The Level Density of Superheavy Nuclei

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Motivation

The experimental trend of nuclear properties (Q_α-values and half-lives) as well as the production cross sections of SHE reveal an increasing stability of nuclei approaching *N*=184 and indicate quite large shell effects beyond *Z*=114.

This is in agreement with the predictions of relativistic and nonrelativistic mean-field models which expect the "island of stability" at

_Z_=120-126 and _N_=172 or 184.

P.G. Reinhard, Rep. Prog. Phys. 52 (1989);P. Ring, Prog. Par. Nucl. Phys. 37 (1996); M. Bender et al., Rev. Mod. Phys. 75 (2003); J Meng et al., Prog. Part. Nucl. Phys. 57 (2006); J.J. Li, Phys. Lett. B 732 (2014).

In the *(N,Z)*-plane, the line, along which all new SHE were discovered in the actinide-based reactions with ^{48}Ca beam, just approaches this region.

Yu. Ts. Oganessian, J. Phys. G34 (2007); Phys. Rev. C 87 (2013)

If the shell-effects at Z=120 is strong, then there is a hope to synthesize new SHE elements with Z ≥120 using the present experimental set up with available stable projectiles and targets.

Motivation

The investigation of the nuclear properties and the shell structure of elements with **100<*Z*<130** is important from the perspective of future experiments.

- The level density, as a function of excitation energy, is required to calculate the survival probability and, correspondingly, the production cross section of SHE.

- The phenomenological values of level density parameters have to be verified in the microscopical calculations.

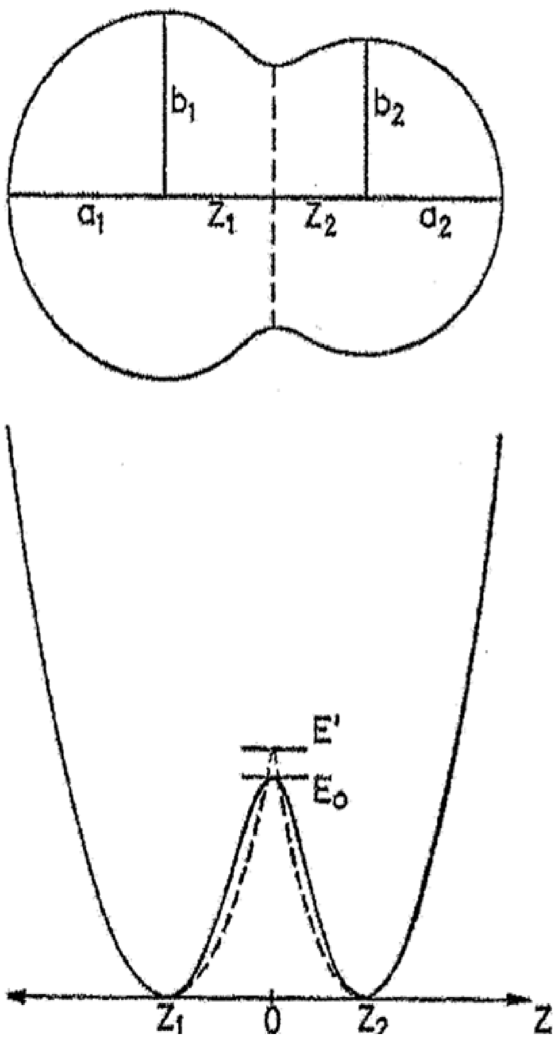
In this work, the modified TCSM is used to investigate the **level density of 100<*Z*<130** nuclei, which belongs to the alpha-chains of $^{296,298,300}120$ nuclei.

Previously, the TCSM was modified to reveal the trends of shell structure, nuclear binding energies, Q_α-values, and quasiparticle states of SHE.

A.N. Kuzmina, G.G. Adamian and N.V. Antonenko, W. Scheid, Phys. Rev. C 85 (2012)

Two Center Shell Model

(J. Maruhn and W. Greiner, Z. Phys. 251 (1972) 431)



The model has five collective coordinates

$\dfrac{\omega_{\rho 2}}{\omega_{\rho 1}}$: mass asymmetry

$\dfrac{\omega_{zi}}{\omega_{\rho i}} = \dfrac{a_i}{b_i} \; (i = 1,2)$: ellipsoidal deformations of the fragments

$z_2 - z_1$: separation of the fragments (elongation)

$\dfrac{E_0}{E'}$: neck parameter, permitting a variable barrier height

$$\left(E' = \frac{1}{2} m_N \omega_{z1} z_1^2 = \frac{1}{2} m_N \omega_{z2} z_2^2 \right)$$

