Damping of quantum vibrations revealed in deep sub-barrier fusion

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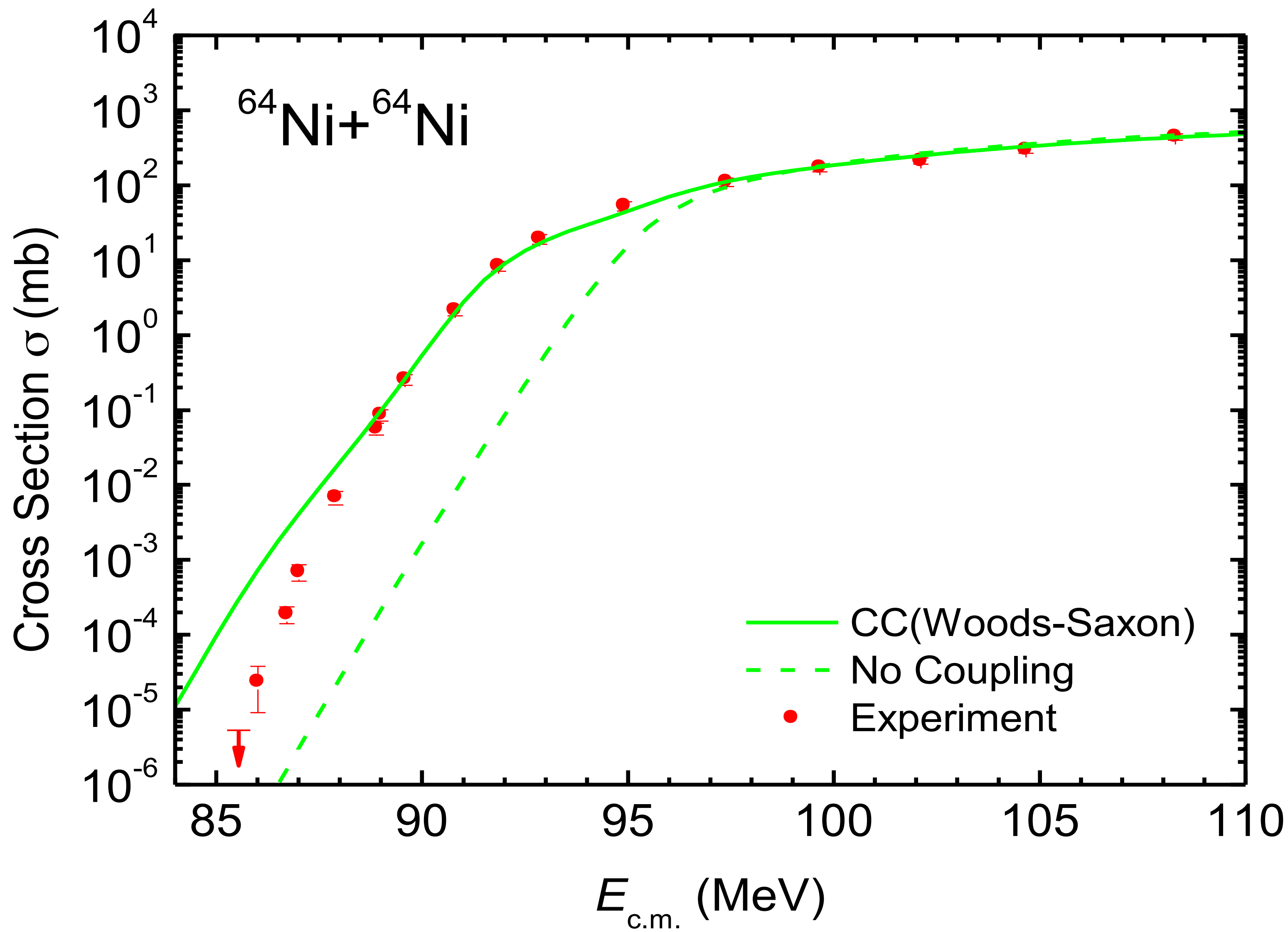
Steep falloff of fusion cross sections

C. L. Jiang *et al.*, Phys. Rev. Lett. **93**, 012701 (2004)



Standard CC calculations largely deviate from experimental data below a certain threshold incident energy

$^{16}O + ^{208}Pb$

Physical Review Focus

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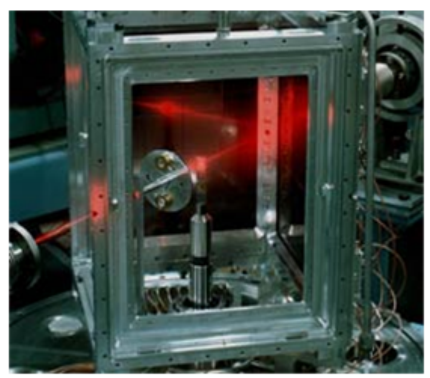
Phys. Rev. Lett. **99**, 192701
(issue of 9 November 2007)
Title and Authors

12 November 2007

Nuclei in Collision

When two large nuclei collide and fuse–rather than flying apart–some of the credit goes to the internal motions of protons and neutrons that result in excited states of the nuclei. The best models account for these states in calculating the fusion rate. But in the 9 November *Physical Review Letters*, Australian physicists say their measurements disagree with even these sophisticated models. The researchers suggest that the internal modes get out of synch even while the collision is underway, so that the nuclei behave more like a macroscopic classical object than a tiny quantum one.



G. Gilmour/Australian National Univ.

Two nuclei can overcome the "Coulomb barrier" of repulsion of like charges and fuse

PRL 99, 192701 (2007) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2007

Beyond the Coherent Coupled Channels Description of Nuclear Fusion

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New measurements of fusion cross sections at deep sub-barrier energies for the reactions $^{16}O + ^{204,208}Pb$ show a steep but almost saturated logarithmic slope, unlike ^{64}Ni-induced reactions. Coupled channels calculations cannot simultaneously reproduce these new data and above-barrier cross-sections with the same Woods-Saxon nuclear potential. It is argued that this highlights an inadequacy of the coherent coupled channels approach. It is proposed that a new approach explicitly including gradual decoherence is needed to allow a consistent description of nuclear fusion.

DOI: 10.1103/PhysRevLett.99.192701 PACS numbers: 25.70.Jj, 03.65.Yz, 24.10.Eq

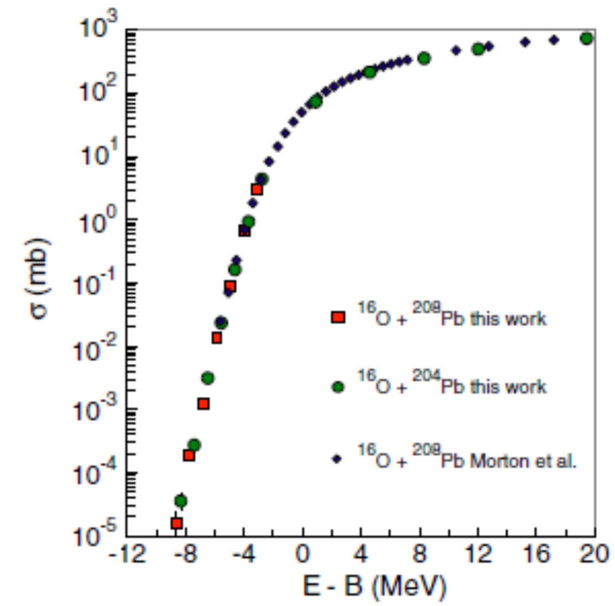


FIG. 2 (color online). Fusion cross sections as a function of the center-of-mass energy with respect to the barrier energies $B = 74.5$ and 74.9 MeV for ^{208}Pb and ^{204}Pb, respectively.

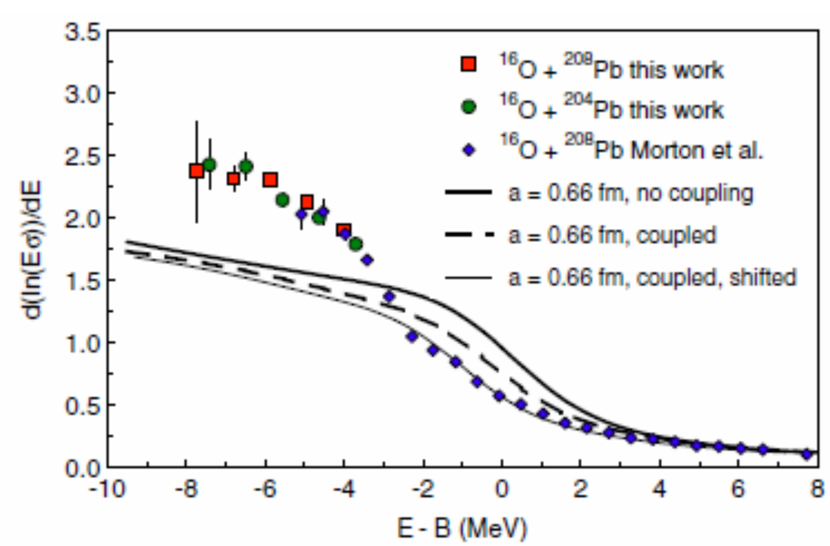
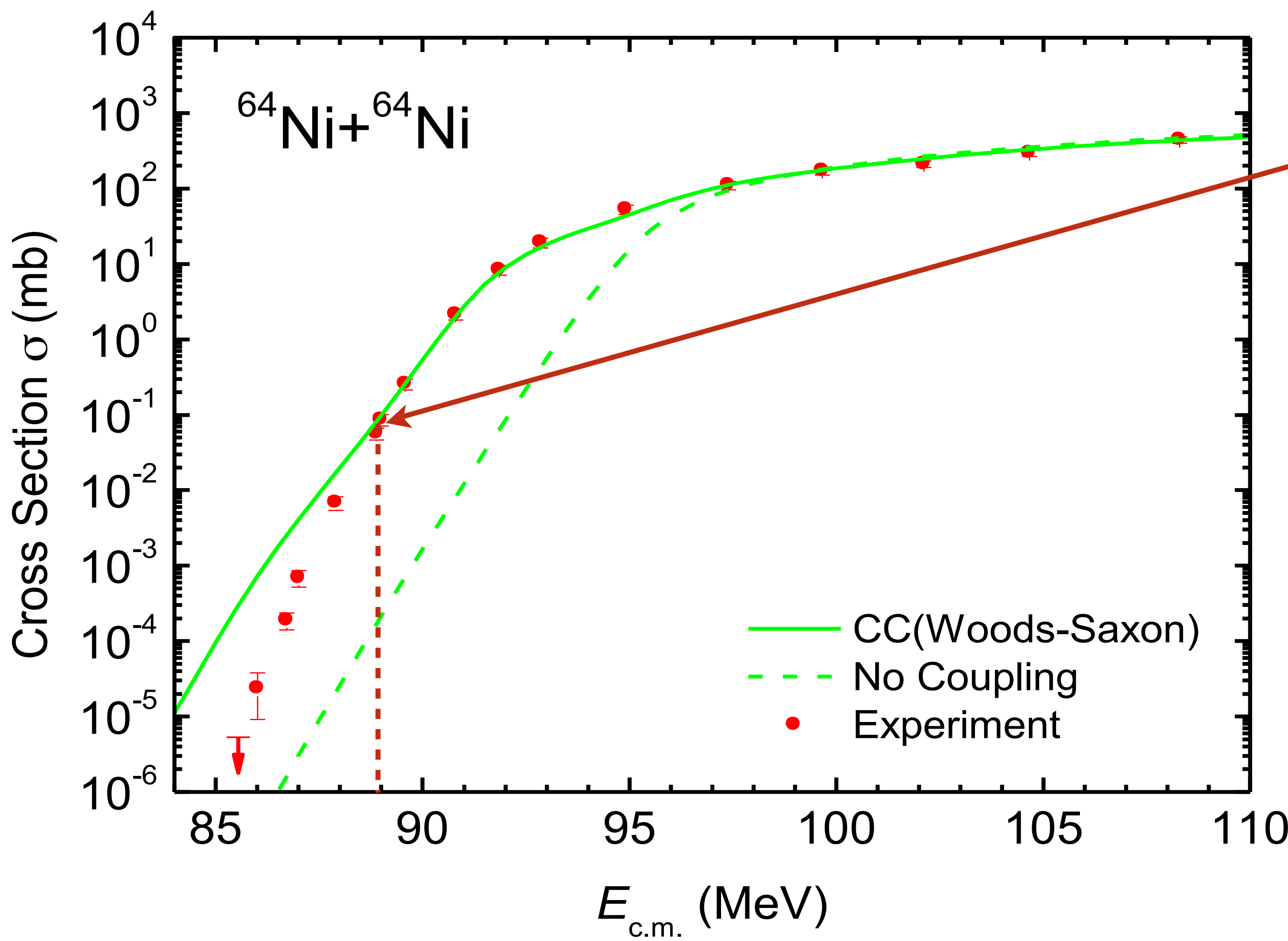
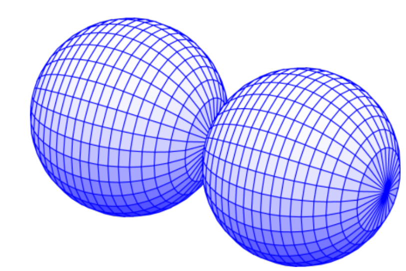


