Fission Dynamics of Low Excited Nuclei with Langevin Approach

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TOKYO INSTITUTE OF TECHNOLOGY
Since 1957

Present study is the results of "Comprehensive study of delayed-neutron yields for accurate evaluation of kinetics of high-burn up reactors" entrusted to Tokyo Institute of Technology by the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT).

16th ASRC International Workshop
Nuclear Fission and Structure of Exotic Nuclei
ASRC, JAEA, Tokai, Japan
18th - 20th. March (2014)

Contains

1. Purpose

2. Model

    Dynamical model with Langevin equation
    Two center shell model

3. Results

    236U, 234U, 240Pu at E*=20 MeV

    Mass distribution of Fission fragments
    TKE
    Charge distribution
    Independent Fission Yields

4. Summary

1. Purpose

- Fission process of low excited nuclei

estimation amounts of heavy elements
radioactive fission products
melted spent nuclear fuel
improving the safety of planned nuclear power plants
obtain the nuclear data that could not be measured
by experiments

- Dynamical model ← Statistical model

- Mass distribution of fission fragments of U, Pu (thermal neutron)
  Charge distribution
  Independent fission yields

Fission process

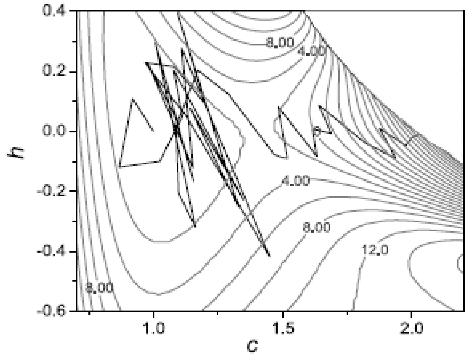


Figure 1. Stochastic Langevin trajectory in the space of the collective coordinates $(c, h, \alpha' = 0)$ is shown against the potential-energy background. The numbers at the isolines specify the values of the potential energy in MeV. The solid line in the right upper corner of the figure is the scission line. The trajectory given in this figure represents a fission event.

Langevin calculation
for fission process
at high excitation energy

LDM

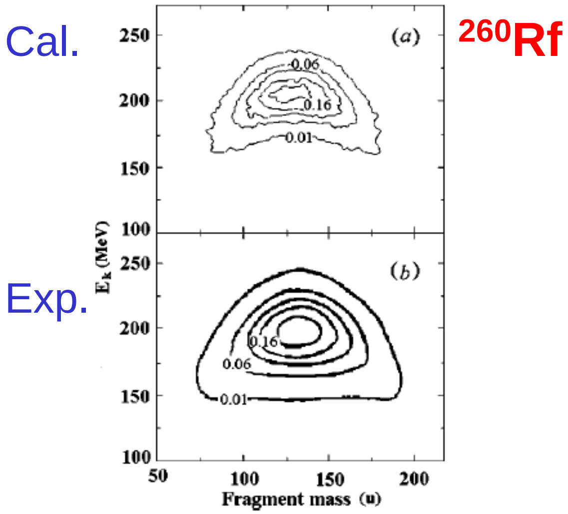


FIG. 6. The theoretical (a) and experimental (b) MED of fission fragments of ^{260}Rf at the total excitation energy $E^* = 74.2$ MeV. The numbers at the contour lines in percents indicate the yield, which is normalized to 200%. The theoretical diagram was calculated with the reduction coefficient $k_s = 0.1$. The experimental diagram was taken from Ref. [57].

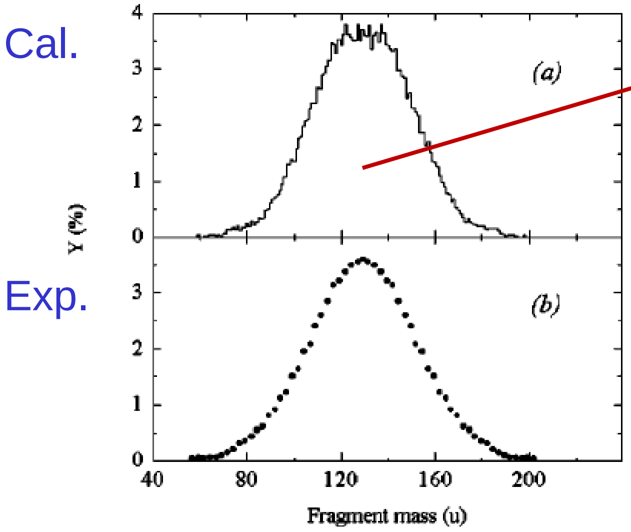


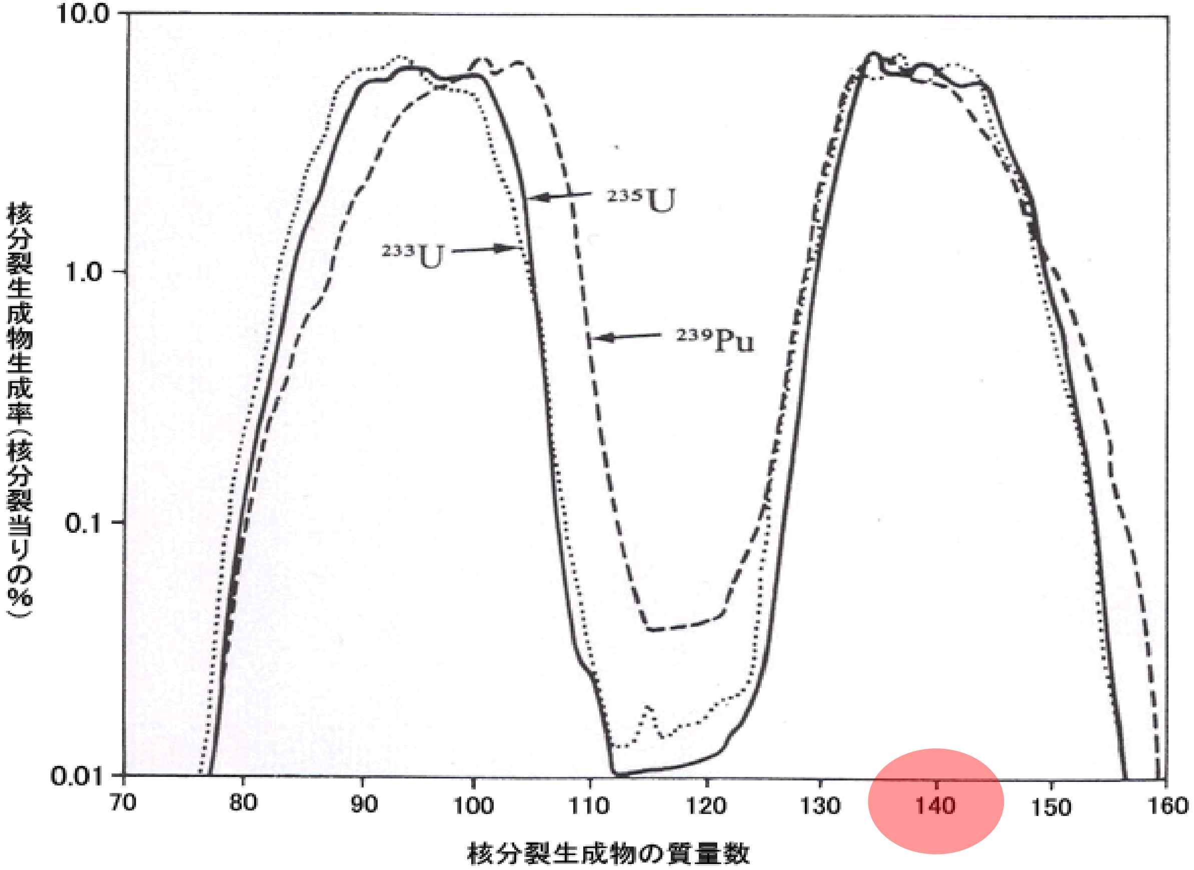
FIG. 8. The theoretical (a) and experimental (b) mass distributions of fission fragments of ^{260}Rf, $E^* = 74.2$ MeV. The theoretical histogram was calculated with the reduction coefficient $k_s = 0.1$. The experimental distribution was taken from Ref. [57].

Mass
Symmetric
Fission events

A.V. Karpov,
P.N. Nadtochy,
D.V. Vanin,
and G.D. Adeev,
PRC 63 (2001) 054610

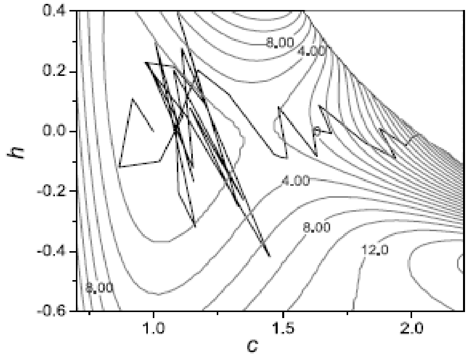
Mass distribution of fission fragments（thermal *n*)

Yield (%)



Fragment Mass (u)

Fission process



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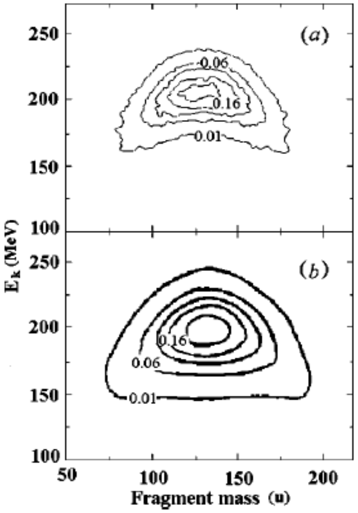
Langevin calculation
for fission process
at high excitation energy

LDM
Shell correction energy

Cal.

^{260}Rf

Exp.



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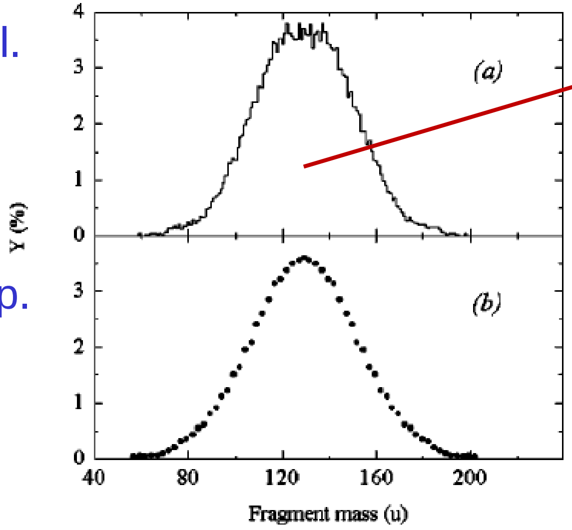
Cal.

Exp.