Description of the Ground State

In the present work we choose the following shape parametrization:

- the elongation $\lambda = L/2R_0$
- the case of deformation $\beta = a/b = \beta_1 = \beta_2$
- the neck parameter $\varepsilon = E_0/E' = 0$
- the mass asymmetry

$\eta = (A_1\text{-}A_2)/(A_1+A_2) = 0$

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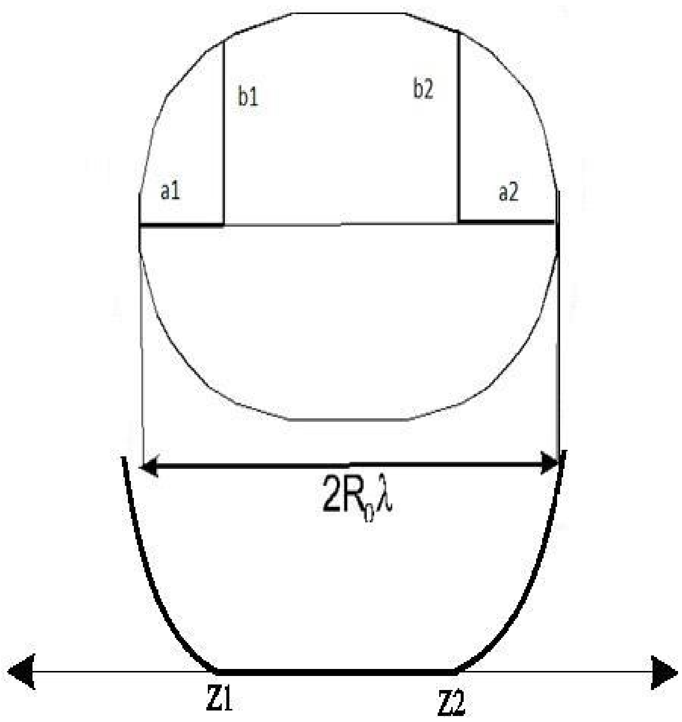


Hamiltonian of model

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(\rho,z) + V_{ls} + V_{l^2},$$

$$V(\rho,z) = \begin{cases} \frac{1}{2}m\omega_z^2(z-z_1)^2 + \frac{1}{2}m\omega_\rho^2\rho^2, & z < z_1, \\ \frac{1}{2}m\omega_\rho^2\rho^2, & z_1 < z < z_2, \\ \frac{1}{2}m\omega_z^2(z-z_2)^2 + \frac{1}{2}m\omega_\rho^2\rho^2, & z > z_2. \end{cases}$$

m- nucleon mass,

$\omega_\rho/\omega_z = a/b = \beta$,

$z_1 - z_2 = 2R_0\lambda - 2a$,

$\omega_\rho = \beta\omega_0 R_0/a$,

$h\omega_0 = 41\ MeV\ A^{-1/3}$

$3a^2R_0\lambda - a^3 = 2R^3_0\,\beta^2$ expression of volume conservation

$$V_{ls} = -\frac{2\hbar\kappa}{m\omega_0'}(\nabla V \times \mathbf{p})\mathbf{s}$$

$$V_{l^2} = -\kappa\mu\hbar\omega_0'\mathbf{l}^2 + \kappa\mu\hbar\omega_0'\frac{\mathcal{N}(\mathcal{N}+3)}{2}\delta_{if}$$

$h\omega'_0 = 41\ MeV\ A'^{-1/3}$

$A' = Aab^2/R^3_0$

Modification:

In order to improve the description of the nuclear spins and parities we introduce the weak dependence of κ, μ – on (N-Z).

Modification of the two center shell model: details

The momentum-dependent part of the Hamiltonian consists of the sl- and l^2-like terms with the parameters $\kappa_{n,p}$ and $\mu_{n,p}$, respectively.

$$\kappa_n = -0.076 + 0.0058\,(N-Z) - 6.53\times10^{-5}\,(N-Z)^2 + 0.002A^{1/3}$$
$$\mu_n = 1.598 - 0.0295\,(N-Z) + 3.036\times10^{-4}\,(N-Z)^2 - 0.095A^{1/3}$$

$$\kappa_p = 0.0383 + 0.00137\,(N-Z) - 1.22\times10^{-5}\,(N-Z)^2 - 0.003A^{1/3}$$
$$\mu_p = 0.335 + 0.01\,(N-Z) - 9.367\times10^{-5}\,(N-Z)^2 + 0.003A^{1/3}$$

G.G.Adamian, N.V. Antonenko, and W. Scheid, Phys. Rev. C 81, 024320 (2010)
G.G.Adamian, N.V. Antonenko, S.N. Kuklin, and W. Scheid, Phys. Rev. C 82, 054304 (2010)

Results of TCSM Calculations for $^{300}120$ chain.