FIG. 3 (color online). Logarithmic slope as a function of energy with respect to the barrier. Calculation with standard parameters fail to match the measurements at low energy.

What is a key physical quantity?



Threshold incident energy
E_s ~ 89 MeV

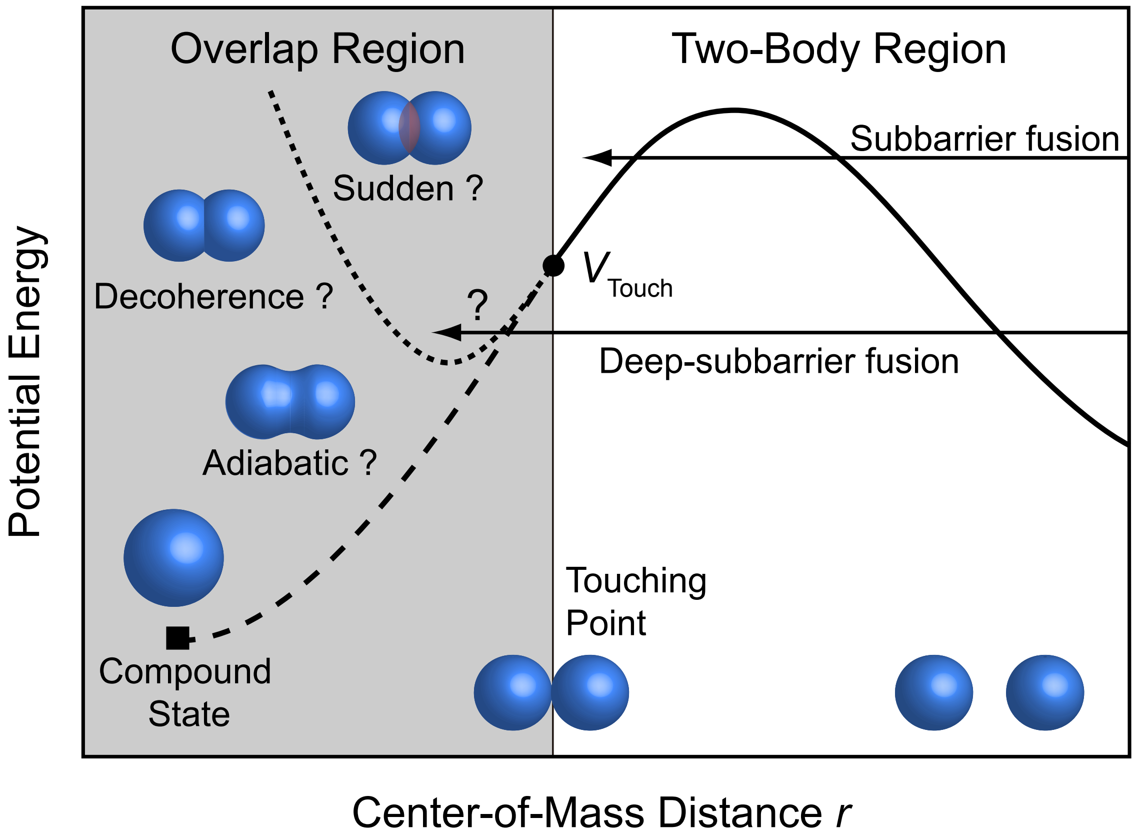


Potential energy at the touching point
V_{Touch} ~ 88.61 MeV

Energy at the touching point strongly correlate with threshold incident energy E_s

Tunneling in overlap region

TI, K. Hagino, and A. Iwamoto, Phys. Rev. C **75**, 064612 (2007)



- **Subbarrier energies ($E > V_{\text{touch}}$)**
  - Inner turning point
    → Outside of touching point
- **Deep subbarrier energies ($E < V_{\text{touch}}$)**
  - Inner turning point
    →In the overlap region

Steep fall-off phenomenon can be attributed to dynamics
after target and projectile touch with each other

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 J(J+1)}{2\mu r^2} + \boxed{V(r)} + \epsilon_n - E\right] u_n(r) + \boxed{\sum_n \left\langle \phi_n \middle| V_{\text{coup}} \middle| \phi_n \right\rangle} u_n(r) = 0$$

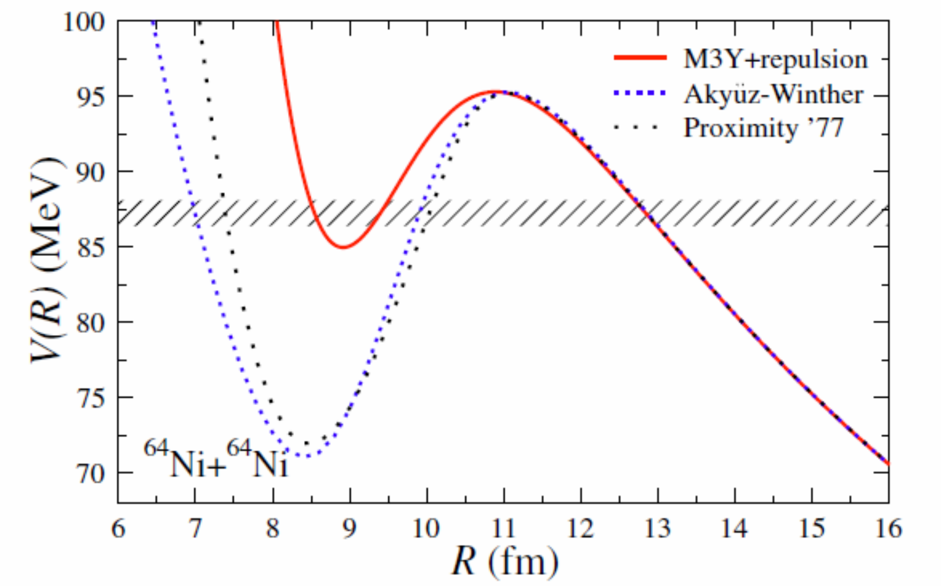
Sudden and adiabatic approaches

Ş. Mişicu and H. Esbensen, Phys. Rev. Lett. **96**, 112701 (2006)

- **Sudden Approach**

  →Shallow potential pocket

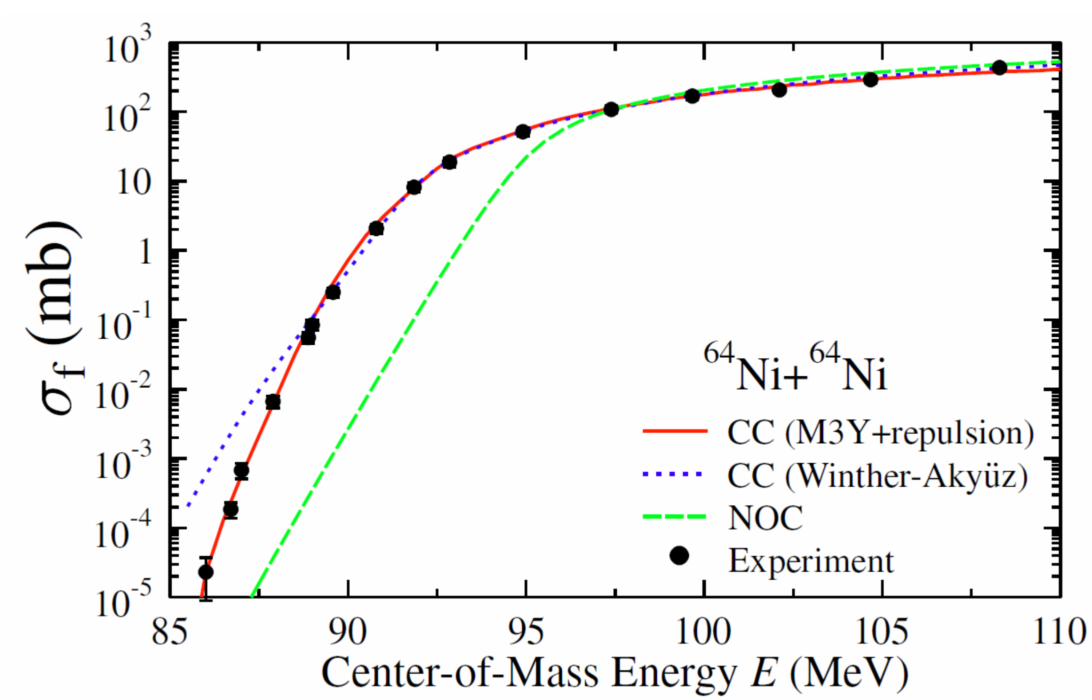
  • Frozen density approximation Mişicu and Esbensen



