

---

# Fission fragment mass distribution of Fm isotopes

- Fluctuation-Dissipation Dynamics of  
Fission -

---

Takahiro Wada

Tomomasa Asano

Kansai University, Osaka, Japan

Talk at JAEA, Tokai, Dec. 3, 2014

---

# Contents

- Observed quantities in fission
  - What is fluctuation-dissipation dynamics?
  - Langevin equation in fission
    - Fragment distribution (mass & TKE)
    - Fm isotopes
    - $^{236}\text{U}$
  - Summary
-

# Key quantities in fission

## ■ Fission rate

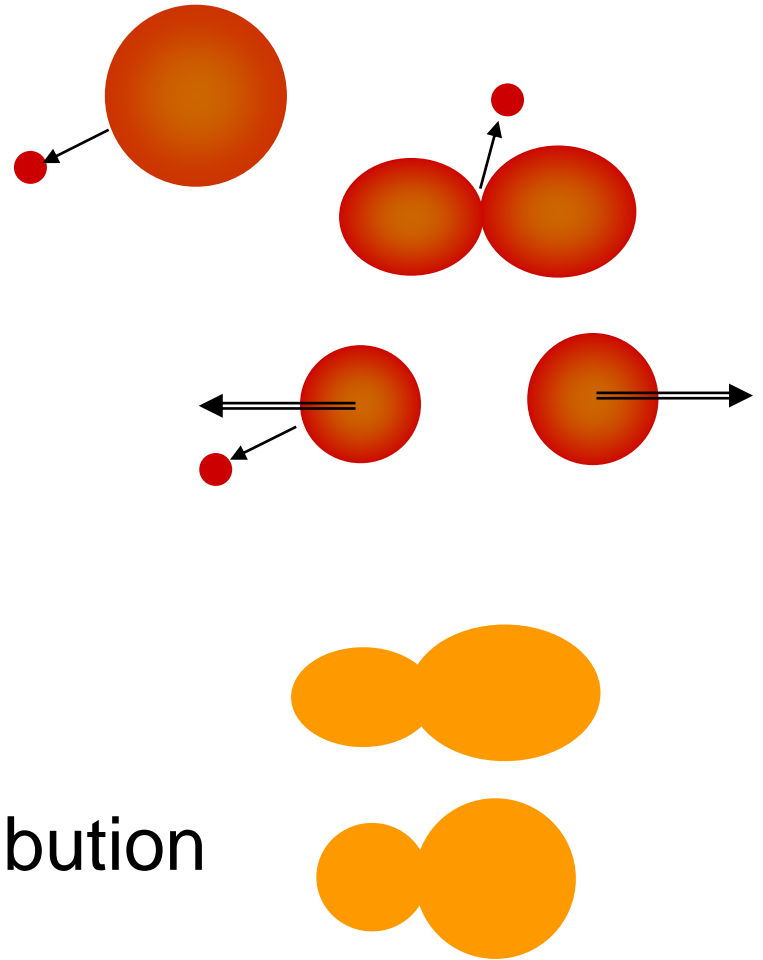
- Height of fission barrier

## ■ Particle emission

- Pre-scission
- At Scission
- Post-scission

## ■ Fission fragments

- Mass distribution
  - Shell correction
- Total kinetic energy distribution
  - Scission configuration



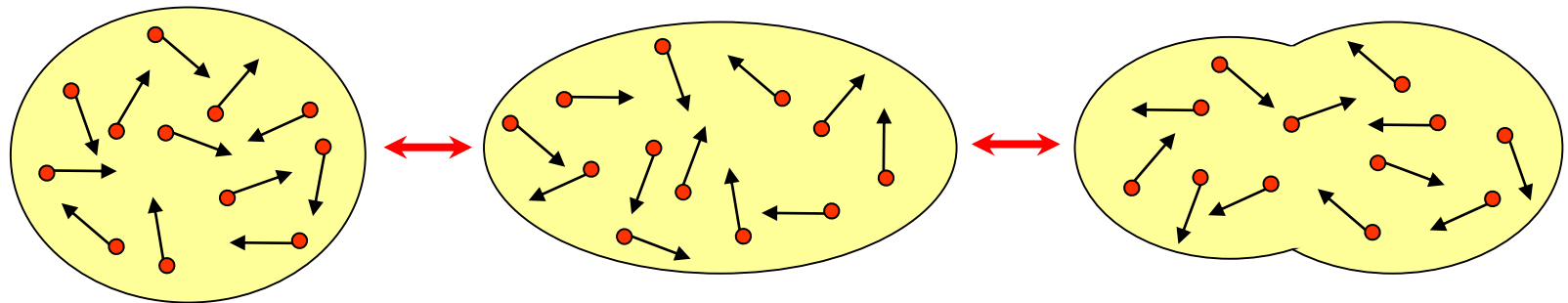
# Fluctuation-dissipation dynamics

- Brownian motion picture

- Macroscopic degree(s) of freedom interacting with microscopic degrees of freedom in thermal motion

Dissipation (collective  $\rightarrow$  nucleonic)  $\Leftarrow$  Friction

Fluctuation (nucleonic  $\rightarrow$  collective)  $\Leftarrow$  Random force



- Macroscopic degrees of freedom

= Nuclear shape

(elongation, deformation, neck, mass asymmetry etc.)

# Two approaches

## Langevin equation

$$\frac{dq}{dt} = \frac{p}{m} \quad \frac{dp}{dt} = -\frac{\partial U}{\partial q} - \frac{\gamma}{m} p + \sqrt{\gamma T} R(t)$$

$$\langle R(t) \rangle = 0, \quad \langle R(t_1)R(t_2) \rangle = 2\delta(t_1 - t_2)$$