A	Z	δE_{sh}	λ	β	β_2	β_4
260	100	-4.13	1.18	1.28	0.257	0.025
264	102	-3.93	1.18	1.28	0.257	0.025
268	104	-3.42	1.16	1.22	0.239	0.014
272	106	-4.29	1.14	1.08	0.245	-0.017
276	108	-5.35	1.12	1.00	0.235	-0.036
280	110	-5.40	1.10	0.96	0.211	-0.044
284	112	-5.40	1.08	0.90	0.193	-0.059
288	114	-6.92	1.06	0.86	0.168	-0.067
292	116	-6.50	0.94	0.90	-0.095	-0.001
296	118	-7.50	1.02	0.94	0.058	-0.028
300	120	-9.20	1.00	0.88	0.036	-0.040
304	122	-9.10	1.00	0.92	0.023	-0.026
308	124	-11.30	0.98	0.90	-0.012	-0.023
312	126	-9.98	0.98	0.90	-0.012	-0.023
316	128	-6.31	0.98	0.90	-0.012	-0.023
320	130	-4.52	0.98	0.90	-0.012	-0.023

Level Density in the Superfluid Nuclear Model

Nuclear level density:

$$\rho = \frac{\exp[S(\beta, \lambda_Z, \lambda_N)]}{(2\pi)^{3/2}\sqrt{D}}.$$

Entropy:

$$S(\alpha_N, \alpha_Z, \beta) = 2\sum_{k=Z,N}\sum_{\nu}\left\{\ln\left[1+\exp(-\beta E_{\nu k})\right] + \frac{\beta E_{\nu k}}{1+\exp(\beta E_{\nu k})}\right\}$$

where:

$$D = \begin{vmatrix} \frac{\partial^2 S}{\partial \beta^2} & \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} \\ \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \mu_Z^2} & 0 \\ \frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 & \frac{\partial^2 S}{\partial \mu_N^2} \end{vmatrix}$$

P. Decowski, W. Grochulski, A. Marcinkowski, K. Siwek, and Z. Wilhelmi, Nucl. Phys. A 110
G.D. Adeev and P.A. Cherdantsev, Yadernaya Fizika 21, 491 (1975)

Level Density in the Superfluid Nuclear Model

N - number of neutrons,
Z - number of protons,
$T=\beta^{-1}$ - temperature,
$\lambda_{N}=\alpha_{N}/\beta$ -chemical potential for neutrons,
$\lambda_{Z}=\alpha_{Z}/\beta$ -chemical potential for protons,
$\epsilon_{\upsilon N}, \epsilon_{\upsilon Z}$ -neutron, proton single- particle energies,
G_N, G_Z -pairing constants.

$E_{\nu k} = \sqrt{(\epsilon_{\nu k} - \lambda_k)^2 + \Delta_k^2}$ -quasiparticle energies of neutrons ($k=N$) and protons ($k=Z$).

Equations determining the saddle point:

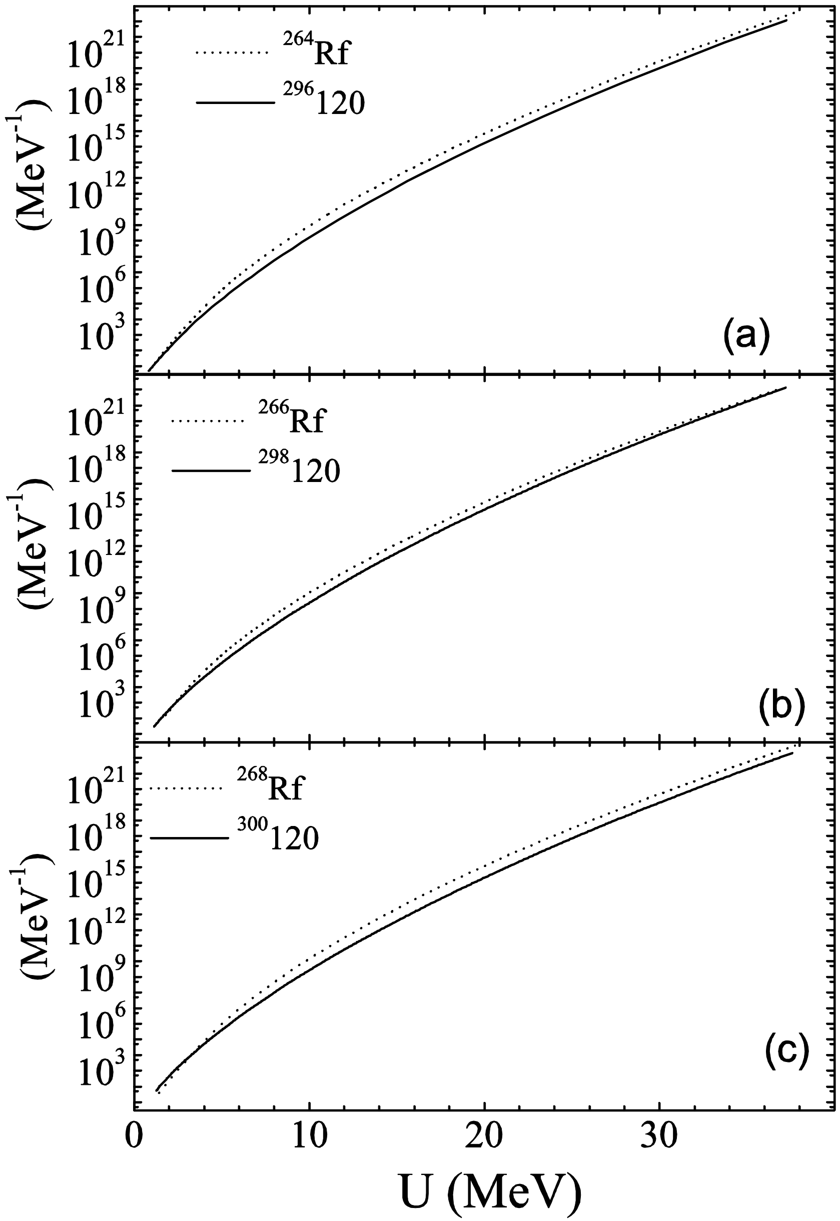
$$N = \sum_{\nu} \left(1 - \frac{\epsilon_{\nu N} - \lambda_Z}{E_{\nu N}} \tanh \frac{\beta E_{\nu N}}{2} \right), \quad Z = \sum_{\nu} \left(1 - \frac{\epsilon_{\nu Z} - \lambda_Z}{E_{\nu Z}} \tanh \frac{\beta E_{\nu Z}}{2} \right)$$

$$\frac{2}{G_N} = \sum_{\nu} \frac{\tanh(\beta E_{\nu N}/2)}{E_{\nu N}}, \quad \frac{2}{G_Z} = \sum_{\nu} \frac{\tanh(\beta E_{\nu Z}/2)}{E_{\nu Z}}$$

$$E(T) = \sum_{k=N,Z} \left\{ \sum_{\nu} \epsilon_{\nu k} \left(1 - \frac{\epsilon_{\nu N} - \lambda_Z}{E_{\nu N}} \tanh \frac{\beta E_{\nu N}}{2} \right) - \frac{\Delta_k^2}{G_k} \right\}$$

$$U = E_{Z,N}(T) - E_{Z,N}(0)$$

Intrinsic Level Densities in Heaviest Nuclei



Due to the smaller densities of the single-particle states near the Fermi surfaces in the nuclei with the closed shell in the ground state, **the LD is smaller for magic or nearly magic nuclei.**

U <10 MeV

the LD of Z=120 and Z=104 isotopes are comparable
(**pairing**).

U ~(10-30) MeV,

the LD in *Z* = *120* isotopes are one order of magnitude smaller than those in Rf
(**shell effects**).

U >30 MeV

the LD of Z=120 and Z=104 isotopes are comparable
(**shell effects faded**).

Fermi Gas Expression for the Level Density

The level density parameter, as a function of excitation energy *a(U)* is required to calculate the survival probability and, correspondingly, the production cross section of the heavies nuclei.

a(U) is extracted by fitting the calculated level density by the Fermi-Gas expression.