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Mass
Symmetric
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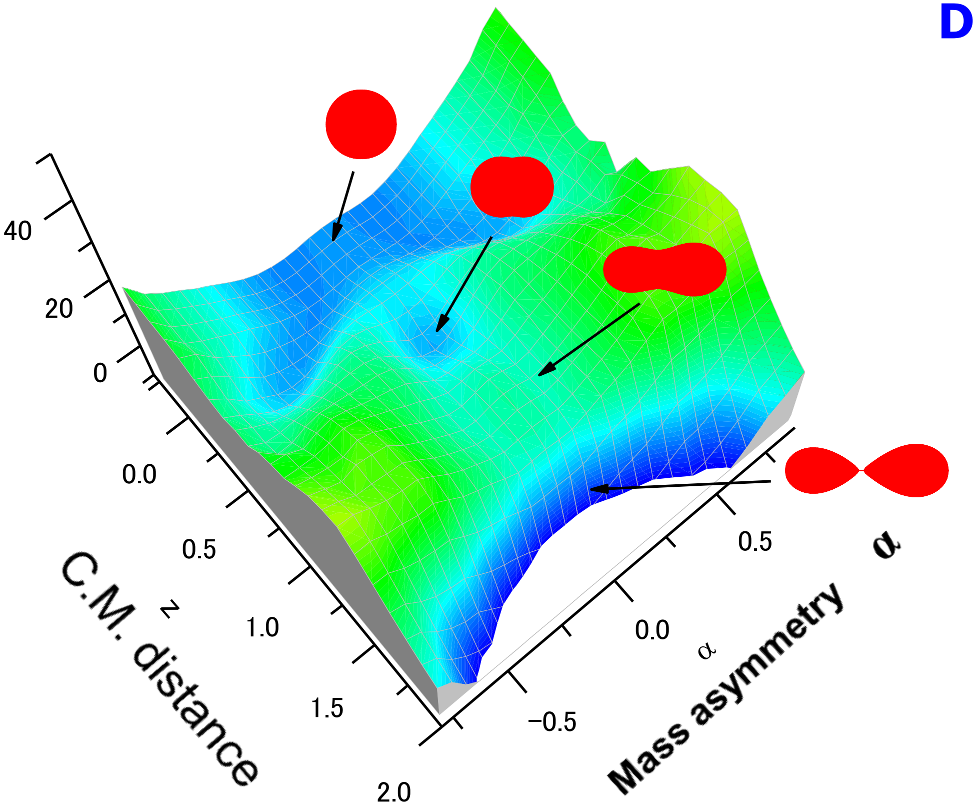
2. Model Fission process ^{240}U E* < 20 MeV



Dynamical calculation

Time-evolution of nuclear shape
in fission process

Two Items

1. Potential energy surface

2. Trajectory ← described by
Equation of Motion

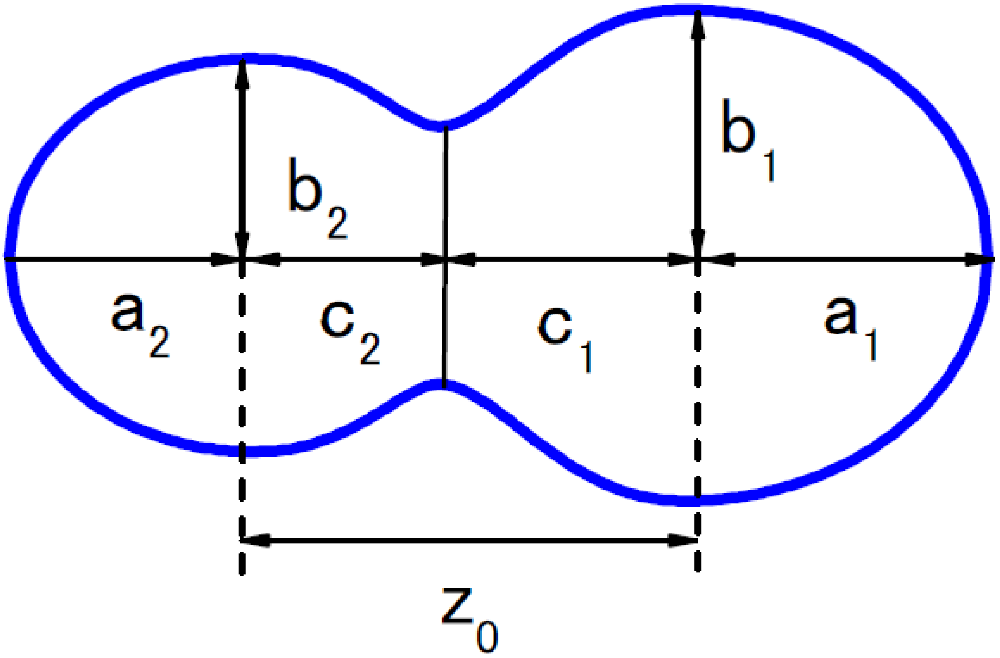
Trajectory on potential energy surface

Nuclear Shape

two-center parametrization (z, δ, α)

(Maruhn and Greiner,
Z. Phys. 251(1972) 431)

$$q(z, \delta, \alpha)$$



$$z = \frac{z_0}{BR}$$

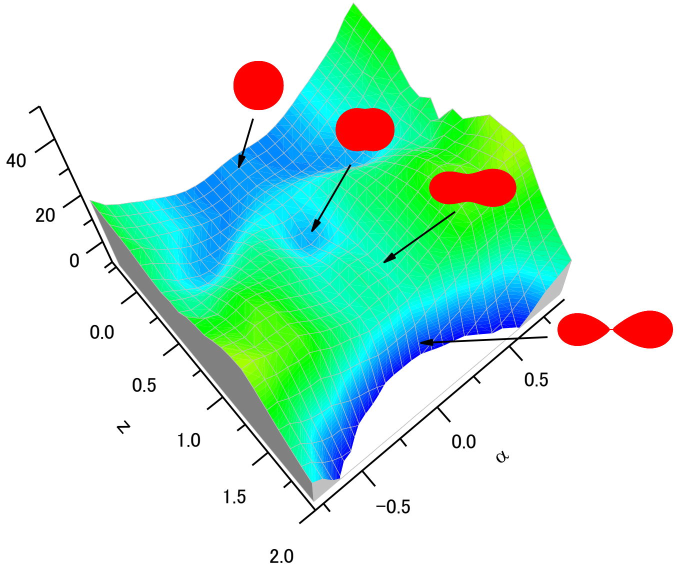
$$B = \frac{3+\delta}{3-2\delta}$$

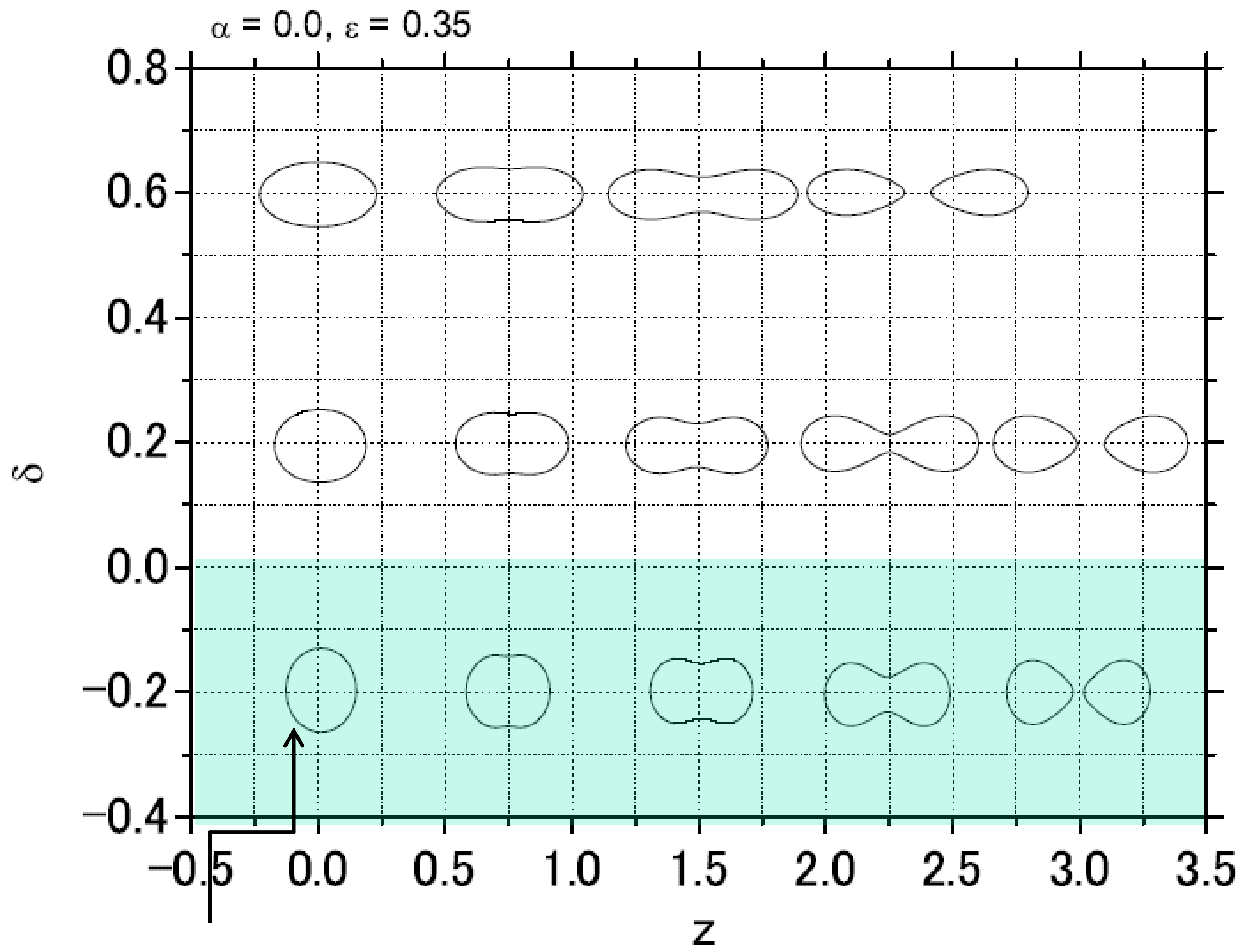
$R:$ Radial of compound nucleus

$$\delta = \frac{3(a-b)}{2a+b}$$

($\delta1=\delta2$)

$$\alpha = \frac{A_1 - A_2}{A_1 + A_2}$$





Curvature: negative

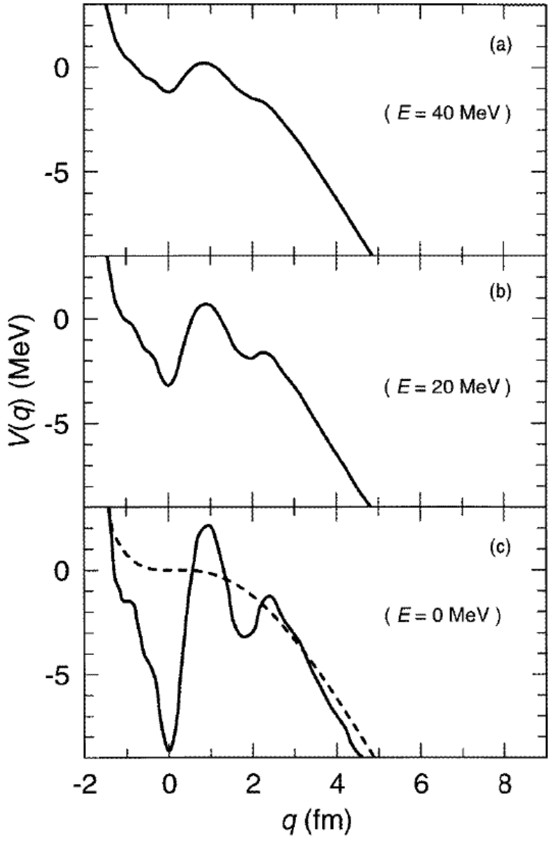
Potential Energy

$$V(q,\ell,T)=V_{DM}(q)+\frac{\hbar^2\ell(\ell+1)}{2I(q)}+V_{SH}(q,T)$$

$$V_{DM}(q)=E_S(q)+E_C(q)$$

$$V_{SH}(q,T)=E^0_{shell}(q)\Phi(T)$$

$^{298}114$



T : nuclear temperature
$E^*=aT^2$ a : level density parameter
Toke and Swiatecki

E_S : Generalized surface energy (finite range effect)
E_C : Coulomb repulsion for diffused surface
E^0_{shell} : Shell correction energy at T=0

I : Moment of inertia for rigid body

$\Phi(T)$: Temperature dependent factor

$$\Phi(T)=\exp\left\{-\frac{aT^2}{E_d}\right\}$$

$$E_d=20\,\mathrm{MeV}$$

Fission barrier recovers at low excitation energy

Dynamical Equation for mean trajectory

Kinetic energy → Intrinsic energy

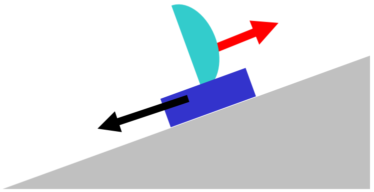
$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k$$

Friction dissipation

Newton equation

$$ma = mg\sin\theta - \gamma v$$



q_i : deformation coordinate （nuclear shape）

two-center parametrization (z,δ,α) (Maruhn and Greiner, Z. Phys. 251(1972) 431)