Eq. of motion of a Brownian particle

## Focker-Planck (Kramers) equation

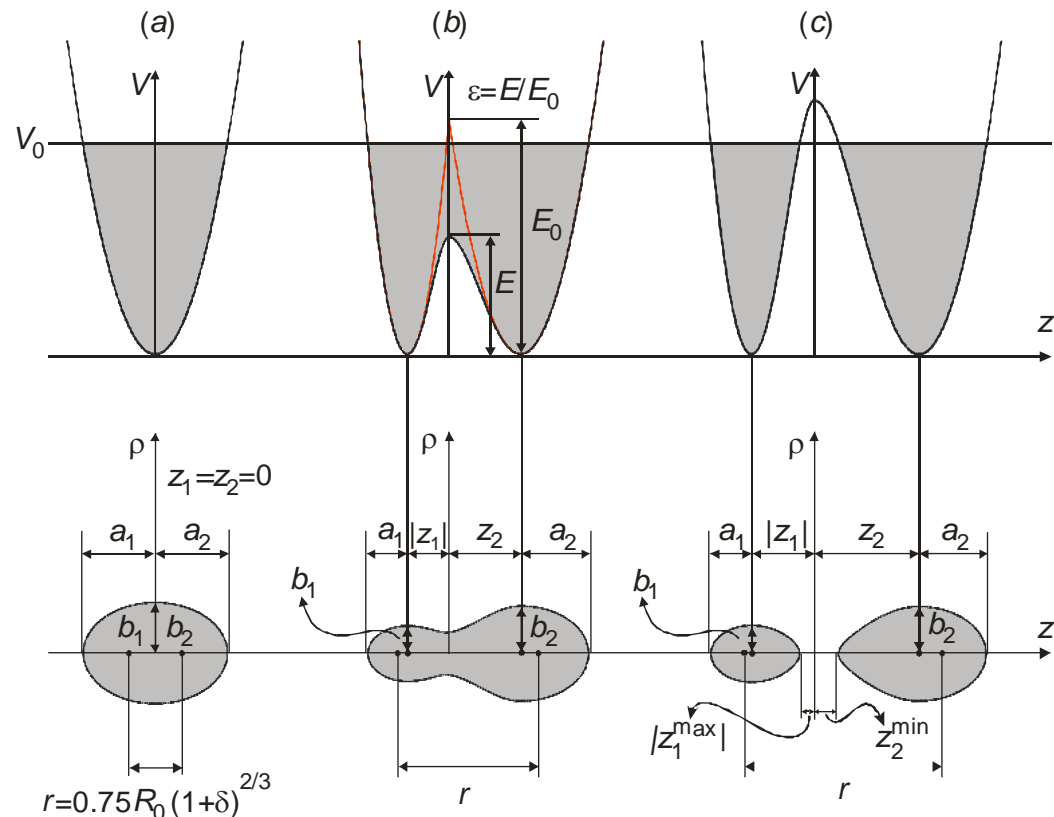
$$\frac{\partial f}{\partial t} = -\frac{p}{m} \frac{\partial f}{\partial q} + \frac{\partial U}{\partial q} \frac{\partial f}{\partial p} + \frac{\gamma}{m} \frac{\partial}{\partial p} (pf) + \gamma T \frac{\partial^2 f}{\partial p^2}$$

Distribution of the Brownian particles

# Two-center shell model parameterization

$$\rho_s^2 = \begin{cases} b_i^2 - (b_i/a_i)^2 (z - z_i)^2 & , z_1 - a_1 \leq z \leq z_1 \text{ or } z_2 \leq z \leq z_2 + a_2 \\ \left\{ b_i^2 - (b_i/a_i)^2 [1 + p_i(z - z_i) + q_i(z - z_i)^2] (z - z_i)^2 \right\} / [1 + g_i(z - z_i)^2] & , z_1 \leq z \leq z_2 \end{cases}$$

elongation ( $Z_0$ )  
 neck parameter ( $\varepsilon$ )  
 deformation ( $\delta_1, \delta_2$ )  
 mass asymmetry ( $\alpha$ )



# TKE systematics

- Viola systematics (1985, V. E. Viola)

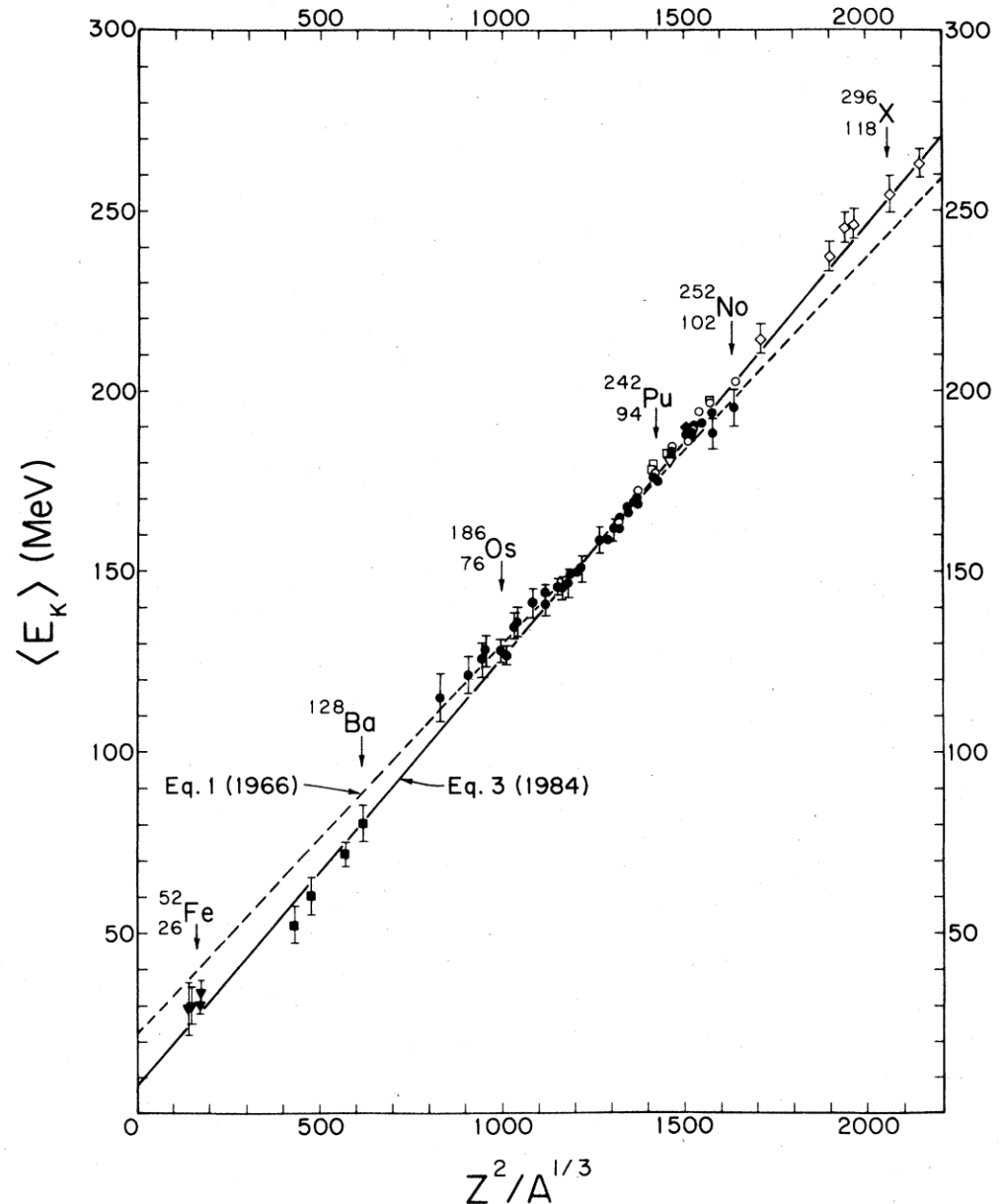
$$\langle E_K \rangle = 0.1071 Z^2 / A^{1/3} + 22.2 \text{ MeV}$$