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}} \exp\left[2\sqrt{aU}\right]$$

$$U = aT^2, \qquad S = 2aT = 2\sqrt{aU}$$

For particles moving in spherical potential well of radius $R = r_0 A^{1/3}, \qquad r_0 = 1.2$ fm, one can estimate:

$$a = \left(\frac{\pi}{3}\right)^{4/3} \frac{2m_N r_0^2}{\hbar^2} A \approx \frac{A}{13.5}$$

In the phenomenological calculations of surviving probabilities one usually takes

a=A/(10-12) MeV^{-1}

Extraction of the Level Density Parameter

We found that the best fit of the calculated LD with the Fermi gas expression is achieved if one uses the level density parameter

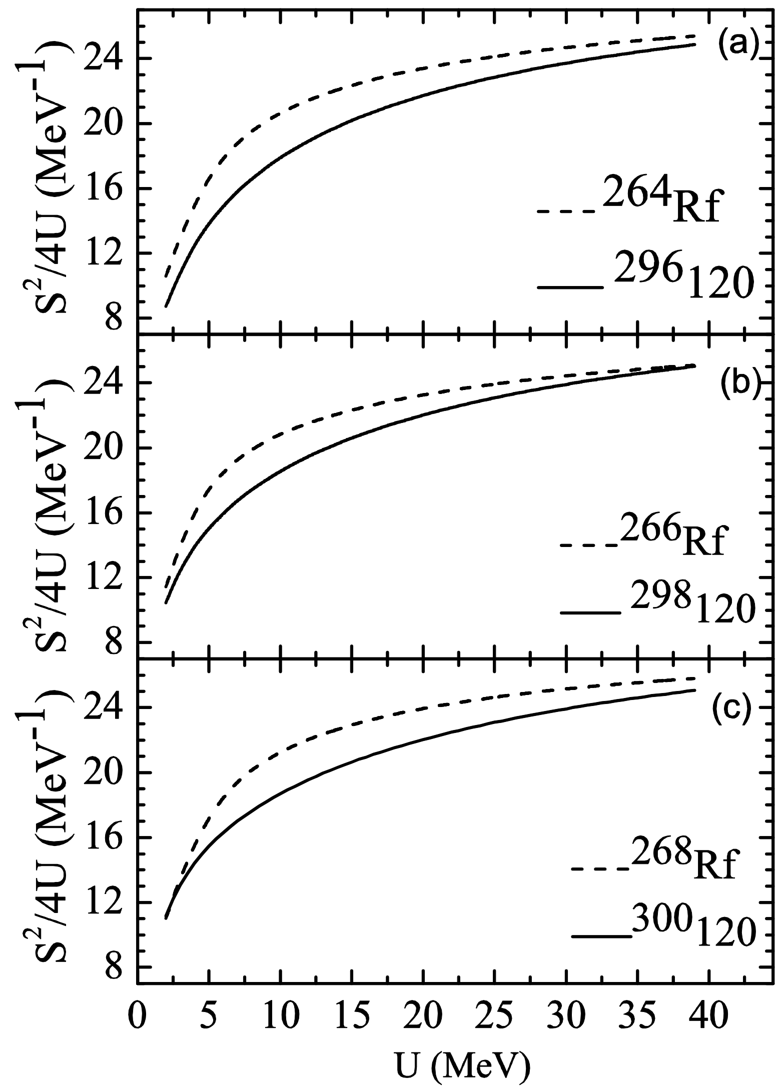
$$a(U) = S^2/4U,$$

where S and U are calculated

$$S = 2 \sum_{k=Z,N} \sum_{\nu} \left\{ \ln[1 + \exp(-\beta E_{k\nu})] + \frac{\beta E_{k\nu}}{1 + \exp(\beta E_{k\nu})} \right\},$$

