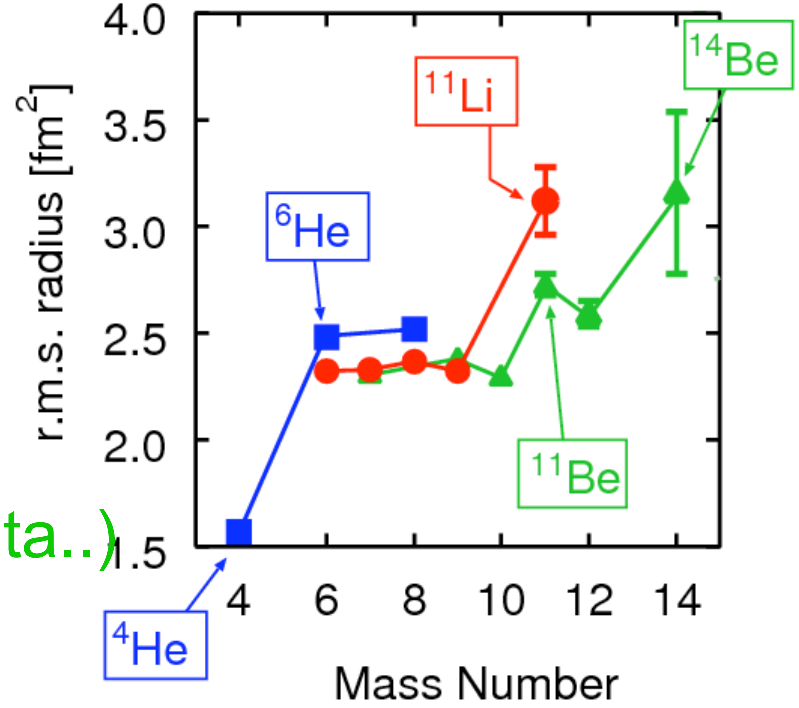
- 80% of attraction is due to pion
- Tensor interaction is particularly important

$$\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q} = \frac{1}{3} q^2 S_{12}(\hat{q}) + \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 q^2 \qquad S_{12}(\hat{q}) = \sqrt{24\pi}\left[Y_2(\hat{q})\left[\sigma_1 \sigma_2\right]_2\right]_0$$

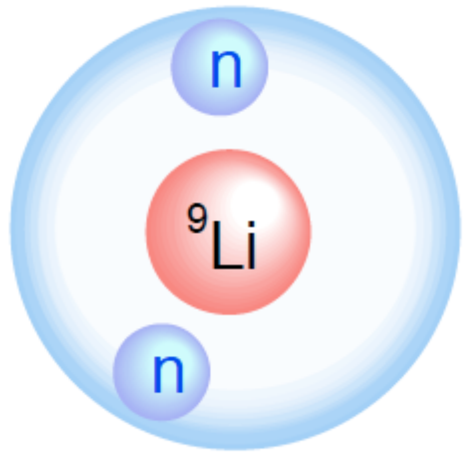
Pion	Tensor	spin-spin

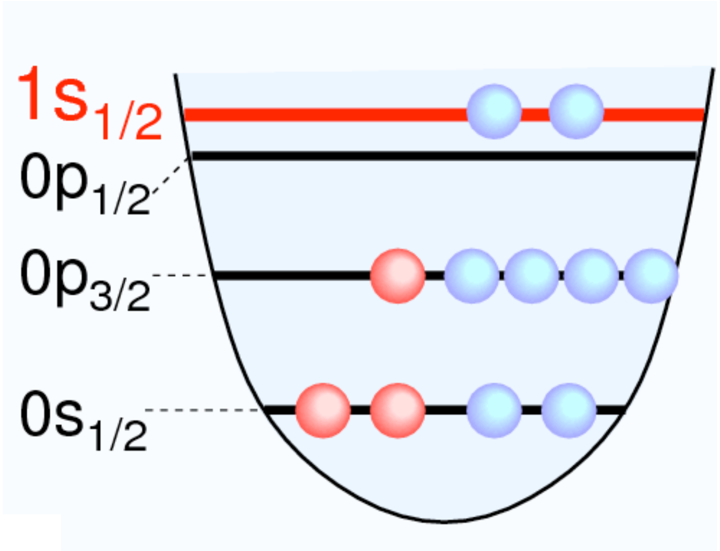
Halo structure in ^{11}Li

Myo Kato Toki Ikeda PRC(2008)



(Tanihata..)





Deuteron wave function

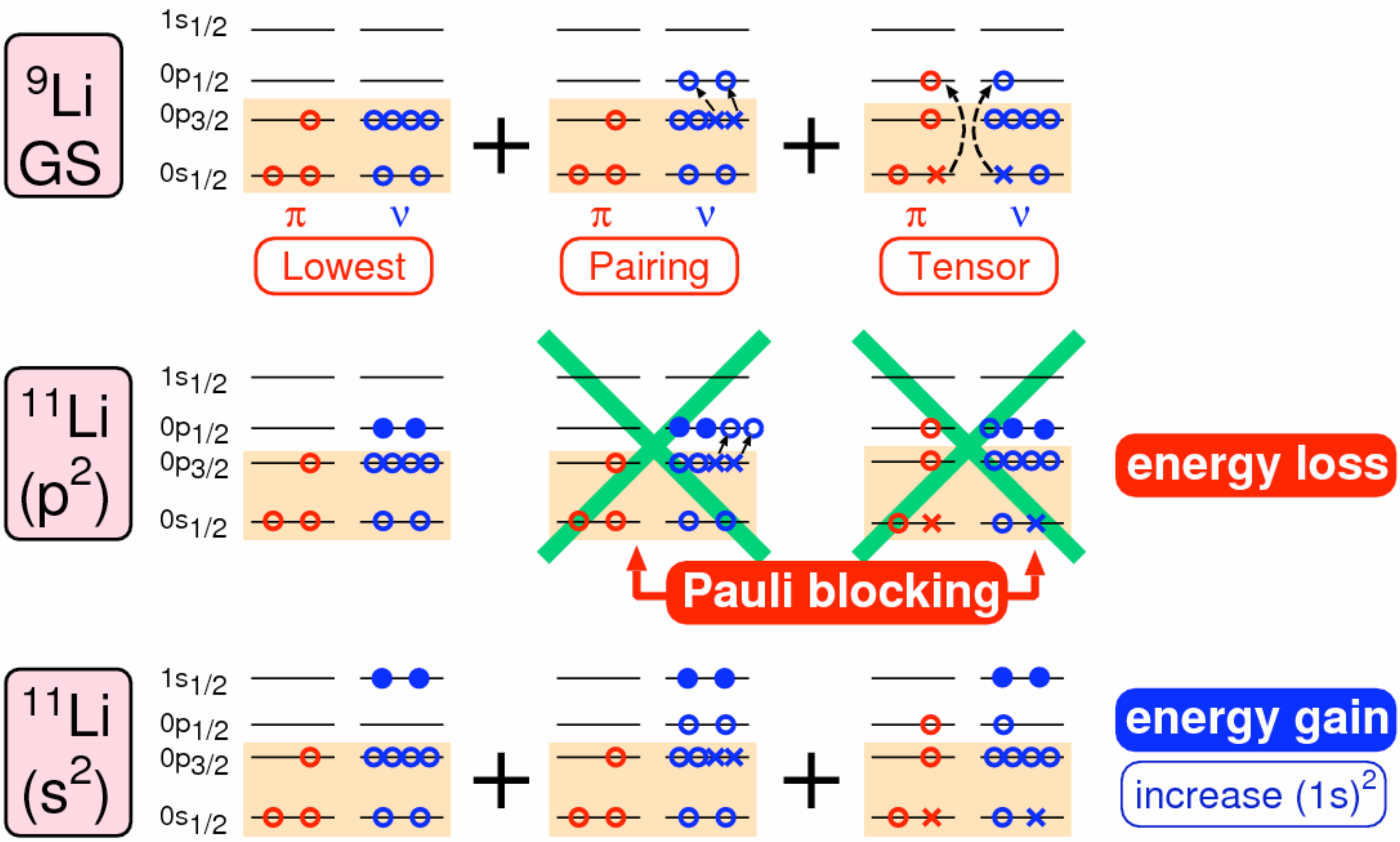
$$\Psi_d = u(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M} + w(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M}$$

$$|[(0s_{1/2})^2]_{1M}\rangle \sim \Psi_{L=0}(R)\psi_{l=0}(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M}$$

$$|[(0p_{1/2})^2]_{1M}\rangle \sim \Psi_{L=0}(R)\psi_{l=2}(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M}$$

Deuteron structure in shell model is produced by 2p-2h states

Expected effects of pairing and tensor correlations in ^{11}Li



Pairing-blocking :

K.Kato,T.Yamada,K.Ikeda,PTP101('99)119, Masui,S.Aoyama,TM,K.Kato,K.Ikeda,NPA673('00)207.

TM,S.Aoyama,K.Kato,K.Ikeda,PTP108('02)133, H.Sagawa,B.A.Brown,H.Esbensen,PLB309('93)1.