Data before 1966

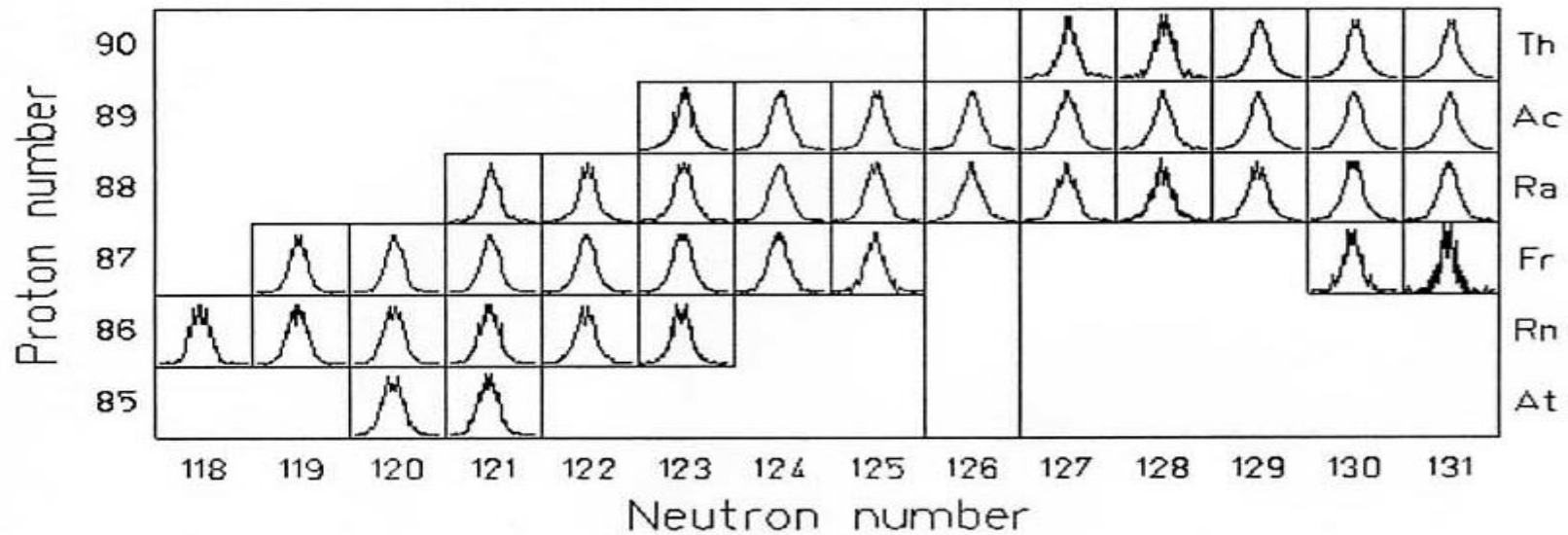
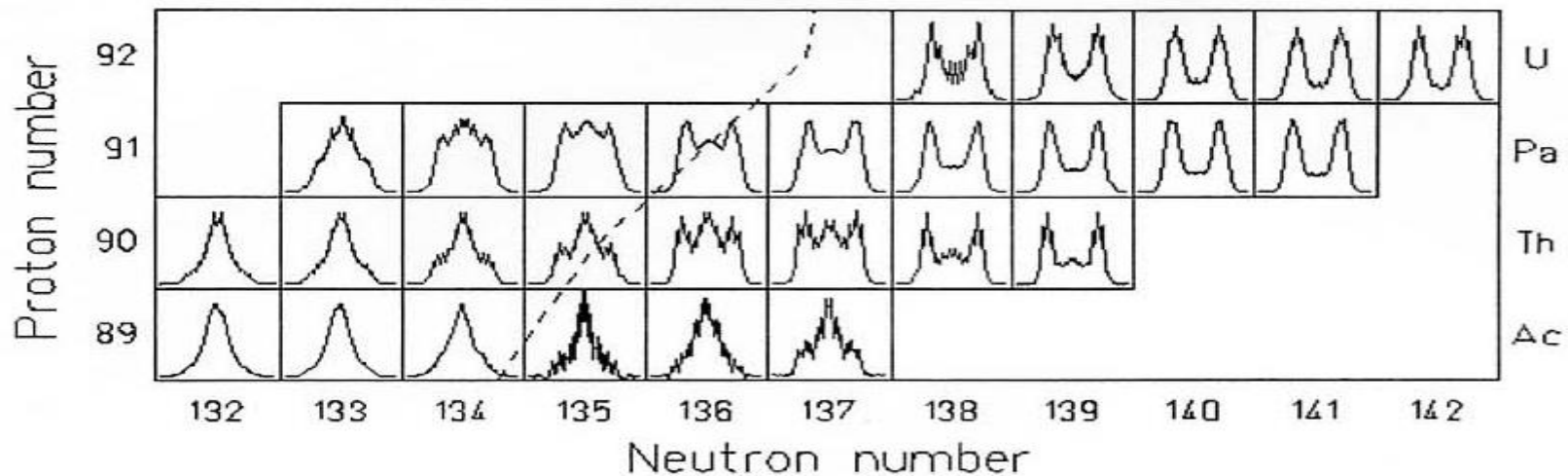
$$\langle E_K \rangle = 0.1189 Z^2 / A^{1/3} + 7.3 \text{ MeV}$$

Data up to 1984

- Main contribution to TKE comes from Coulomb repulsion
- Measure of the fragment deformation



# Fission modes





# Extension to multi-dimension

## Multi-dimensional Langevin equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j \quad i, j, k = 1, \dots, N$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} \left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij} \left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

$$\langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2 \delta_{ij} \delta(t_1 - t_2) \quad \sum_k g_{ik} g_{jk} = T \gamma_{ij}$$