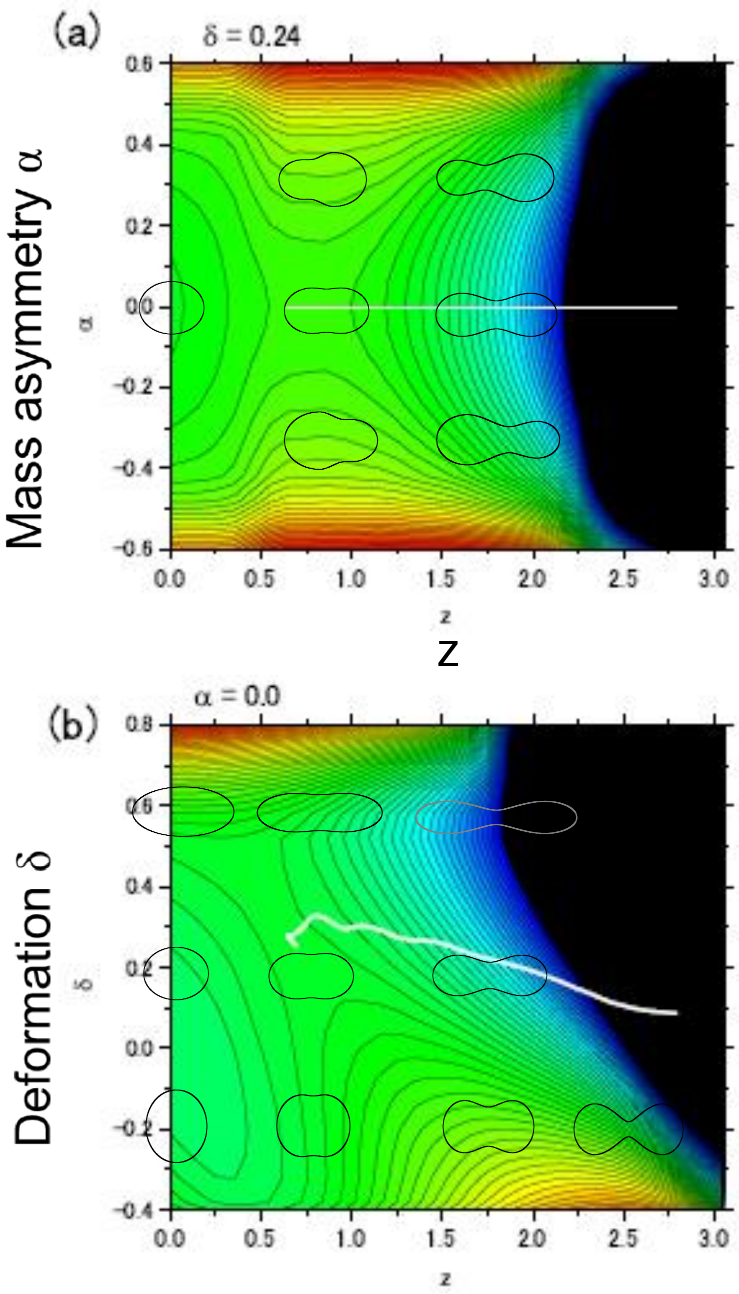
p_i : momentum

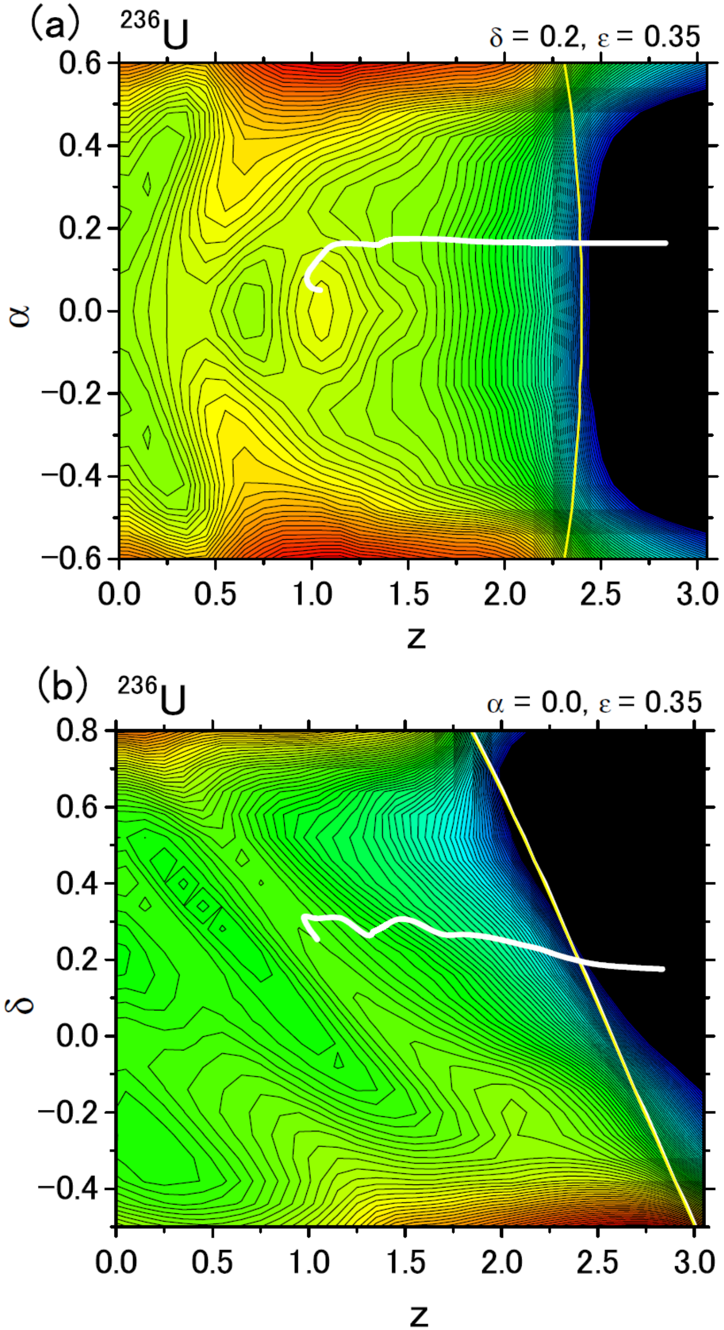
m_{ij} : Hydrodynamical mass (inertia mass)

γ_{ij}: Wall and Window (one-body) dissipation （friction ）

$$E_{\text{int}} = E^* - \frac{1}{2}\left(m^{-1}\right)_{ij} p_i p_j - V(q)$$

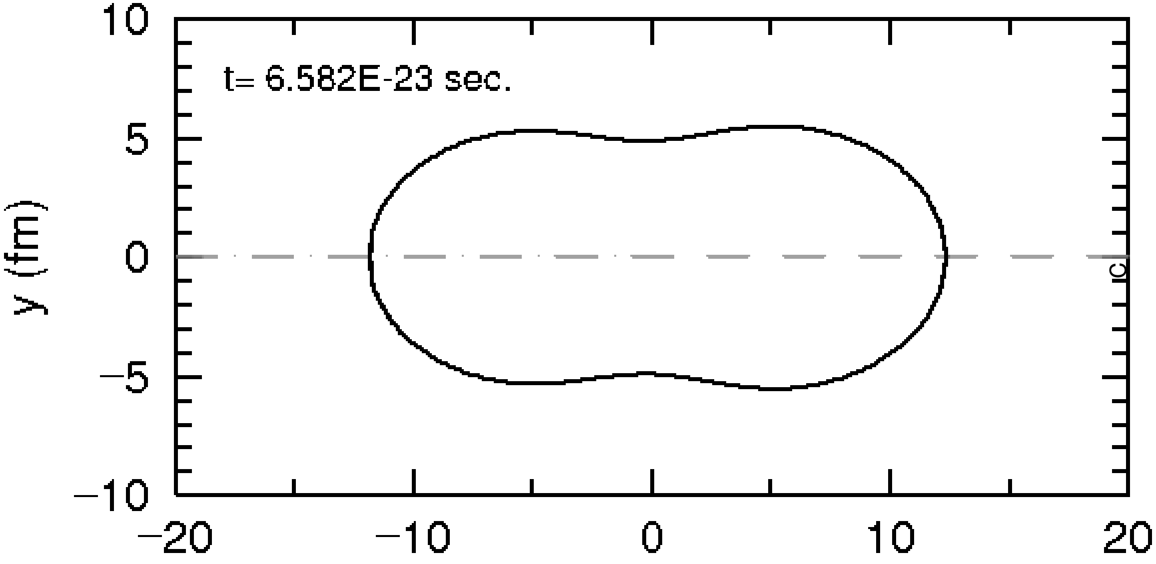
E_{int} : intrinsic energy, E^* : excitation energy

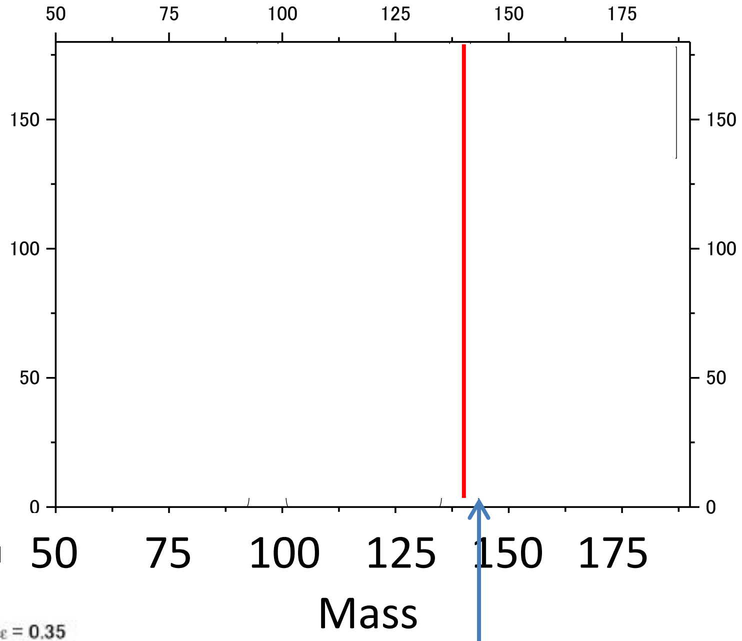

(a)
δ = 0.24
Mass asymmetry α
α
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z
z
(b)
α = 0.0
Deformation δ
δ
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z


(a)
^{236}U
δ = 0.2, ε = 0.35
α
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z
(b)
^{236}U
α = 0.0, ε = 0.35
δ
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z

236U E*=20 MeV
All off

Mass distribution
of fission fragments





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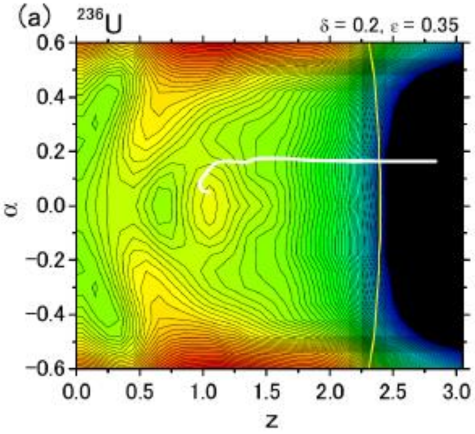
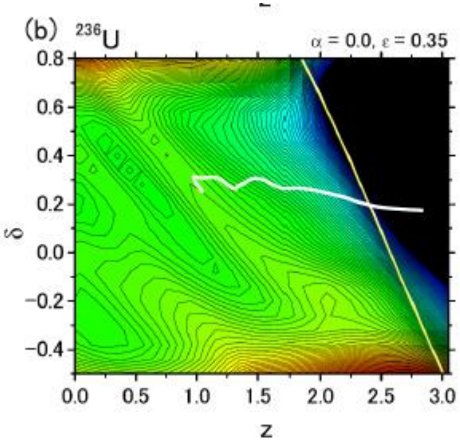
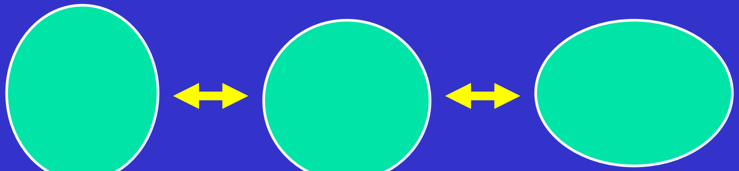