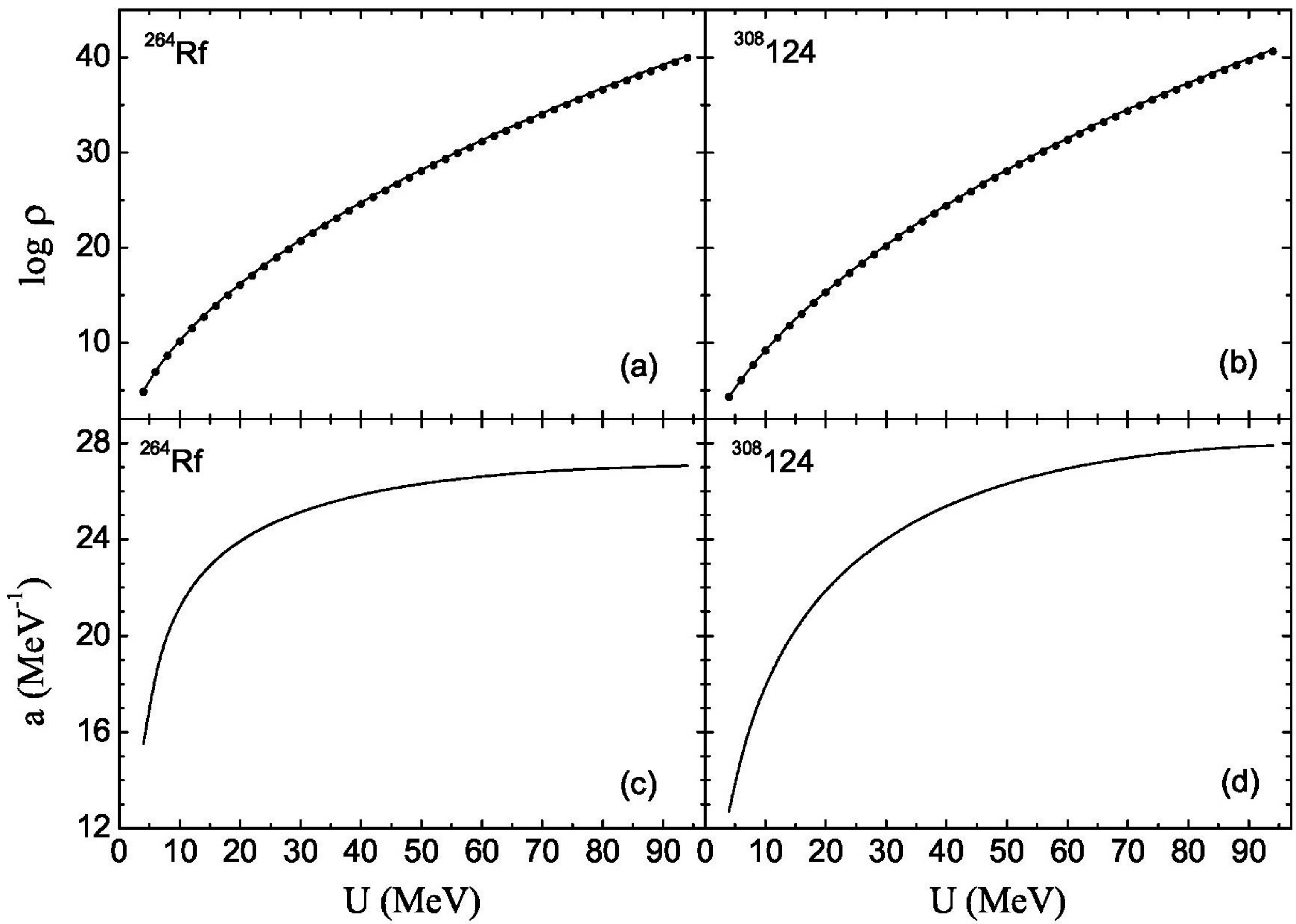
$$E_{Z,N}(T) = \sum_{k=Z,N} \left\{ \sum_{\nu} \varepsilon_{k\nu} \left(1 - \frac{\varepsilon_{k\nu} - \lambda_k}{E_{k\nu}} \tanh \frac{1}{2} \beta E_{k\nu} \right) - \frac{\Delta_k^2}{G_k} \right\}$$