$m_{ij}(q)$  Hydrodynamical inertial mass

$\gamma_{ij}(q)$  Wall-and-Window (one-body) friction

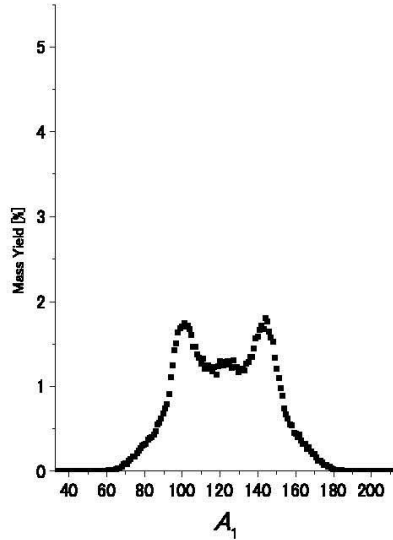
$V(q)$  Macro-microscopic potential

$$V(q, E_x) = V_{LD}(q) + \Delta E_{\text{shell}}(q, E_x = 0) \exp\left(-\frac{E_x}{E_d}\right)$$

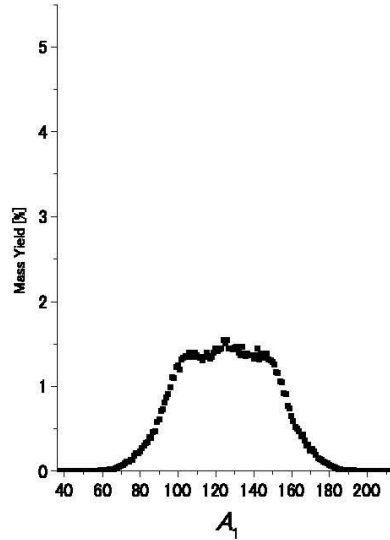
$\{q_{ij}\}$  : collective parameters

# Mass distribution for Fm isotopes

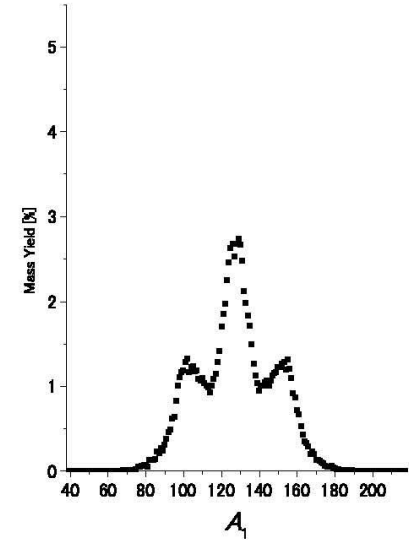
$^{246}\text{Fm}$ ,  $E=10$  MeV



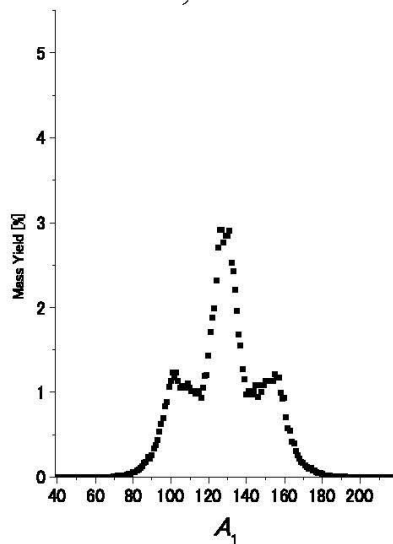