Tensor optimized shell model
(TOSM)

Myo, Toki, Ikeda, Kato, Sugimoto, PTP 117 (2006)

0p-0h + 2p-2h

$$\Phi(^4\mathrm{He}) = \Sigma_i\, C_i\, \psi_i(\{b_\alpha\}) \;=\; C_1\,(0s)^4 + C_2\,(0s)^2(\overline{0p}_{1/2})^2 + \cdots$$

$b_{0s} \neq b_{\overline{0p}}$ (size parameter)

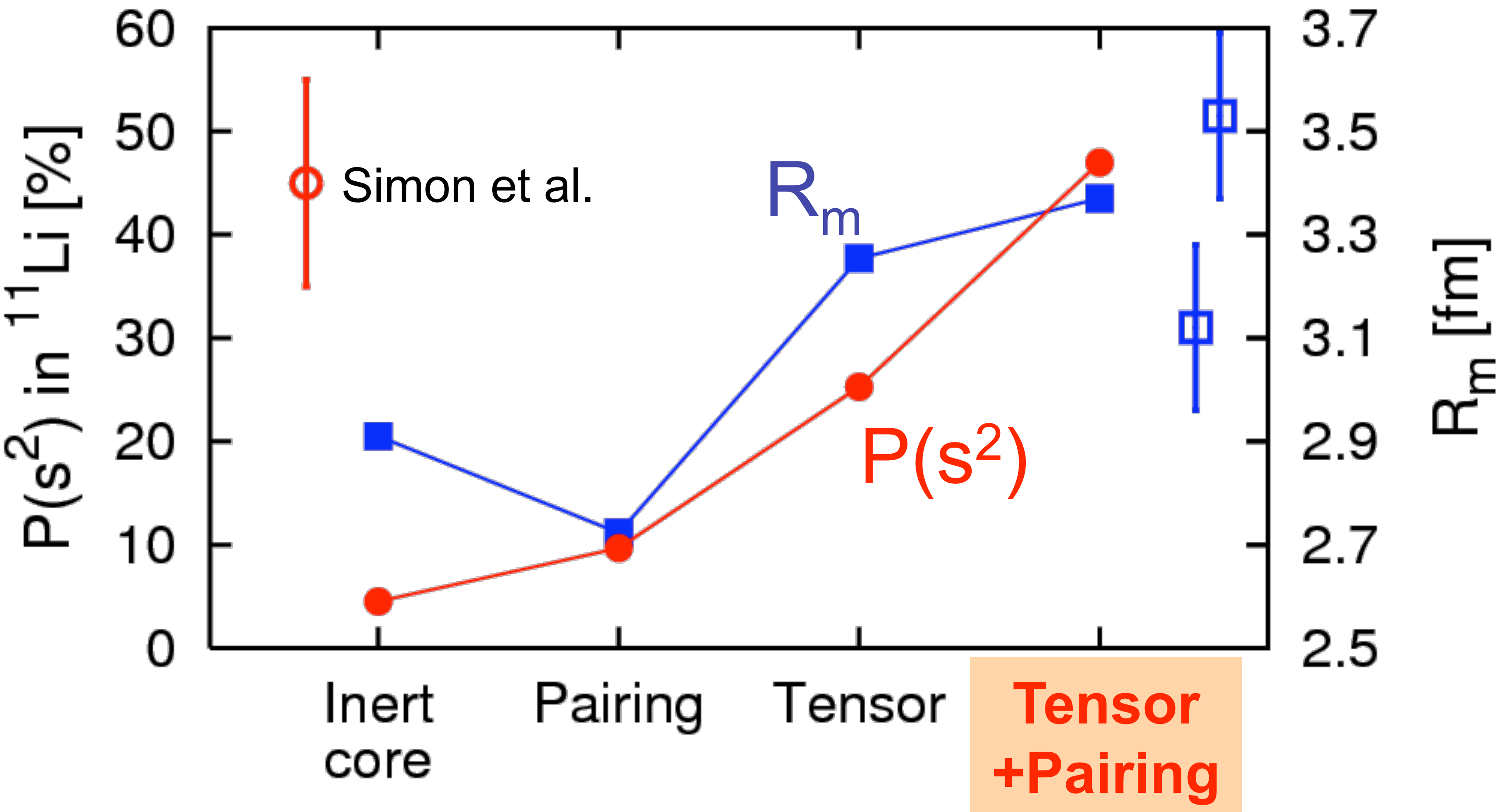
Energy variation

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij}, \qquad v_{ij} = v_{ij}^{C} + v_{ij}^{T} + v_{ij}^{LS} + v_{ij}^{Clmb},$$

G^c

$$\delta \frac{\langle\Phi|\, H \,|\Phi\rangle}{\langle\Phi|\Phi\rangle} = 0 \quad \Rightarrow \quad \frac{\partial\langle H-E\rangle}{\partial b_\alpha} = 0\,, \quad \frac{\partial\langle H-E\rangle}{\partial C_i} = 0.$$

^{11}Li G.S. properties (S_{2n}=0.31 MeV)



$E(s^2)-E(p^2)$	2.1	1.4	0.5	-0.1	[MeV]

Pairing interaction couples $(0p)^2$ and $(1s)^2$ states.

Unitary Correlation Operator Method

(UCOM)

$$\Psi_{\text{corr.}} = C \cdot \Phi_{\text{uncorr.}} \quad \leftarrow \text{SM, HF, FMD}$$

short-range correlator

$$C^{\dagger} = C^{-1} \quad \text{(Unitary trans.)}$$

$$H\Psi = E\Psi \qquad C^{+}HC\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance r

$$C = \exp(-i\sum_{i<j} g_{ij}), \qquad g = \tfrac{1}{2}\{p_r s(r) + s(r) p_r\} \qquad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

$$g = g^{\dagger} \ : \text{Hermitian generator}$$

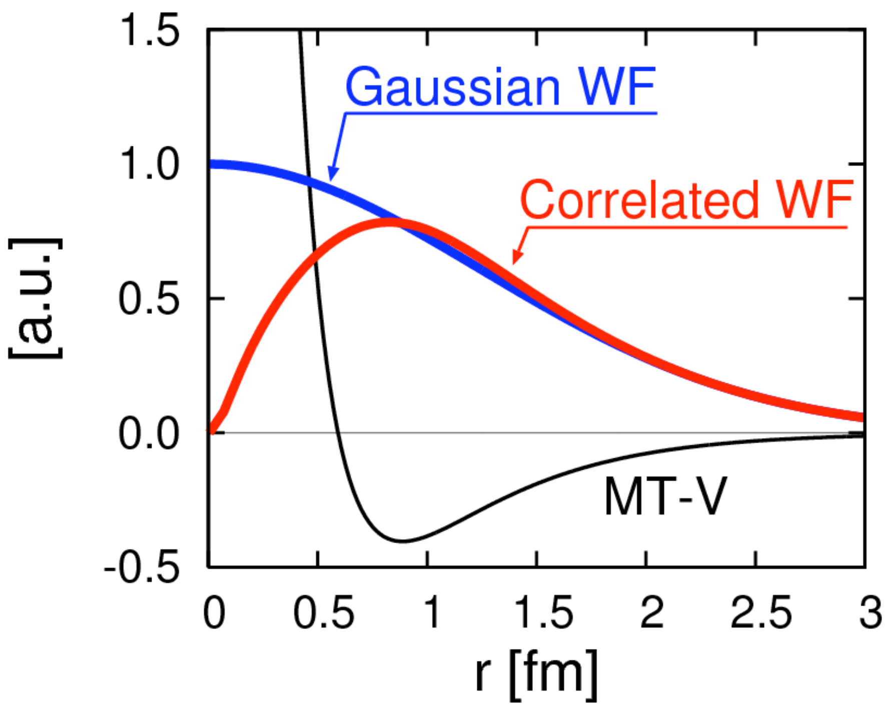
$$R'_{+}(r) = \frac{s(R_{+}(r))}{s(r)}$$

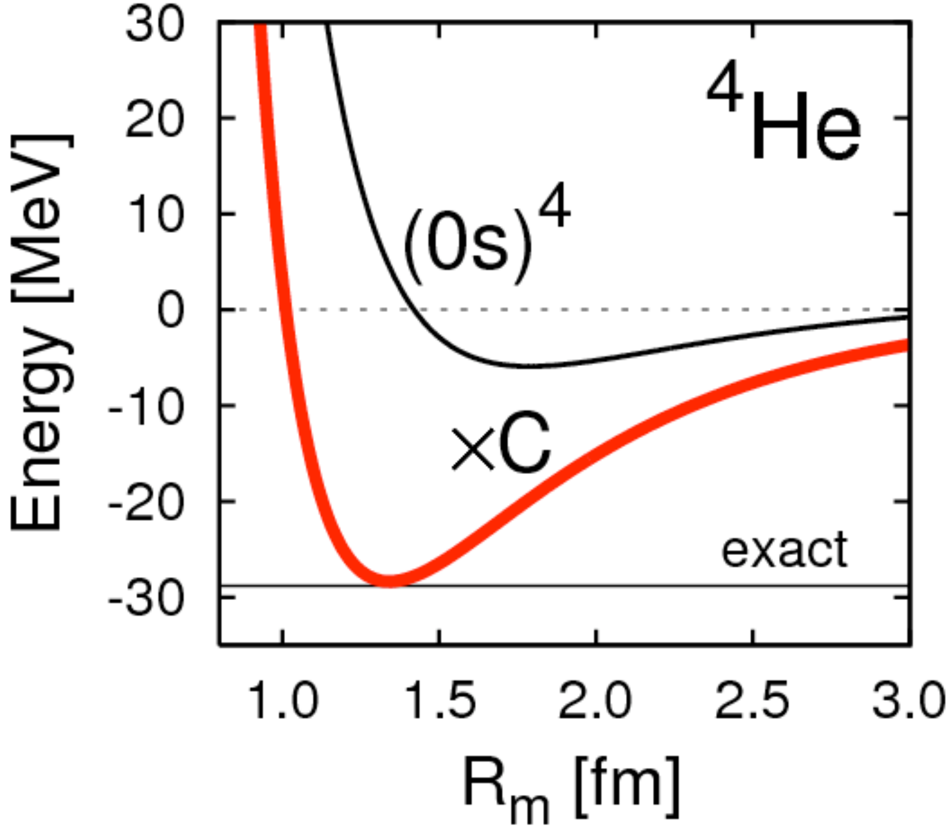
H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

