Microscopic description of extra-push energy and fusion dynamics in heavy systems

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- What is fusion hindrance?
- Analysis by microscopic TDHF
- Nucleus-nucleus potential
- Energy dissipation

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Introduction: What is fusion hindrance?

- $P_{fus}$ is hindered in $Z_p Z_T > 1600$ (superheavy elements)
- Extra-push energy is needed for fusion

C.-C. Sahm et al., NPA441 (1985) 316

Swiatecki, NPA376 (1982) 275
Quasi-fission in heavy systems

Deformation energy

LOW Z
Fusion

Saddle

HIGH Z

Saddle

Quasi-fission

Taken from Swiatecki et al., PRC71(2005)014602
Quasi-fission in heavy systems

- Strong energy dissipation
- Potential energy

**Analysis with Langevin equation**

Zagrebaev, Greiner, JPG31(2005) 825
Aritomo et al., PRC85 (2012) 044614

Taken from Swiatecki et al., PRC71(2005)014602
Quasi-fission in heavy systems

Aim

Understand the origin of fusion hindrance with a microscopic reaction model

TDHF

• Strong energy dissipation
• Potential energy

Deformation

Taken from Swiatecki et al., PRC71(2005)014602
Time-dependent Hartree-Fock (TDHF)

\[ i\hbar \frac{\partial}{\partial t} \phi_i(r, t) = \hat{h}[\rho(r, t)]\phi_i(r, t) \]

- Self-consistent description for both static and dynamical properties
- Shell effects
- Single-particle: Quantum; Many-body (collective): Classical
- Low energy: mean-field approximation

Estimate of extra-push energy with TDHF

Simenel, EPJA48(2012)152
Guo, Nakatsukasa, EPJ. Conf. 38 (2012) 09003
Method: Extract potential and dissipation

TDHF simulation $\rightarrow \rho(t)$

$\rho(t) \rightarrow R(t), P(t)$

Assume that $R(t), P(t)$ obey a Newton equation

\[
\frac{dR}{dt} = \frac{P}{\mu(R)}
\]

\[
\frac{dP}{dt} = -\frac{dV(R)}{dR} - \frac{d}{dR} \frac{P^2}{2\mu(R)} - \gamma(R) \frac{dR}{dt}
\]

Extraction stops when $\frac{dR}{dt} \sim 0$
(Overlap of densities becomes large)

$R$: Relative distance
$P$: Momentum
$V$: Potential
$\gamma$: Friction coefficient

Washiyama, Lacroix, PRC78(2008)024610
Result: Extracted potential

$^{96}\text{Zr}^+ \, ^{124}\text{Sn}$  $Z_p Z_T = 2000$

Density at $R=10.0, 12.1, 14.0$ fm for $E_{cm} = 250$ MeV

Fusion threshold $= 228$ MeV

SLy4d Skyrme force
dx = 0.8fm
dt = 0.45fm/c

Washiyama, in preparation
Potential barrier vanishes by dynamical effect

Energy dependence of potential is less around the "barrier"
Potentials of heavy systems

\[ ^{96}\text{Zr} + ^{132}\text{Sn} \]

\[ ^{70}\text{Zn} + ^{208}\text{Pb} \]

\[ ^{96}\text{Zr} + ^{136}\text{Xe} \]

\[ ^{124}\text{Sn} + ^{136}\text{Xe} \]
Origin of fusion hindrance

\[ E_{\text{extra}} = E_{\text{thres}} - V_{\text{FD}}: \text{ Extra-push energy for TDHF} \]

- \( E_{\text{thres}} \): Fusion threshold energy
- \( V_{\text{FD}} \): Frozen density barrier
Extra-push energy

\[ E_{\text{extra}} = E - V_B \]

\[ E = E_{\text{expt}}, \quad V_B = V_{\text{Bass}} \text{ for Expt.} \]

\[ E = E_{\text{thres}}, \quad V_B = V_{\text{FD}} \text{ for TDHF} \]

Data from Schmidt, Morawek, Rep.Prog.Phys.54(1991)949
E_{extra} \sim \Delta V + E_{diss}

(E_{cm} \sim E_{kin} + V + E_{diss})

\Delta V : \text{Increase in potential from } V_{FD}

\Delta V \text{ is calculated at } E_{cm} = E_{thres}
Origin of fusion hindrance

\[ E_{\text{extra}} \sim \Delta V + E_{\text{diss}} \]

\[ \Delta V > E_{\text{diss}} \text{ in more systems} \]

Main contribution to fusion hindrance comes from dynamical increase in potential.
Summary