- **Quantum decoherence of channel wave function**

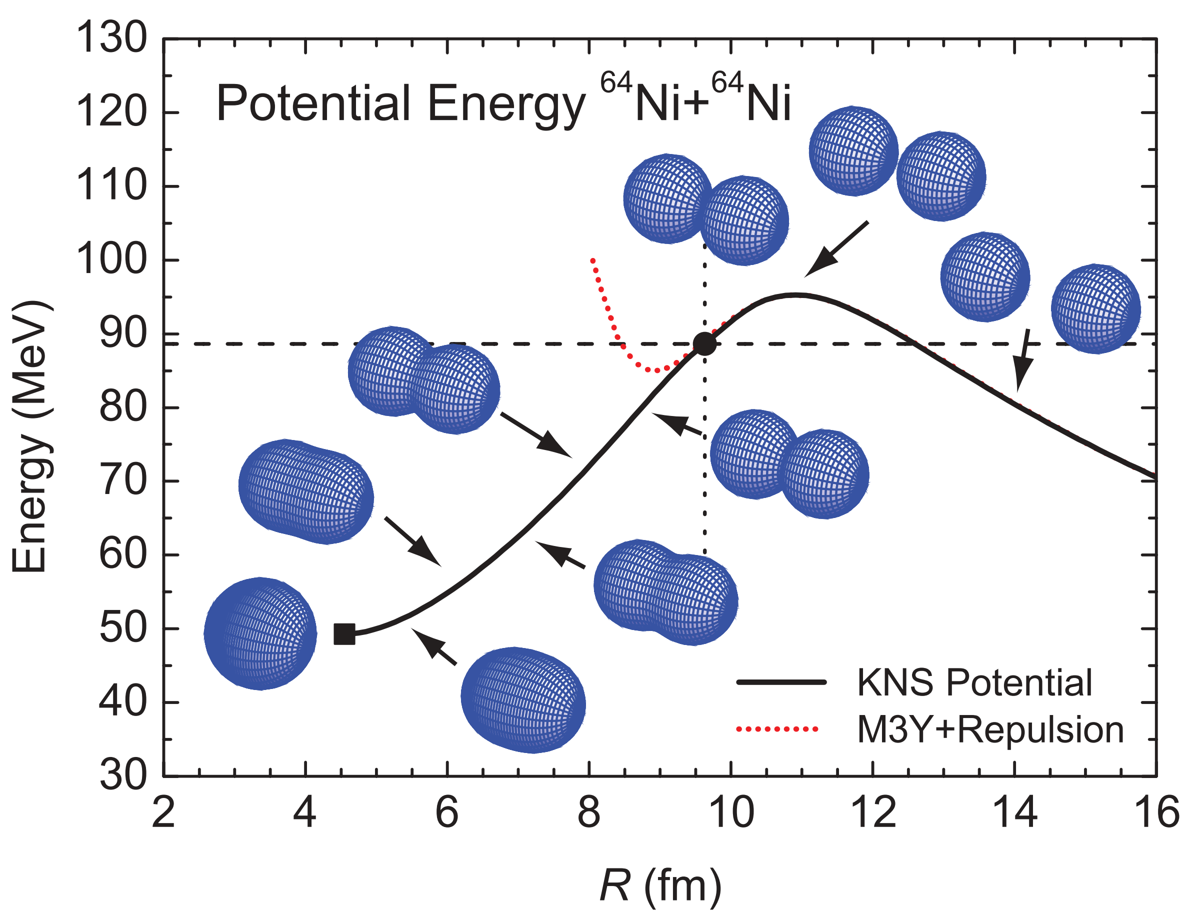
  →Coupling to thermal bath

  • Dasgupta *et al.* and Diaz-Torres



Adiabatic potential energy

- Assuming that neck formations between colliding two nuclei occur after the touching, we smoothy joint between the two and one body potential energies
  - describe the one-body shapes by the Lemniscatoid parametrization



- **Total Potential Energy**

$$E(r) = E_V + E_C(r) + E_N(r)$$

- **Yukawa-plus-Exponential(YPE) Model**

$$E_N = -\frac{c_s}{8\pi^2 r_0 a^3}\iint\left(\frac{\sigma}{a}-2\right)\frac{e^{-\sigma/a}}{\sigma}d^3r d^3r'$$

$$c_s = a_s(1-\kappa_s I^2) \quad I = (N-Z)/A$$

Problems in coupling potential

the calculated adiabatic potential

○ ×

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2 J(J+1)}{2\mu r^2}+V(r)+\epsilon_n-E\right]u_n(r)+\boxed{\sum_n \left\langle\phi_n\middle|V_{\mathrm{coup}}\middle|\phi_n\right\rangle}u_n(r)=0$$

How should we calculate the coupling potential around overlap region?

■ Double counting of CC effects

- Adiabatic one-body potential with neck formations already includes a large part of the channel coupling effects

We need an extension of the standard coupled-channel equation

Coupling potential（Collective model）

- **Calculation of** $\langle \phi_n | V_{\mathrm{coup}} | \phi_{n'} \rangle$

$$R(\theta,\phi) = R_0\left(1 + \sum_{\mu} \alpha_{\lambda\mu} Y^{*}_{\lambda\mu}(\theta,\phi)\right)$$

- **Intrinsic eigenstate** $\hat{O}|\alpha\rangle = \lambda_\alpha |\alpha\rangle$

$$R_0 \rightarrow R_0 + \hat{O} = R_0 + \beta_2 R_0 Y_{20} + \beta_3 R_0 Y_{30}$$

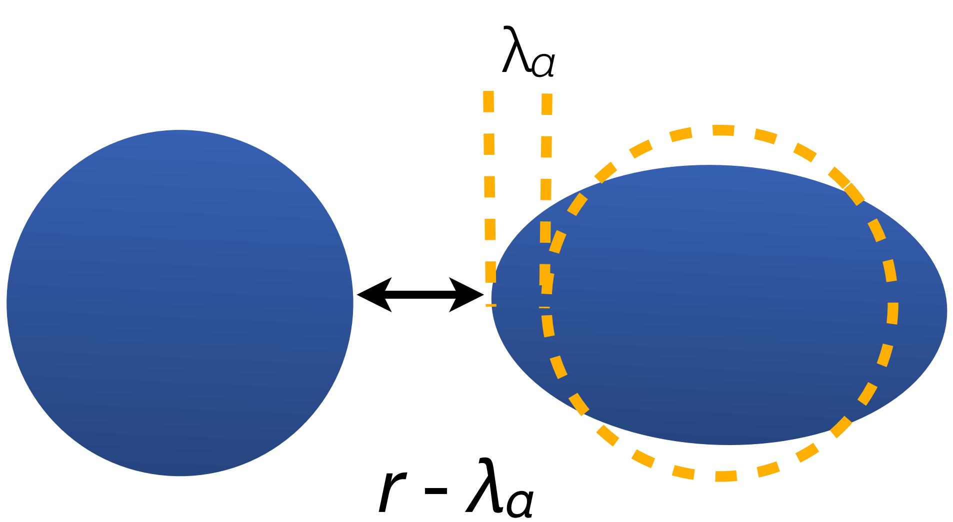
$$\hat{h}_0 = \hbar\omega_0 \sum_{\mu} a^{\dagger}_{\lambda\mu} \alpha_{\lambda\mu} \qquad \alpha_{\lambda\mu} = \frac{\beta_\lambda}{\sqrt{2\lambda+1}}\left(a^{\dagger}_{\lambda\mu} + (-)^{\mu} \alpha_{\lambda\mu}\right)$$

$$\boxed{\beta_\lambda = \frac{4\pi}{3zR^{\lambda}}\sqrt{\frac{B(E\lambda)}{e^2}}}$$

input: $B(E\lambda)$

$\beta_\lambda \rightleftarrows B(E\lambda)$

- **Coupling potential** $V_{\mathrm{coup}}(r,\hat{O}) = V^{(\mathrm{N})}_{\mathrm{coup}}(r,\hat{O}) + V^{(\mathrm{C})}_{\mathrm{coup}}(r,\hat{O})$



$$V^{(\mathrm{N})}_{nm} = \langle I0 | V_N(r,\hat{O}) | I'0 \rangle - V^{(0)}_N(r)\delta_{nm}$$

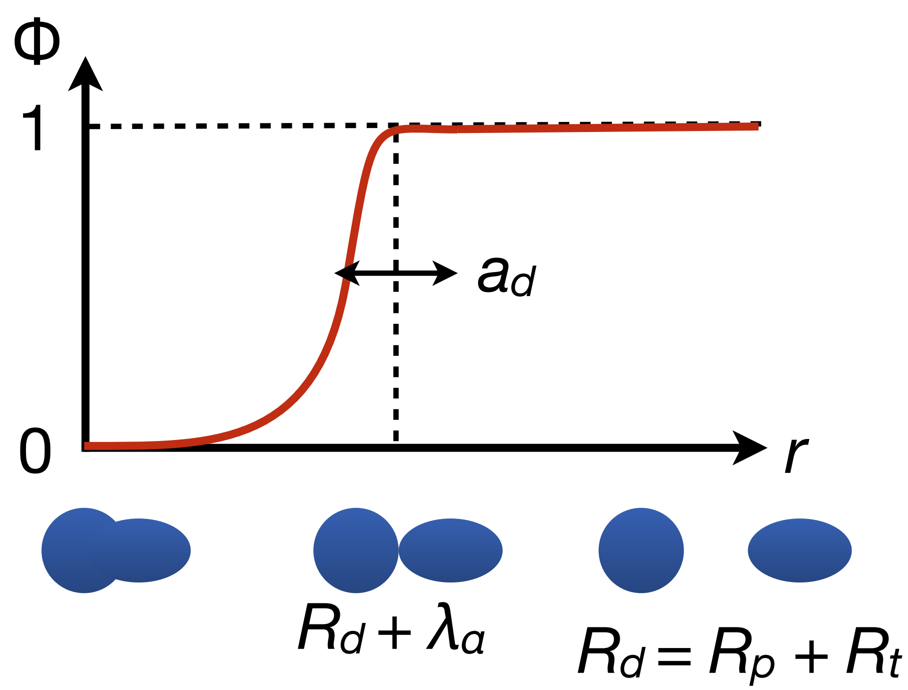
$$= \sum_{\alpha} \langle I0|\alpha\rangle\langle\alpha|I'0\rangle \boxed{V_N(r,\lambda_\alpha)} - V^{(0)}_N(r)\delta_{nm}$$

$$\boxed{V_N(r,\lambda_\alpha) \approx V^{(0)}_N(r) - \frac{dV^{(0)}_N(r)}{dr}\lambda_\alpha + \frac{1}{2}\frac{d^2V^{(0)}_N(r)}{dr^2}\lambda_\alpha^2}$$