$$C^{\dagger} r\, C = R_{+}(r)$$

^{4}He with UCOM

$^{A}\text{He} \;:\; (0s)^{A}$ with b-parameter

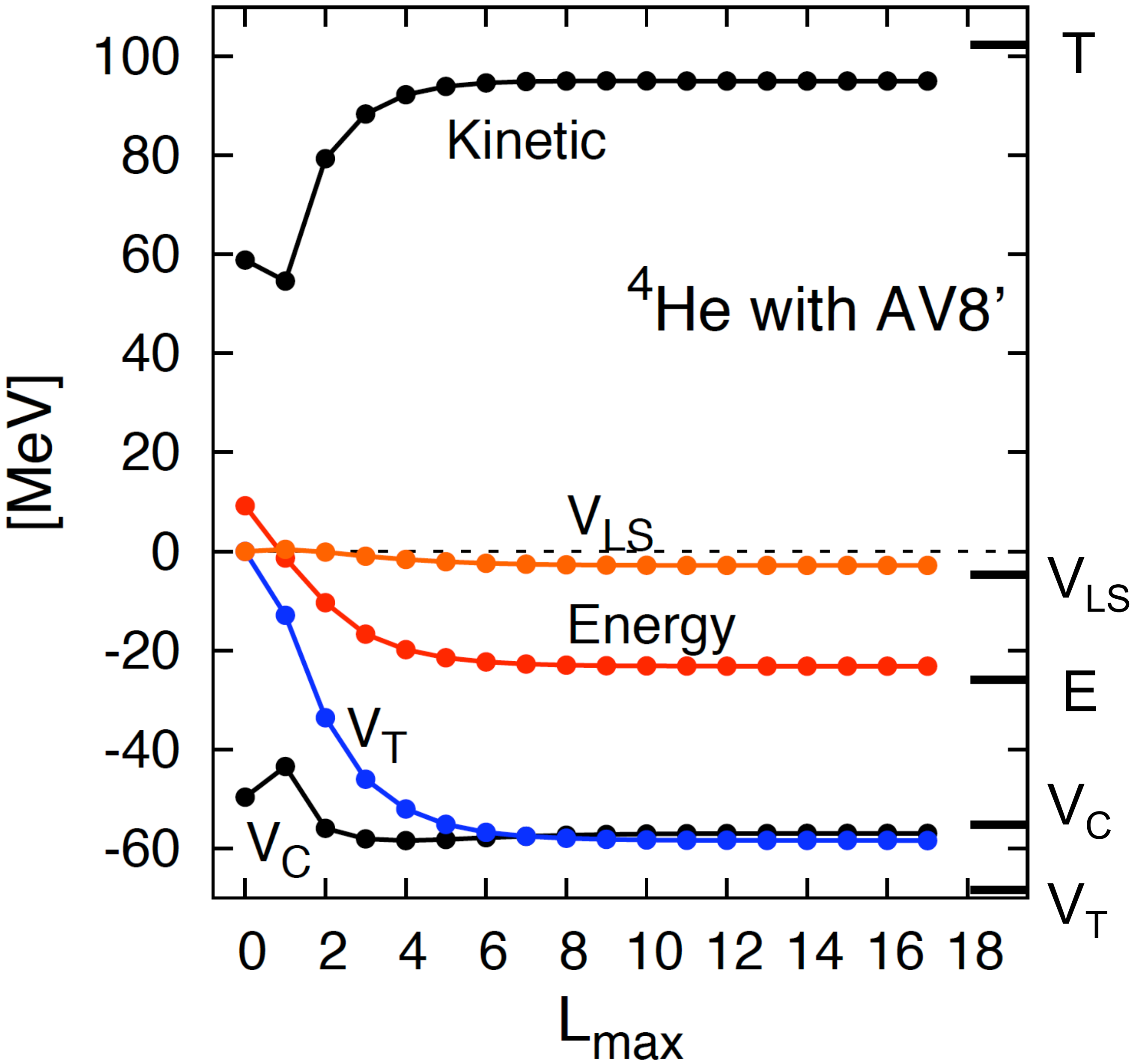




TOSM+UCOM with AV8'

$$\Psi = C_0|0\rangle + \sum_{\alpha} C_\alpha |2p2h:\alpha\rangle$$

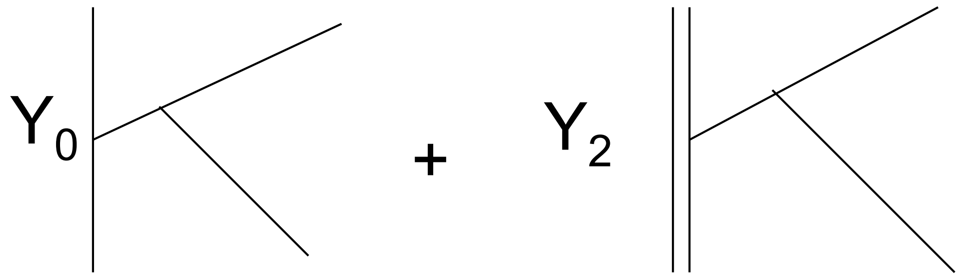
(Myo Toki Ikeda)



Few body Calculation (Kamda et al)

Tensor-optimized few-body model for s-shell nuclei

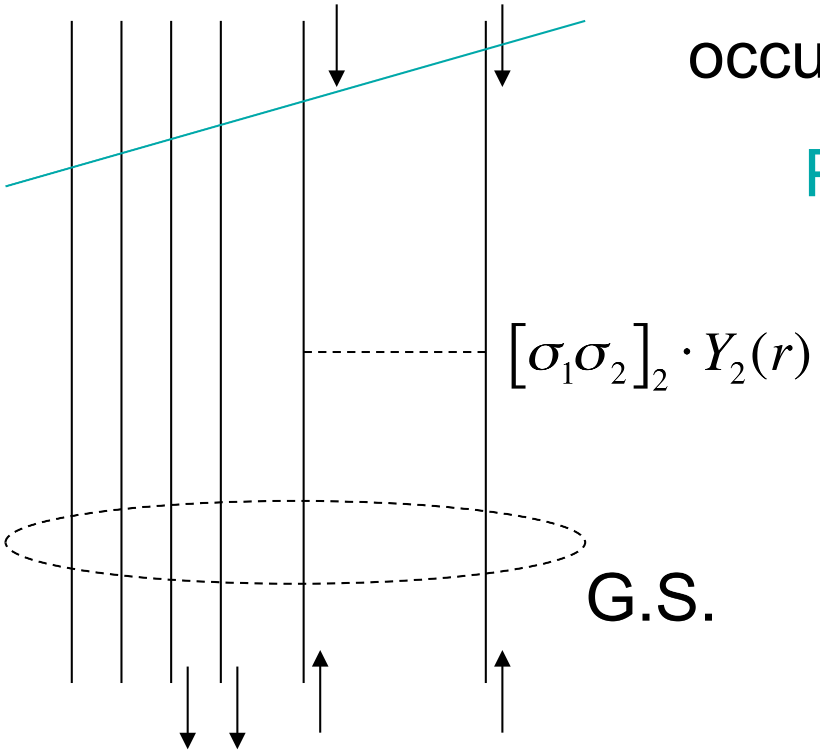
K. Horii,[1,*] H. Toki,[1,†] T. Myo,[2,‡] and K. Ikeda[3,§]



$$\langle D | S_{12} | S \rangle \neq 0$$

Nucleus	Energy	Kinetic	Central	Tensor	LS
deuteron	-2.23	19.95	-4.49	-16.64	-1.03
^{3}H(TOFM)	-7.54	46.67	-21.98	-30.47	-1.95
SVM[7]	-7.76	47.57	-22.49	-30.84	-2.00
^{4}He(TOFM)	-24.05	95.37	-54.58	-60.79	-4.05
TOSM[4]	-22.30	90.50	-55.71	-54.55	-2.53
SVM[1]	-25.92	102.35	-55.23	-68.32	-4.71

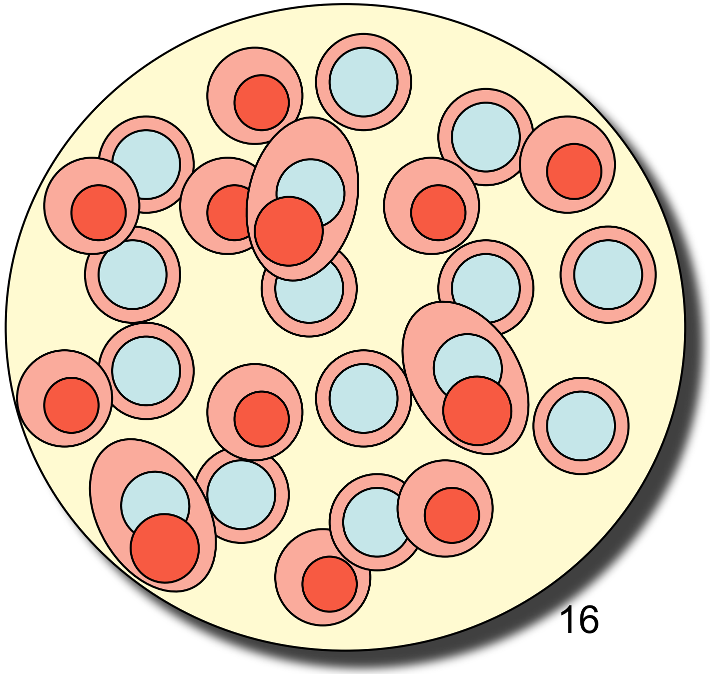
TOSM should be used for nuclear many body problem
2p-2h excitation is essential for treatment of pion



Spin-saturated

The spin flipped states are already occupied by other nucleons.