Taking into account the <u>fluctuation around the mean trajectory</u>

<u>Thermal fluctuation</u> of nuclear shape
→ thermal fluctuation of collective motion



Multi-dimensional Langevin Equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

Friction dissipation

Random force fluctuation

Newton equation

ordinary differential equation

$$\langle R_i(t)\rangle = 0,\ \langle R_i(t_1) R_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2) : \text{white noise (Markovian process)}$$

$$\sum_k g_{ik} g_{jk} = T\gamma_{ij}$$

Einstein relation

Fluctuation-dissipation theorem

q_i : deformation coordinate （nuclear shape）

two-center parametrization (z, δ, α) (Maruhn and Greiner, Z. Phys. 251(1972) 431)

p_i : momentum

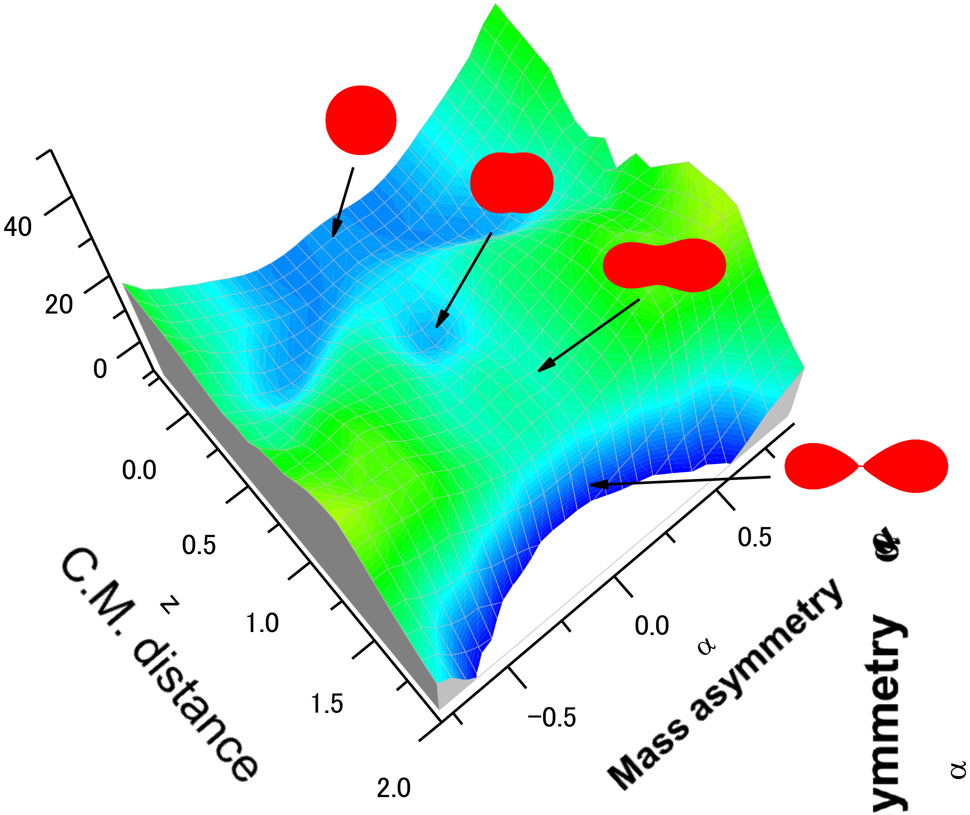
m_{ij} : Hydrodynamical mass (inertia mass）

γ_{ij}: Wall and Window (one-body) dissipation （friction ）

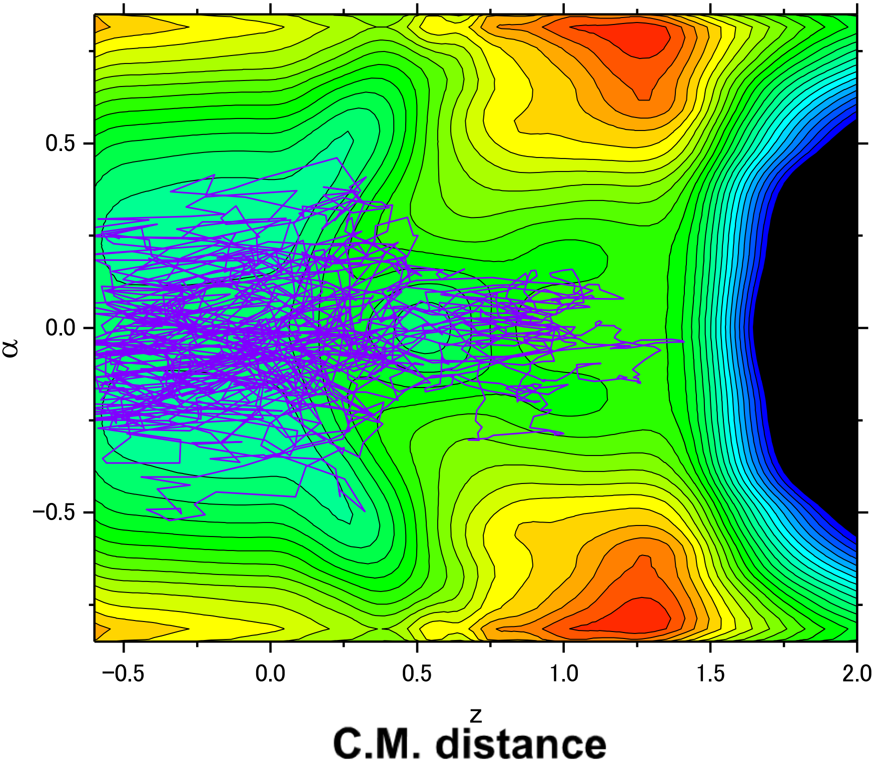
$$E_{\mathrm{int}} = E^* - \frac{1}{2}\left(m^{-1}\right)_{ij} p_i p_j - V(q)$$

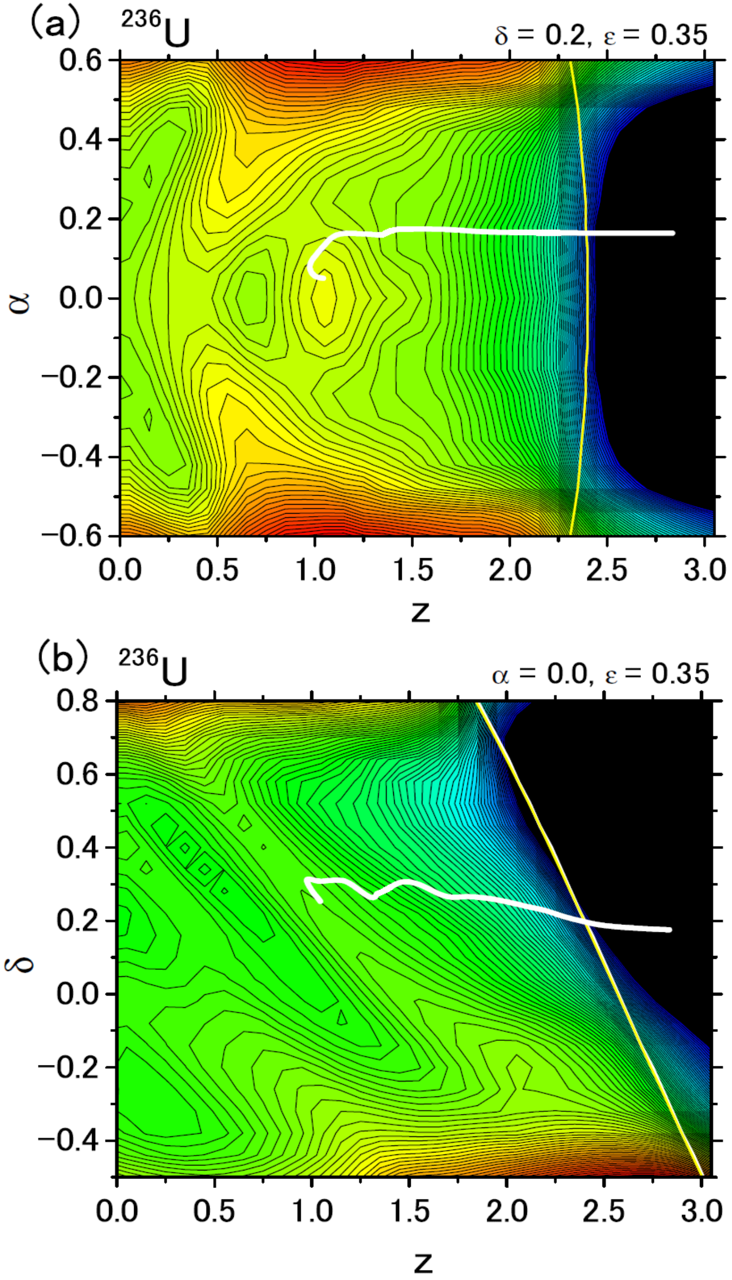
E_{int} : intrinsic energy, E^* : excitation energy