Extension of coupled-channel model

- **Damping factor**

TI, K. Hagino, and A. Iwamoto, Phys. Rev. Lett. **103**, 202701 (2009)



$$\Phi(r,\lambda_\alpha) = \begin{cases} 1 & (r \geq R_d + \lambda_\alpha) \\ e^{-(r-R_d-\lambda_\alpha)^2/2a_d^2} & (r < R_d + \lambda_\alpha) \end{cases}$$

$$R_d = r_d(A_T^{1/3} + A_P^{1/3})$$ a_d: Damping factor

$$\boxed{V_N(r,\lambda_\alpha) \sim V_N^{(0)}(r) + \left[-\frac{dV_N^{(0)}(r)}{dr}\lambda_\alpha + \frac{1}{2}\frac{d^2V_N^{(0)}(r)}{dr^2}\lambda_\alpha^2 \right] \Phi(r,\lambda_\alpha)}$$

Two body

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 J(J+1)}{2\mu r^2} + V(r) + \epsilon_n - E \right] u_n(r) + \sum_n \left\langle \phi_n \middle| V_{\rm coup} \middle| \phi_n \right\rangle u_n(r) = 0$$

↓

Touching point

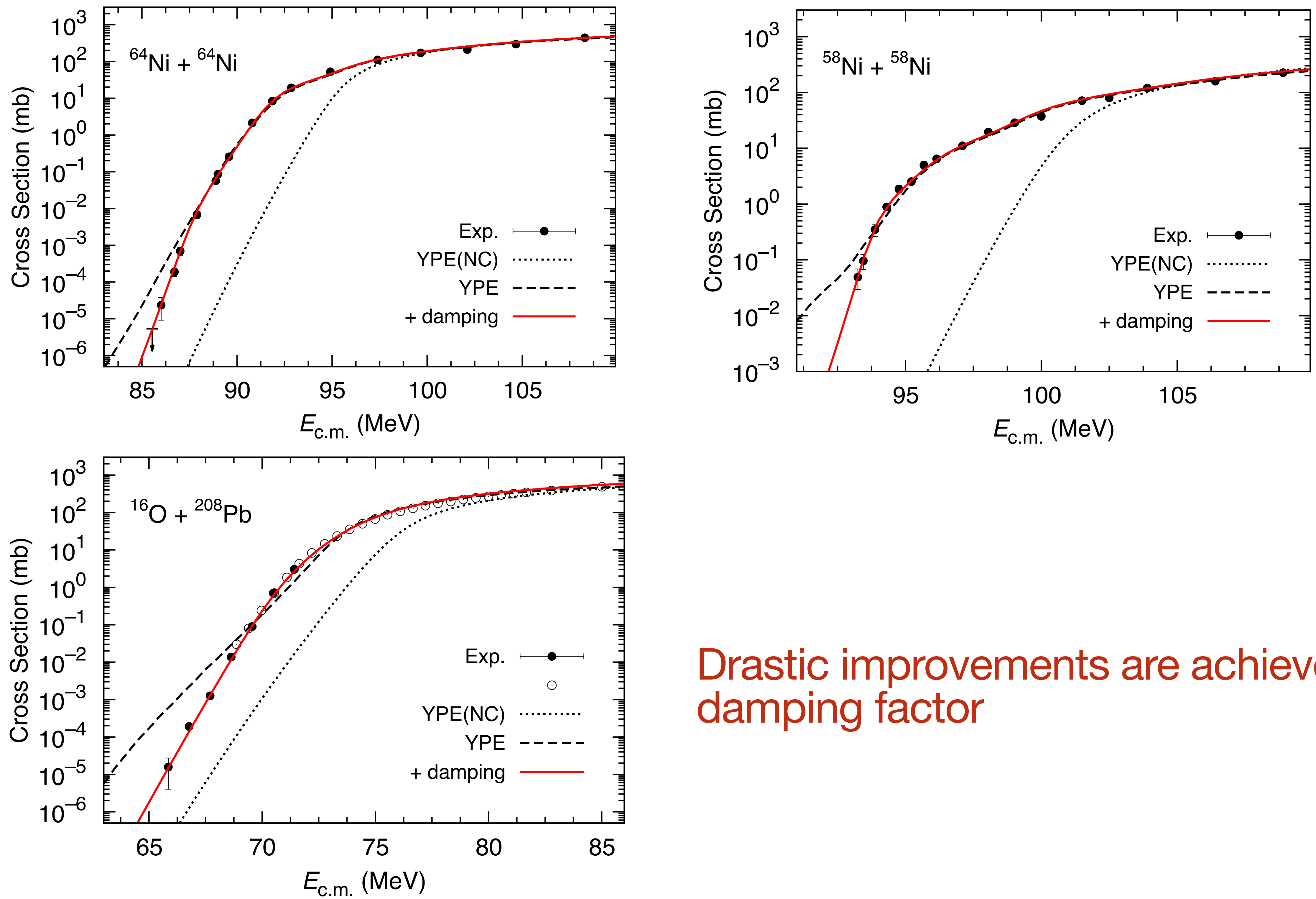
$$V_{nm}^{(N)} = \left\langle I0 \middle| V_N(r,\hat{O}) \middle| I'0 \right\rangle - V_N^{(0)}(r)\delta_{nm} \to 0$$

↓

One body

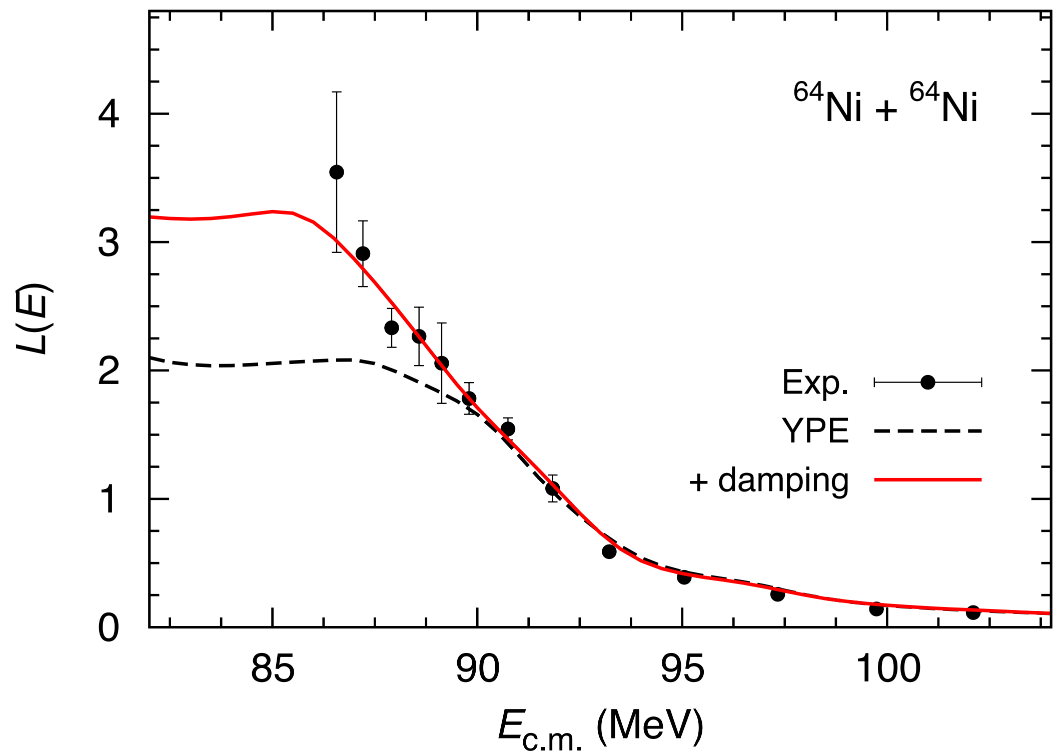
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 J(J+1)}{2\mu r^2} + V(r) + \epsilon_n - E \right] u_n(r) = 0$$

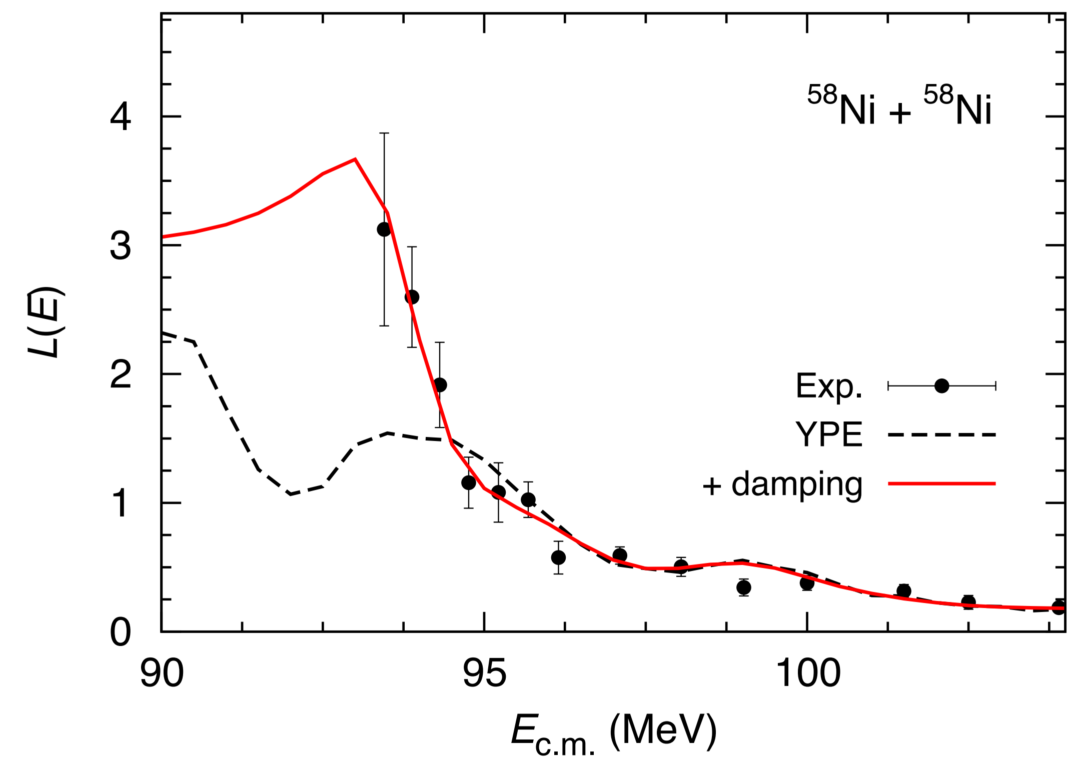
Calculated results: fusion cross section