$^{252}\text{Fm}$ ,  $E=10$  MeV



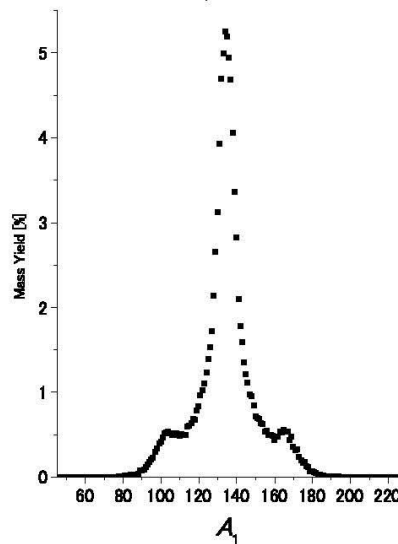
$^{256}\text{Fm}$ ,  $E=10$  MeV



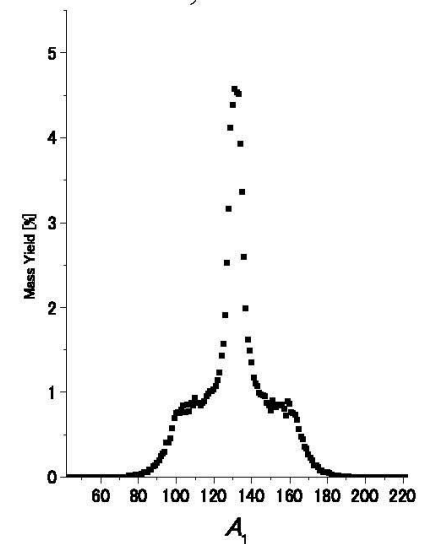
$^{258}\text{Fm}$ ,  $E=10$  MeV



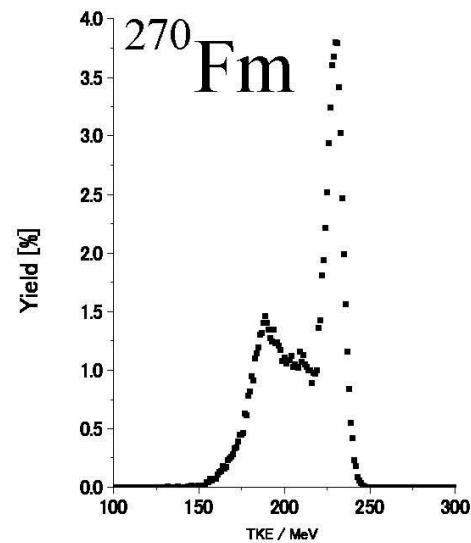
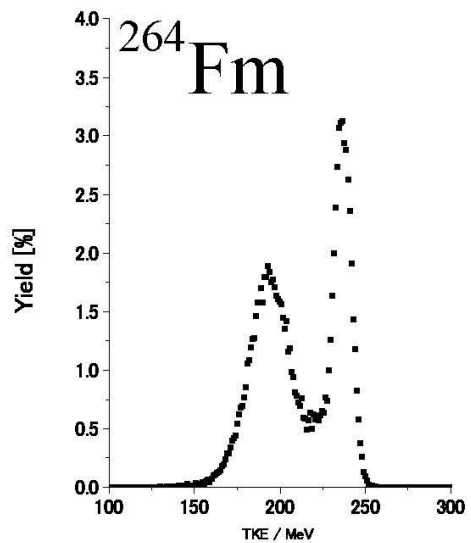
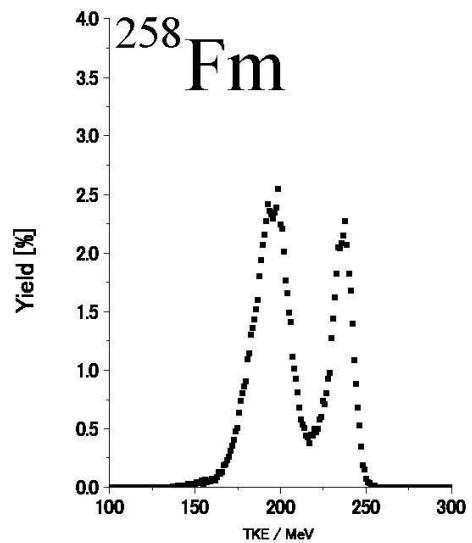
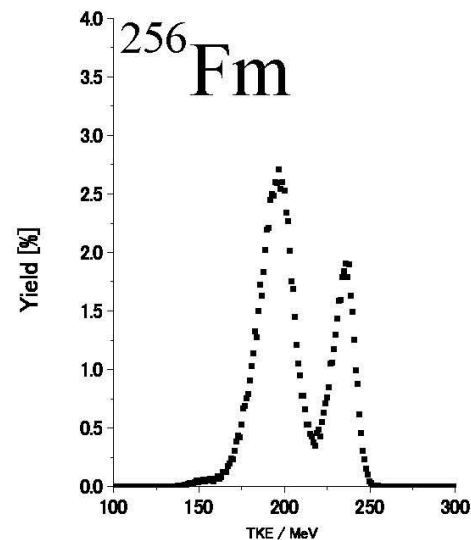
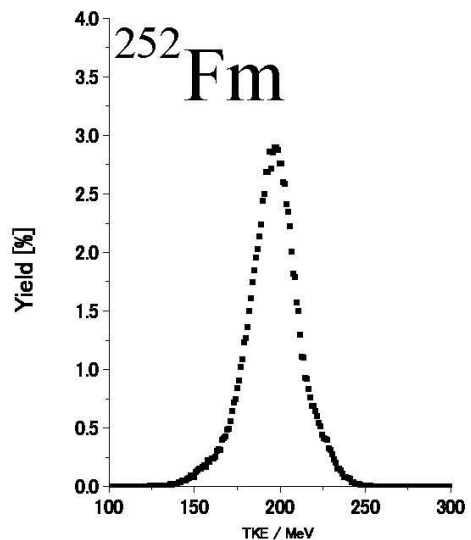
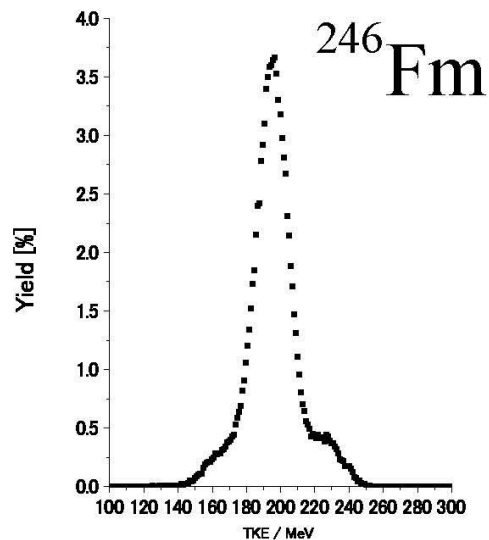
$^{264}\text{Fm}$ ,  $E=10$  MeV