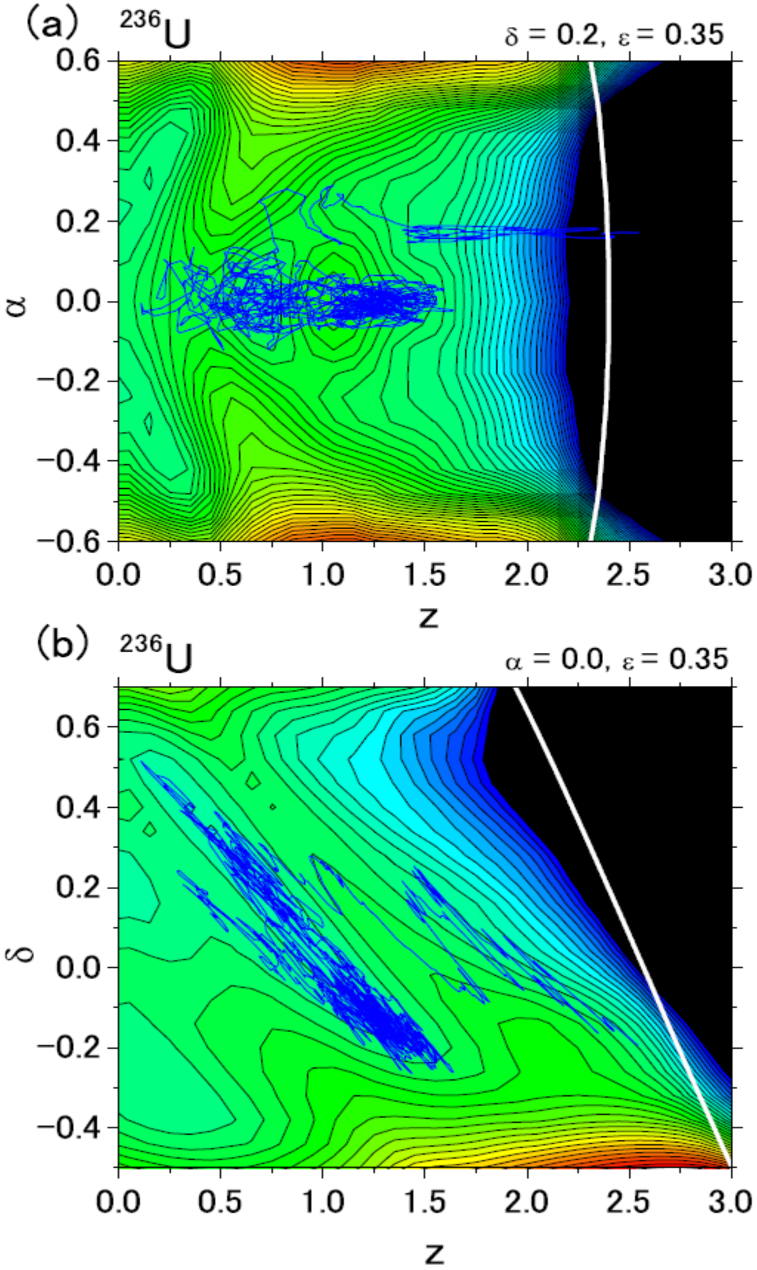
Fission process ^{240}U E* < 20 MeV



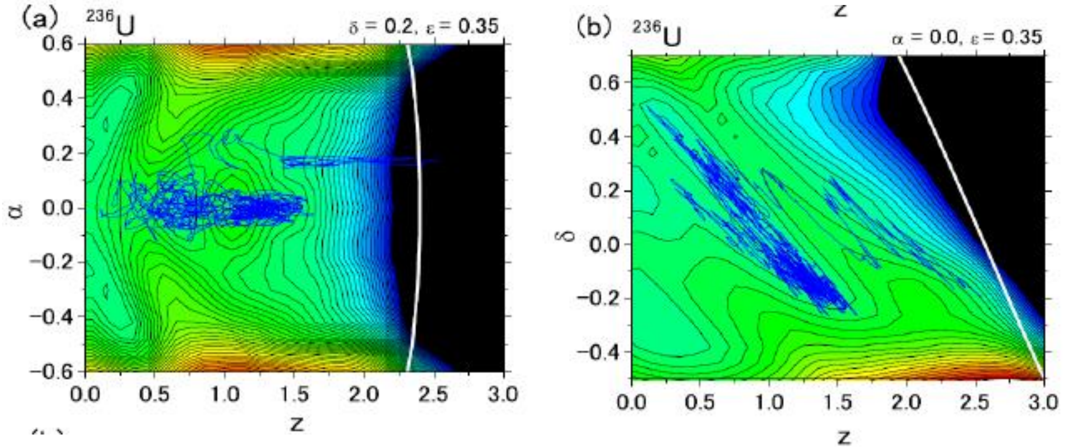
Trajectory on potential energy surface

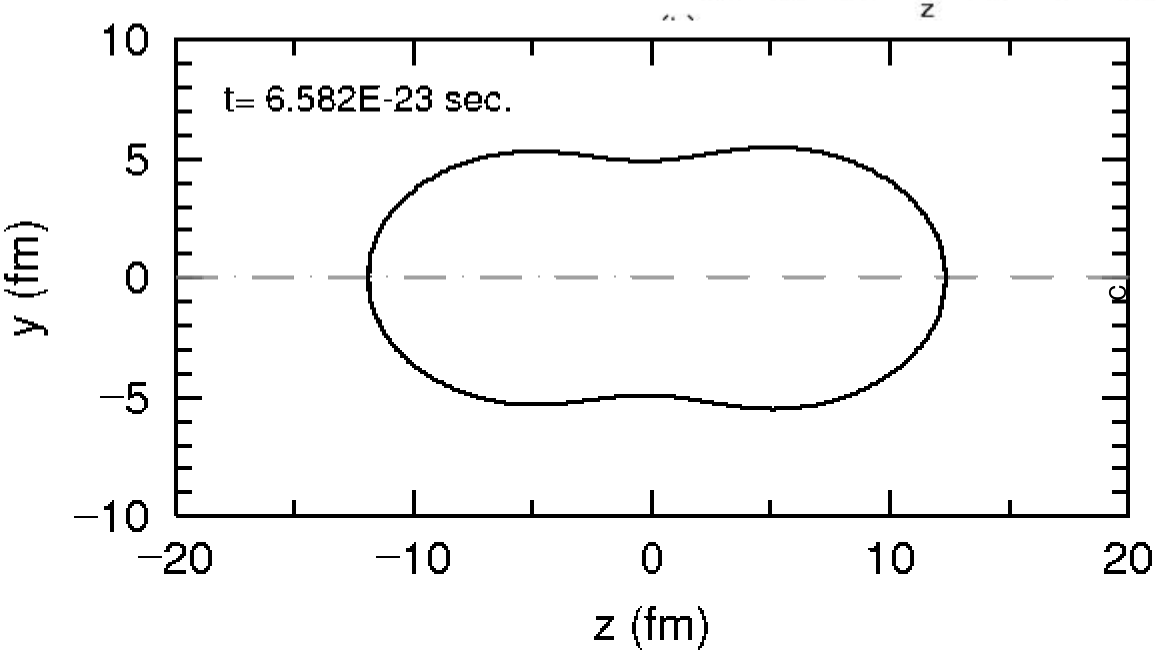


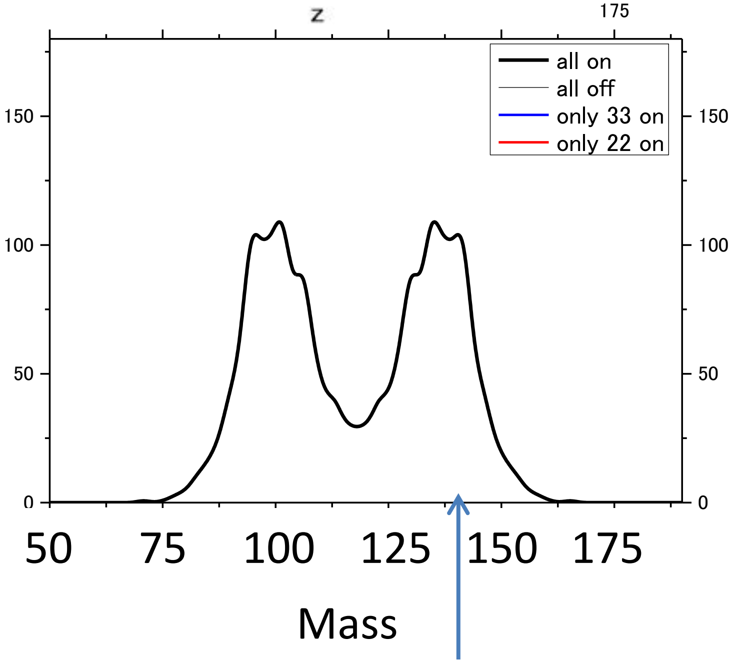

(a)
^{236}U
δ = 0.2, ε = 0.35
α
z
(b)
^{236}U
α = 0.0, ε = 0.35
δ
z


(a)
^{236}U
δ = 0.2, ε = 0.35
α
z
(b)
^{236}U
α = 0.0, ε = 0.35
δ
z

236U E*=20 MeV
All on







140

摩擦力
散逸

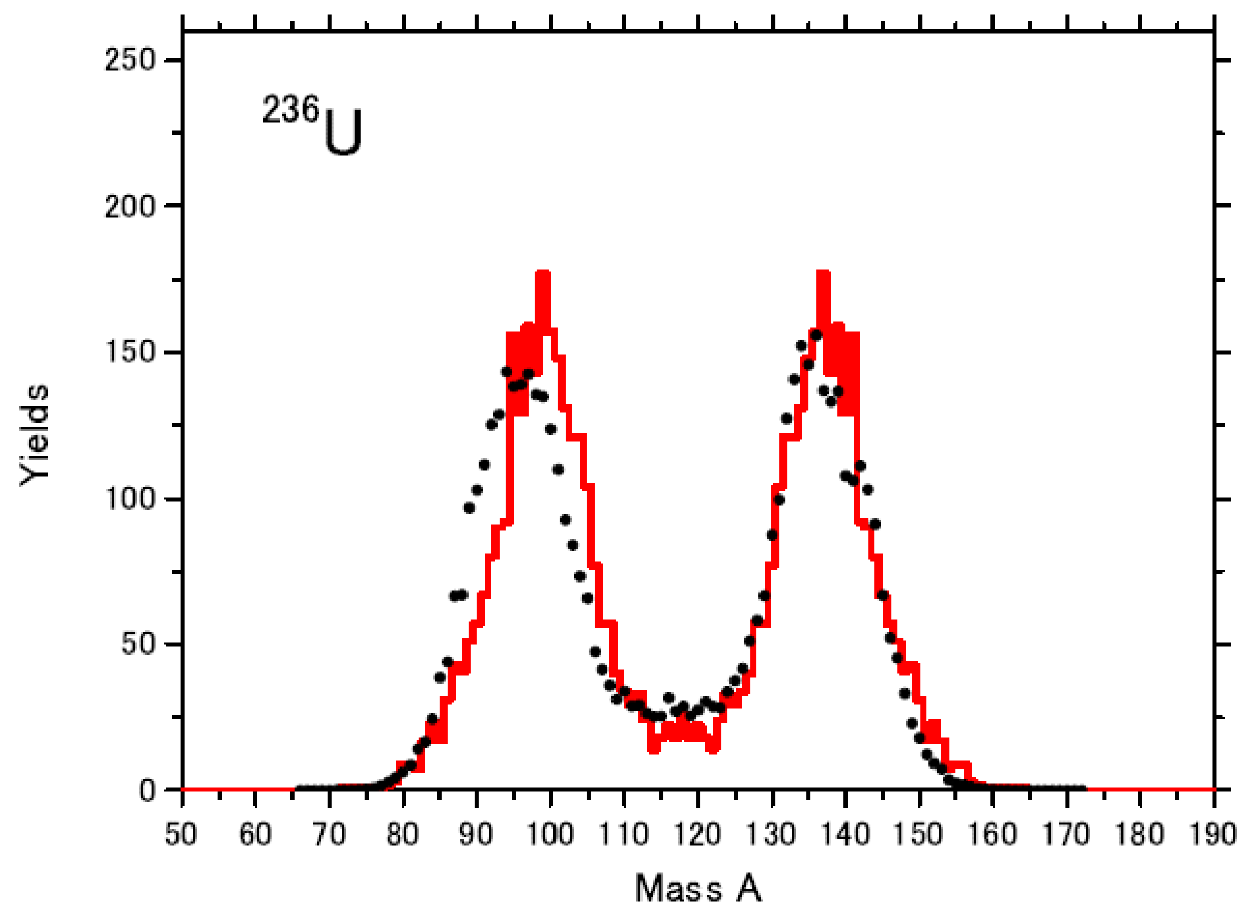
ランダム力
揺動

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

Mass distribution
of fission fragments

^{236}U Mass distribution of fission fragments $E^* = 20$ MeV

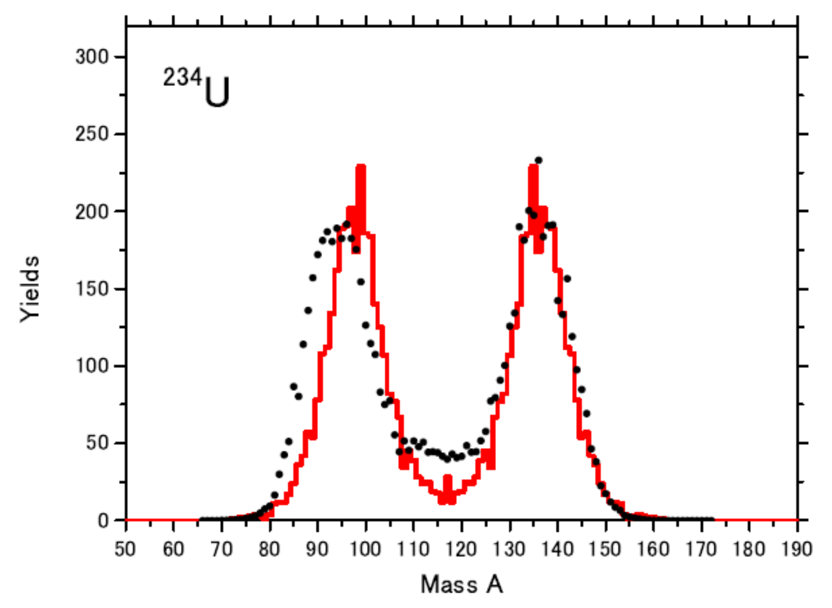


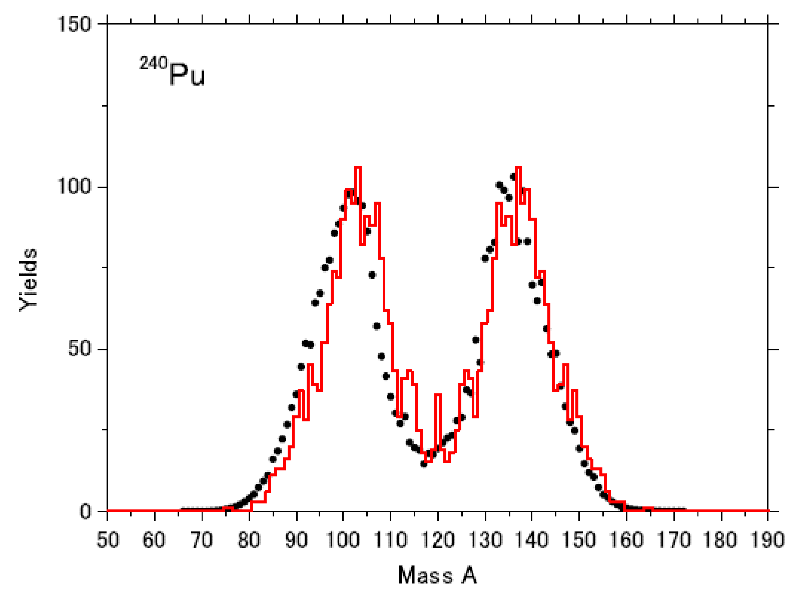
100%
Shell correction
energy

J.Katakura, JENDL FP Decay Data File 2011 and Fission Yields Data File 2011, JAEA-Data/Code 2011-025 (Mar 2012).