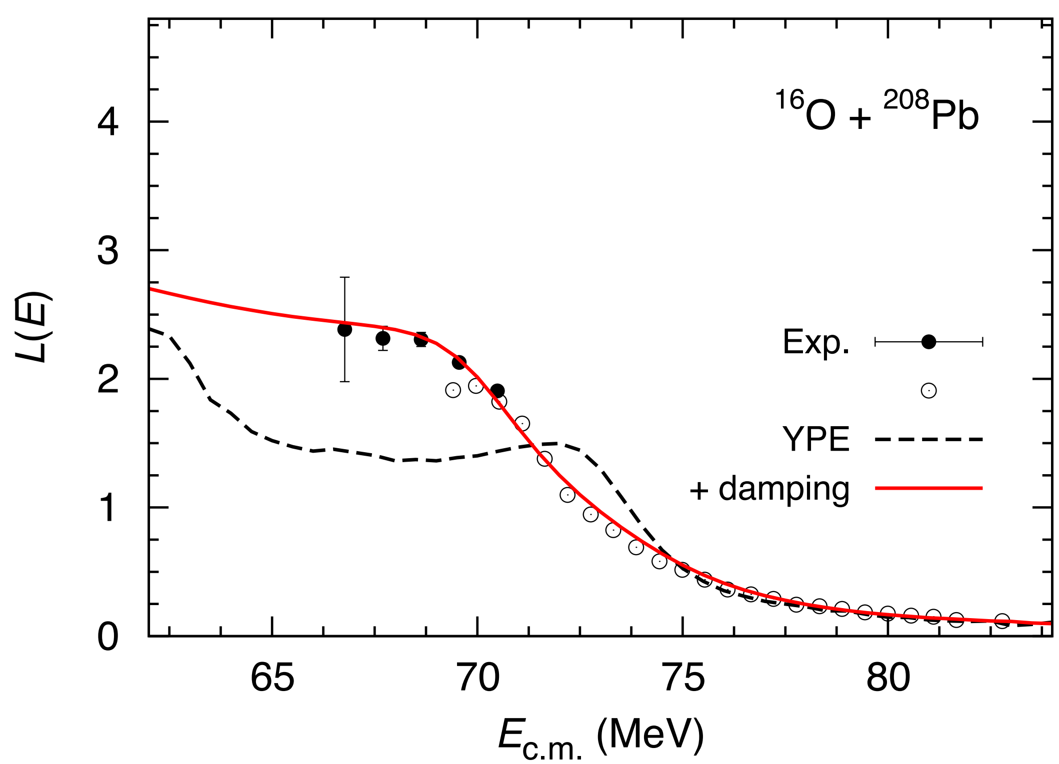


Drastic improvements are achieved by damping factor

First derivative of fusion cross section

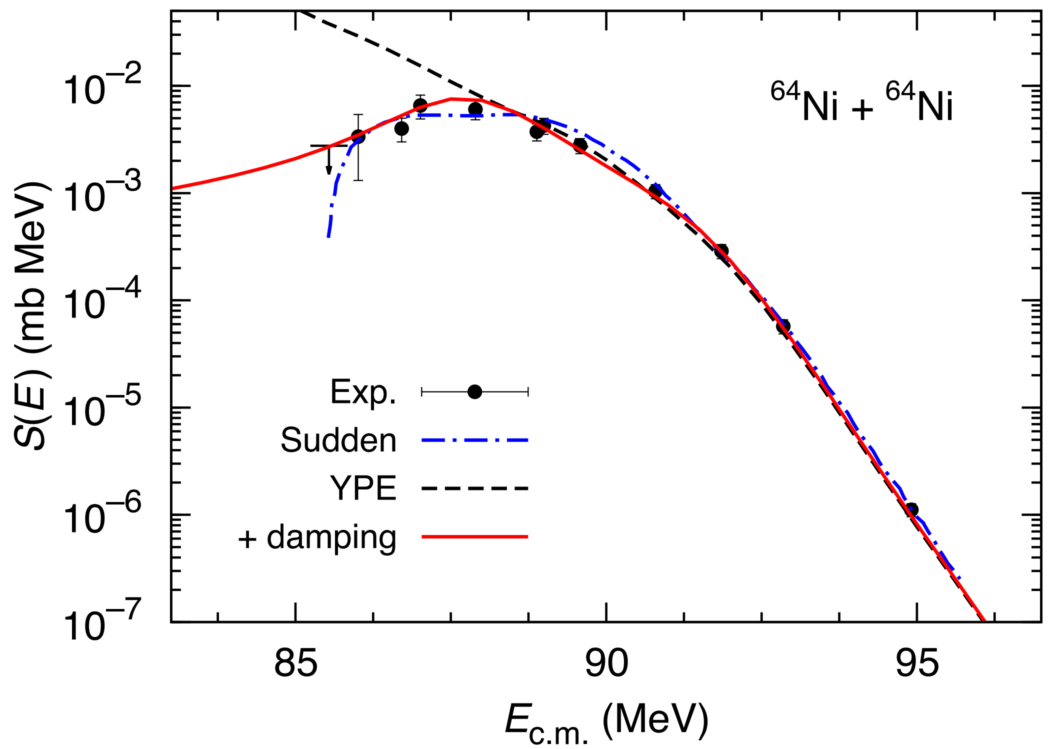


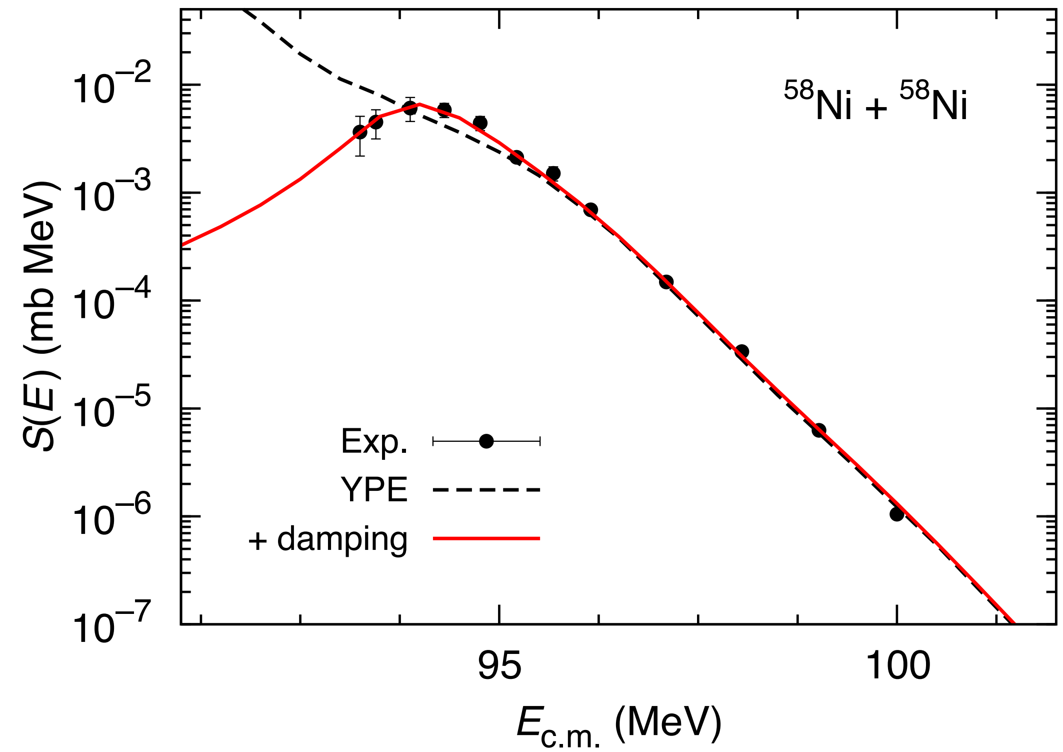


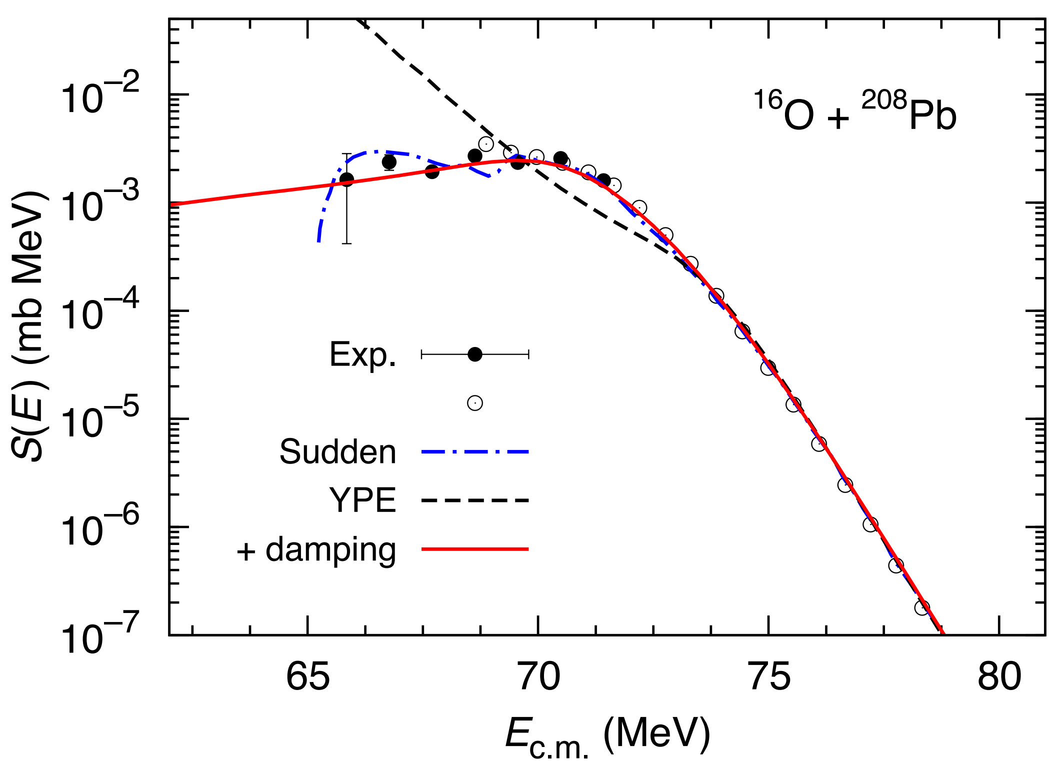


Reproduce the saturation at extremely low incident energies

Astrophysical S-factor







Differs considerably from sudden model

Motivation

- **What is the microscopic origin of the damping factor phenomenologically introduced?**
  - Coupling potential varnishes around the touching of colliding two nuclei
  - Coupling potential strongly correlates with the transition strength

    → Transitions between channels decrease due to the damping of the vibrational excitation?