$^{270}\text{Fm}$ ,  $E=10$  MeV

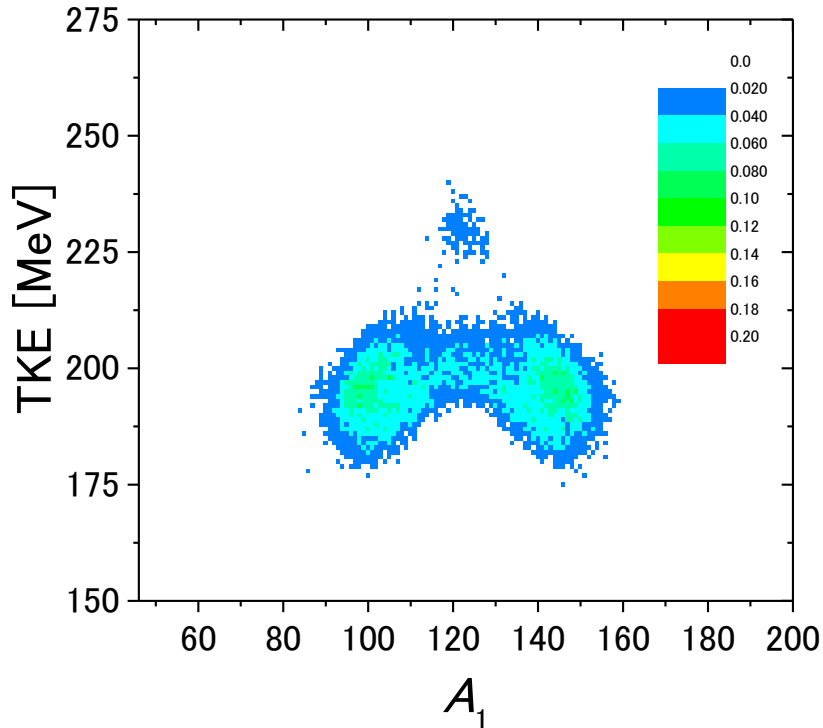


# TKE distribution for Fm isotopes

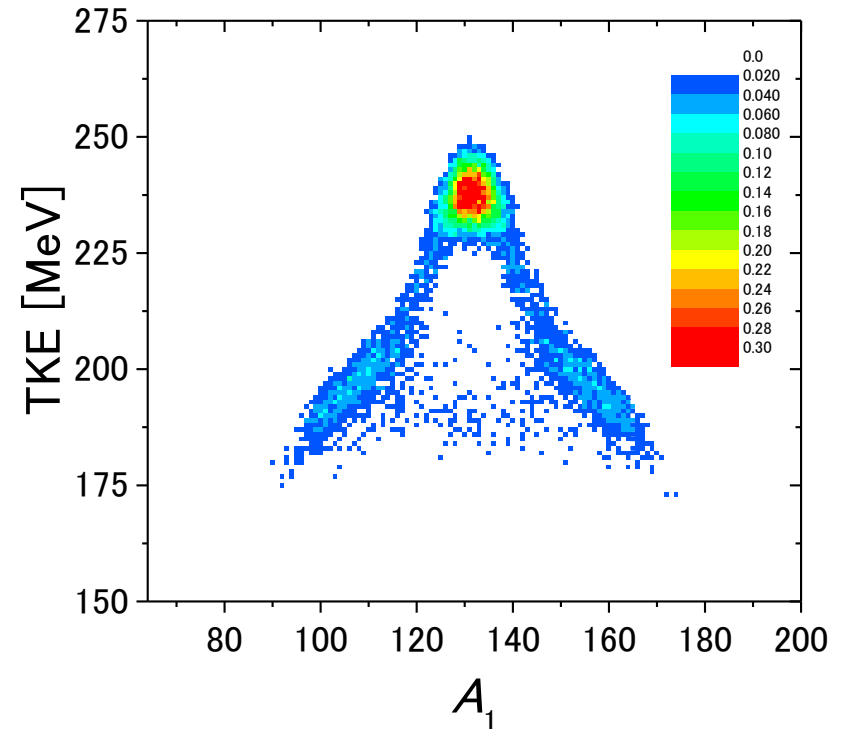


# Mass and TKE distribution

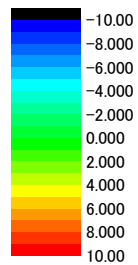
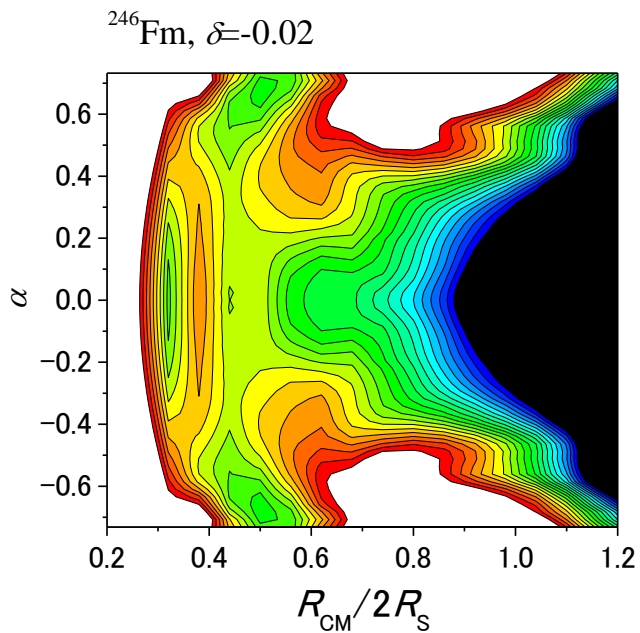
$^{246}\text{Fm}$



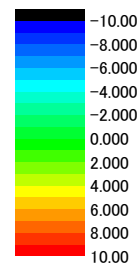
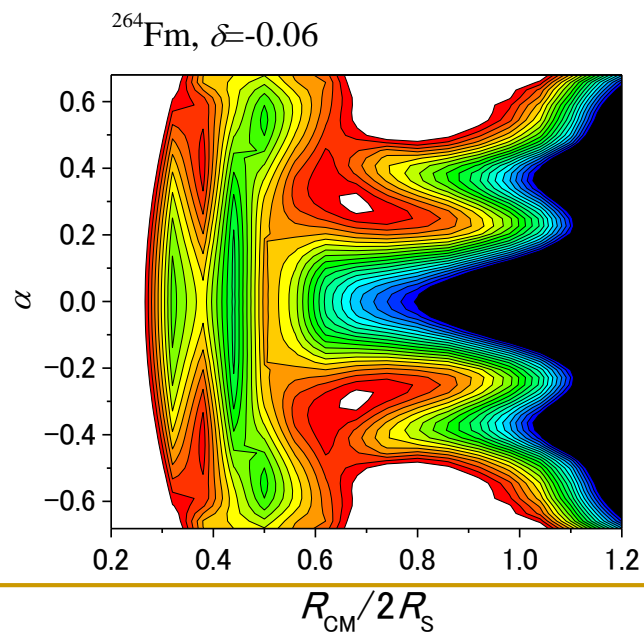
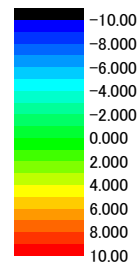
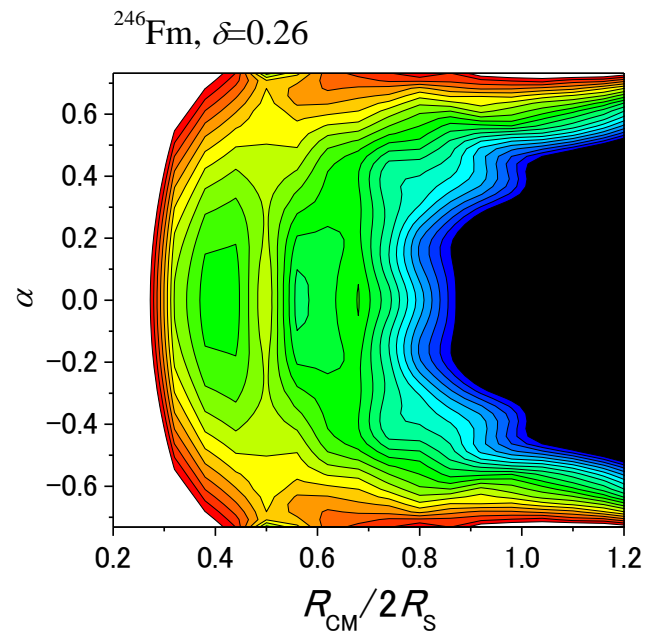
$^{264}\text{Fm}$



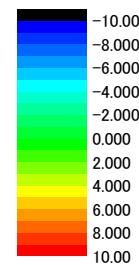
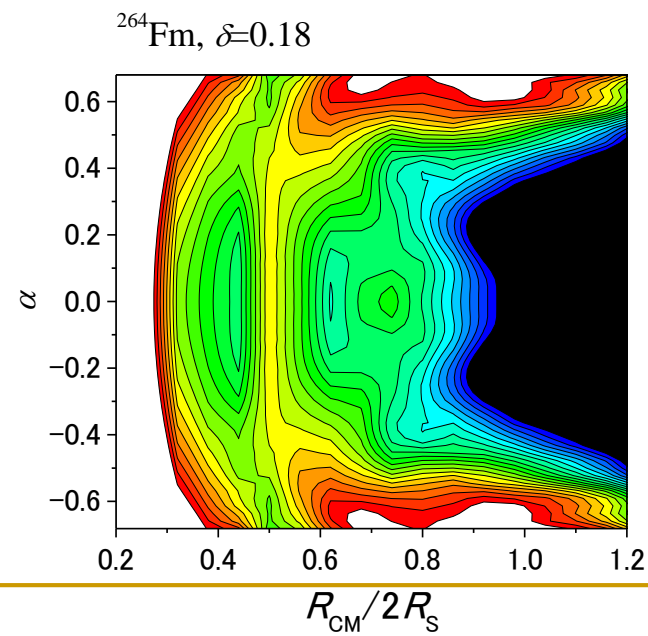
- Strong correlation between mass distribution and TKE distribution



