

# Heavy-ion fusion and quasi-elastic scattering around the Coulomb barrier

Kouichi Hagino, *Tohoku University*  
Neil Rowley, *IPN Orsay*



TOHOKU  
UNIVERSITY

## 1. *Introduction*

*: fusion and quasi-elastic barrier distributions*

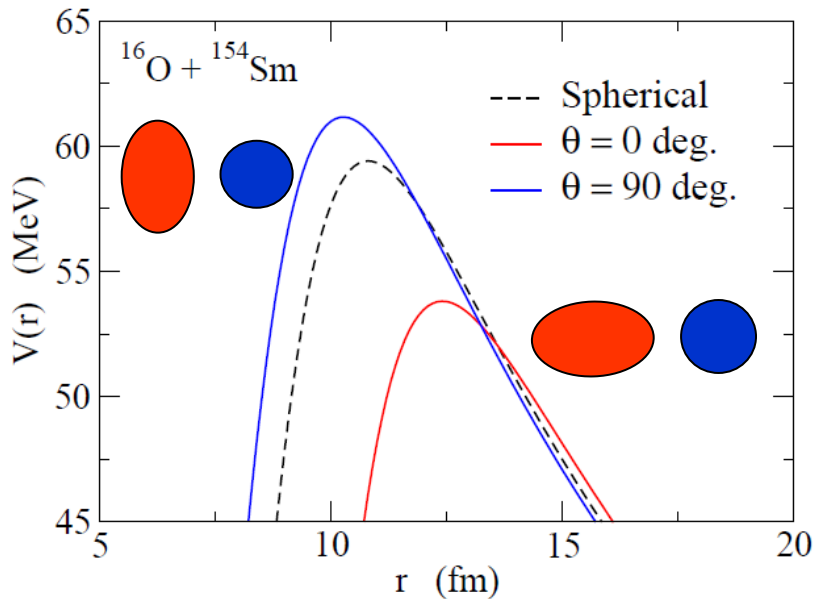
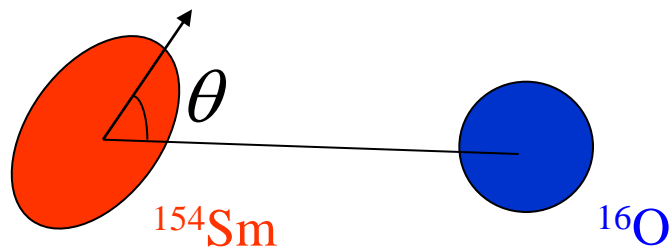
## 2. *Sum-of-differences (SOD) method*