Check the transition strengths *B*(*E*2 or *E*3) of target and projectile when colliding two nuclei approach each other

To investigate quantum-mechanical vibrational spectrum, we for the first time apply the random-phase approximation (RPA) method to the two-body system

Mean-field potential

- **Total Hamiltonian**

expanded by deformed harmonic-oscillator basis

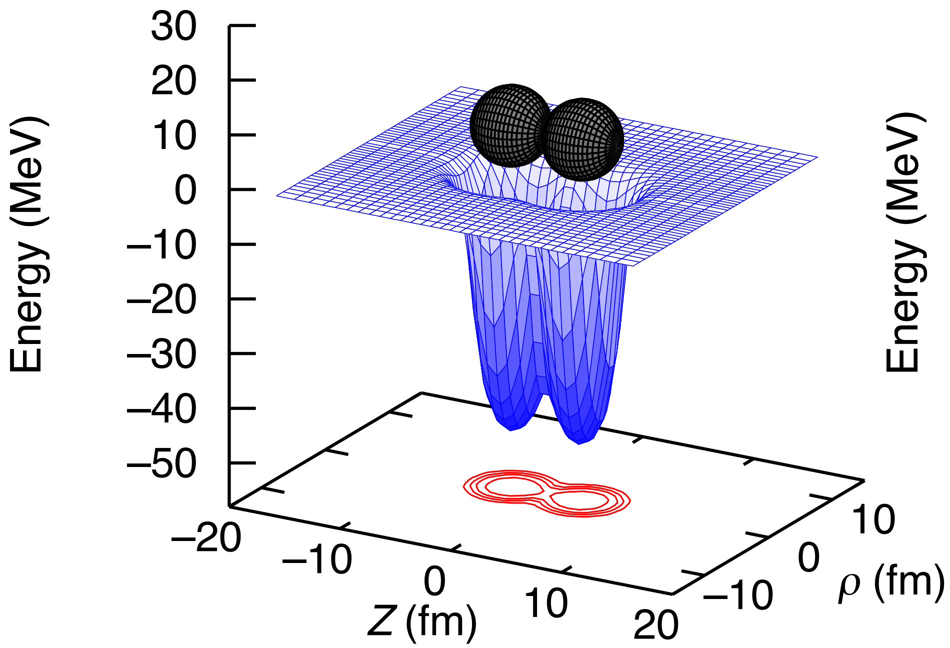
$$H = -\frac{\hbar^2}{2m}\Delta + V_N(\vec{r}) + V_{\mathrm{S.O.}}(\vec{r}) + V_C(\vec{r})(1-\tau_3)/2$$

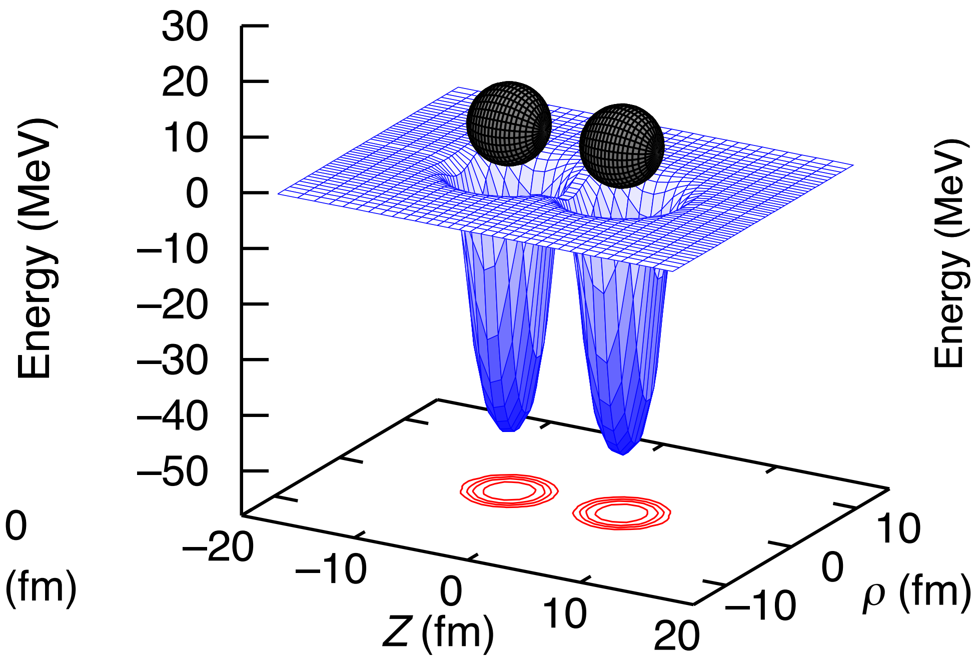
$$V_{\mathrm{S.O.}} = -\lambda\left(\frac{\hbar}{2m_{\mathrm{nuc}}c}\right)^2 \frac{\vec{\sigma}\cdot\nabla V\times\vec{p}}{\hbar}$$

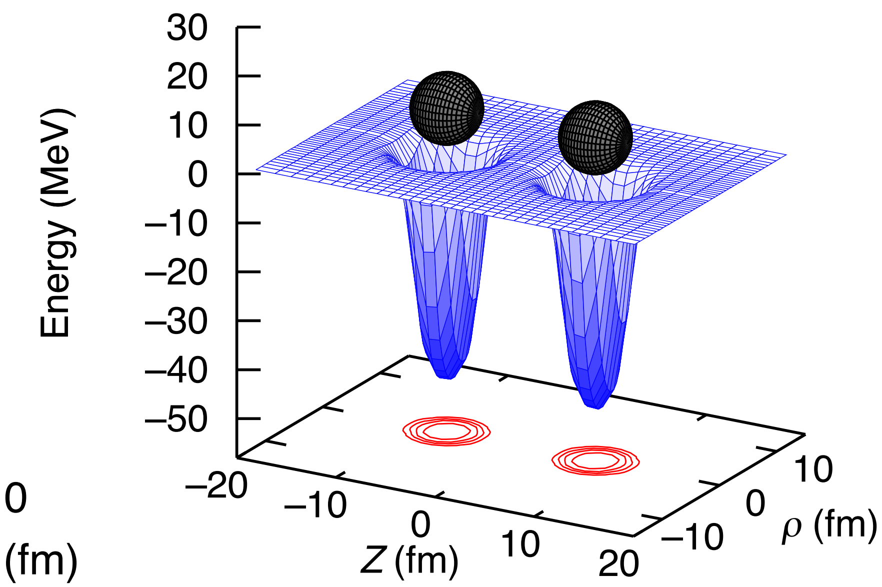
- **Folded Yukawa potential**

One-body shape
→Lemniscatoid parametrization

$$V_{\mathrm{N}}(\vec{r}) = -\frac{V_0}{4\pi a_{\mathrm{pot}}}\int_V \frac{e^{-\left|\vec{r}-\vec{r'}\right|/a_{\mathrm{pot}}}}{\left|\vec{r}-\vec{r'}\right|/a_{\mathrm{pot}}}d\vec{r'}$$







Random-phase approximation (RPA) method

- Since we describe the two-body system by a Slater determinant, it is easy to apply the RPA method to the two-body system

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - Y_{mi}^\nu a_i^\dagger a_m \qquad Q_\nu|\mathrm{RPA}\rangle = 0$$

$$\langle \mathrm{RPA}| \left[a_i^\dagger a_m, [H, Q_\nu^\dagger]\right] |\mathrm{RPA}\rangle = \hbar\Omega_\nu \langle \mathrm{RPA}| \left[a_i^\dagger a_m, Q_\nu^\dagger\right] |\mathrm{RPA}\rangle$$

$$\langle \mathrm{RPA}| \left[a_m^\dagger a_i, [H, Q_\nu^\dagger]\right] |\mathrm{RPA}\rangle = \hbar\Omega_\nu \langle \mathrm{RPA}| \left[a_m^\dagger a_i, Q_\nu^\dagger\right] |\mathrm{RPA}\rangle$$

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\Omega_\nu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$A_{minj} = \langle \mathrm{RPA}| \left[a_m^\dagger a_i, [H, a_n^\dagger a_j]\right] |\mathrm{RPA}\rangle = (\epsilon_m - \epsilon_n)\delta_{mn}\delta_{ij} + \bar{v}_{mjin}$$

$$B_{minj} = -\langle \mathrm{RPA}| \left[a_i^\dagger a_m, [H, a_j^\dagger a_n]\right] |\mathrm{RPA}\rangle = \bar{v}_{mnij}$$

Residual interaction

■ Density-dependent δ type residual interaction

(Shlomo-Bertsch)

- neutron-neutron, proton-proton

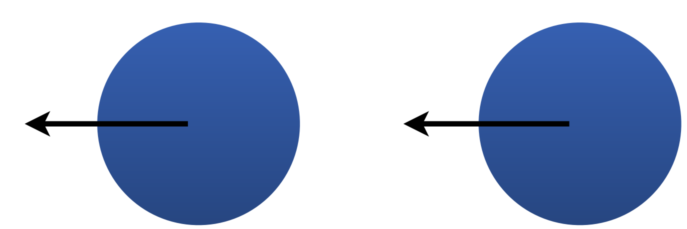
$$v_{ph}(\mathbf{r}_1, \mathbf{r}_2) = \left[\frac{t_0}{2}(1 - x_0) + \frac{t_3}{12}(4 - x_3)\rho(\mathbf{r}_1)\right]\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