http://wwwndc.jaea.go.jp/cgi-bin/FPYfig

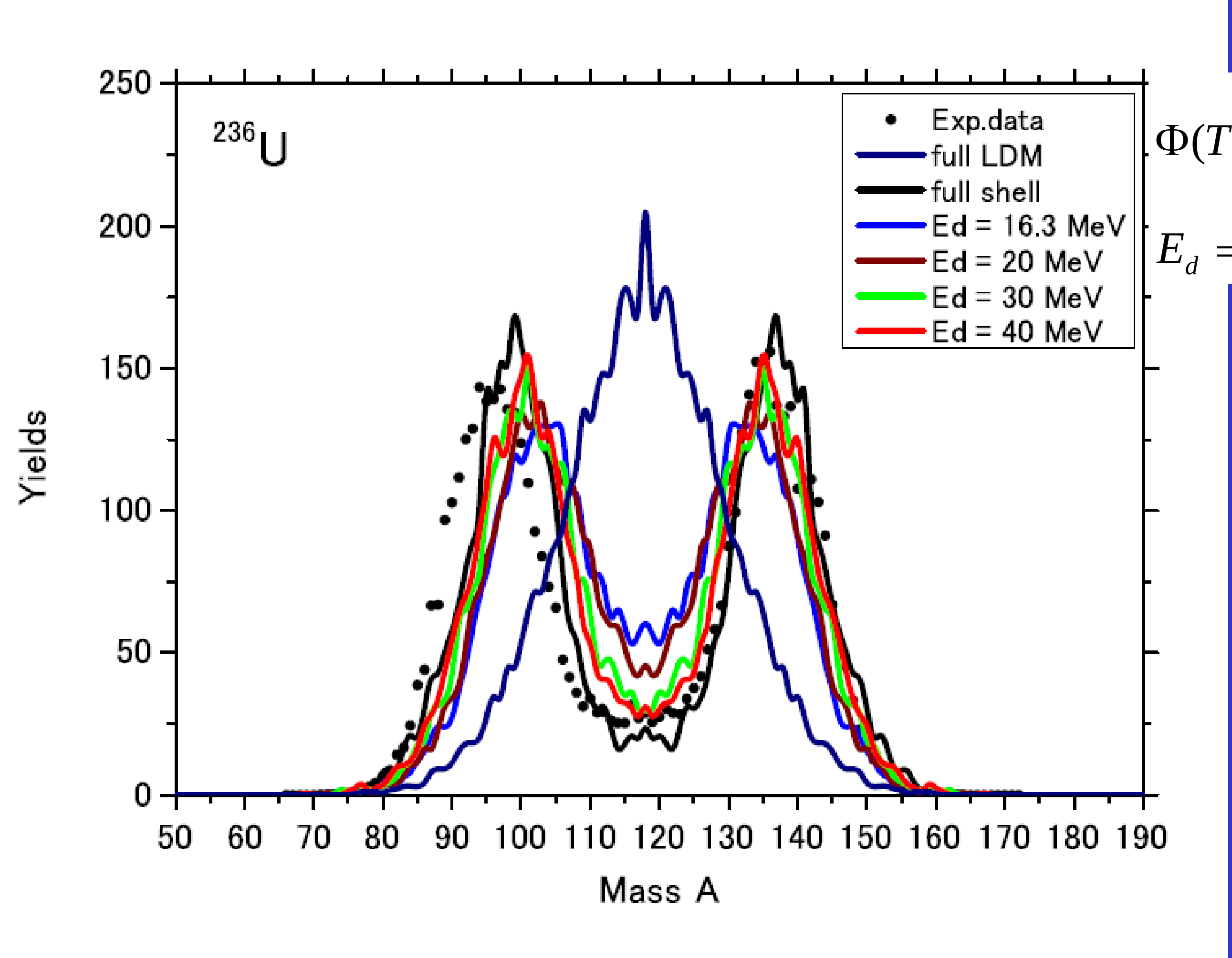
Mass distribution of fission fragments $E^* = 20$ MeV





^{236}U **Mass Distribution of Fission Fragments** $E^* = 20$ MeV

Dependence of Shell damping energy

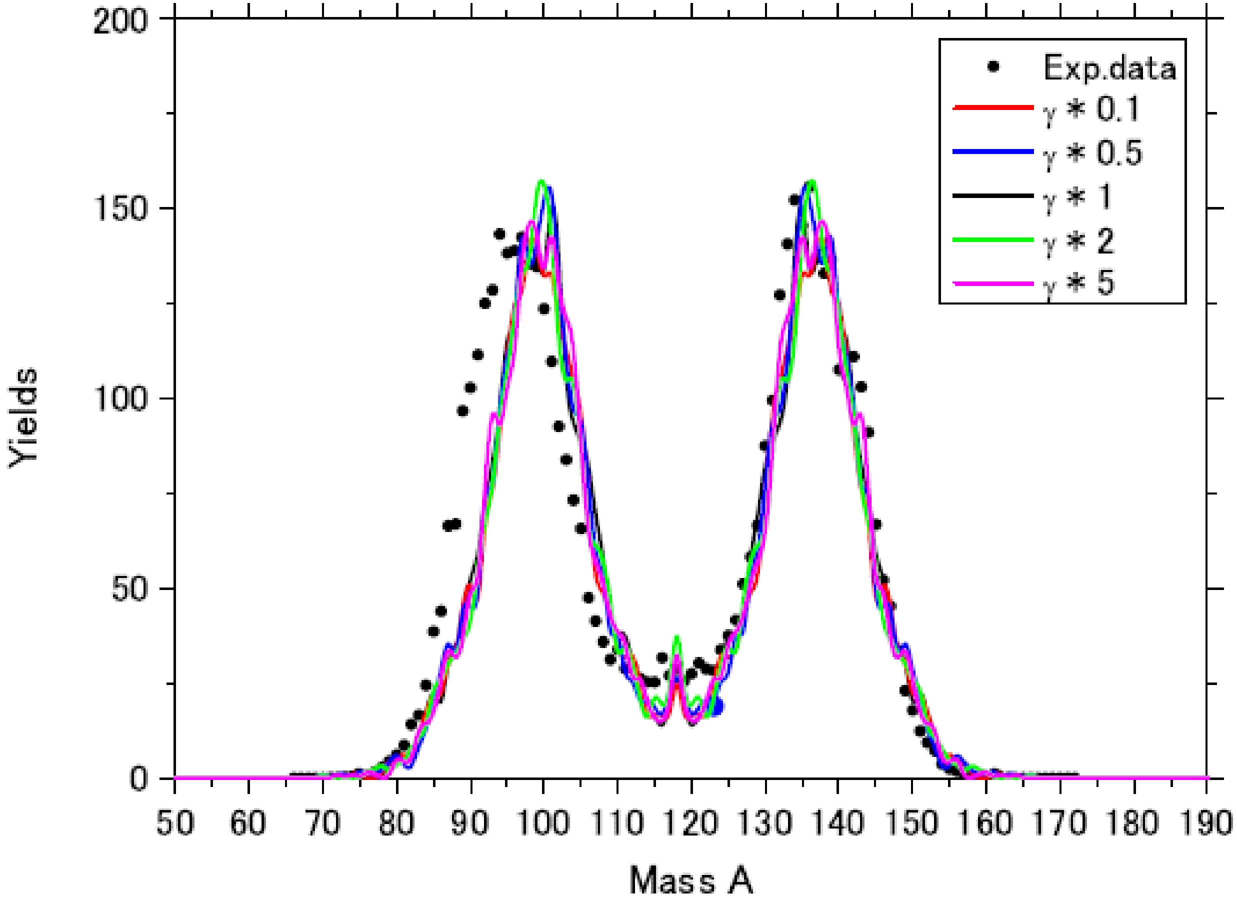


$$\Phi(T) = \exp\left\{ -\frac{aT^2}{E_d} \right\}$$

$$E_d = 20\,\mathrm{MeV}$$

^{236}U **Mass distribution of fission fragments** E* = 20 MeV

Friction -dependence



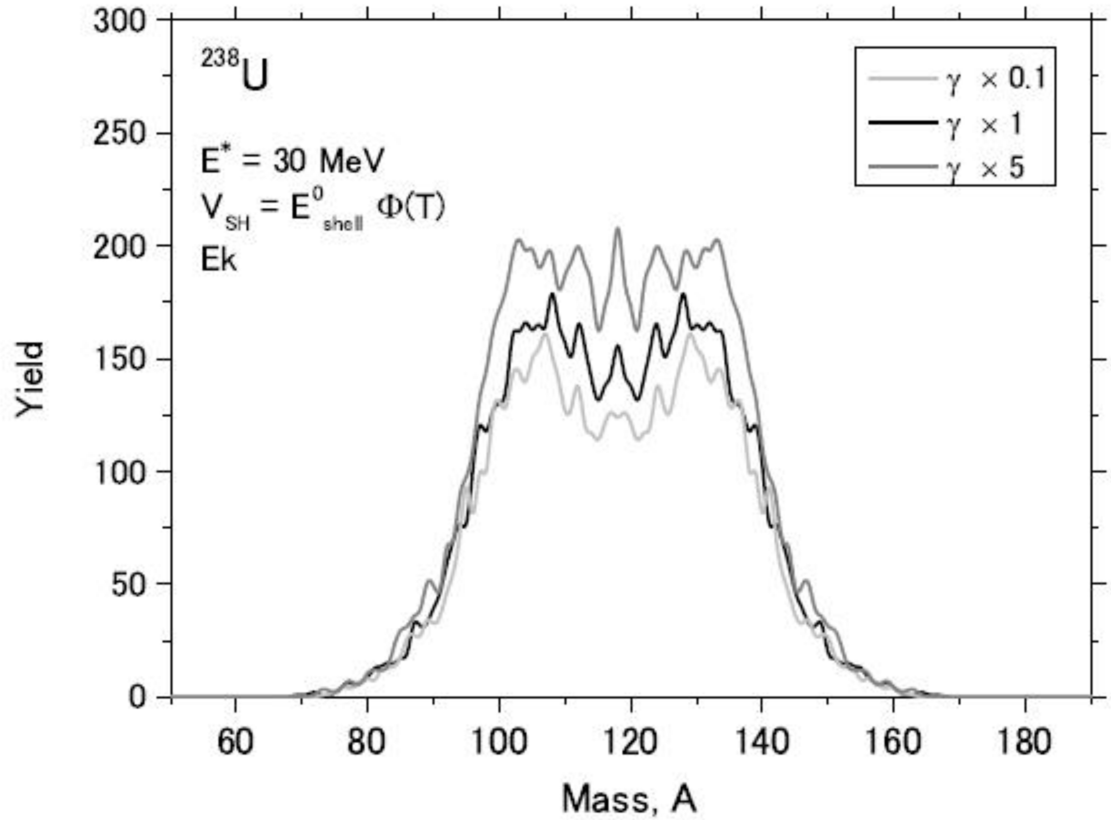
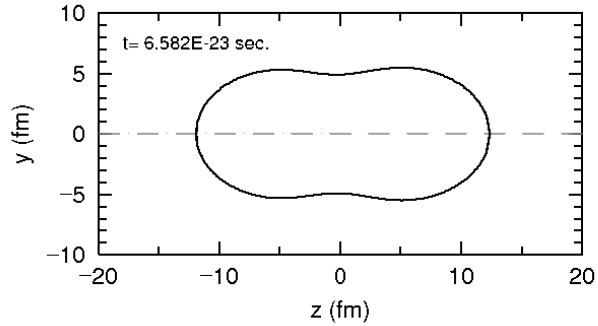
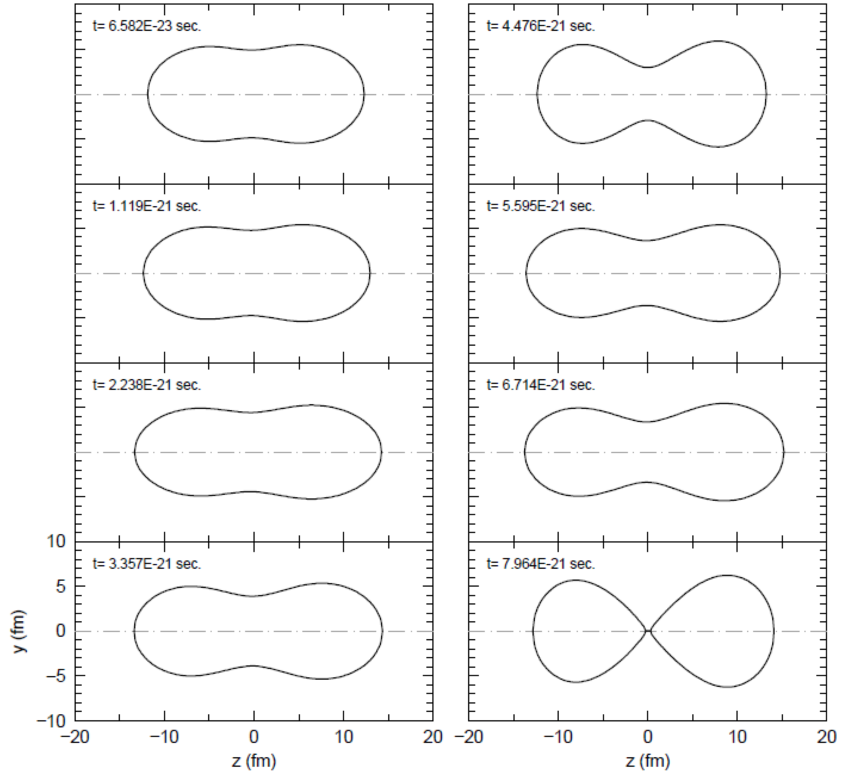