## 3. *Summary*

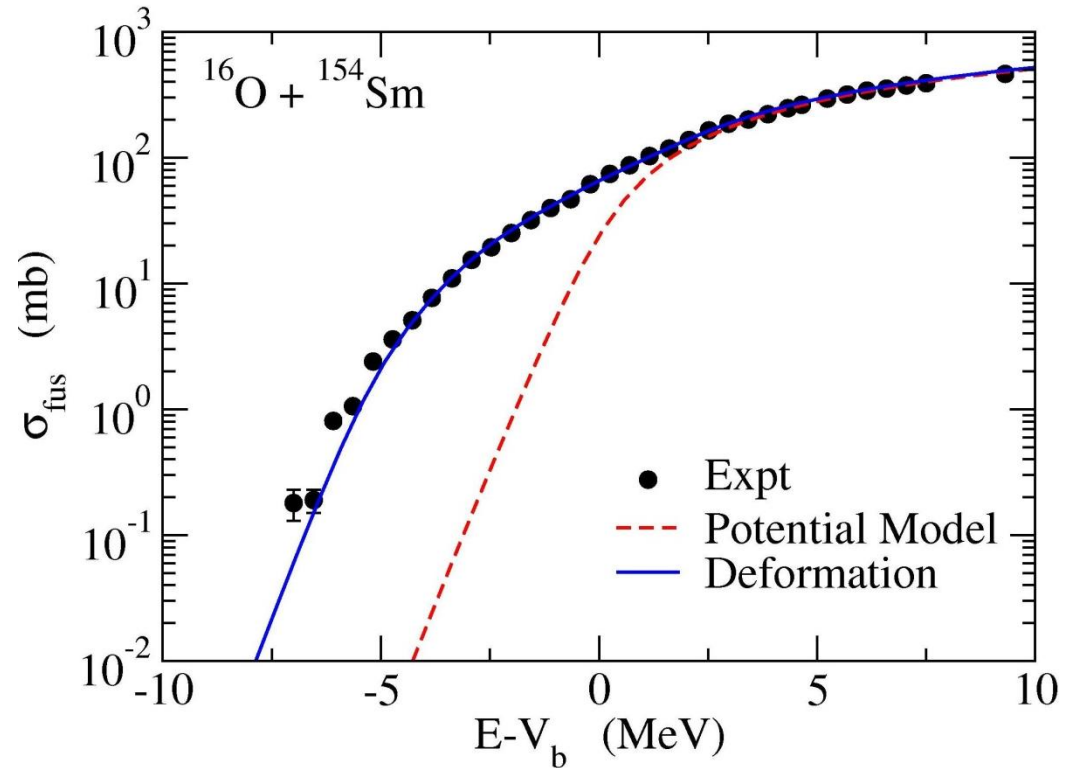
# Introduction

**H.I. Sub-barrier fusion:**  
strong interplay between  
reaction and structure



coupled-channels equations

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

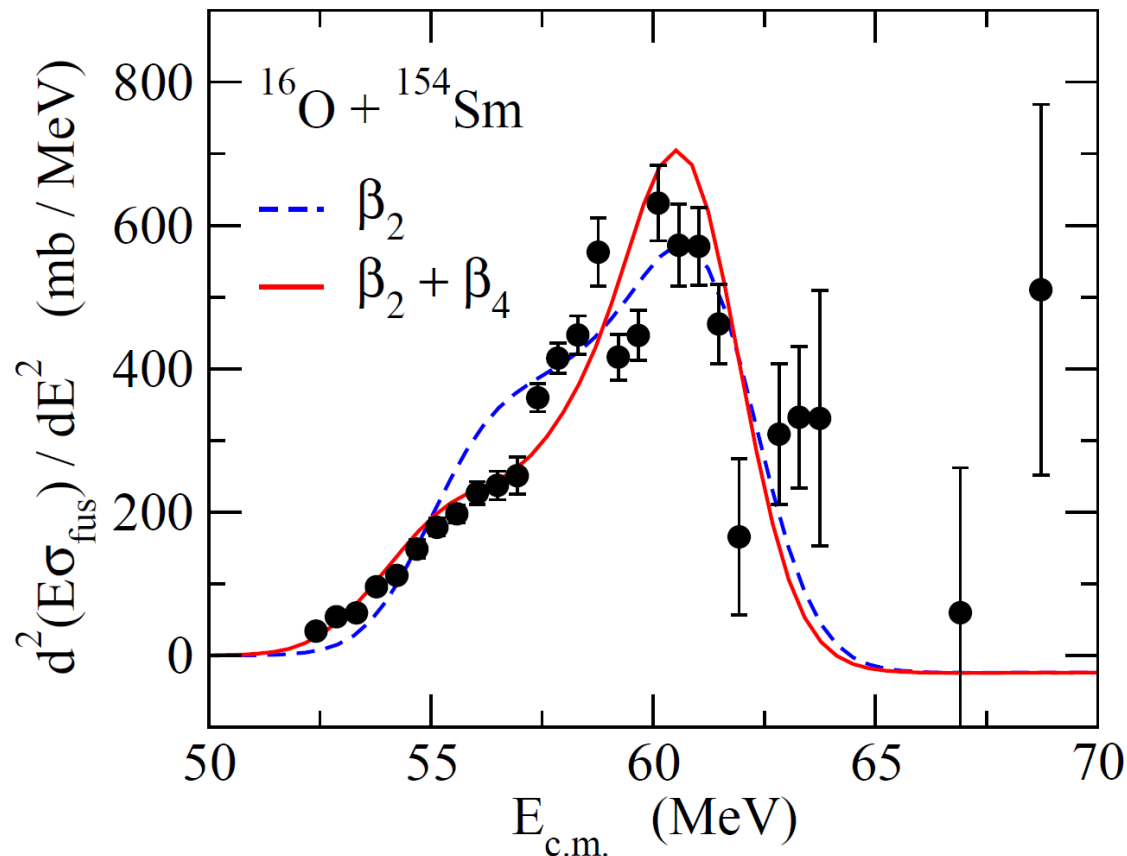


**Def. Effect:** enhances  $\sigma_{\text{fus}}$  by a factor  
of 10 ~ 100

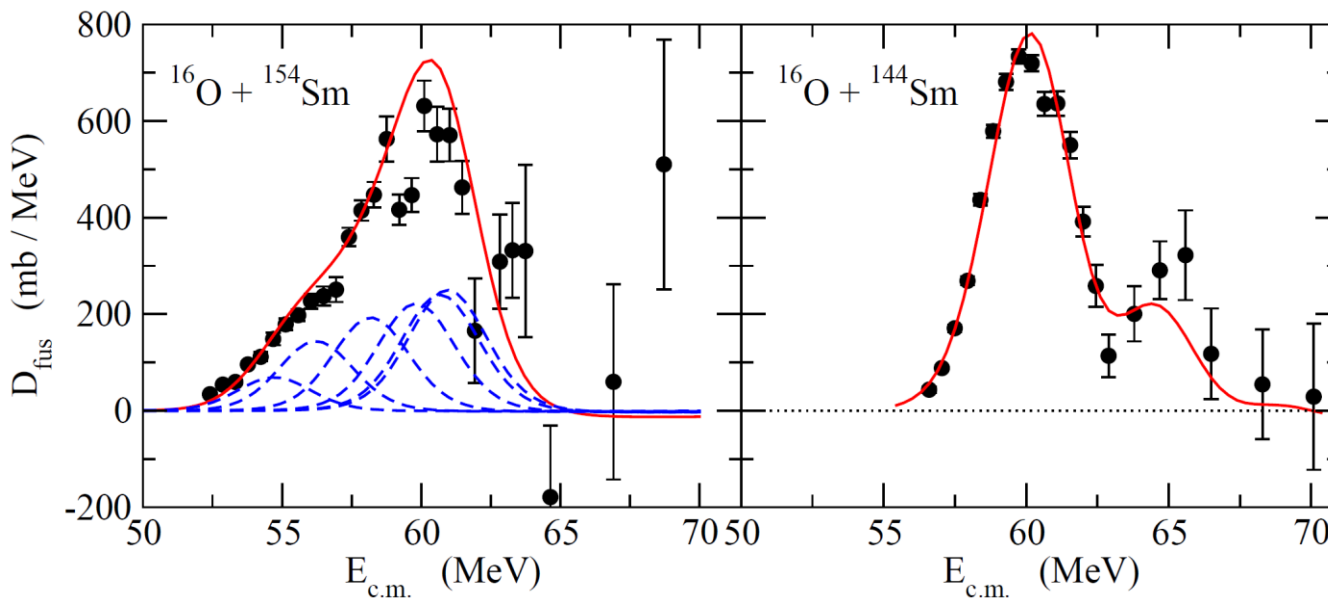
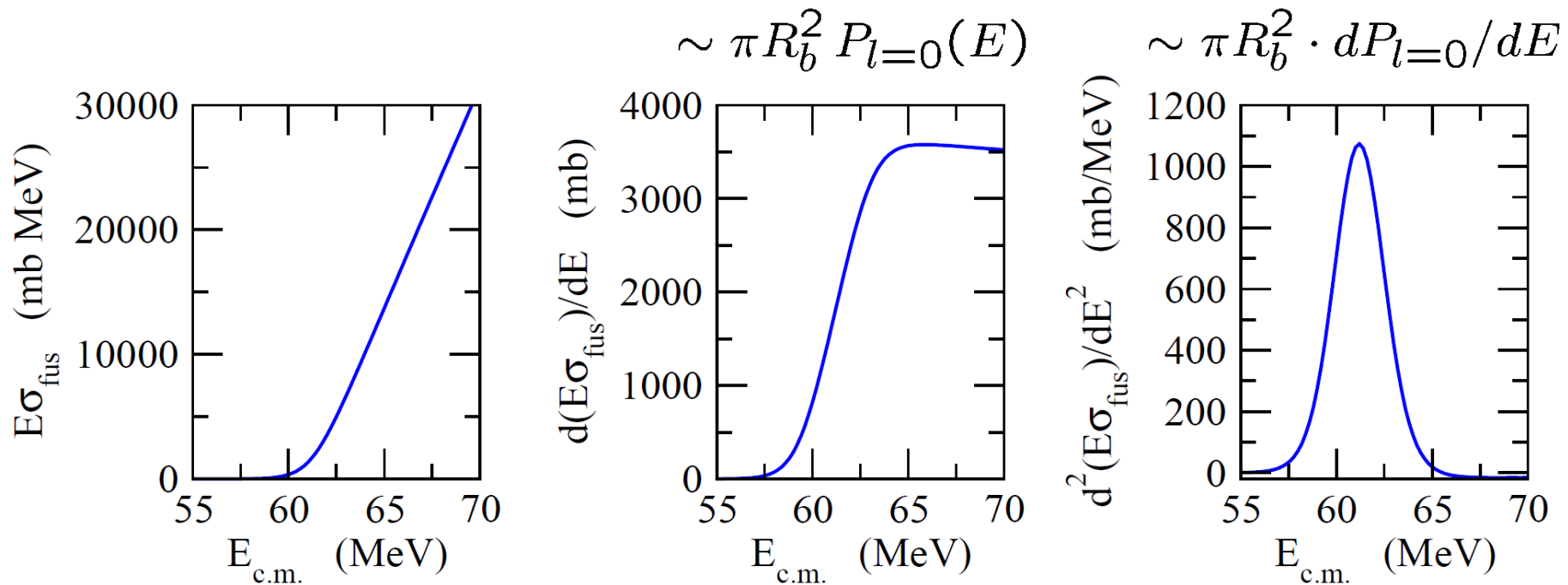
## Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254('91) 25  
J.X. Wei, J.R. Leigh et al., PRL67('91) 3368



M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98)401



## Quasi-elastic barrier distribution

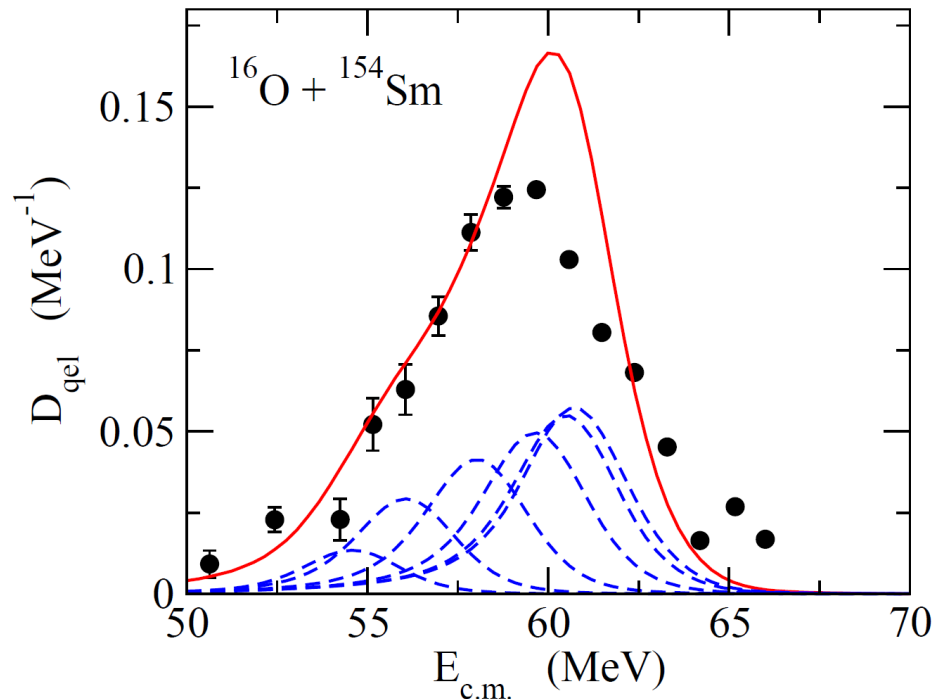
$$D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)} \right)$$

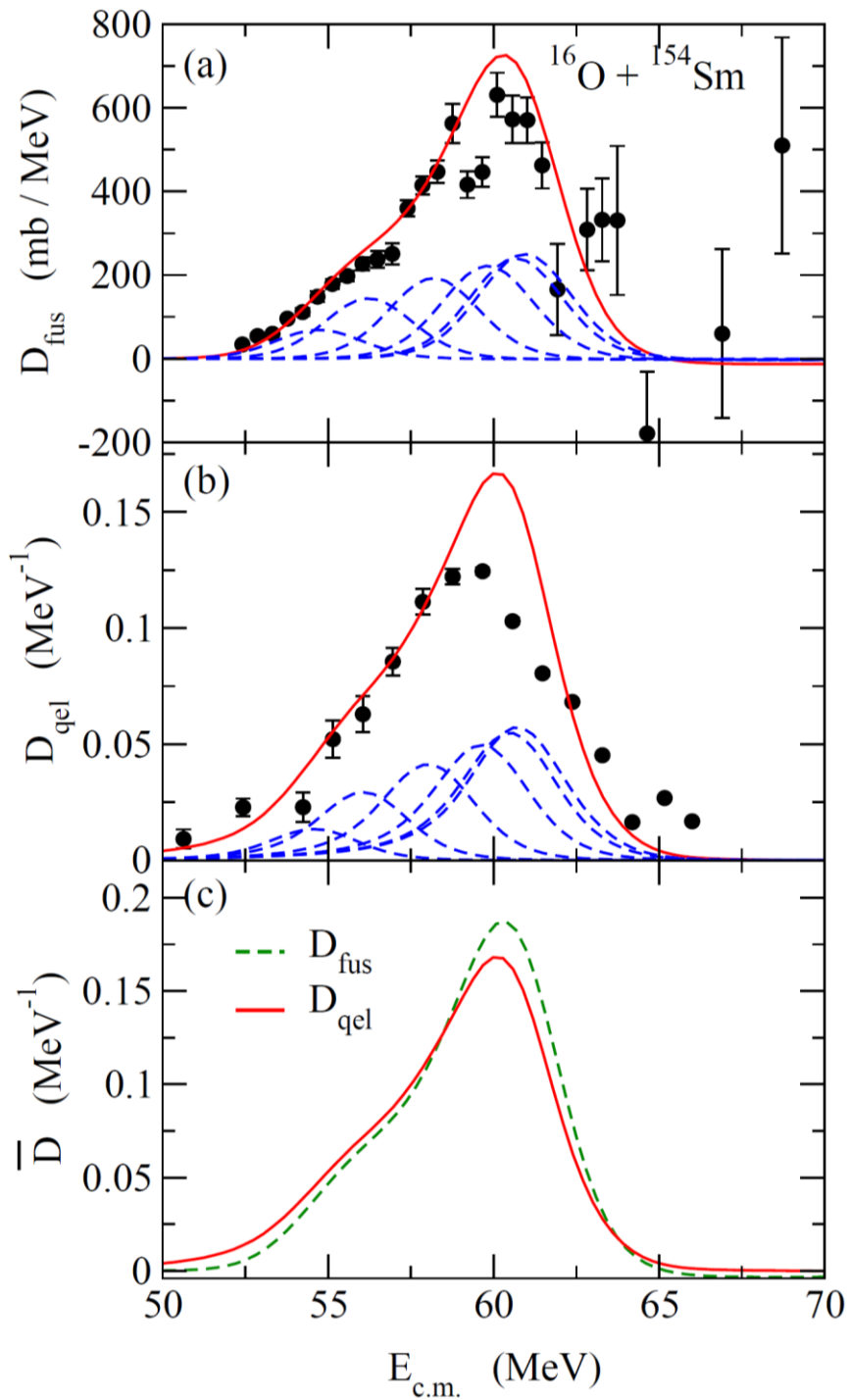
H. Timmers et al., NPA584('95)190

## Quasi-elastic scattering:

A sum of all the reaction processes other than fusion  
(elastic + inelastic + transfer + .....)