- neutron-proton

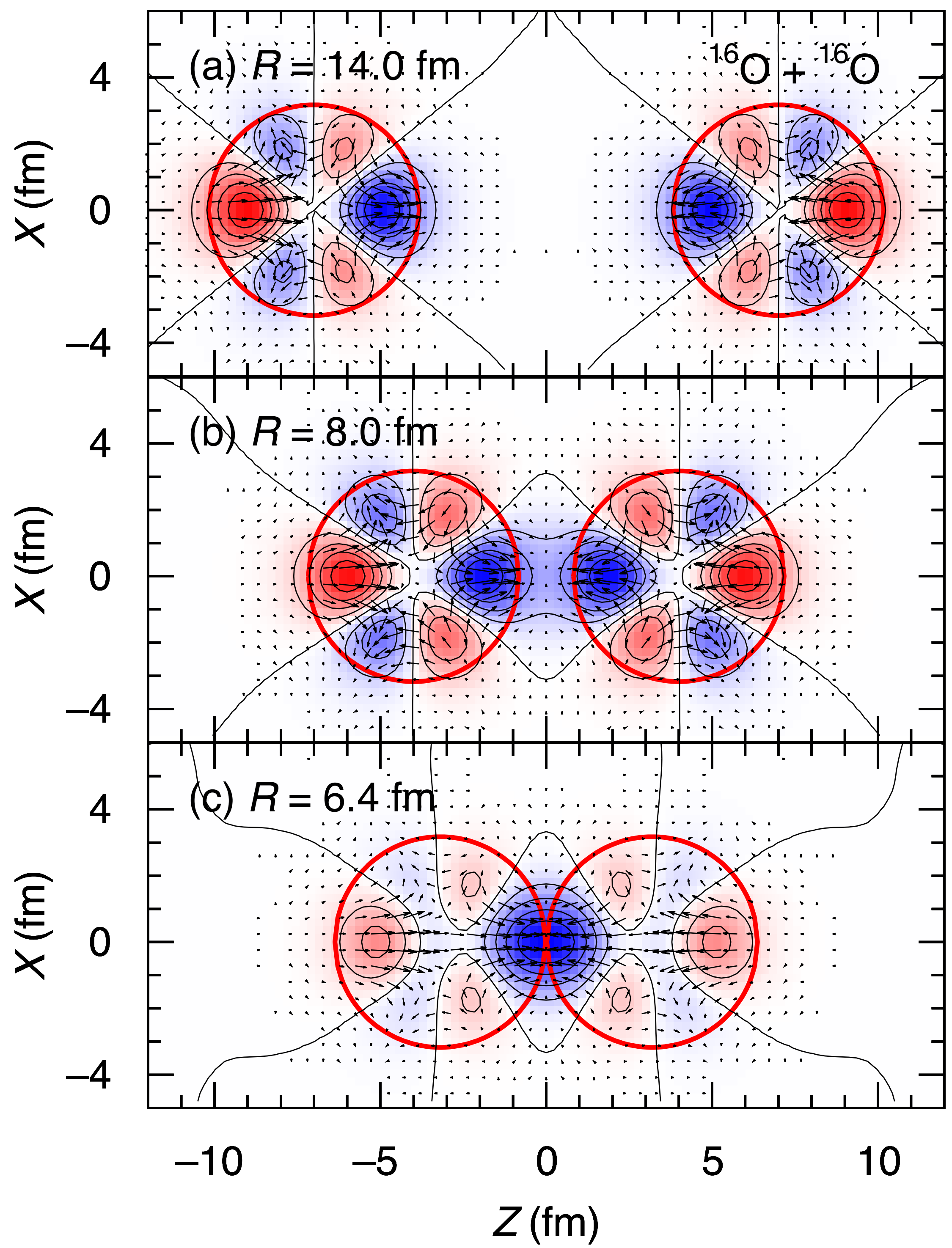
$$v_{ph}(\mathbf{r}_1, \mathbf{r}_2) = \left[t_0(1 + \frac{x_0}{2}) + \frac{t_3}{12}(5 + x_3)\rho(\mathbf{r}_1)\right]\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

t_0 = -1100 MeV fm^3, t_3 = 16000 MeV fm^6, x_0 = 0.5, x_3 = 1.0

Fine-tune the strength of the residual interaction so that the eigen energy of $K = 0^-$ mode (center-of-mass motion) becomes zero



Transition density and current



The first 3^- excited state of the RPA solution with $K = 0^+$

- **Transition density**

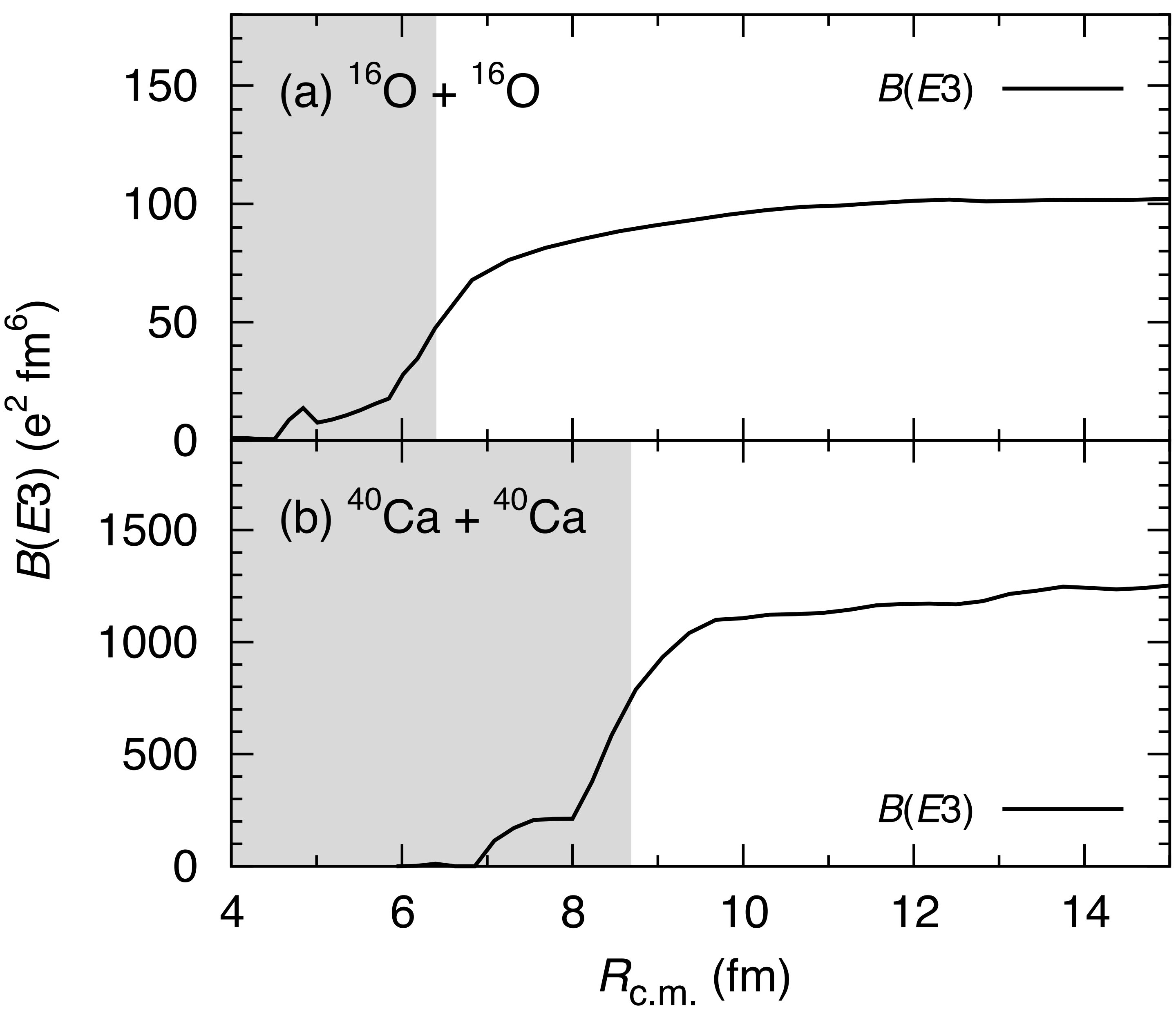
$$\rho^{\nu}(\mathbf{r}) = -\frac{i}{\hbar}\sqrt{\frac{\hbar}{2M_{\nu}\Omega_{\nu}}}\langle 0|\,[\hat{\rho}^{\nu}(\mathbf{r}), P_{\nu}]\,|0\rangle$$

- **Transition current**

$$\mathbf{j}^{\nu}(\mathbf{r}) = \sqrt{\frac{M_{\nu}\Omega_{\nu}}{2\hbar}}\langle 0|\left[\hat{\mathbf{j}}^{\nu}(\mathbf{r}), Q_{\nu}\right]|0\rangle$$

Amplitude of the vibrational excitation becomes small around the touching point

$B(E3)$ strength of the right-sided nucleus



$$\langle R, 3^- | \hat{Y}_{30}^{(R)} | R, 0^+ \rangle$$

$$= \frac{1}{\sqrt{2}} \left(\langle 3^-, + | \hat{Y}_{30}^{(R)} | \phi_0 \rangle + \langle 3^-, - | \hat{Y}_{30}^{(R)} | \phi_0 \rangle \right)$$

$$\hat{Y}_{30}^{(R)} = \hat{Y}_{30}(r - R)$$

cf.

$$\left| \Psi^+ \right\rangle = \frac{1}{\sqrt{2}} \left(|R\rangle + |L\rangle \right)$$

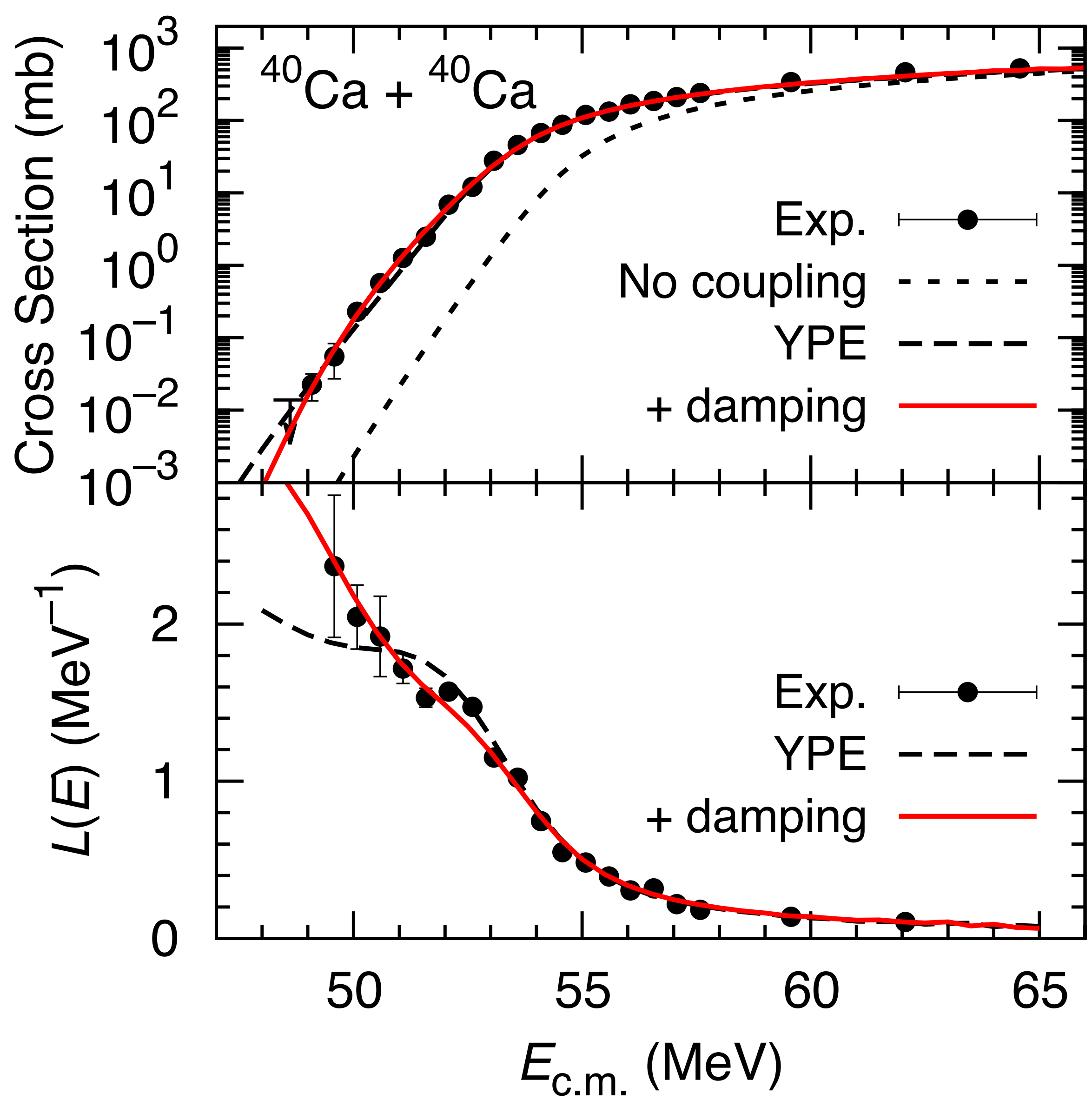
$$\left| \Psi^- \right\rangle = \frac{1}{\sqrt{2}} \left(|R\rangle - |L\rangle \right)$$