Pauli forbidden



Extended Brueckner–Hartree–Fock theory with pionic correlation in finite nuclei

Yoko Ogawa *, Hiroshi Toki

Annals of Physics (2011)

$$\langle 0|S_{12}|0\rangle = 0, \qquad S_{12} = \sqrt{\frac{24\pi}{5}}[Y_2(\hat{r}) \times [\sigma_1 \times \sigma_2]_2]^{(0)}.$$

HF state cannot handle the tensor interaction

$$|\Psi\rangle = C_0|0\rangle + \sum_{\alpha\mu} C_{\alpha\mu}|2p-2h;\alpha,\mu\rangle$$

$$\delta\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} = 0 \qquad \langle\Psi|\Psi\rangle = C_0^* C_0 + \sum_{\alpha\mu} C_{\alpha\mu}^* C_{\alpha\mu} = 1$$

Total energy

$$E = \langle \Psi | \widehat{H} | \Psi \rangle = C_0^* C_0 \langle 0 | \widehat{H} | 0 \rangle + C_0^* \sum_{\beta\nu} C_{\beta\nu} \langle 0 | \widehat{H} | 2p-2h; \beta, \nu \rangle + C_0 \sum_{\alpha\mu} C_{\alpha\mu}^* \langle 2p-2h; \alpha, \mu | \widehat{H} | 0 \rangle$$
$$+ \sum_{\alpha\mu\beta\nu} C_{\alpha\mu}^* C_{\beta\nu} \langle 2p-2h; \alpha, \mu | \widehat{H} | 2p-2h; \beta, \nu \rangle .$$

$$\langle 2p-2h; \alpha, \mu | \widehat{H} | 2p-2h; \beta, \nu \rangle = \langle 0 | \widehat{H} | 0 \rangle \delta_{\alpha\mu,\beta\nu} + \langle 2p-2h; \alpha, \mu | \widetilde{H} | 2p-2h; \beta, \nu \rangle$$

$$E = \langle 0 | \widehat{H} | 0 \rangle + C_0^* \sum_{\beta\nu} C_{\beta\nu} \langle 0 | \widehat{H} | 2p-2h; \beta, \nu \rangle + C_0 \sum_{\alpha\mu} C_{\alpha\mu}^* \langle 2p-2h; \alpha, \mu | \widehat{H} | 0 \rangle$$
$$+ \sum_{\alpha\mu\beta\nu} C_{\alpha\mu}^* C_{\beta\nu} \langle 2p-2h; \alpha, \mu | \widetilde{H} | 2p-2h; \beta, \nu \rangle .$$

Energy variation

$$\frac{\partial}{\partial C_{\alpha\mu}^*} \langle \Psi | \widehat{H} - E | \Psi \rangle = 0,$$

$$C_0 \langle 2p-2h; \alpha, \mu | \widehat{H} | 0 \rangle + \sum_{\beta\nu} C_{\beta\nu} \langle 2p-2h; \alpha, \mu | \widehat{H} | 2p-2h; \beta, \nu \rangle = E C_{\alpha\mu} .$$

Variation with HF state

$$\frac{\partial}{\partial \psi_b^*(\mathbf{x})}\left\{\langle \Psi|\widehat{H}|\Psi\rangle - \sum_b \varepsilon_b \langle \psi_b|\psi_b\rangle\right\} = 0$$

$$\frac{\partial}{\partial \psi_b^*(\mathbf{x})}\langle 0|\widehat{H}|0\rangle + C_0^* \sum_{\alpha\mu} C_{\alpha\mu} \frac{\partial}{\partial \psi_b^*(\mathbf{x})}\langle 0p-0h|\widehat{H}|2p-2h;\alpha,\mu\rangle$$
$$+\sum_{\alpha\mu\beta\nu} C_{\alpha\mu}^* C_{\beta\nu}\frac{\partial}{\partial \psi_b^*(\mathbf{x})}\langle 2p-2h;\alpha,\mu|\widetilde{H}|2p-2h;\beta,\nu\rangle = \varepsilon_b \psi_b(\mathbf{x}).$$

$$T\psi_b(\mathbf{x}) + \sum_d \int d\mathbf{x}' \psi_d^*(\mathbf{x}') V(\mathbf{x},\mathbf{x}')[\psi_b \otimes \psi_d]_{\mathscr{A}}(\mathbf{x},\mathbf{x}') - C_0^* \sum_{\alpha\mu} C_{\alpha\mu} N\widehat{J}\widehat{T}\langle [\cdot d]_{JT}|V|[ac]_{JT}\rangle_{\mathscr{A}}(\mathbf{x})$$
$$+\sum_{\alpha\mu\beta\nu} C_{\alpha\mu}^* C_{\beta\nu} E_{2p-2h}(\alpha\mu,\beta\nu : \mathbf{x}) = \varepsilon_b \psi_b(\mathbf{x}).$$

$$\frac{\partial}{\partial \psi_b^*(\mathbf{x})}\left\langle 2p-2h;\alpha,\mu|\widetilde{H}|2p-2h;\beta,\nu\right\rangle = E_{2p-2h}(\alpha\mu,\beta\nu : \mathbf{x})$$

We can solve finite nuclei by solving the above equation.

EBHF equation

$$C_{\beta\nu} = \sum_{\alpha\mu} \left[E - \langle 2p-2h;\alpha,\mu|\hat{H}|2p-2h;\beta,\nu\rangle \right]^{-1} \times \langle 2p-2h;\alpha,\mu|\hat{V}|0\rangle C_0$$

$$= \sum_{\alpha\mu} \frac{1}{E - \langle \alpha,\mu|\hat{H}|\beta,\nu\rangle} \langle \alpha,\mu|\hat{V}|0\rangle C_0.$$

$$T\psi_b(x) + \sum_d \int d^3x' \psi_d^*(x') V(x,x') \left[\psi_b \psi_d\right]_A + |C_0|^2 \sum_{\alpha\mu,\beta\nu} \frac{\partial \langle 0|\hat{V}|\alpha\mu\rangle}{\partial \psi_b^*(x)} \frac{1}{E - \langle \beta\nu|\hat{H}|\alpha\mu\rangle} \langle \beta\nu|\hat{V}|0\rangle$$

$$+ |C_0|^2 \sum_{\alpha\mu,\beta\nu,\alpha'\mu',\beta'\nu'} \langle 0|\hat{V}|\alpha\mu\rangle \frac{1}{E - \langle \alpha'\mu'|\hat{H}|\alpha\mu\rangle} \frac{\partial \langle \alpha'\mu'|\tilde{H}|\beta'\nu'\rangle}{\partial \psi_b^*(x)} \frac{1}{E - \langle \beta\nu|\hat{H}|\beta'\nu'\rangle} \langle \beta\nu|\hat{V}|0\rangle = \varepsilon_b \psi_b(x)$$

$$V_{eff} = |C_0|^2 V + |C_0|^2 \sum_{\alpha\mu,\beta\nu} \langle 0|\hat{V}|\alpha,\mu\rangle \frac{1}{E - \langle \beta,\nu|\hat{H}|\alpha,\mu\rangle} \langle \beta,\nu|\hat{V}|0\rangle.$$

$$\frac{\partial}{\partial \psi_b^*(x)} \langle 0|V_{eff}|0\rangle$$

Similar to BHF equation

Feshbach projection method

$$H(P+Q)\Psi = E(P+Q)\Psi.$$

$$\begin{cases} PHP\Psi + PHQ\Psi = EP\Psi, \\ QHP\Psi + QHQ\Psi = EQ\Psi. \end{cases}$$

$$Q\Psi = \frac{1}{E-QHQ} QHP\Psi$$

$$PHP\Psi + PHQ\frac{1}{E-QHQ}QHP\Psi = EP\Psi.$$

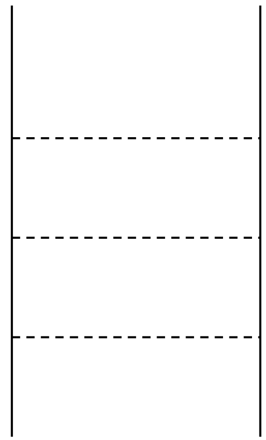
$$V_{eff} = PVP + PVQ\frac{1}{E-QHQ}QVP.$$

$$P = |0\rangle\langle 0| \qquad Q = \sum_{\alpha} |2p2h:\alpha\rangle\langle 2p2h:\alpha|$$

$$V_{eff} = |C_0|^2 V + |C_0|^2 \sum_{\alpha\beta} \langle 0|V|\alpha\rangle \frac{1}{E - \langle\beta|H|\alpha\rangle}\langle\beta|V|0\rangle$$

Comparison to BHF theory

$$G = V - V\frac{Q}{e}G = V - V\frac{Q}{e}V + V\frac{Q}{e}V\frac{Q}{e}V - \ldots$$



$$V_{eff} = |C_0|^2 V + |C_0|^2 \sum_{\alpha\beta} \langle 0|V|\alpha\rangle \frac{1}{E - \langle\beta|H|\alpha\rangle} \langle\beta|V|0\rangle$$

$$= |C_0|^2 \left[V - \sum_{\alpha} \langle 0|V|\alpha\rangle \frac{1}{E_\alpha} \langle\alpha|V|0\rangle + \sum_{\alpha\beta} \langle 0|V|\alpha\rangle \frac{1}{E_\alpha} \langle\alpha|V|\beta\rangle \frac{1}{E_\beta} \langle\beta|V|0\rangle + .. \right]$$

$$E - \langle\beta|H|\alpha\rangle \approx -E_\alpha \delta_{\alpha\beta} - \langle\beta|V|\alpha\rangle$$

- There appears $|C_0|^2$.
- $|C_0|^2$ is not normalized to 1.

$$|C_0|^2 + \sum_{\alpha} |C_\alpha|^2 = 1$$

Relativistic Chiral Mean Field Model for Finite Nuclei

Yoko OGAWA[1] and Hiroshi TOKI[2]

Nucl. Phys (2011)