$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)}$$





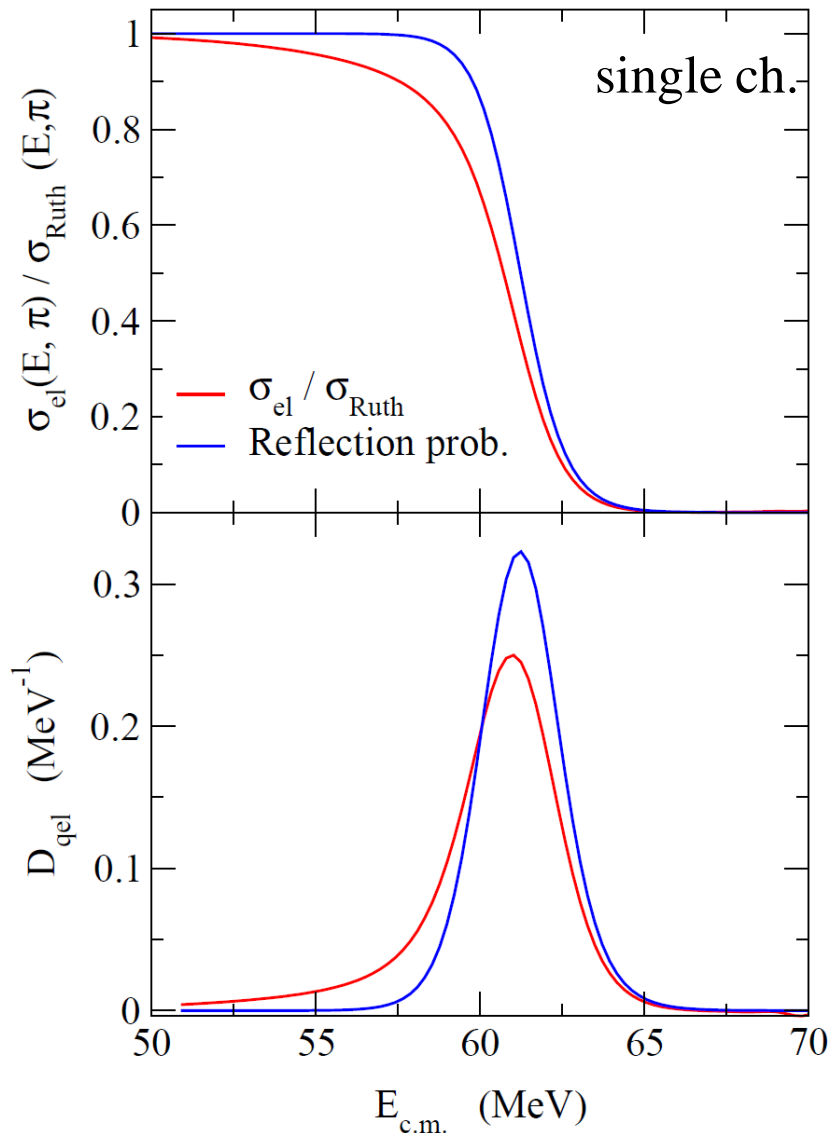
$D_{\text{fus}}$  and  $D_{\text{qel}}$ : behave similarly to each other

cf. Eryk Piasecki's talk on Friday

cf. Application to reactions relevant to SHE

[S. Mitsuoka et al., PRL99('07)182701]

# Problems with quasi-elastic barrier distributions



$$D_{qel}(E) = -\frac{d}{dE} \left( \frac{\sigma_{qel}(E, \pi)}{\sigma_{Ruth}(E, \pi)} \right)$$

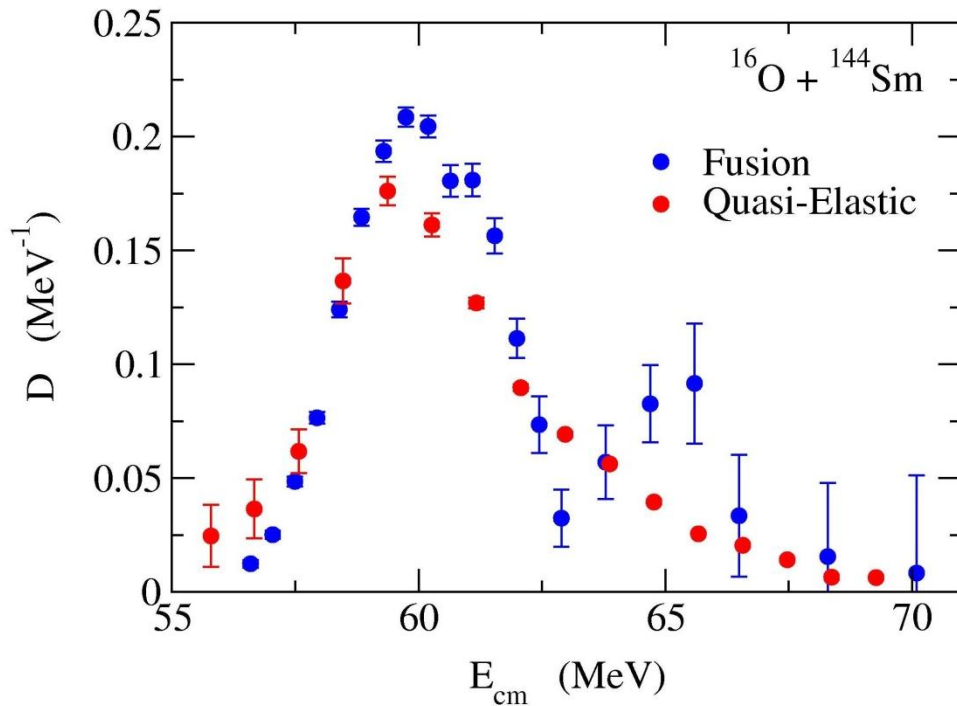
$D_{qel}$  and  $D_{fus}$ : behave similarly,  
but not identically



the effect of nuclear distortion  
of the classical trajectory

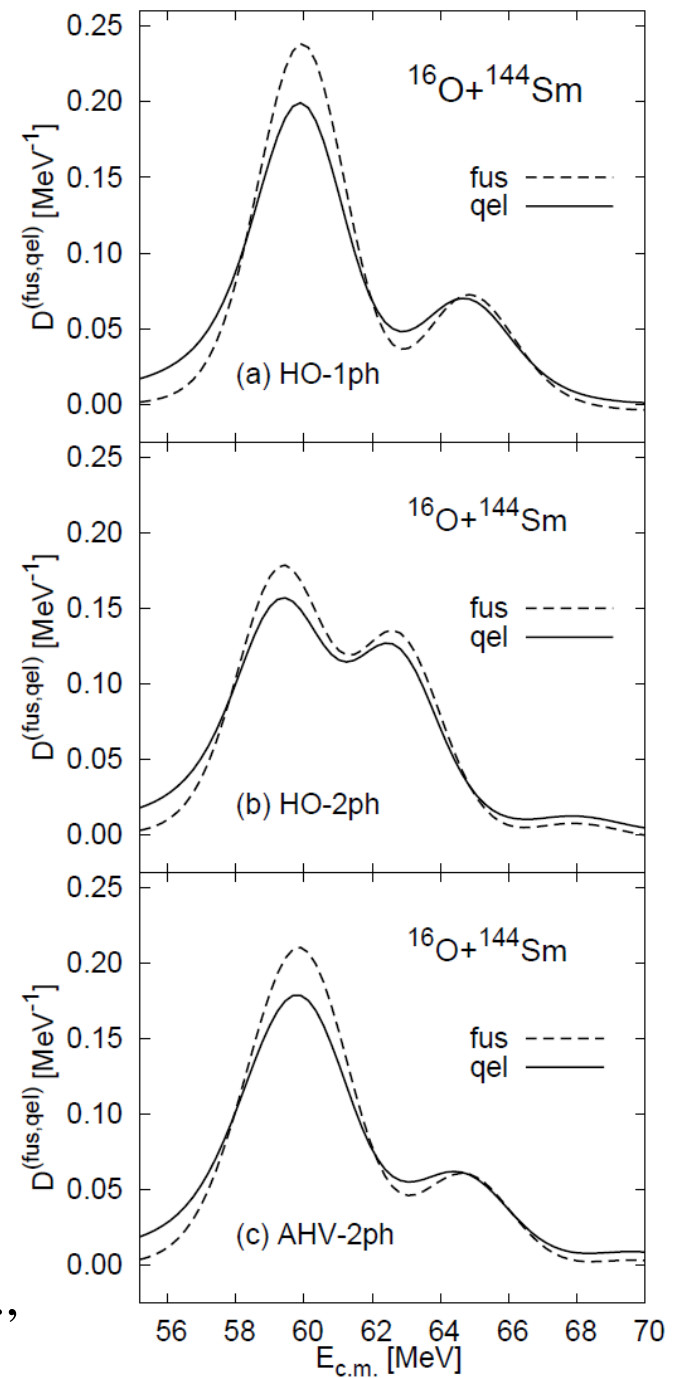
$$R_{l=0} \neq \frac{\sigma_{qel}(E, \pi)}{\sigma_{Ruth}(E, \pi)} = \alpha \cdot R_{l=0}$$