- Fusion hindrance in heavy systems
- Potential and energy dissipation are extracted from TDHF
- Change the property of potential
  - No barrier structure
  - Dynamical increase
- Analysis of extra-push energy
- As system becomes heavier, dynamical increase in potential is more significant for the fusion hindrance
- The strength and behaviour are similar to lighter systems
- Friction shows a strong R-dependence at $dR/dt \sim 0$
As $x_{\text{eff}}$ becomes larger, contribution from $\Delta V$ becomes larger than dissipated energy, $\Delta V > E_{\text{diss}}$
Extra-push energy

\[ X_{\text{eff}}: \text{size of the system} \]

\[ x_{\text{eff}} = \frac{4Z_1Z_2}{A_1^{1/3}A_2^{1/3}(A_1^{1/3} + A_2^{1/3})} \left/ \left( \frac{Z^2}{A} \right)_{\text{crit}} \right. \]

\[ (Z^2/A)_{\text{crit}} = 50.833(1 - 1.7826f^2) \]
Origin of fusion hindrance

$E_{\text{thres}}$: Fusion threshold energy

$E_{\text{extra}} = E_{\text{thres}} - V_{\text{FD}}$: Extra-push energy

$\Delta V$: Increase in potential from $V_{\text{FD}}$

$\Delta V$ is calculated at $E_{\text{cm}} = E_{\text{thres}}$

$E_{\text{extra}} \sim \Delta V + E_{\text{diss}}$

$(E_{\text{cm}} \sim E_{\text{kin}} + V + E_{\text{diss}})$
Dissipated energy

Dissipation from the relative motion to the internal excitations

\[ E_{\text{diss}}(R) = \int_0^t dt' \gamma[R(t')] \left( \frac{dR}{dt} \right)^2 \]
Dissipated energy

Dissipation from the relative motion to the internal excitations

\[ E_{\text{diss}}(R) = \int_0^t dt' \gamma[R(t')] \left( \frac{dR}{dt} \right)^2 \]

![Graph for 96Zr + 124Sn](image)

![Graph for 40Ca + 40Ca](image)
\[ E_{\text{extra}} = E_{\text{thres}} - V_{FD} \text{ [MeV]} \]

Fusion threshold energy from TDHF

Guo, Nakatsukasa, EPJ.Conf. 38 (2012) 09003
Our method vs. Density-constrained

Washiyama, Lacroix, PRC78 (2008) 024610

$^{16}$O + $^{16}$O

- Agree with each model
- Validation of our model
Density-constrained TDHF

\[ \mathcal{E}_{DC}[\rho(t)](R) \] from minimization of \( \hat{H} - \int d^3r \lambda(r) \hat{\rho}(r) \)

Potential is obtained as a function of R

Potential

Umar, Oberacker, PRC74(2006)021601

Umar et al., PRC81(2010)064607
Energy density functional

- Skyrme energy density functional

\[ E_{\text{Sk}} \equiv \int d^3r \sum_{t=0,1} \mathcal{H}_t(r) \]

\[ \mathcal{H}_t = C^\rho_t \rho_t^2 + C^{\Delta \rho}_t \rho_t \Delta \rho_t + C^{\tau}_t \rho_t \tau_t + C^\nabla \cdot J_t \rho_t \nabla \cdot J_t - C^T_t \sum_{\mu, \nu = x, y, z} J_{t, \mu \nu} J_{t, \mu \nu} \cdots \]

\[ + C^s_t s_t^2 + C^{\Delta s}_t s_t \cdot \Delta s_t - C^\tau_t j_t^2 + C^\nabla \cdot J_t s_t \cdot \nabla \times j_t + C^T_t s_t \cdot T_t + \cdots \]

\[ \rho_g(r) = \rho_g(r, r')|_{r=r'}, \quad \tau_g(r) = \nabla \cdot \nabla' \rho_g(r, r')|_{r=r'}, \quad J_{q, \mu \nu}(r) = -\frac{i}{2}(\nabla_\mu - \nabla'_\mu) s_{q, \nu}(r, r')|_{r=r'} \]

\[ s_q(r) = s_q(r, r')|_{r=r'}, \quad T_q(r) = \nabla \cdot \nabla' s_q(r, r')|_{r=r'}, \quad j_q(r) = -\frac{i}{2}(\nabla - \nabla') \rho_g(r, r')|_{r=r'} \]

\[ \rho_g(r, r') = \sum_{\sigma = \pm 1} \rho_g(r \sigma, r' \sigma) = \sum_{\sigma = \pm 1} \sum_k n_k \phi_k(r \sigma q) \phi_k^*(r' \sigma q) \]

\[ s_q(r, r') = \sum_{\sigma, \sigma' = \pm 1} \rho_g(r \sigma, r' \sigma') \langle \sigma' | \hat{\sigma} | \sigma \rangle \]

Bender, Heenen, Reinhard, Rev. Mod. Phys. 75 (2003) 121