Direct terms only

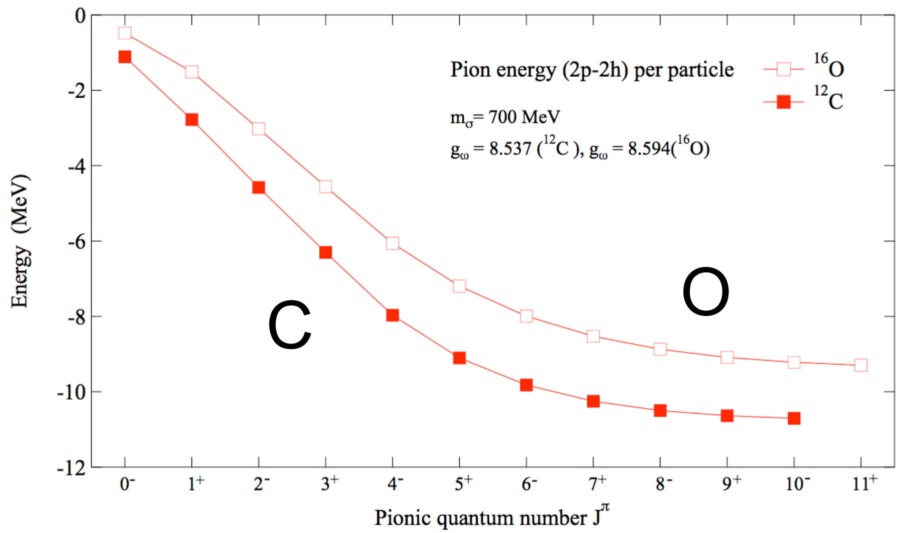
$$\mathcal{L} = \mathcal{L}_{\sigma,\omega} + \mathcal{L}_{\pi},$$

$$\begin{aligned}
\mathcal{L}_{\sigma,\omega} &= \bar{\psi}(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu)\psi \\
&+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}{m_\sigma}^2\sigma^2 - \lambda f_\pi\sigma^3 - \frac{\lambda}{4}\sigma^4 \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu \\
&+ \tilde{g_\omega}^2 f_\pi\sigma\omega_\mu\omega^\mu + \frac{1}{2}\tilde{g_\omega}^2\sigma^2\omega_\mu\omega^\mu,
\end{aligned}$$

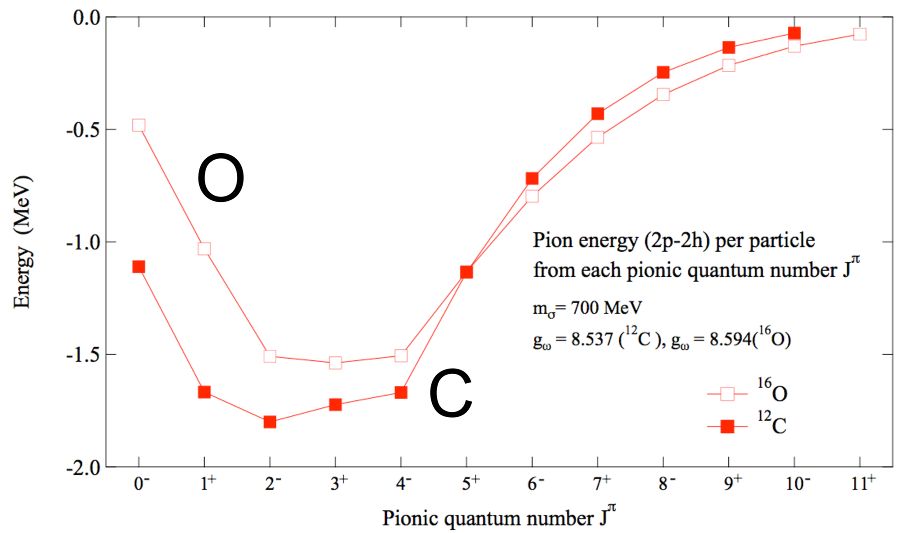
$$\mathcal{L}_\pi = -\frac{g_A}{2f_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\pi^a\tau^a\psi + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}{m_\pi}^2{\pi^a}^2$$

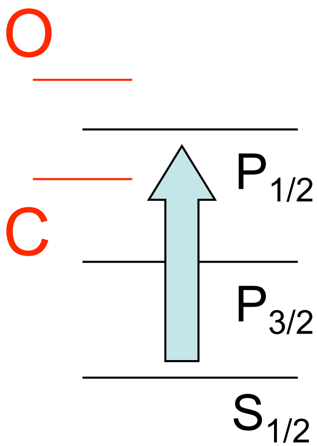
Relativistic chiral mean field model

Pion contribution



Individual contribution





The difference between ^{12}C and ^{16}O is 1.5 MeV/N.

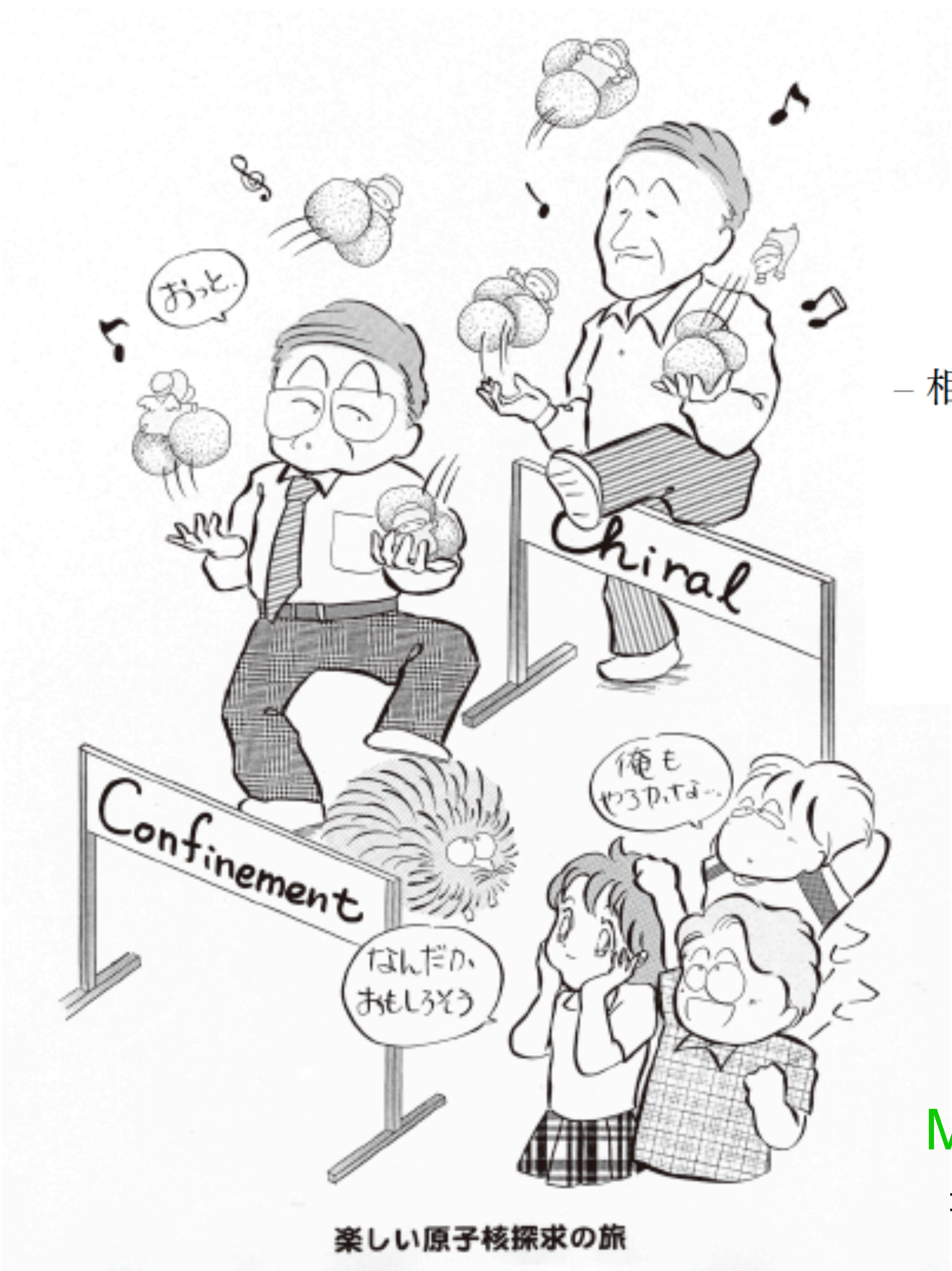
The difference comes from low pion spin states (J<3).
This is the Pauli blocking effect.

Pion energy

Pion tensor provides large attraction for ^{12}C

Renaissance in Nuclear Physics by pion

- We have a EBHF theory with pion
- Unification of hadron and nuclear physics
- Pion is the main player—Pion renaissance
- High momentum components are produced by pion（Tensor interaction）—Nuclear structure renaissance
- Nuclear Physics is truly interesting!!



相対論的多体系としての原子核

– 相対論的平均場理論とカイラル対称性 –

土岐　博、保坂　淳

平成 21 年 12 月 28 日

Monden

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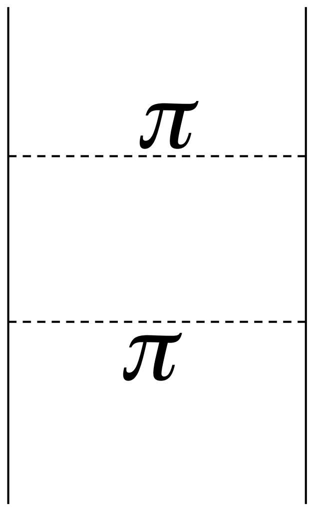
Relativistic Brueckner-Hartree-Fock theory