$$\alpha \sim 1 + \frac{V_N(r_c)}{ka} \frac{\sqrt{2a\pi k\eta}}{E}$$



↑  
**discrepancy: open problem**

M. Zamrun F. and K.H.,  
 PRC77('08)014606

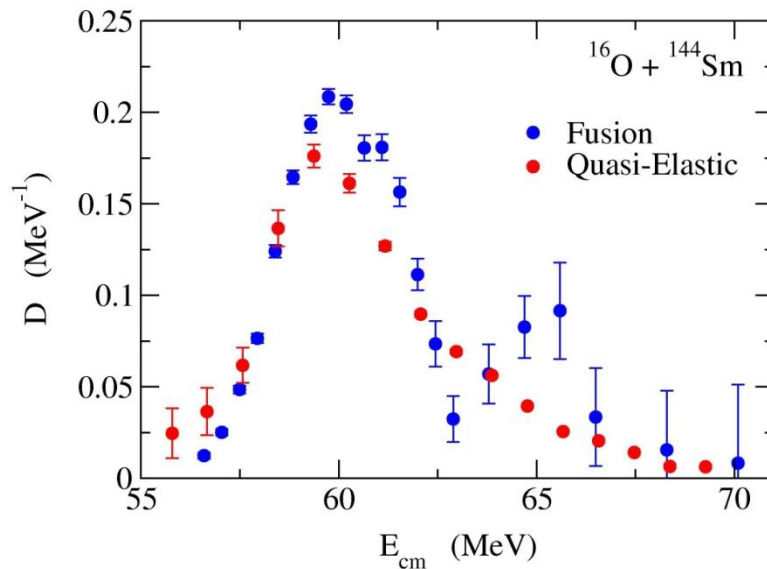




# Problems with quasi-elastic barrier distributions

$$D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)} \right)$$

➤  $D_{\text{qel}}$  and  $D_{\text{fus}}$ : behave similarly, but not identically



➤  $D_{\text{qel}}$ : not applicable to symmetric systems

$$\sigma(\theta) = |f(\theta) \pm f(\pi - \theta)|^2$$

—————> diverges at  $\theta = \pi$

# Sum-of-differences (SOD) method

J.T. Holdeman and R.M. Thaler, PRL14('65)81, PR139('65)B1186

C. Marty, Z. Phys. A309('83)261, A322('85)499


$$\sigma_R \sim 2\pi \int_{\theta_{\min}}^{\pi} \sin \theta d\theta (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{el}}(\theta))$$

expt.: H. Wojciechowski et al., PRC16('77)1767

T. Yamaya et al., PLB417('98)7 etc.

generalization (K.H. and N. Rowley, in preparation)

$$\sigma_R = \sigma_{\text{fus}} + \sigma_{\text{inel}} + \sigma_{\text{tr}}$$

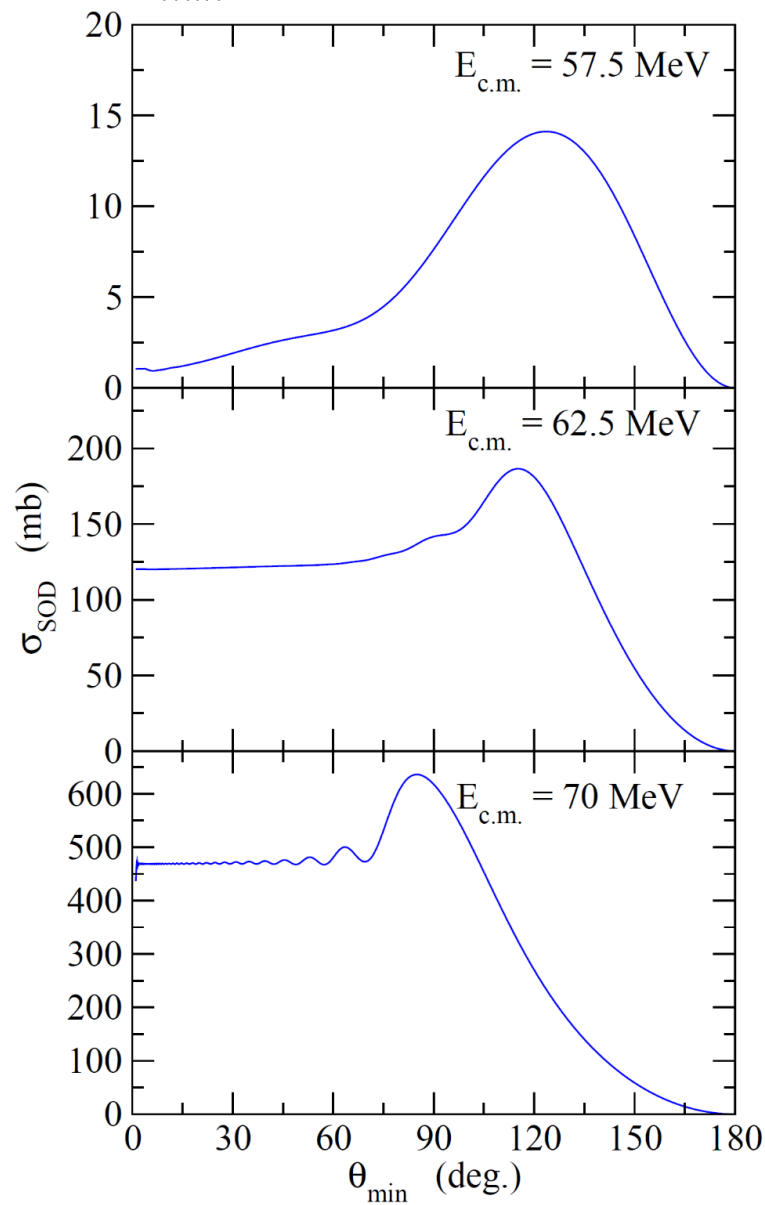
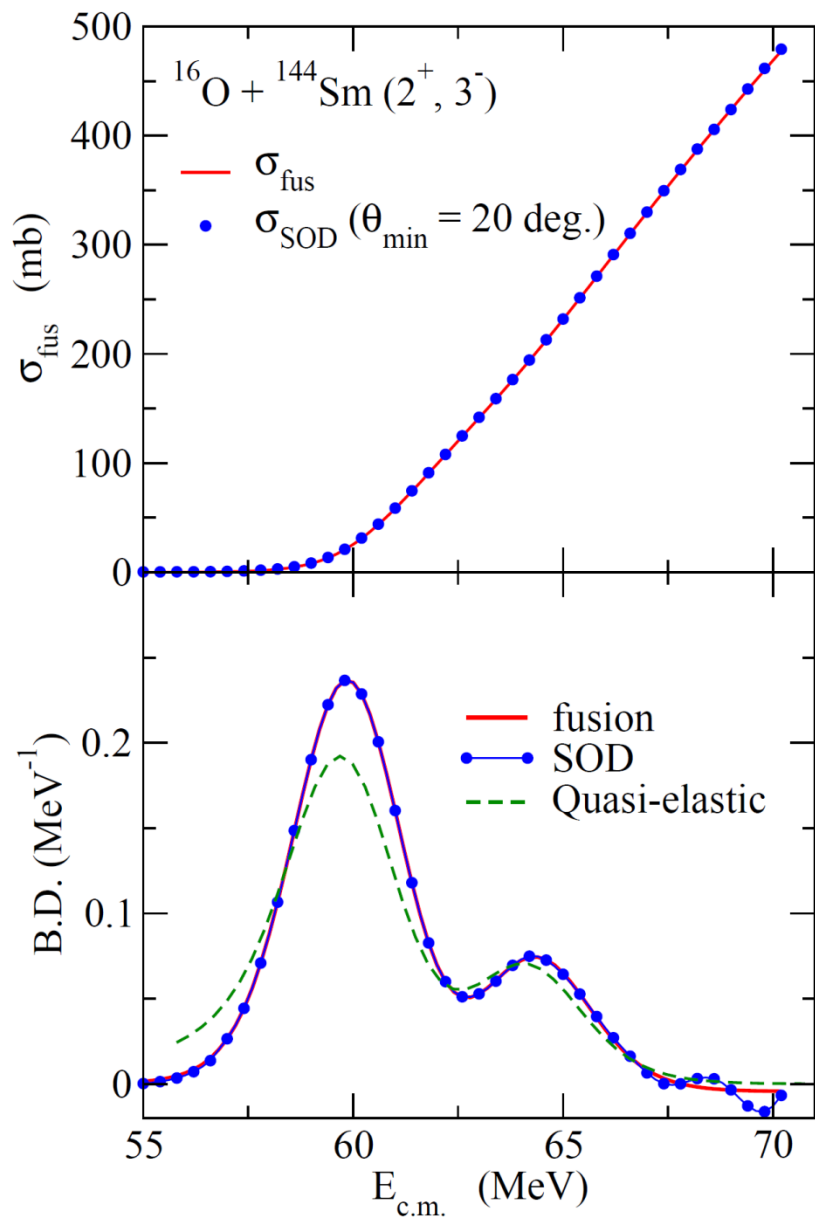

$$\sigma_{\text{fus}} \sim 2\pi \int_{\theta_{\min}}^{\pi} \sin \theta d\theta (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$

$$= 2\pi \int_{\theta_{\min}}^{\pi} \sin \theta d\theta \sigma_{\text{Ruth}}(\theta) \left( 1 - \frac{\sigma_{\text{qel}}(\theta)}{\sigma_{\text{Ruth}}(\theta)} \right)$$

→  $D_{\text{fus}}$  from  $\sigma_{\text{qel}}$ ?