$$U = E_{Z,N}(T) - E_{Z,N}(0).$$

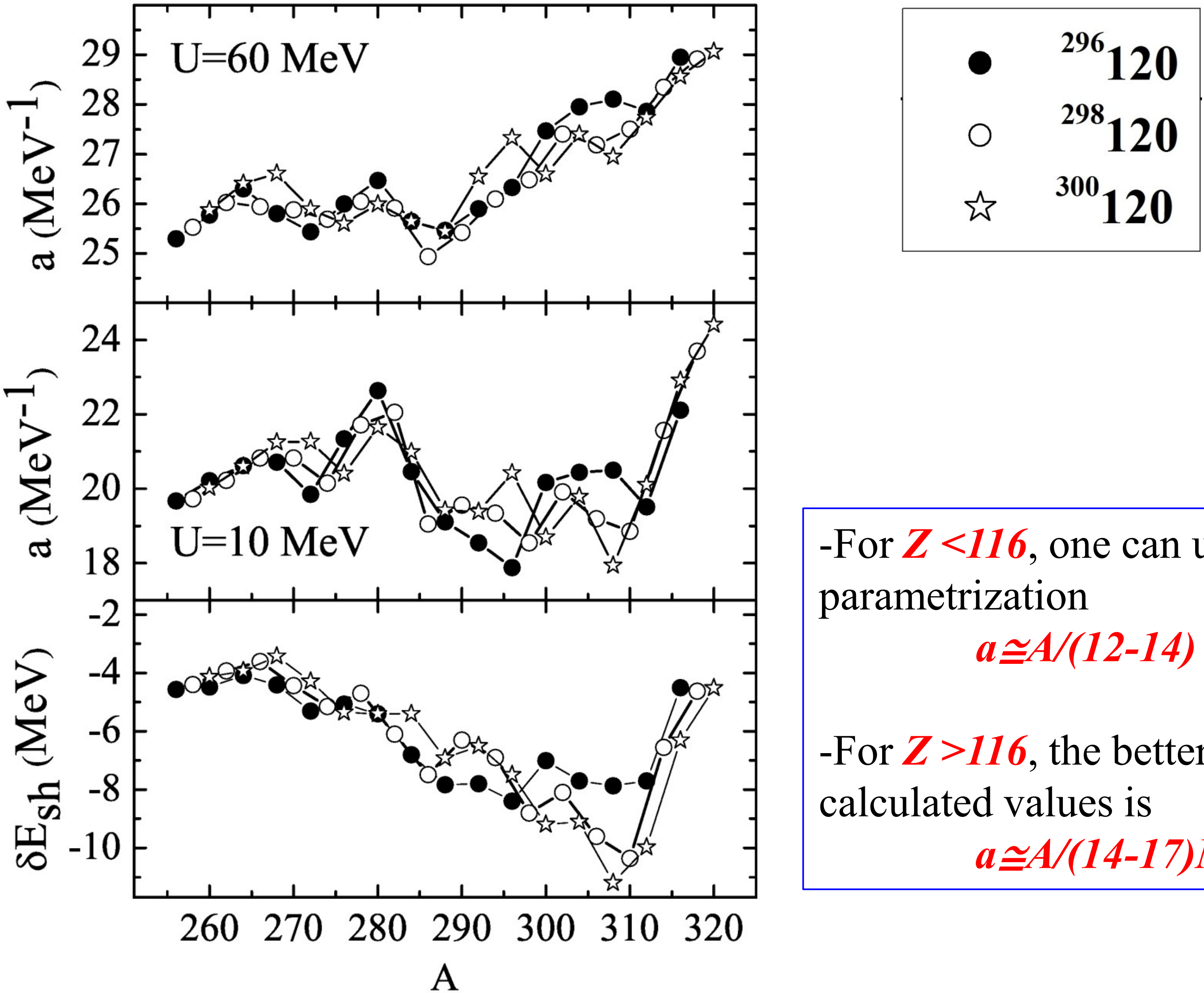


Note, that one does not need to make a back-shift of excitation energy as in the phenomenological treatment.

Fit of the calculated level density with the Fermi-Gas Expression.



Dependence of Level Density Parameter on Shell Corrections



-For ***Z <116***, one can use the parametrization

$$a \cong A/(12\text{-}14)\ MeV^{-1}$$

-For ***Z >116***, the better fit of the calculated values is

$$a \cong A/(14\text{-}17) MeV^{-1}.$$

Level Density Parameter and Shell Corrections

❖With increase of excitation energy, the correlation between level density parameter *a* and shell corrections δE_{sh} is destroyed and the dependence of ***a*** parameter on mass number becomes rather smooth.

❖Based on the study of the dependencies of **a** on E_{sh} and U, one can use the following parametrization of the level density parameter:

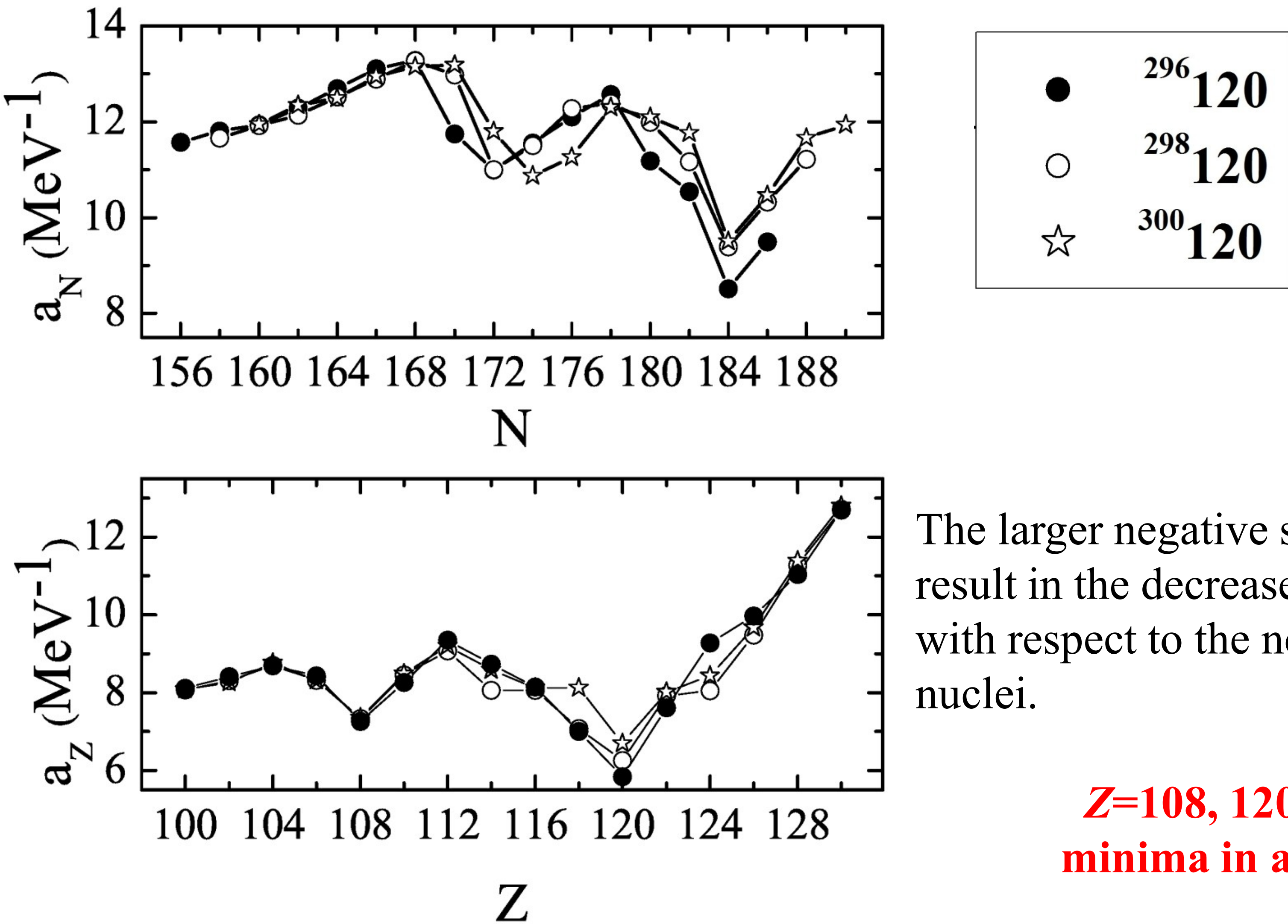
- $U = E^* - \Delta_Z - \Delta_N$ - for even-even nuclei
- $a(A,U) = \tilde{a}(A)\left[1 + \frac{1-\exp\{-\gamma U/E'_D\}}{U} E_{sh}\right]$
- $\tilde{a} = \alpha A + \beta A^2$

A.B. Ignatyuk, G.N. Smirenkin, and A.S. Tishin, Yadernaya Fizika 21, 485(1975)

$$\alpha = 0.118 \text{ MeV}^{-1},\ \beta = -0.53 \times 10^{-4} \text{ MeV}^{-1},\ E'_D = 27 \text{ MeV}$$

Level Density Parameter as a function of *N* and *Z*

$$a = S_N^2/(4U_N) + S_Z^2/(4U_Z) = a_N + a_Z$$



The larger negative shell corrections result in the decrease of the value of *a* with respect to the neighborhood nuclei.

***Z*=108, 120 & *N*=184**
minima in all chains.

Summary

- **The intrinsic level densities** have been microscopically calculated in the region of superheavy nuclei. The method of calculations have been tested for light nuclei with known experimental data.

- **The level density parameters**, which are used in the Fermi-gas model, have been determined for the nuclei of alpha-decay chains containing **^{296}120,^{298}120,^{300}120**.

- The dependencies of the level density parameter on the shell correction and excitation energy have been studied. **The damping factor** in the dependence of the level density parameter on excitation energy is found to be

$$E'_D = 27 \text{ МэВ}$$

- For the superheavy nuclei considered, the level density parameters are approximately ***A*/(11-13) MeV** at excitation energies corresponding to the (3-5) neutron evaporation channels. Thus, calculations of survival probabilities with these values of a seem to be microscopically justified.