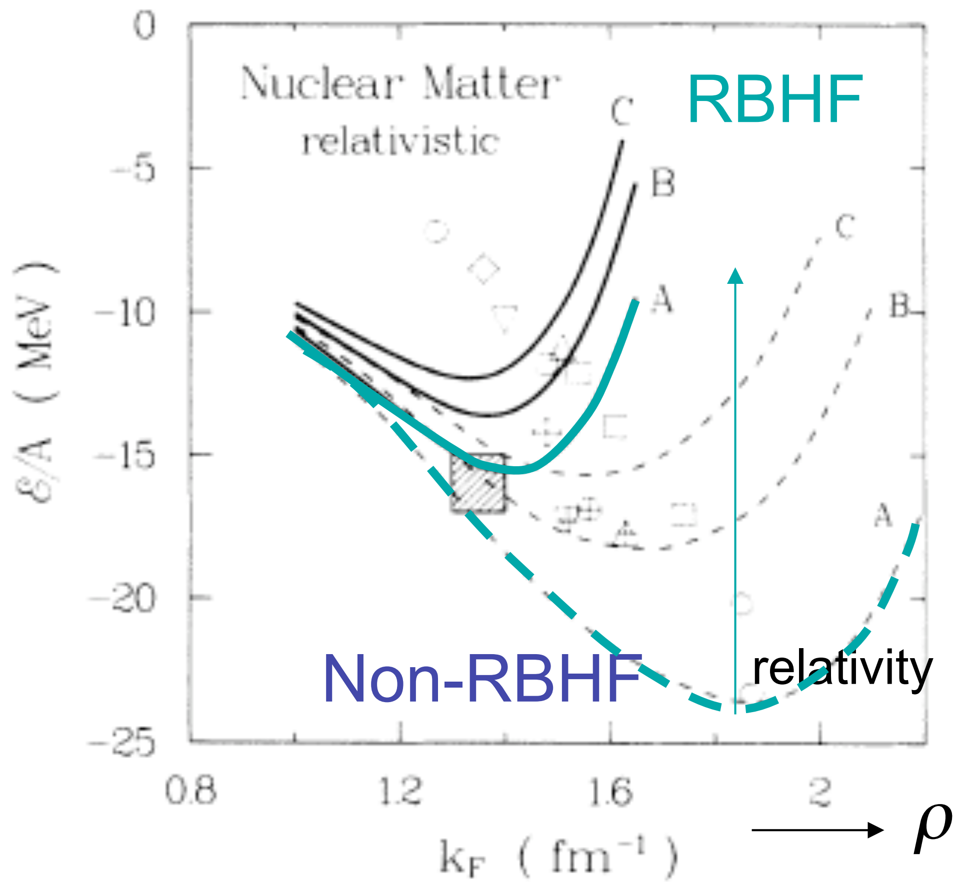
Brockmann-Machleidt (1990)

$$G = V + V\frac{Q}{e}G$$



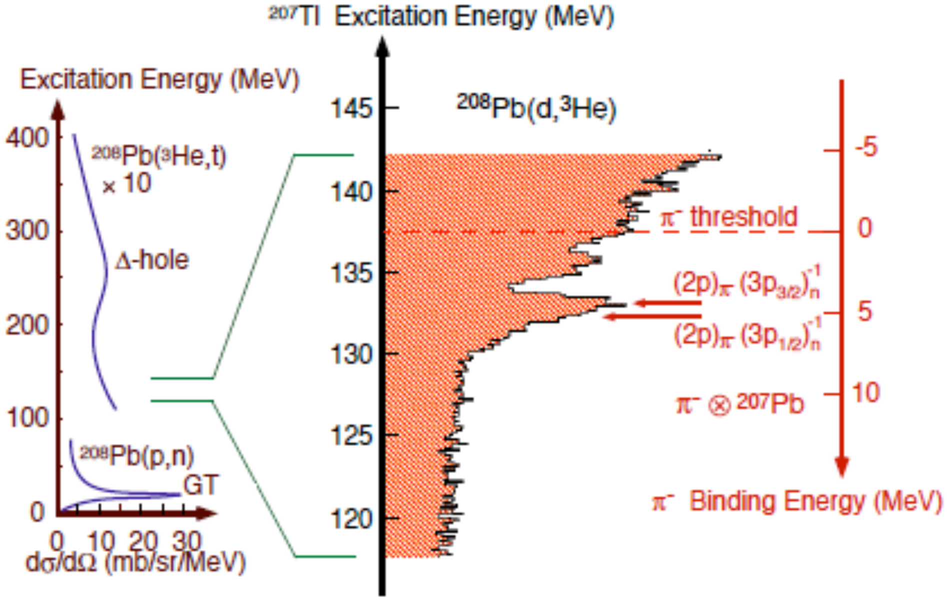
U_s~ -400MeV

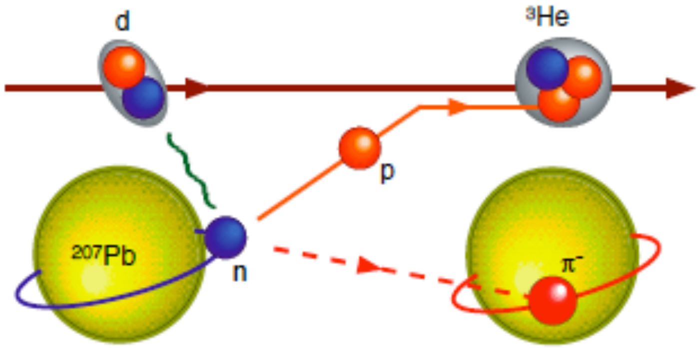
U_v~ 350MeV



Important experimental data

Deeply bound pionic atom





Prediction

Toki Yamazaki, PL(1988)

Found by (d,^{3}He) @ GSI

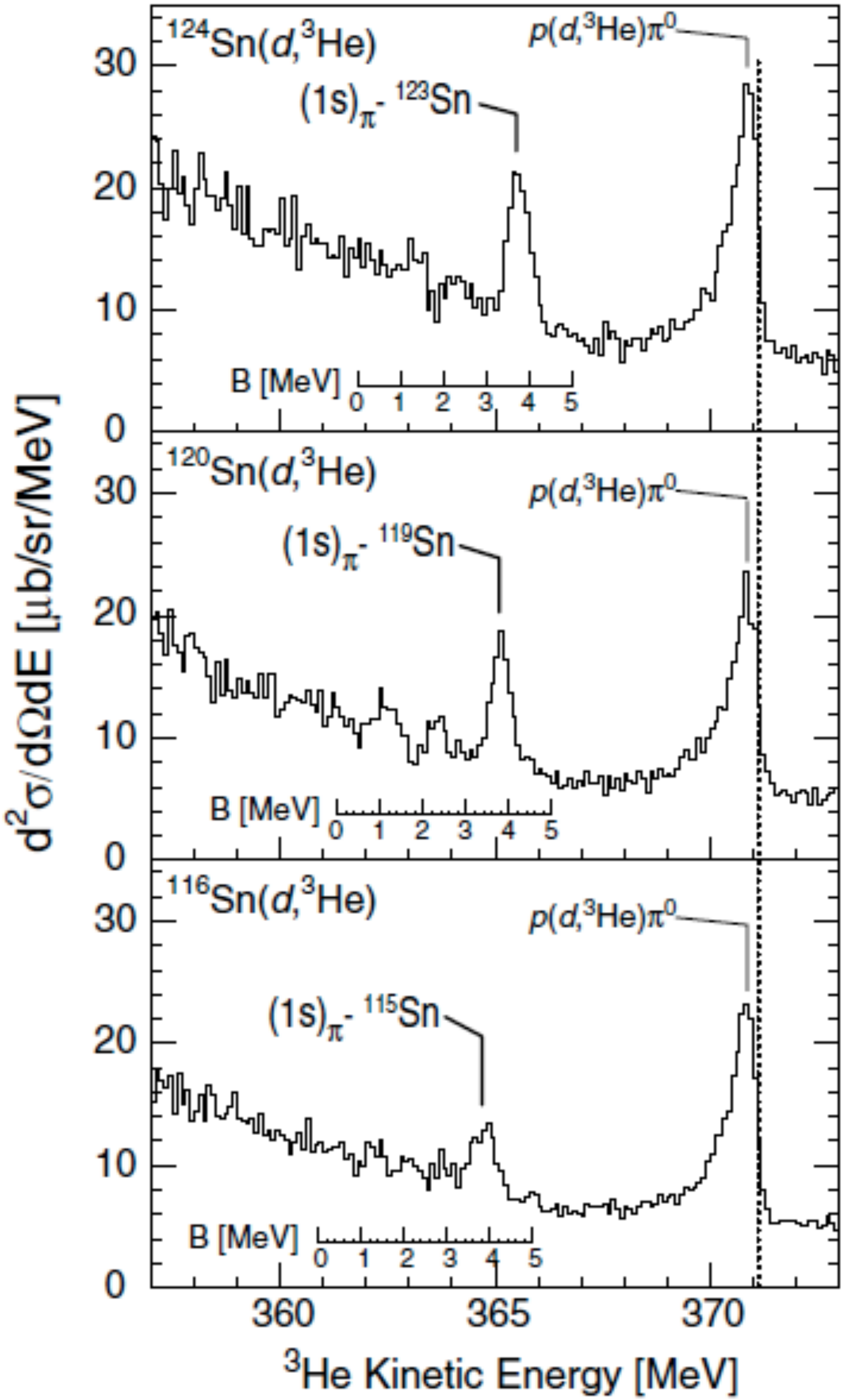
Itahashi, Hayano, Yamazaki..
Z. Phys.(1996), PRL(2004)

Physics : isovector s-wave

$$\frac{b_1}{b_1(\rho)} = 1 - 0.37\frac{\rho}{\rho_0}$$

$$f_\pi^2 m_\pi^2 = -2m_q \langle \overline{\psi}\psi \rangle$$

$$b_1 \propto \frac{1}{f_\pi^2} \qquad \frac{\langle \overline{\psi}\psi \rangle_\rho}{\langle \overline{\psi}\psi \rangle} = 1 - 0.37\frac{\rho}{\rho_0}$$



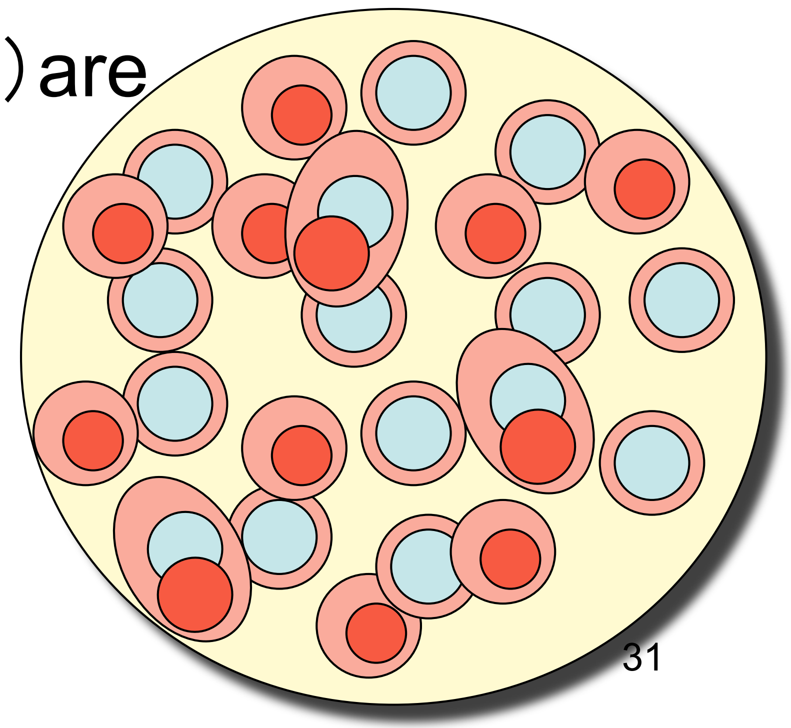
Suzuki, Hayano, Yamazaki..
PRL(2004)

Optical model analysis
for the deeply bound state.