# Does SOD work for fusion barrier distributions?

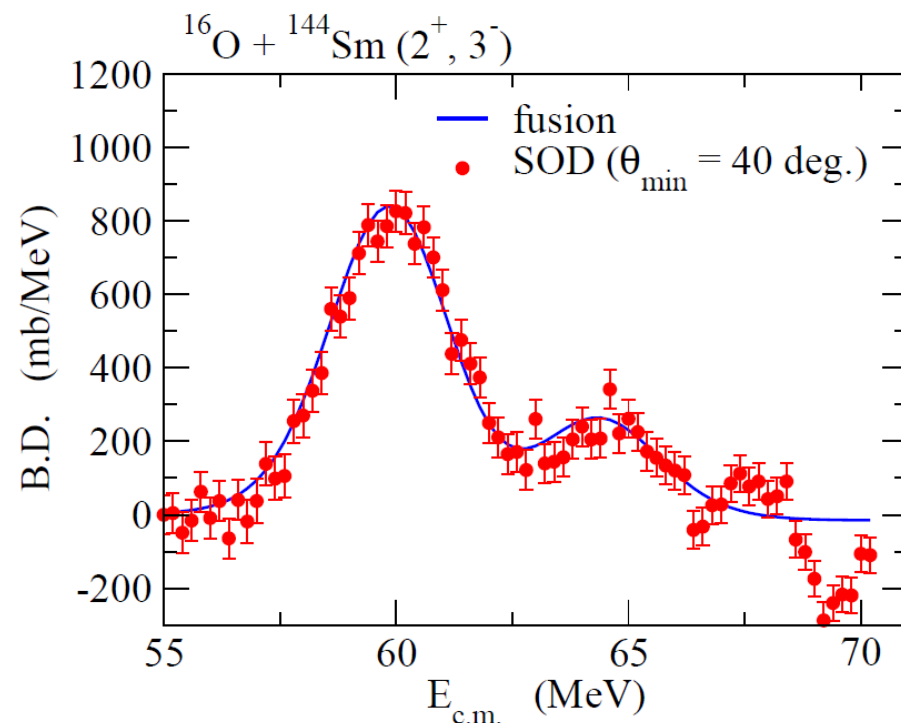
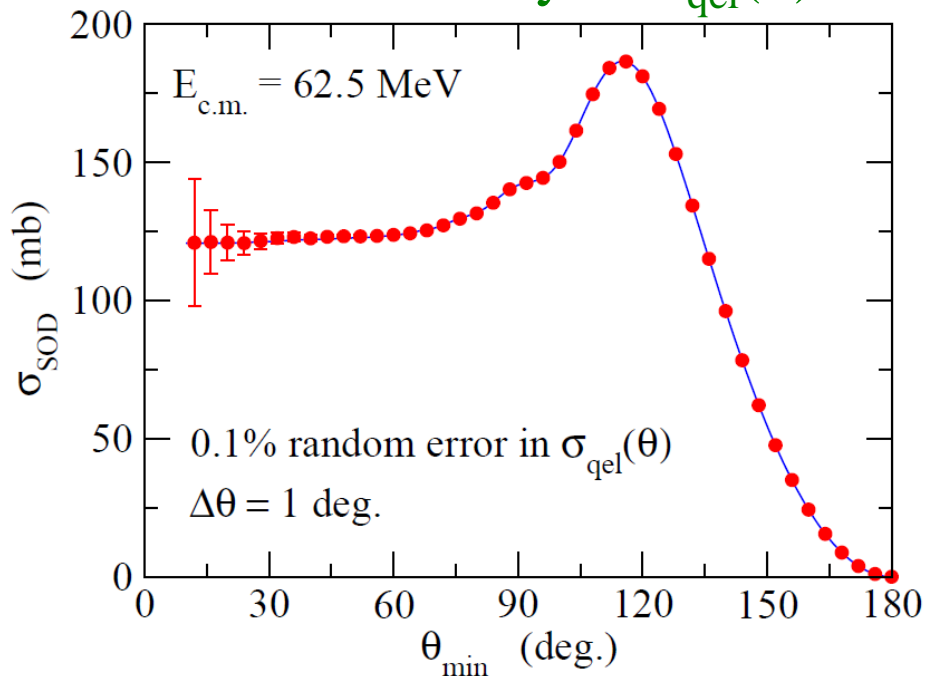
$$\sigma_{\text{SOD}} = 2\pi \int_{\theta_{\text{min}}}^{\pi} \sin \theta d\theta (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$



# SOD with “experimental” quasi-elastic cross sections

$$\sigma_{\text{qel}}^{(\text{exp})}(E, \theta) \sim \sigma_{\text{qel}}^{(\text{th})}(E, \theta) + \Delta\sigma_{\text{qel}}^{(\text{th})}(E, \theta) \leftarrow \text{randomly generated}$$

0.1% accuracy in  $\sigma_{\text{qel}}(\theta)$

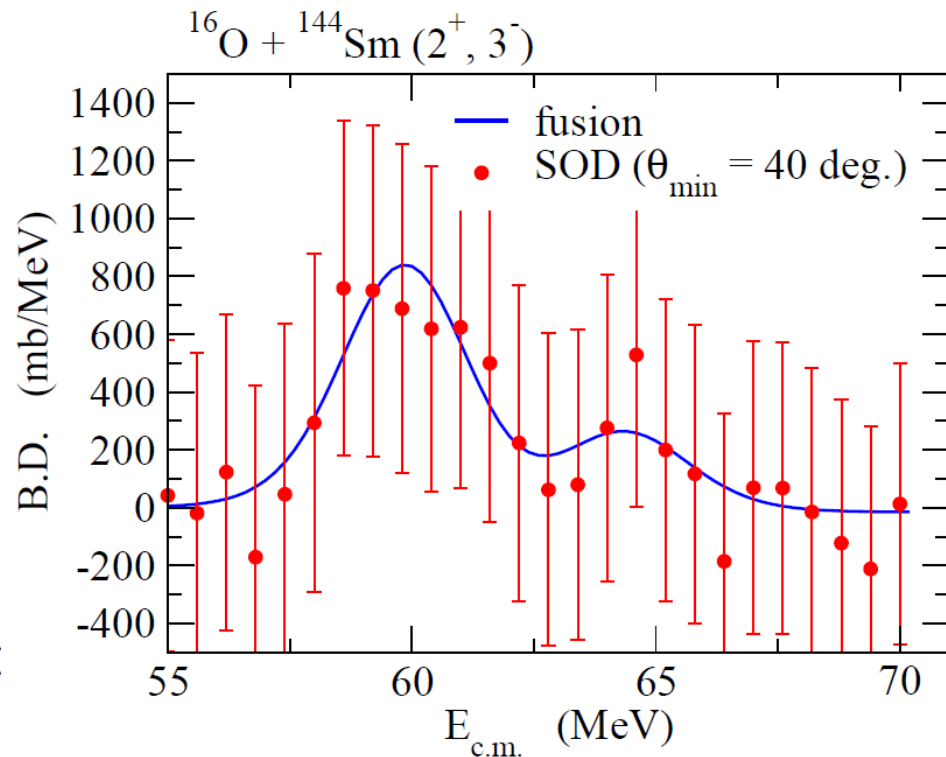
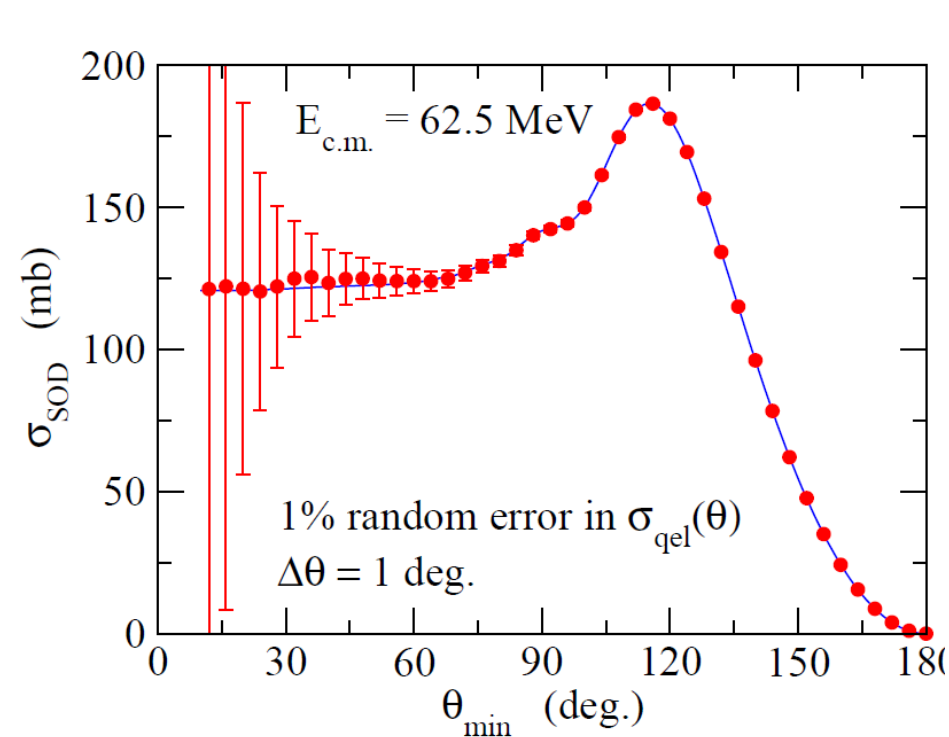


uncertainty in  $\sigma_{\text{SOD}}$

$\theta_{\text{min}} = 40 \text{ deg.}$	0.95%
30 deg.	1.96%
20 deg.	5.41%

# SOD with “experimental” quasi-elastic cross sections

$$\sigma_{\text{qel}}^{(\text{exp})}(E, \theta) \sim \sigma_{\text{qel}}^{(\text{th})}(E, \theta) + \Delta\sigma_{\text{qel}}^{(\text{th})}(E, \theta) \leftarrow \text{randomly generated}$$