FIG. 13. MDFF of ^{236}U at $E^* = 30$ MeV for the friction tensor γ multiplied by 0.1, 1, and 5. The temperature dependence of shell correction energy is considered as $E_d = 20$ MeV. At the staring point, the system has the initial boost in the $-z$ direction.

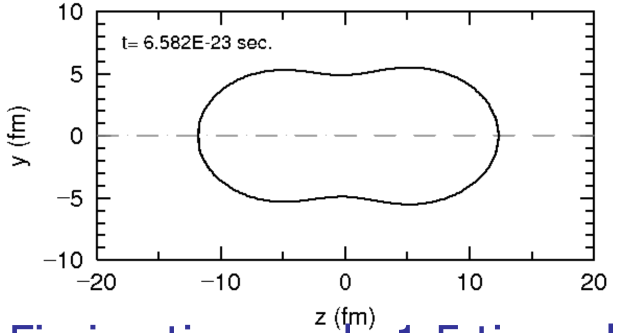
^{236}U $E^* = 20$ MeV

with random force

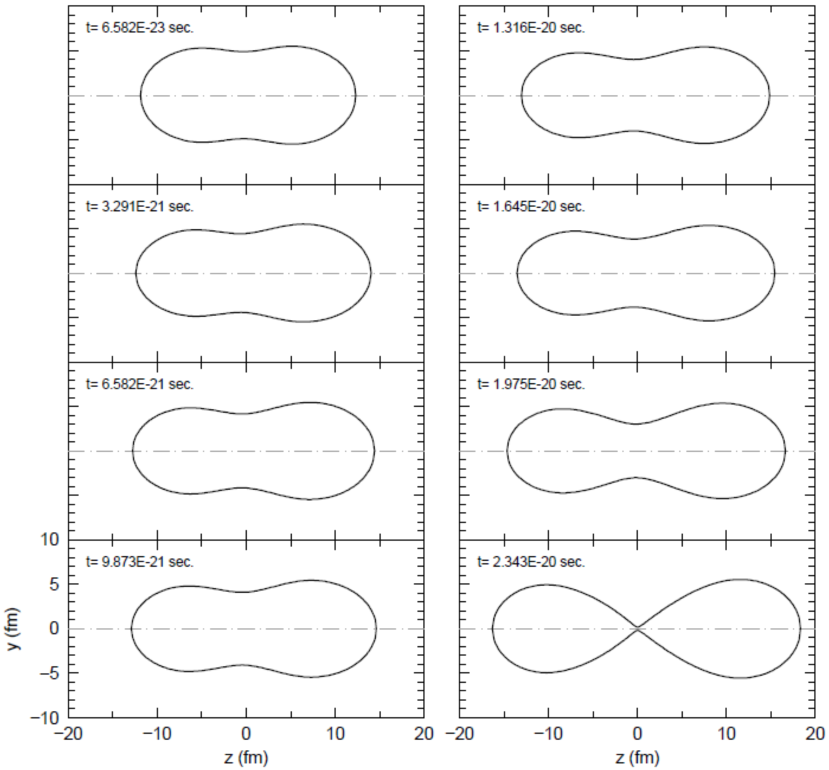




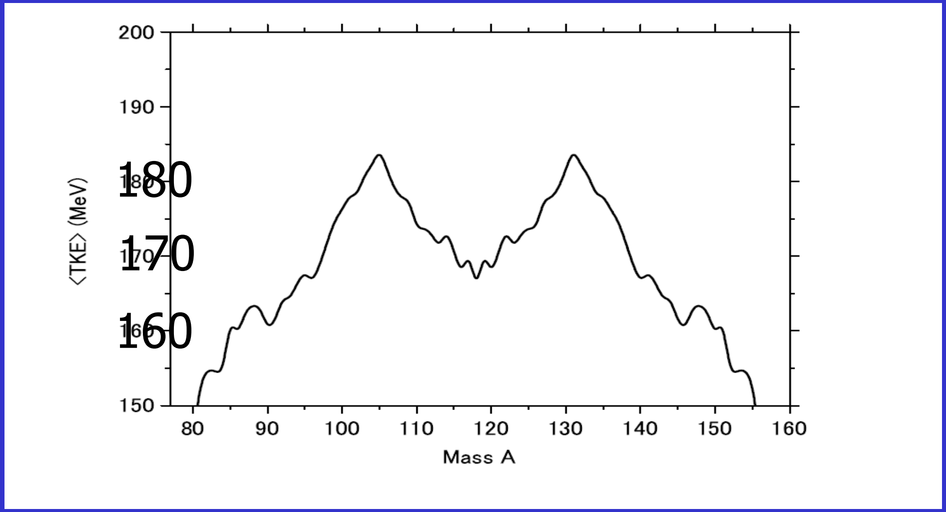
without random force



Fission time scale 1.5 times longer



^{236}U **Mass Distribution of Fission Fragments** E^{*} = 20 MeV



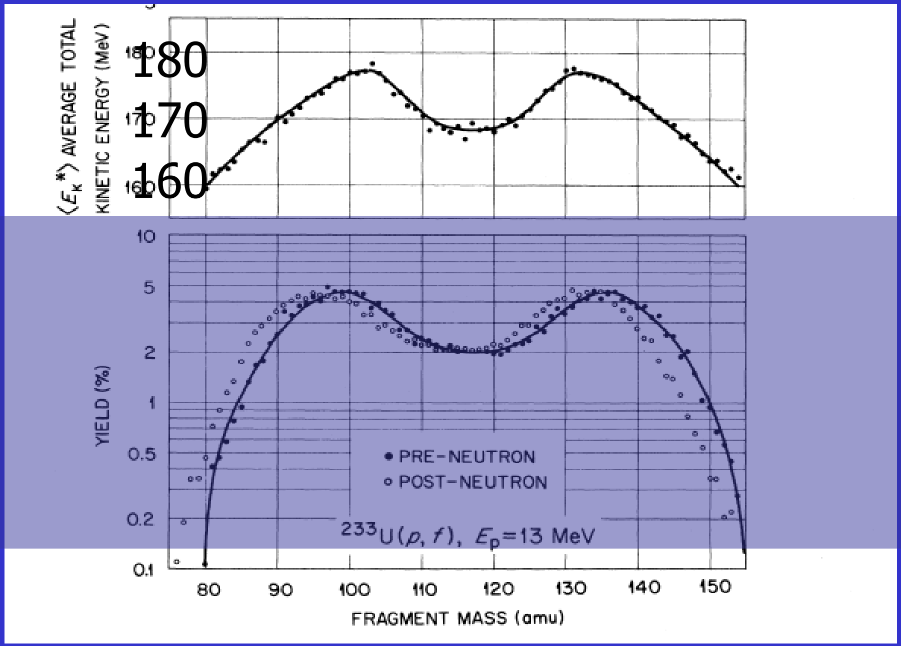
^{236}U E^{*}=20 MeV

<TKE>$_{cal}$=171.8 MeV

(=151.8 MeV ← no fluctuation)

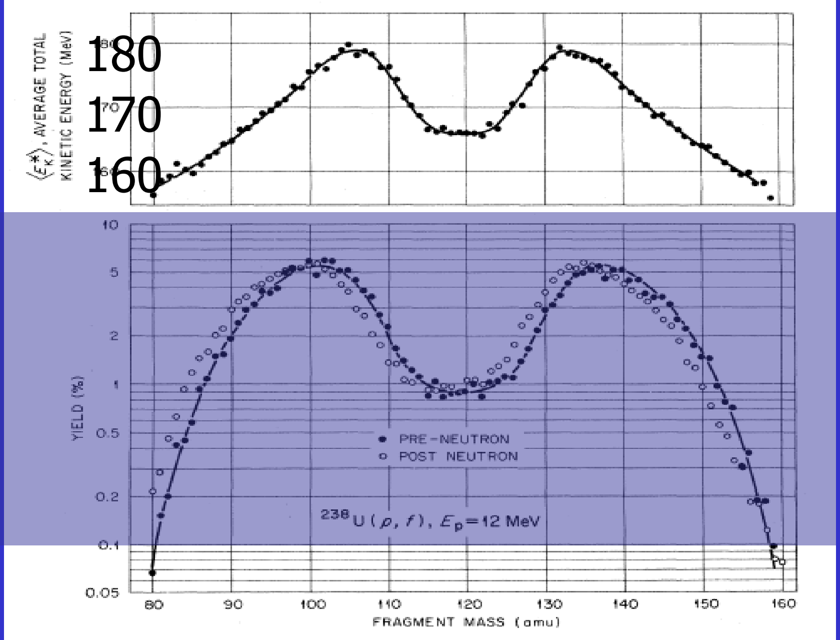
<TKE>$_{exp}$=168.2~171.3 MeV
(E*=21 MeV)