$^{246}\text{Fm}$

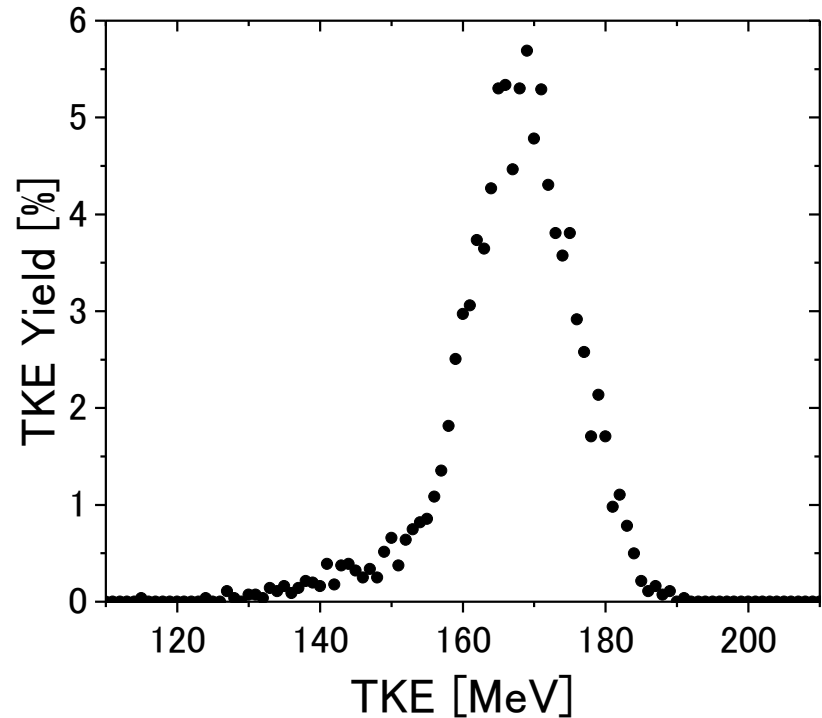
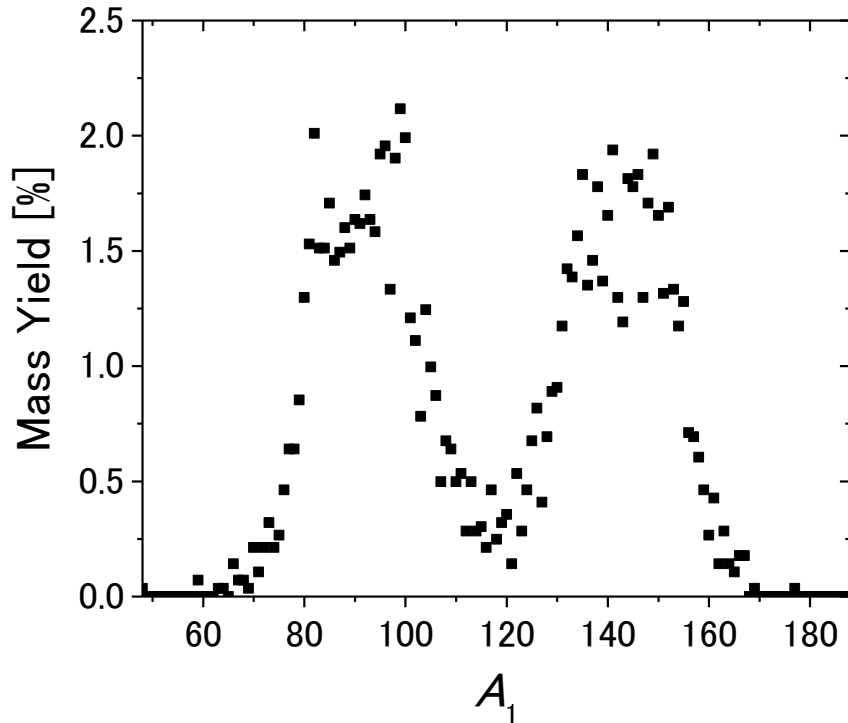