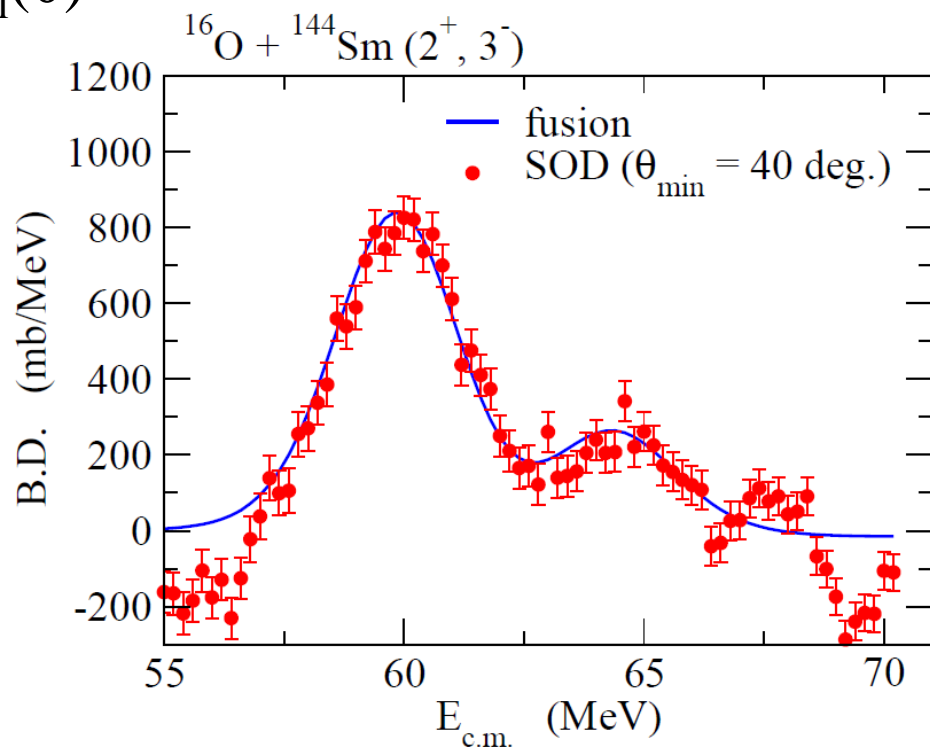
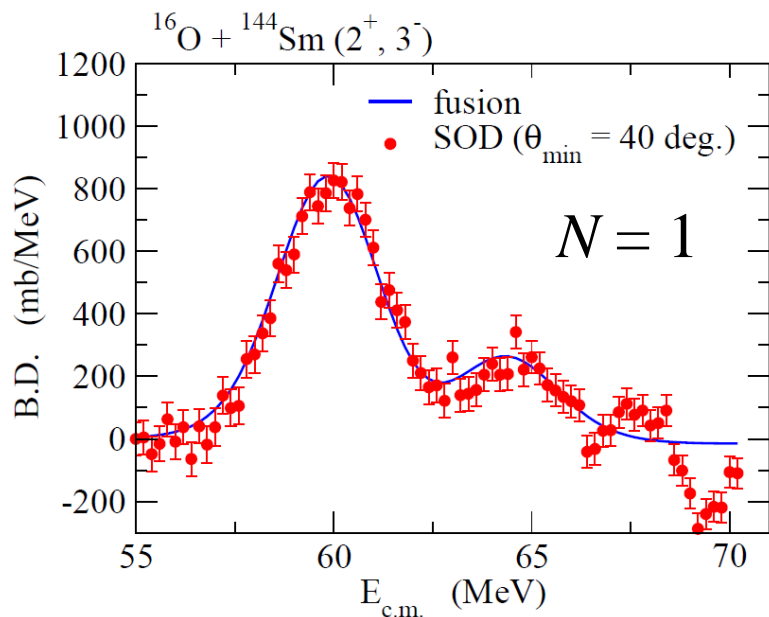
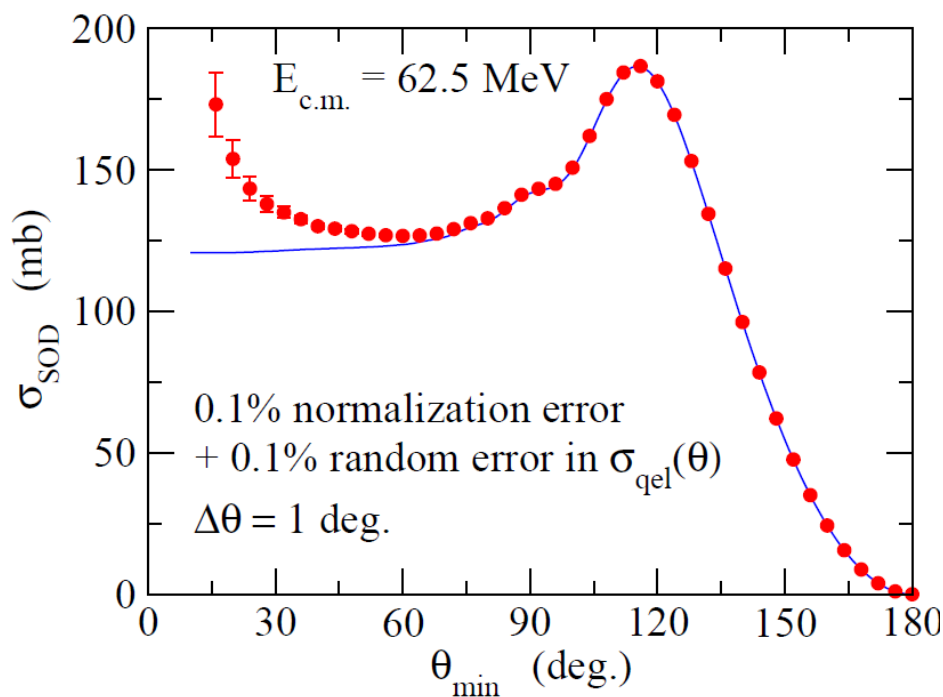
uncertainty in  $\sigma_{\text{SOD}}$

$\theta_{\text{min}} = 40 \text{ deg.}$	9.48%
30 deg.	19.3%
20 deg.	54.0%

## Effect of normalization error

$$\sigma_{\text{qel}}^{(\text{exp})}(E, \theta) \sim N \sigma_{\text{qel}}^{(\text{th})}(E, \theta) + \Delta \sigma_{\text{qel}}^{(\text{th})}(E, \theta)$$

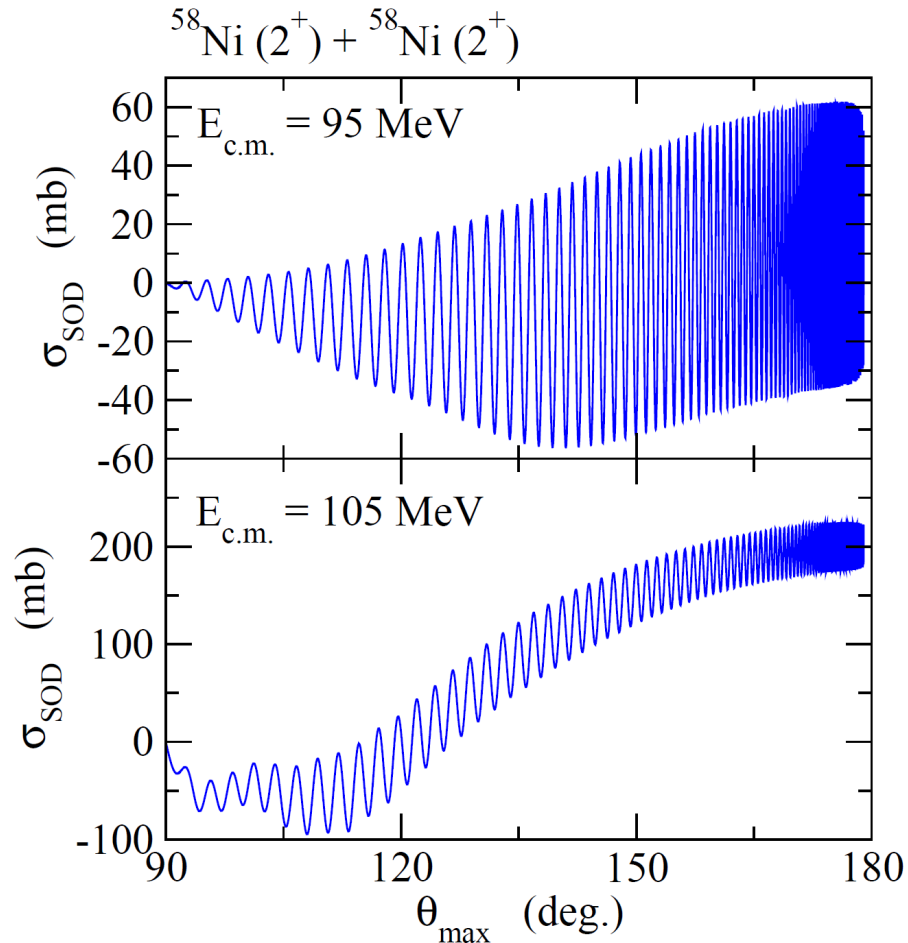
$N = 0.999 \pm 0.1\%$  accuracy in  $\sigma_{\text{qel}}(\theta)$