$$\rightarrow \quad |R\rangle = \frac{1}{\sqrt{2}} \left(\left| \Psi^- \right\rangle + \left| \Psi^+ \right\rangle \right)$$

$B(E3)$ considerably decreases around the touching point

Coupled-channel calculation with damping factor

■ Check correlation between the calculated $B(E3)$ and the damping factor which well reproduce the experimental data of fusion cross sections



low-lying 3^- and 2^+
single-phonon, all mutual excitations

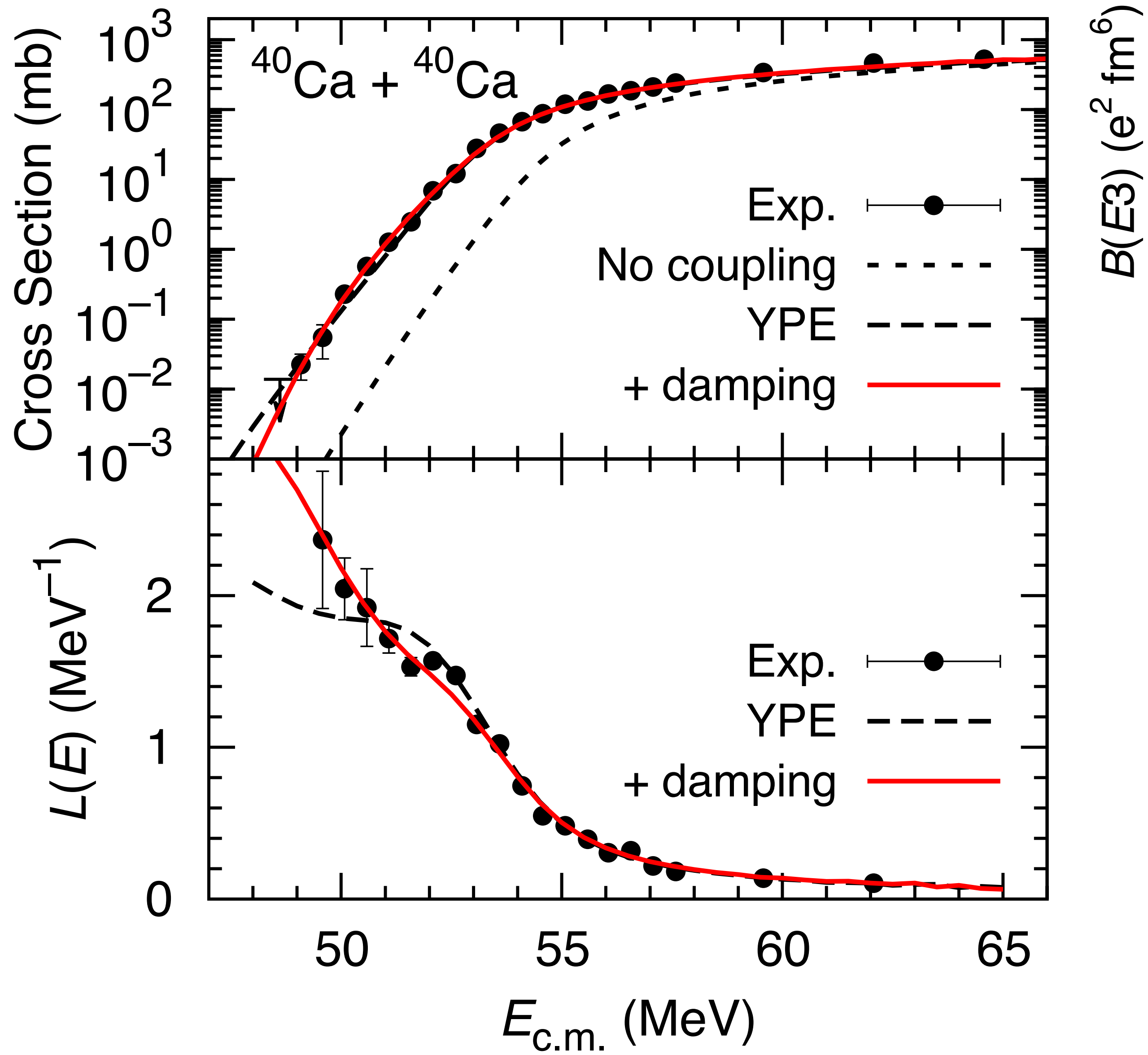
3^-: 3.737 MeV, $\beta = 0.401$, $\beta_c = 0.401$
2^-: 3.905 MeV, $\beta = 0.112$, $\beta_c = 0.112$

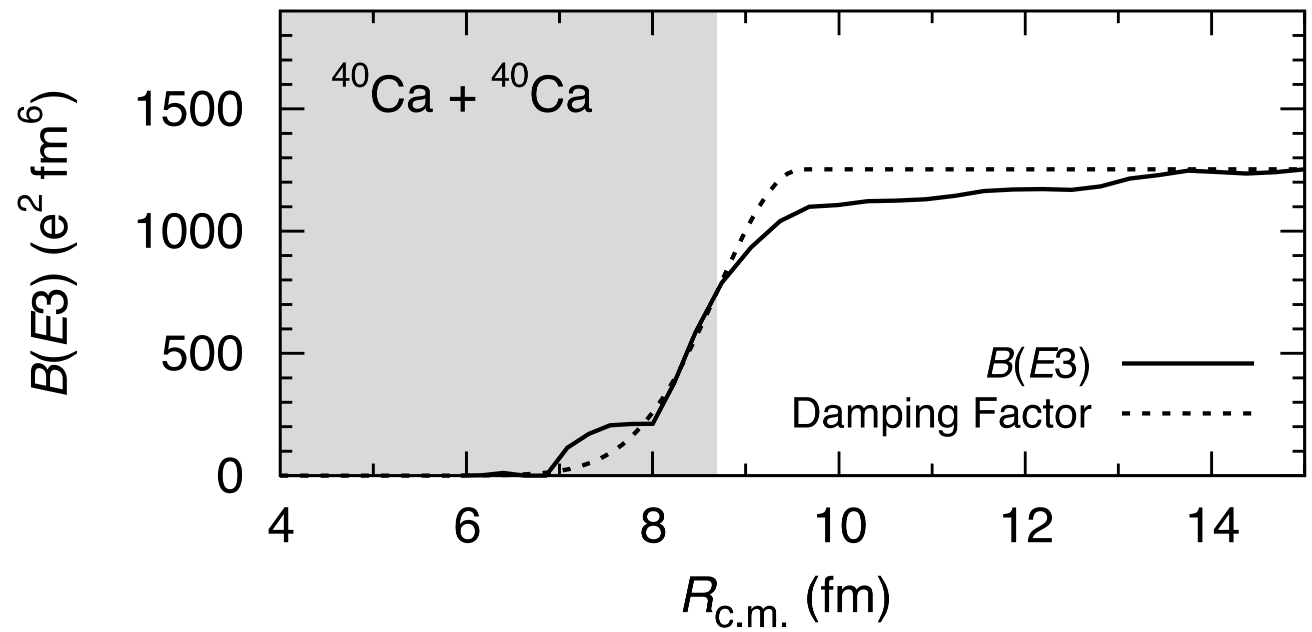
YPE parameters
$r_0 = 1.191$ fm, $a = 0.6525$ fm

$$\Phi(r, \lambda_\alpha) = \begin{cases} 1 & (r \geq R_d) \\ e^{-\frac{(r-R_d)^2}{2a_d^2}} & (r \leq R_d) \end{cases}$$

$R_d = 9.6$ fm, $a_d = 0.9$ fm

Correlation between $B(E3)$ and damping factor





$$\Phi(r,\lambda_\alpha) = \begin{cases} 1 & (r \geq R_d) \\ e^{-\frac{(r-R_d)^2}{2a_d^2}} & (r \leq R_d) \end{cases}$$

$R_d = 9.6$ fm, $a_d = 0.9$ fm

The damping factor simulates the damping of the quantum vibrations when colliding nuclei approach each other

Summary

- We, for first time, apply the RPA method to the two-body $^{16}O+^{16}O$ and $^{40}Ca+^{40}Ca$ systems and calculate the vibrational excitation when two colliding nuclei approach each other
- The transition strength *B*(*E*3) largely decreases when colliding two nuclei approach each other due to the change of their wave functions and each 3^- excitation mode vanishes
- The large reduction of *B(E3)* around the touching point strongly correlates with the damping factor which reproduces well the experimental fusion cross section
- The vanishing of the coupling between the relative and the intrinsic degree of freedoms is responsible for the fusion hindrance in deep sub-barrier reactions

TI, K. Hagino, and A. Iwamoto, Phys. Rev. C **75**, 064612 (2007)
TI, K. Hagino, and A. Iwamoto, Phys. Rev. Lett. **103**, 202701 (2009)
TI and K. Matsuyanagi, Phys. Rev. C**88**, 011602(R) (2013)