$^{264}\text{Fm}$



# Mass and TKE distribution of $^{236}\text{U}$

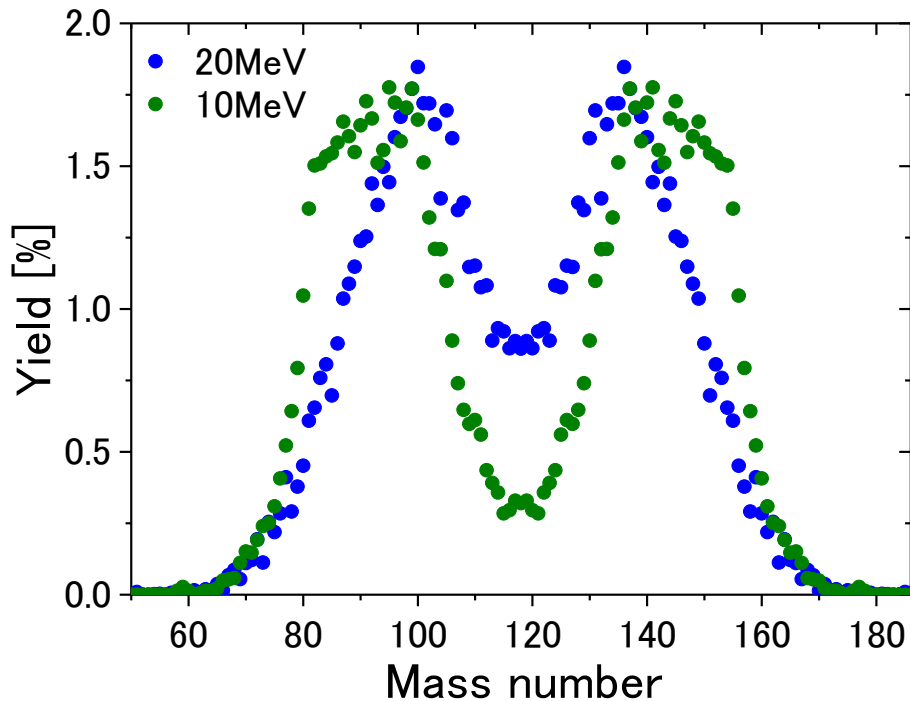
$E^* = 10 \text{ MeV}$



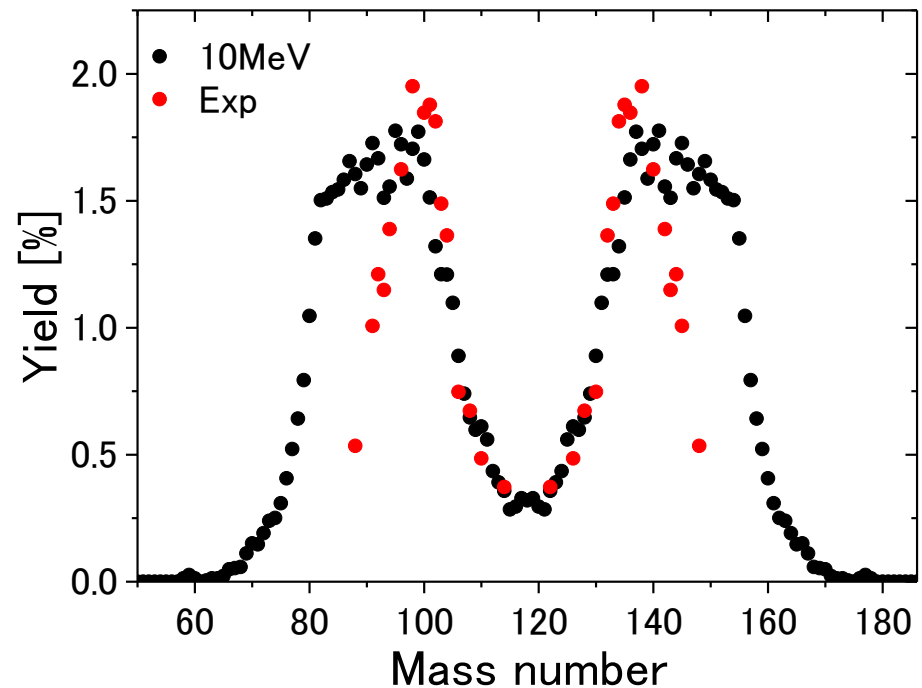
- Statistics is poor
  - High fission barrier and low excitation energy

# Mass distribution for $^{236}\text{U}$

## Energy dependence

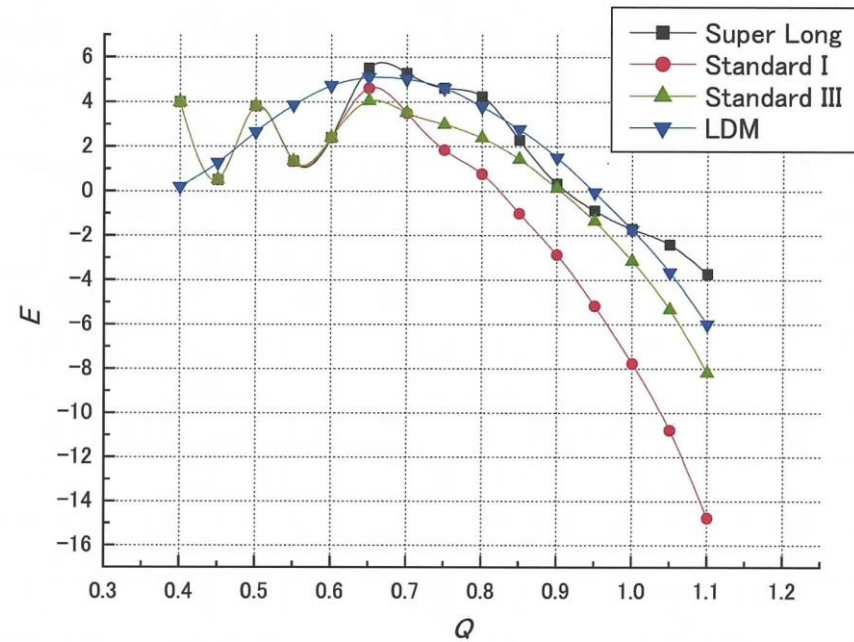
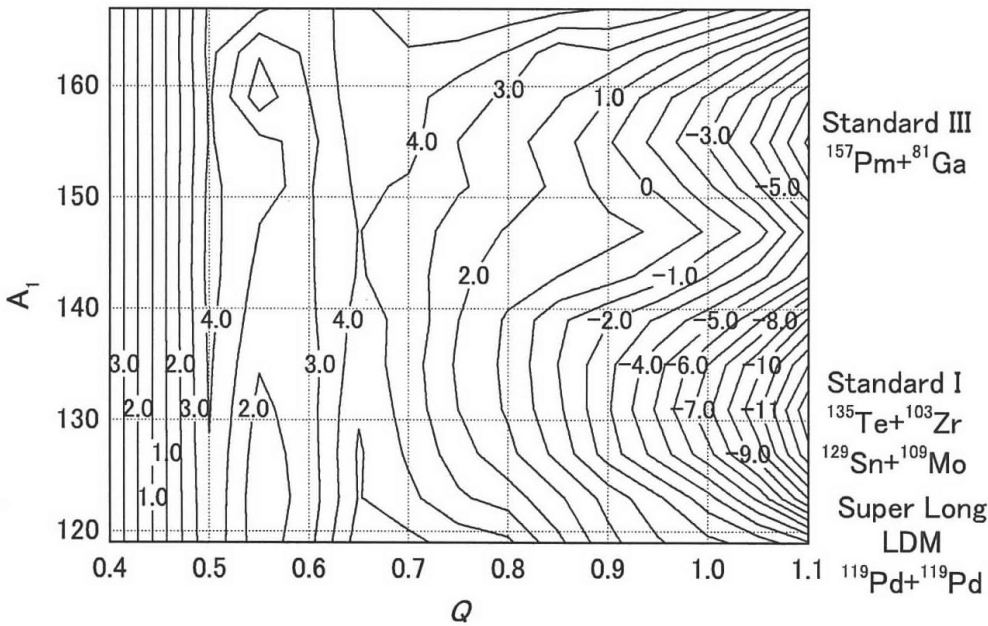


## Exp. vs 3D-Langevin



- Symmetric component increases with  $E^*$
- Two mass asymmetric components

# Fission modes



- Symmetric : super long
- Asymmetric
  - Standard I : Heavy fragment is “magic”
  - Standard III : Heavy fragment is deformed



# Summary

- The fluctuation-dissipation dynamics is a general framework to describe the dynamics of a few slow (collective) variables interacting with many fast variables that can be treated as a heat bath.
- Powerful tool to study dynamics of fission
  - Fragment mass and TKE distributions
- Fm isotopes
  - Symmetric vs Asymmetric fission
  - Isotope dependence
- $^{236}\text{U}$
- More flexible shape parameterization
  - Independent deformation of fragments, Neck