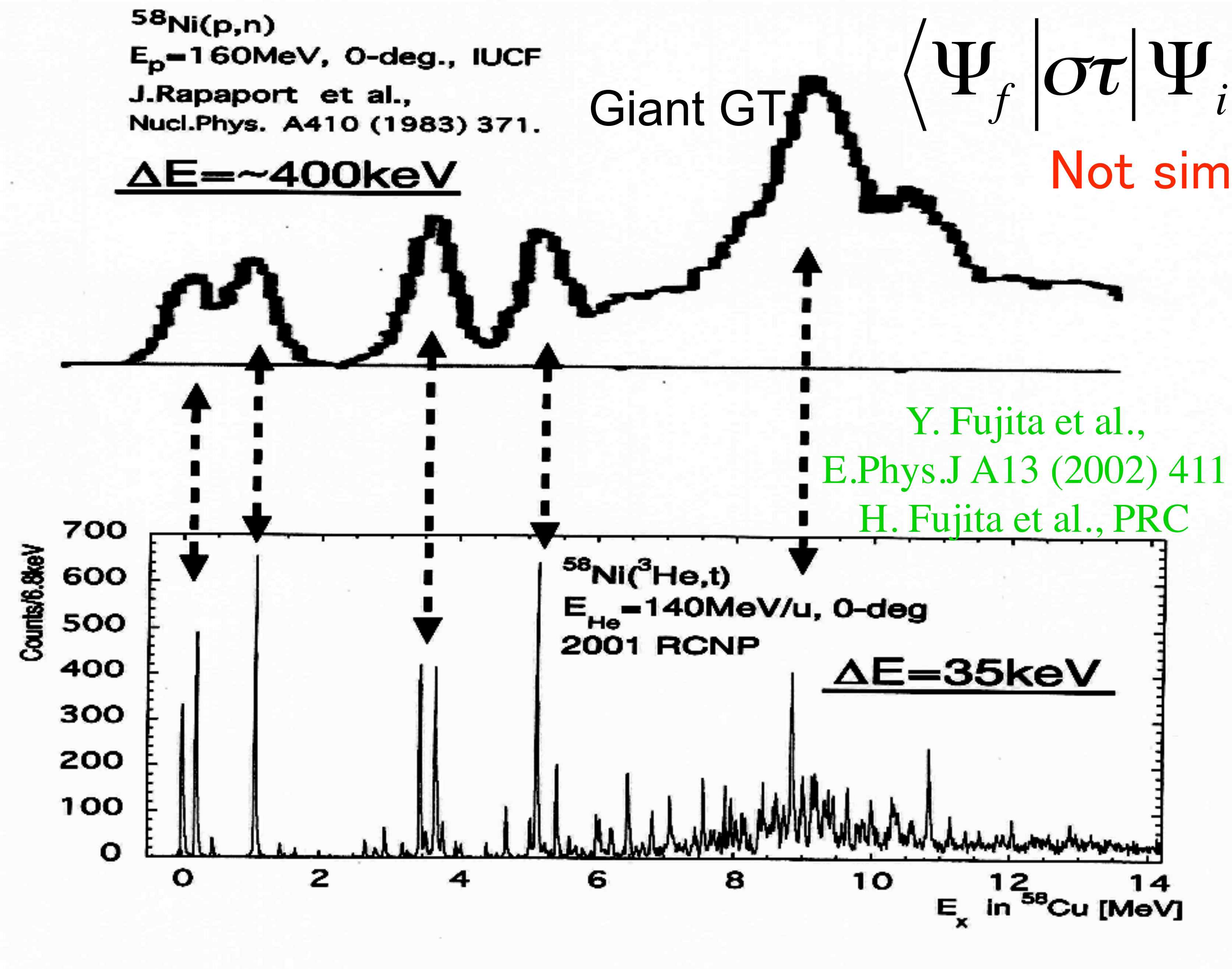
Nuclear structure caused by pion

- 2p-2h excitation is 20%
- High momentum components
- Low momentum components（Shell model）are reduced

  by 20%



RCNP experiment (high resolution)