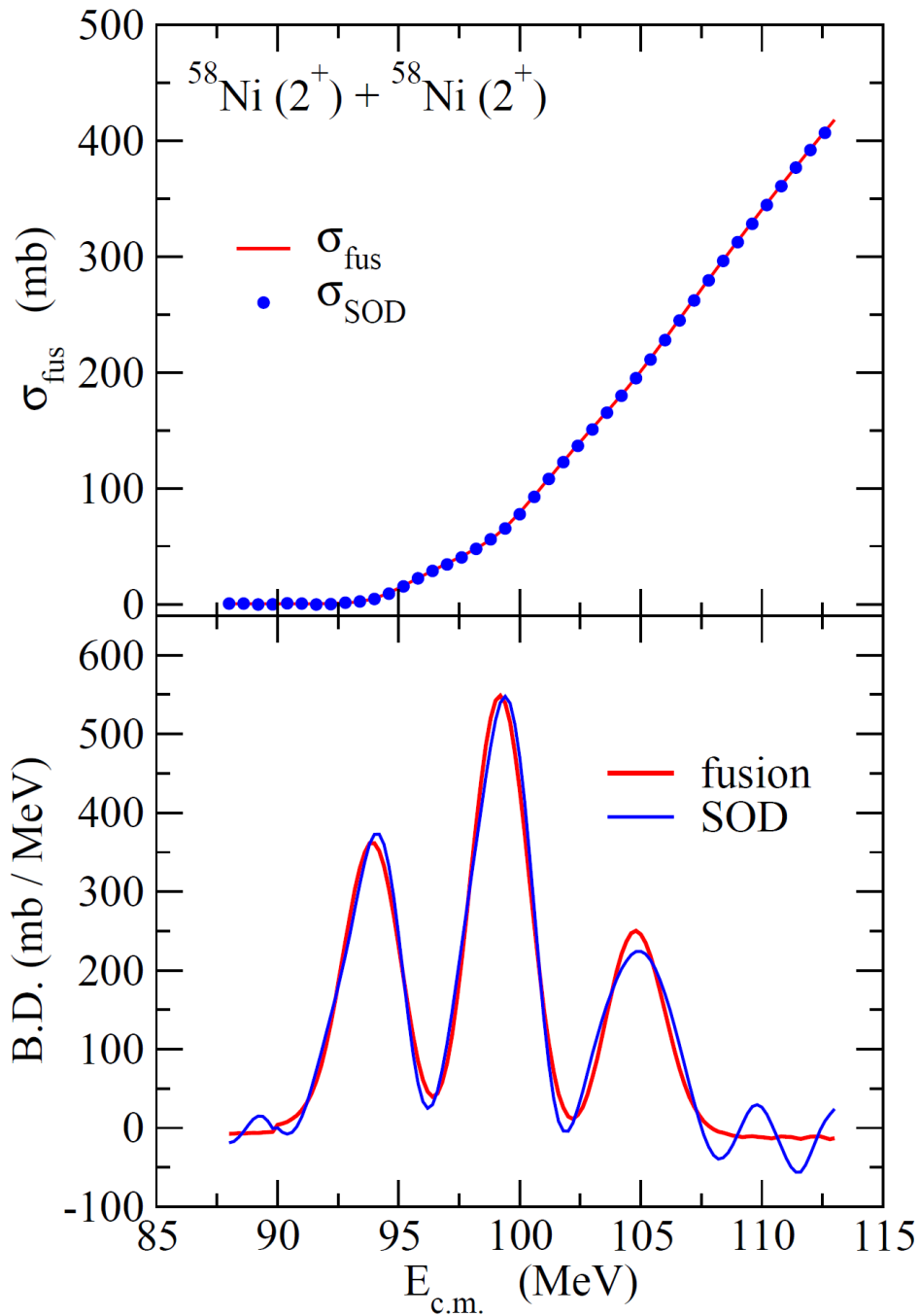


## Symmetric sysmtes

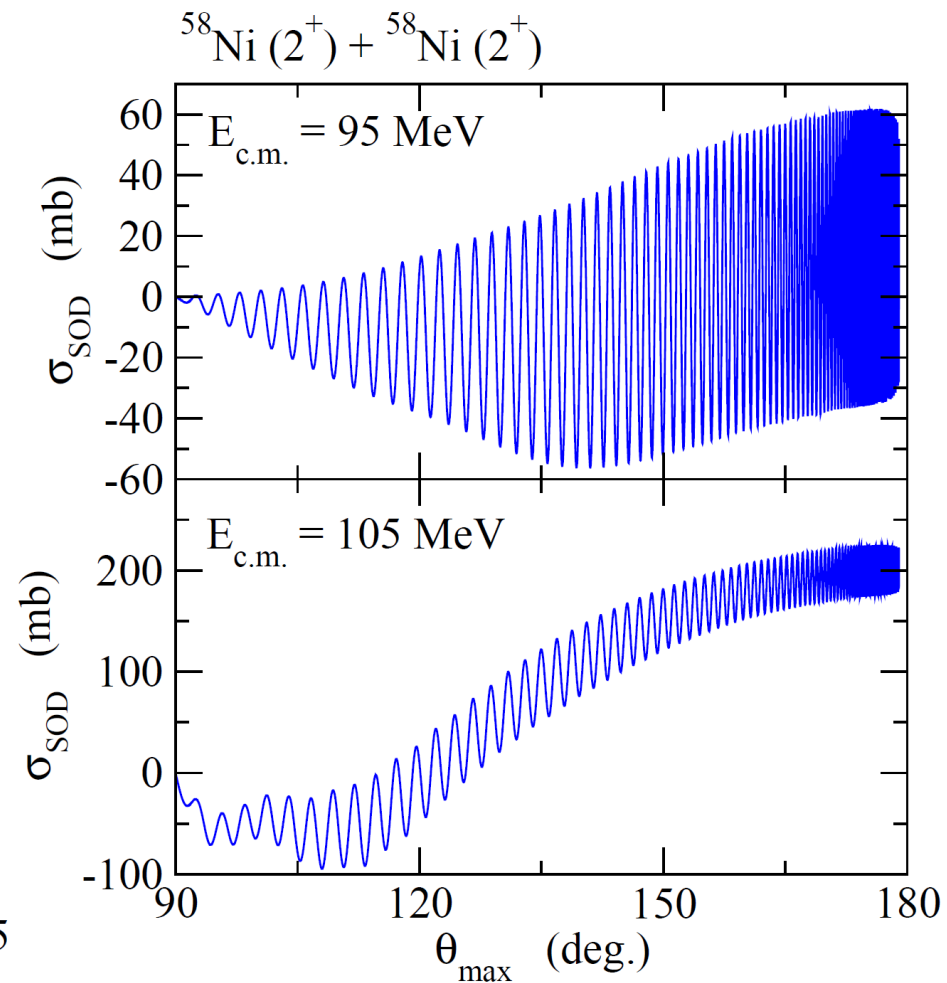
$$\sigma_{\text{fus}} \sim 2\pi \int_{\theta_{\text{min}}}^{\pi} \sin \theta d\theta (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$

$$\longrightarrow \sigma_{\text{fus}} \sim 2\pi \int_{\pi/2}^{\theta_{\text{max}}} \sin \theta d\theta (\sigma_{\text{Mott}}(\theta) - \sigma_{\text{qel}}(\theta))$$