^{233}U

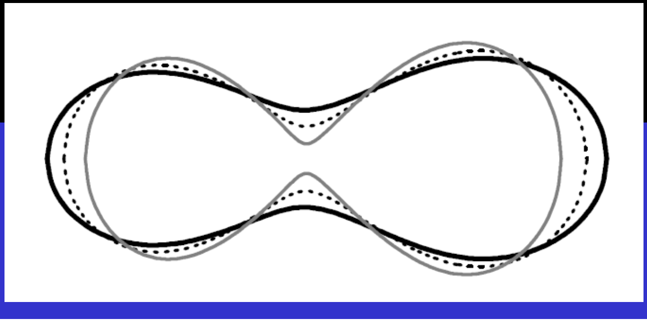


^{238}U

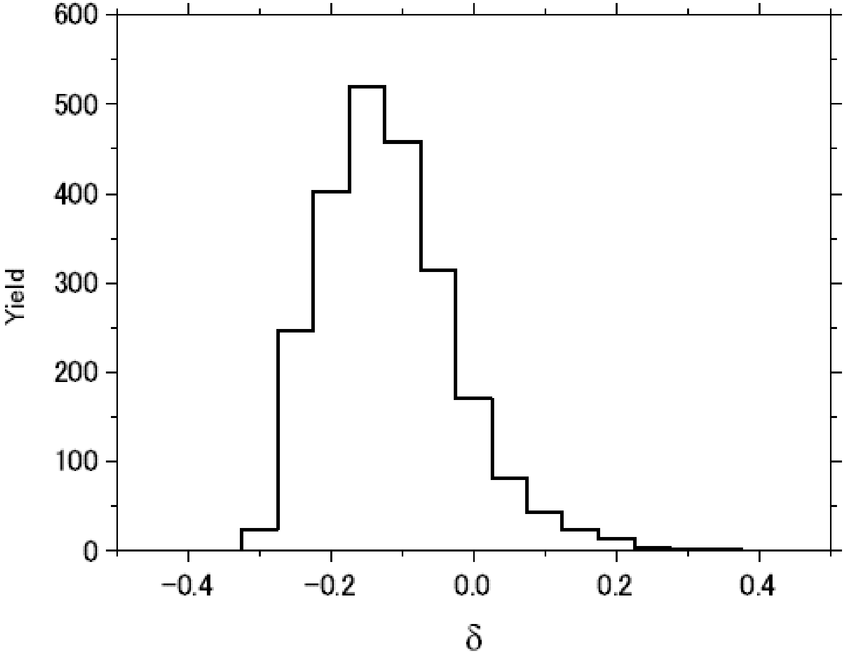
S.C. Burnett et. al., Phys. Rev. C 3, 2034 (1971)

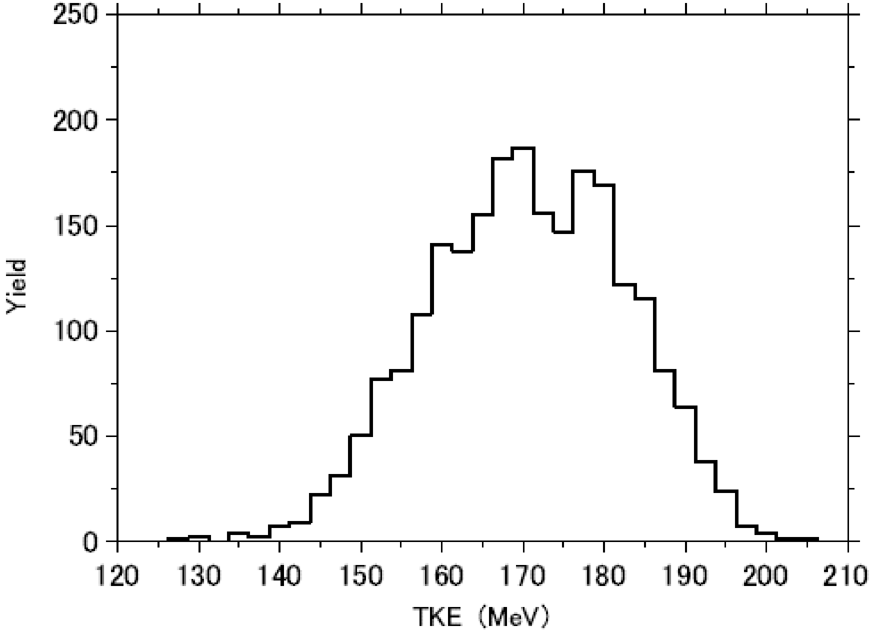


^{236}U   E* = 20 MeV



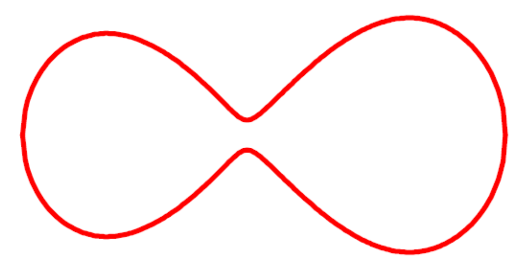
Fission fragments





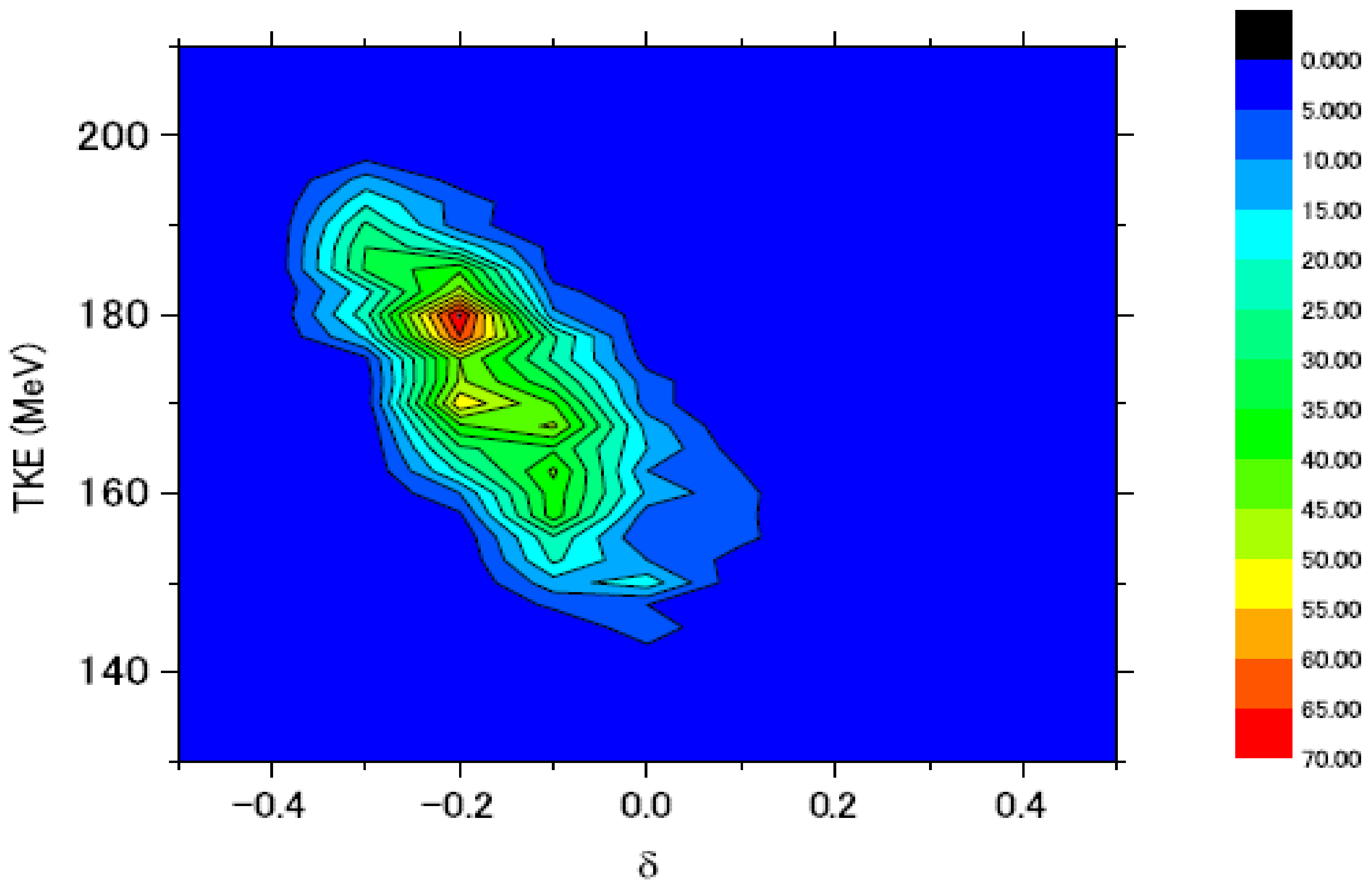
δ distribution

TKE distribution



$\delta = -0.2$

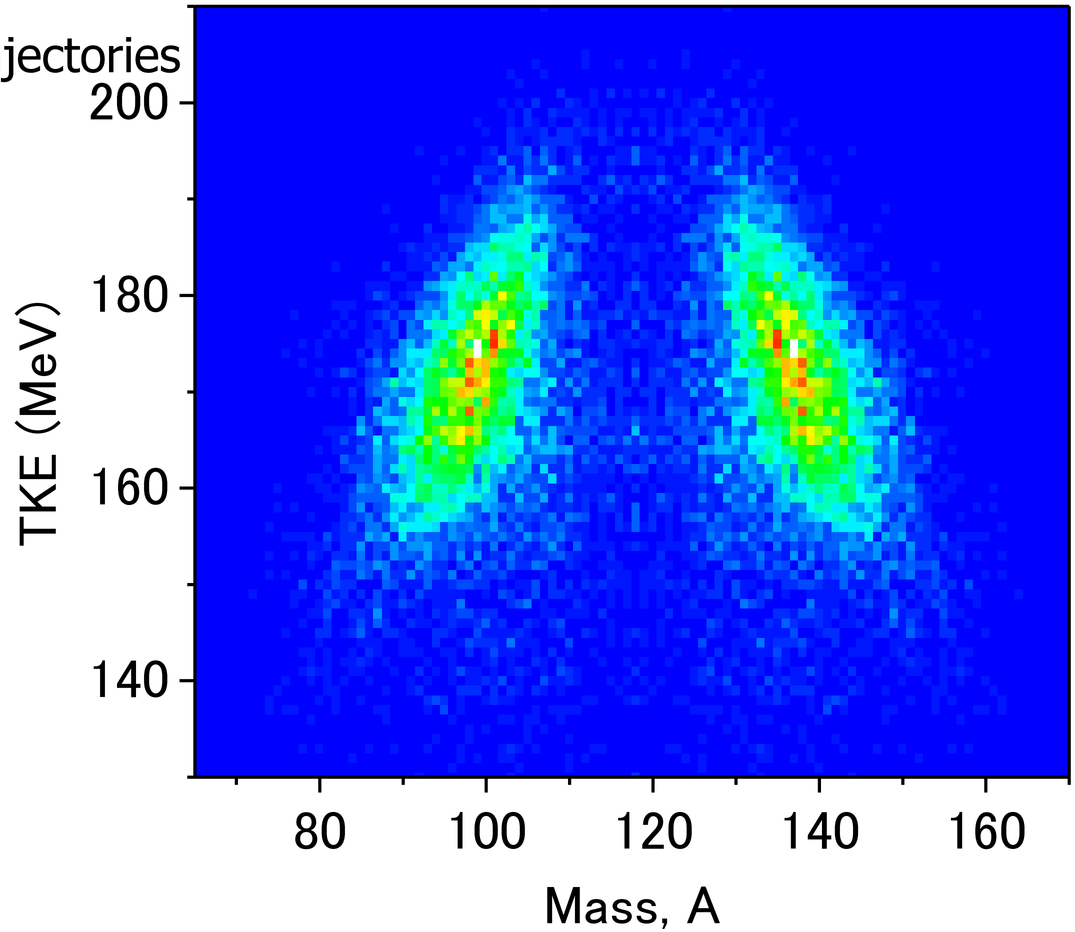
$^{236}U \quad E^{*} = 20$ MeV



^{236}U E* = 20 MeV Mass-TKE distribution

Calculation with Langevin equation

100,000 trajectories



4. Summary

1. Development of calculation method for independent fission yields

2. Dynamical calculation

   Langevin equation and two-center shell model

3. Mass distribution of fission fragments (^{234,236}U, ^{240}Pu)

4. Charge distribution and total kinetic energy distribution of fission fragments

5. Independent fission yields

Further study

improve the model to decrease the difference between the calculation and Exp.

increase the number of variables δ1 and δ2

estimation of nuclear transfer rate

taking into account neutron emission

....AND....

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^{236}U   E*=20 MeV