$$\left\langle \Psi_f \left| \sigma\tau \right| \Psi_i \right\rangle$$

Not simple

Y. Fujita et al.,
E.Phys.J A13 (2002) 411
H. Fujita et al., PRC

^{15}O

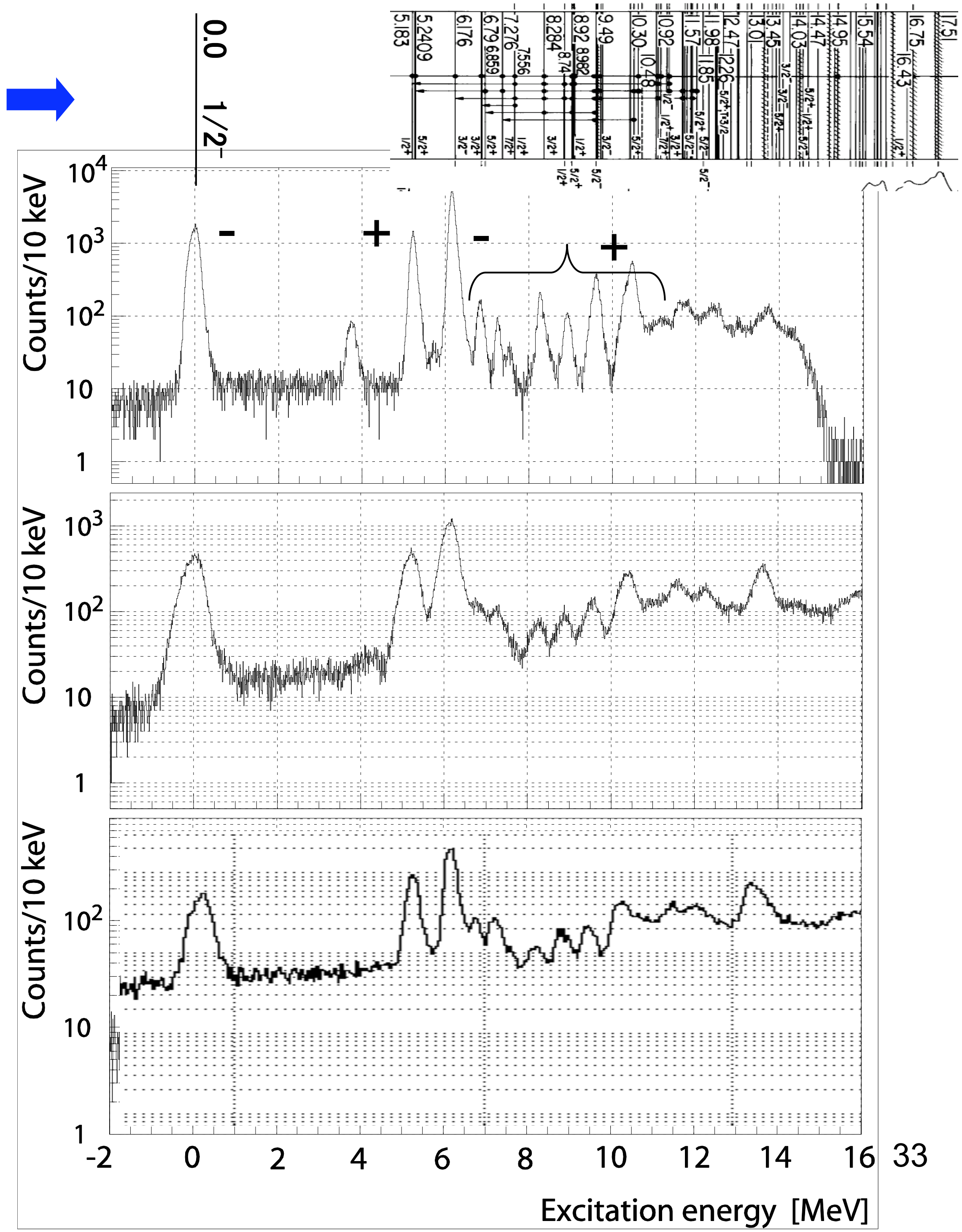
Level scheme

Ong, Tanihata et al

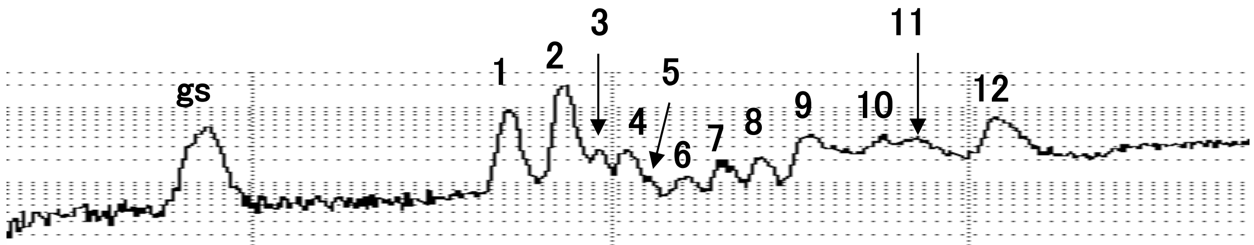
^{16}O (p,d)
E_p = 198 MeV
Θ_d = 10°

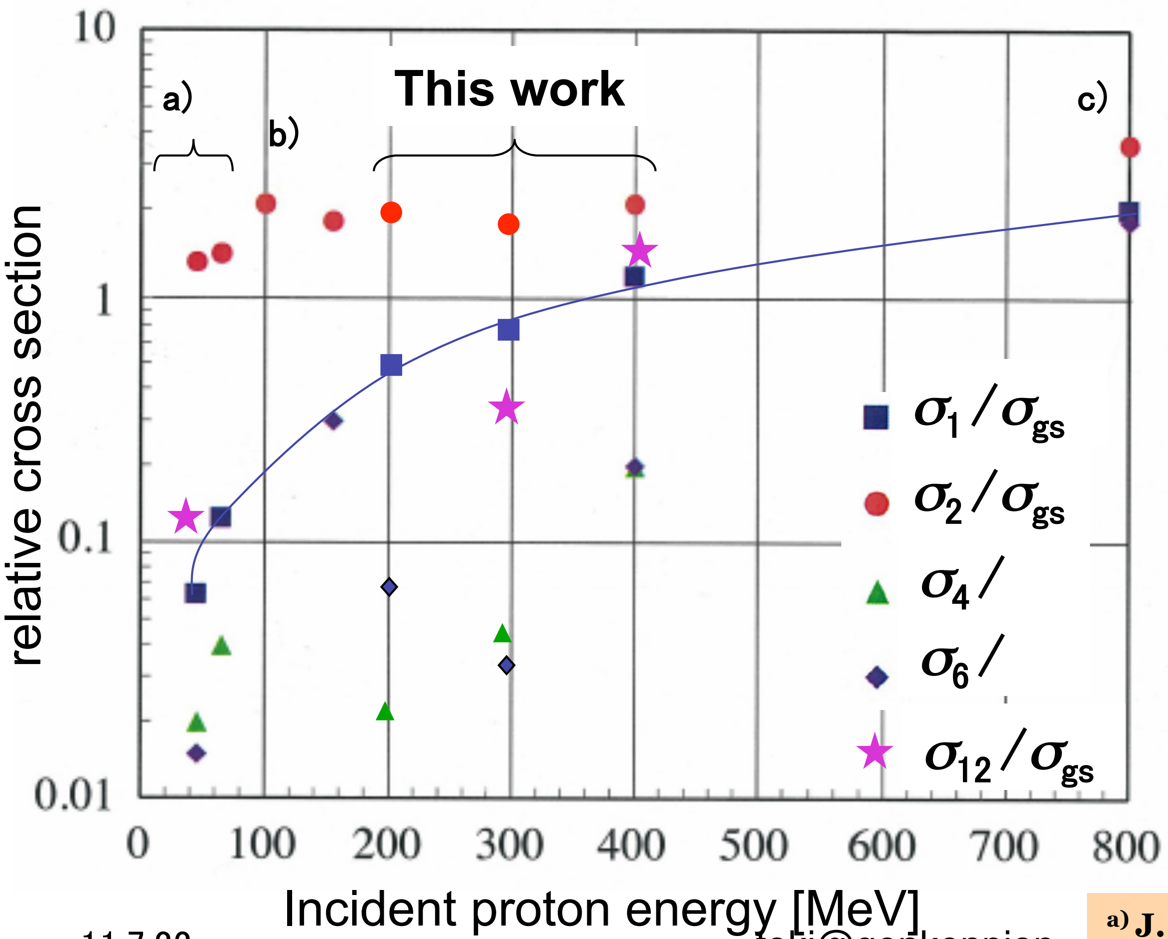
^{16}O (p,d)
E_p = 295 MeV
Θ_d = 10°

^{16}O (p,d)
E_p = 392 MeV
Θ_d = 10°



Relative Cross Section





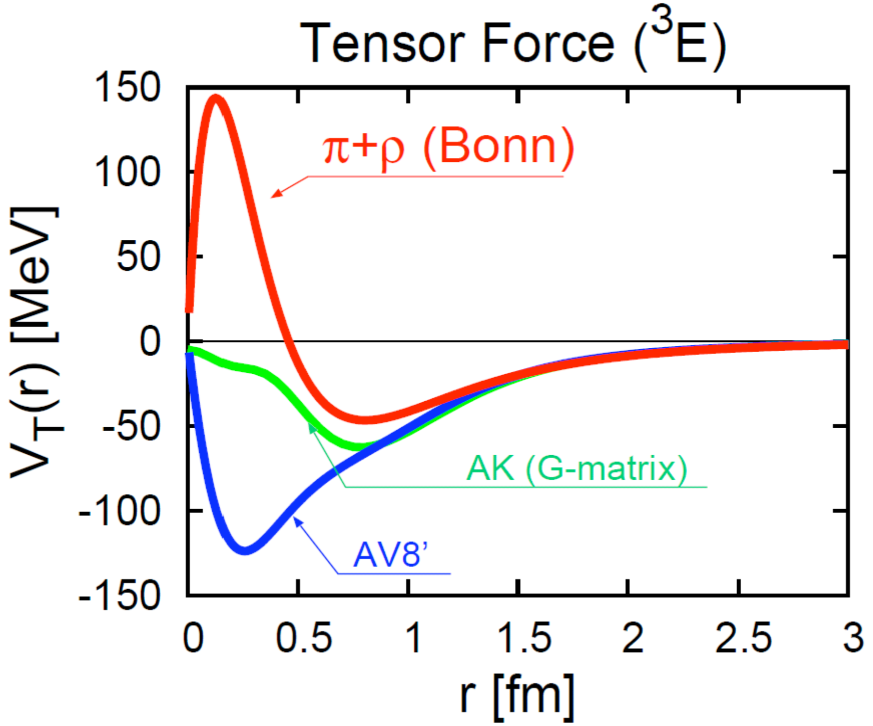
	Ex[MeV]	J^π
gs	0.0	$1/2^-$
1	**5.2**	**$1/2^+$, $5/2^+$**
2	6.2	$3/2^-$
3	**6.8**	**$3/2^+$, $5/2^+$**
4	**7.3**	**$7/2^+$**
5	**7.6**	**$1/2^+$**
6	**8.3**	**$3/2^+$**
7	**8.9**	**$5/2^+$,$1/2^+$**,$(1/2)^-$
8	9.5	$(3/2)^+$,$5/2^-$
9	10.5	$(9/2^+)$,$(3/2^-)$,$(3/2)^+$
10	11.6	$5/2^-$
11	11.9	$5/2^-$, $5/2^-$
12	**13.6**	**$5/2^+$**
	13.7	$3/2^-$

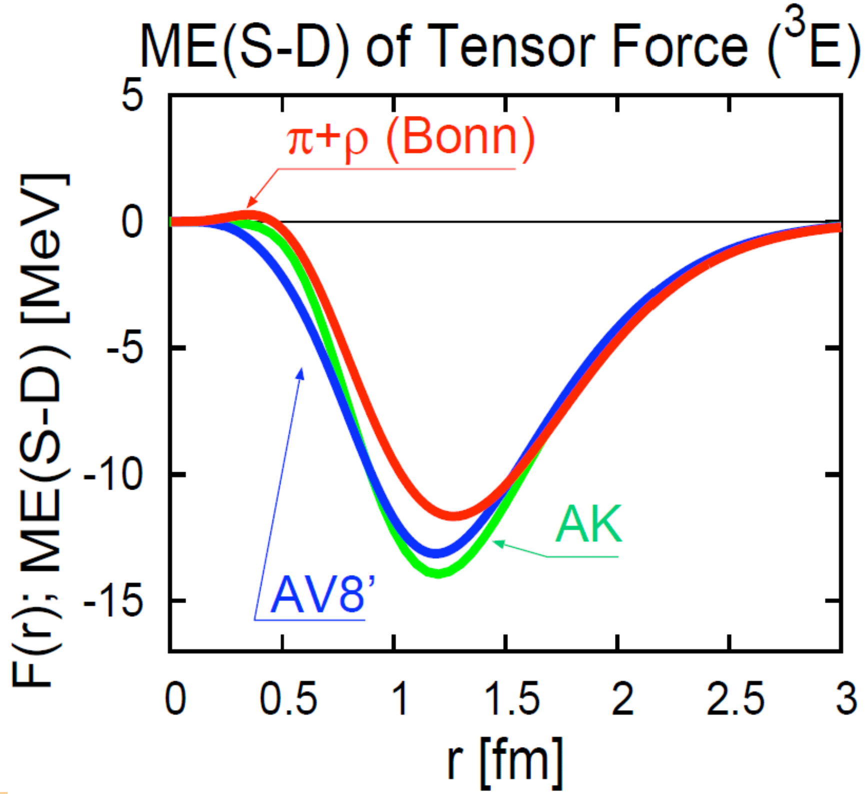
a) J. L. Snelgrove *et al.*, PR187, 1246(1969)
b) J.K.P. Lee *et al.*, NPA106, 357(1968)
c) G.R. Smith *et al.*, PRC30, 593(1984)

The property of tensor interaction

$V_{tensor} = V_T(r) \cdot S_{12}$

$F(r) = r^2 \cdot \phi_{0s}(r, b_{0s}) \cdot V_T(r) \cdot \phi_{0d}(r, b_{0d})$





Centrifugal potential (800MeV@0.5fm) pushes away the L=2 wave function