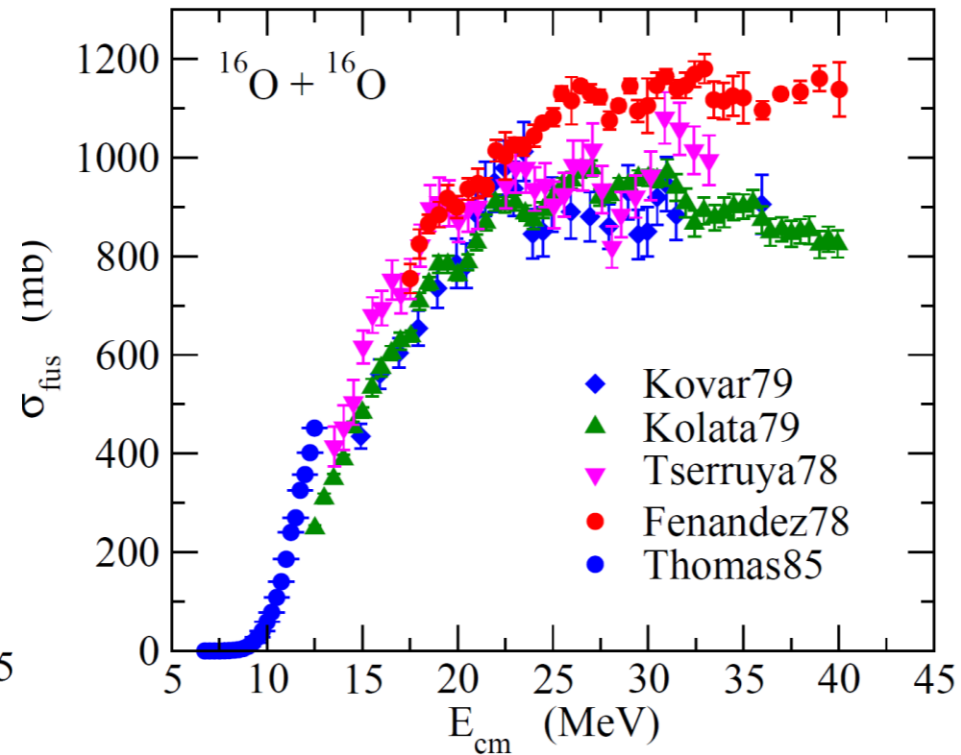
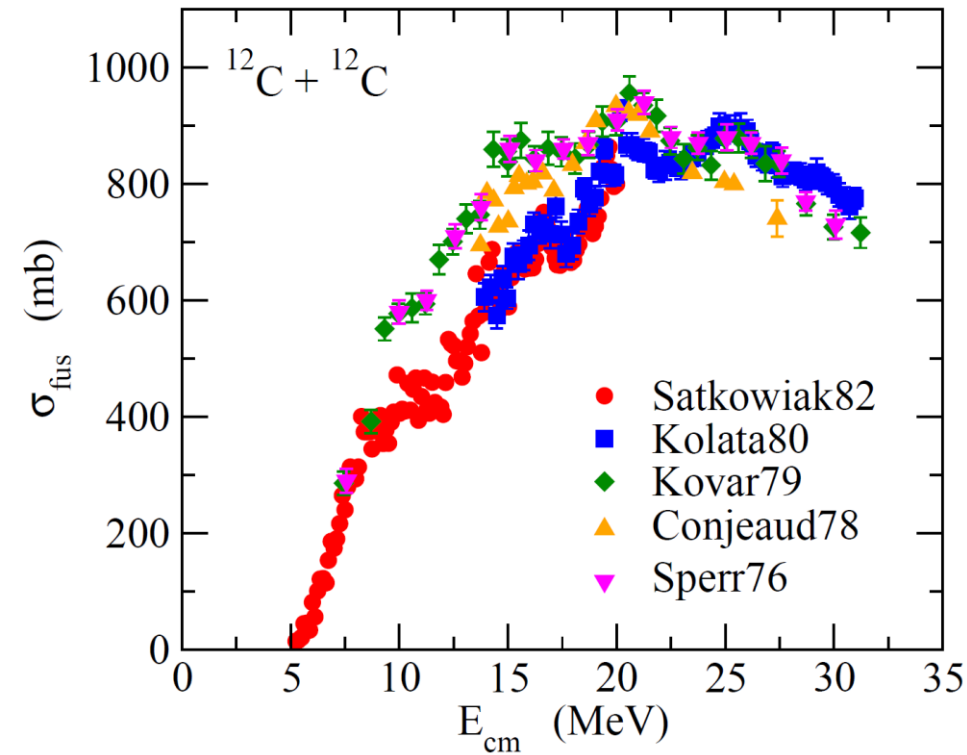


← average in the range of  
 $\theta_{\text{max}} = (176.5, 179.5)$  deg.





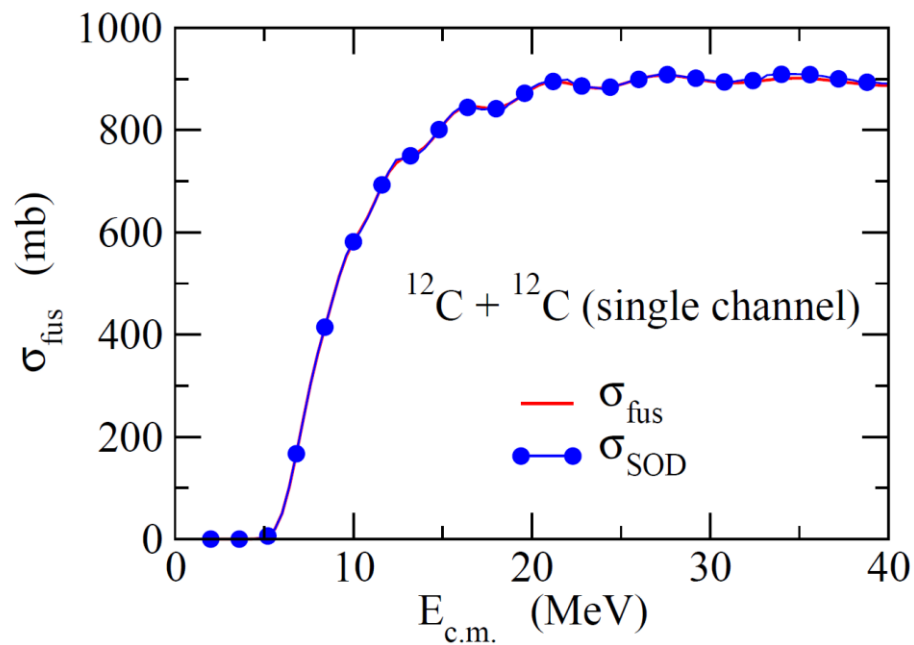
# Fusion of light symmetric systems: fusion oscillations



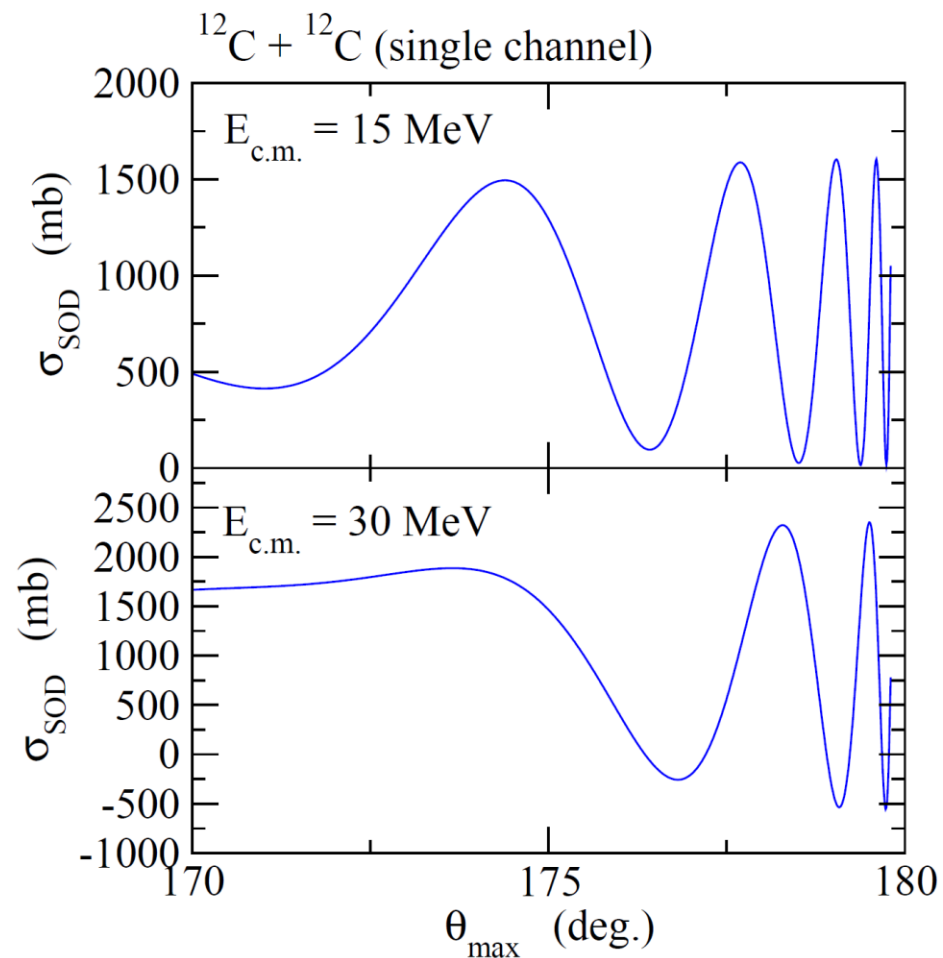
The expt. data: rather scattered

- ✓ systematic errors
- ✓ missing evaporation channels

→  $\sigma_{\text{fus}}$  from SOD?



average of a maximum and  
 a minimum in  $\sigma_{\text{SOD}}$



# Summary

- ✓ Fusion barrier distribution  $D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$
- ✓ Quasi-elastic barrier distribution  $D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)} \right)$
- ✓ Sum-of-differences (SOD) method

$$\sigma_{\text{fus}} \sim 2\pi \int_{\theta_{\text{min}}}^{\pi} \sin \theta d\theta (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$

## $D_{\text{SOD}}$ :

- closer correspondence to  $D_{\text{fus}}$  compared to  $D_{\text{qel}}$
  - need an accuracy of  $\Delta\sigma_{\text{qel}} \sim 0.1\%$
  - applicable also to symmetric systems
- application to light symmetric systems?  
(fusion oscillations)

