54th ASRC International Workshop Sakura-2019:
Nuclear Fission and Structure of Exotic Nuclei

JAEA, Tokai, Japan, 25-27 March 2019

Fission studies with microscopic level densities
I: Fragment mass distributions and kinetic energies

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in collaboration with

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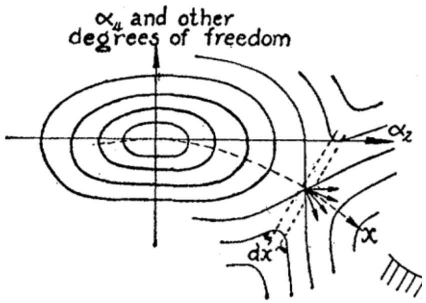
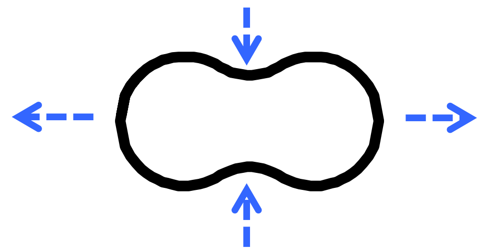
Fission shape dynamics



Bohr & Wheeler 1939

Time evolution of the nuclear shape parameters, $\chi(t)$:



3QS (Nix 1969):

Elongation Q_2

Neck radius c

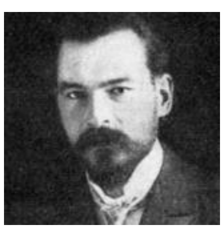
Defs ε_{f1} & ε_{f2}

Asymmetry α

Paul Langevin
(1872-1946)

Langevin equation: $d\pi/dt = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

$\boldsymbol{M}(\chi)$	$U(\chi)$	$\gamma(\chi)$
inertial tensor	potential energy	dissipation tensor

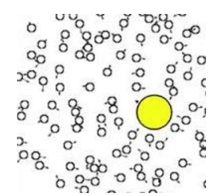


Marian Smoluchowski
(1872 – 1917)

The shape motion is *highly dissipative* => Brownian motion

Smoluchowski equation: $\boldsymbol{0} = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

[1st application: Kramers 1940] $U(\chi)$ $\gamma(\chi)$



Phase space dominance: Metropolis walk on $U(\chi)$

J. Blocki *et al.* (1978)

A. Bulgac *et al.* (2018)

Macroscopic:

Wall formula

$$\dot{Q} = \rho \bar{v} \oint V^2 d\sigma$$

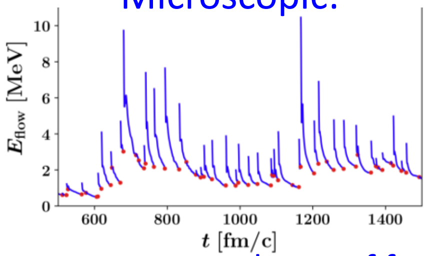
$t_{damp} \approx$ 20-40 fm/c

The nuclear *shape evolution* …

… is *highly dissipative* …

… so it is akin to *Brownian motion*

Microscopic:



$t_{damp} \approx$ several tens of fm/c

Phase space dominance:

$\nu(\chi \rightarrow \chi') \sim \rho(\chi')$

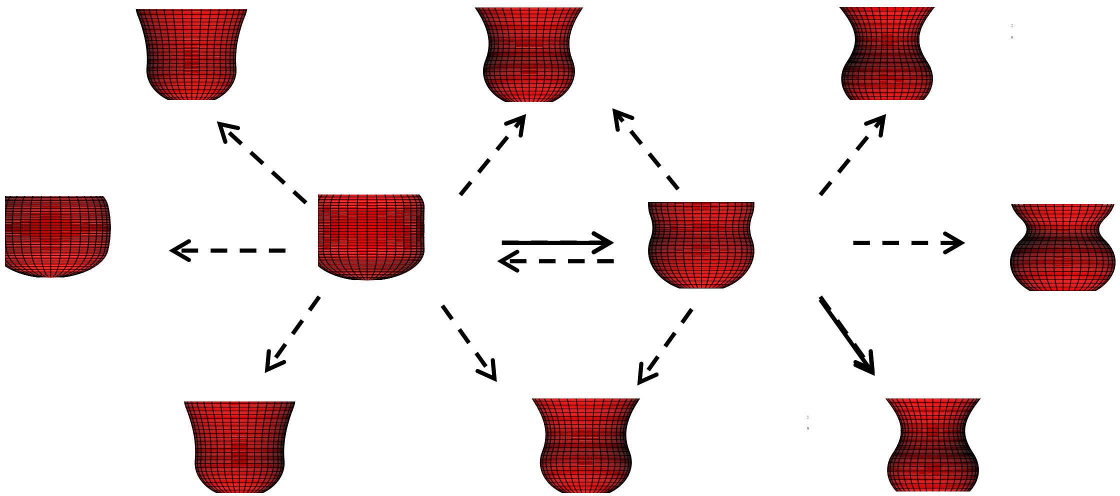
=>

The fission evolution can be simulated by *random walks* among the shapes …

Metropolis:

$P(\rho' \geq \rho) = 1$

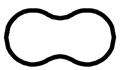
$P(\rho' \leq \rho) = \rho'/\rho$



… generating a *ensemble of paths* through the space of shapes

Calculation of the shape-dependent microscopic nuclear level density

H. Uhrenholt, S. Åberg, A. Dobrowolski, Th. Døssing, T. Ichikawa, P. Möller: NPA **913** (2013) 127



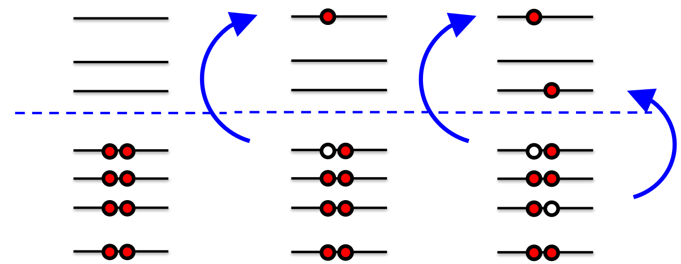
$$\rho(E,I,\pi) = \frac{1}{\Delta E}\int_{E}^{E+\Delta E} \sum_i \delta(E' - E_i(I,\pi))dE' \quad => \quad \rho(E) = \sum_{I\pi} \rho(E,I,\pi)$$

$$E = \mathbb{E}_p + \mathbb{E}_n + E_{rot}$$

Intrinsic states:

Consider all multiple p-h excitations for protons and neutrons separately

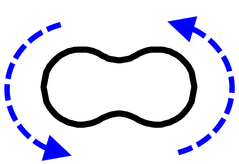
$$|i\rangle = \prod_{\alpha=1}^{n} a^{+}_{\nu_\alpha} a_{\nu'_\alpha} |0\rangle,$$



Pairing

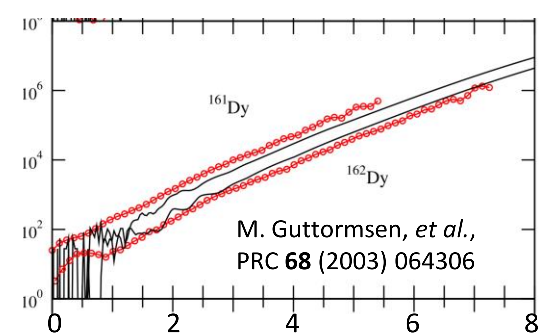
Calculate BCS pairing for each state: $\mathbb{E}_p$ & $\mathbb{E}_n$

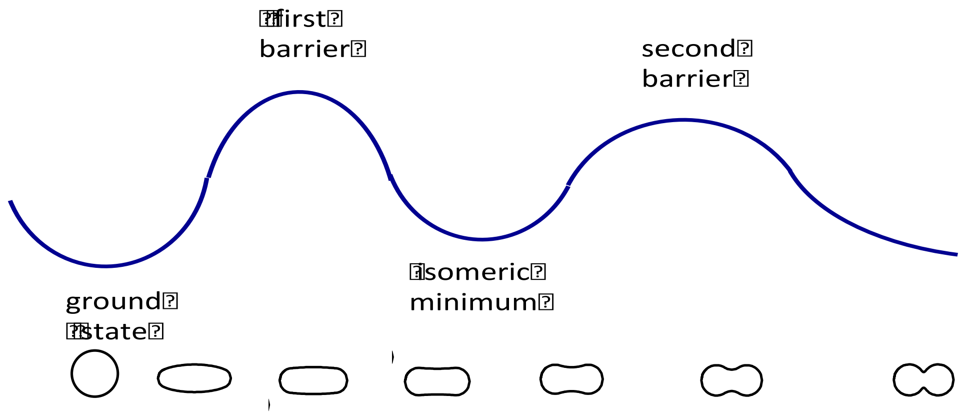
Rotation



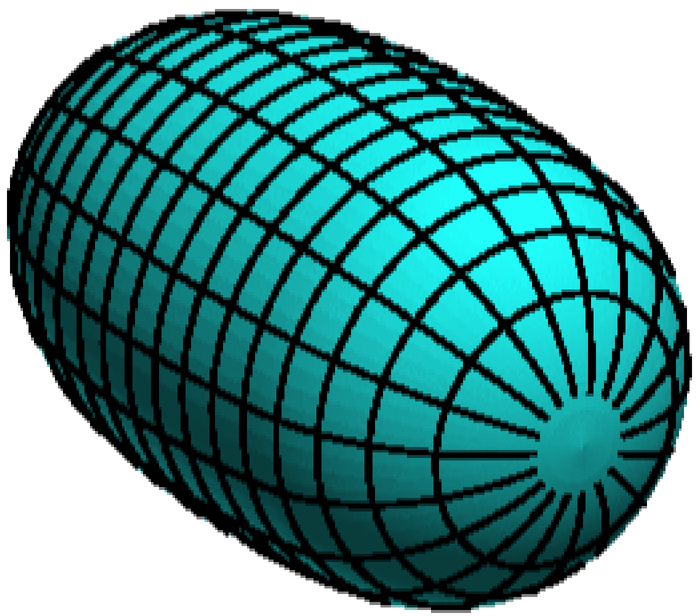
Rotational band built on each intrinsic state: E_{rot}

$$E_{rot}(I,K) = [I(I+1)-K^2]/2\mathbb{I}_{perp}(\chi,\Delta_p,\Delta_n)$$



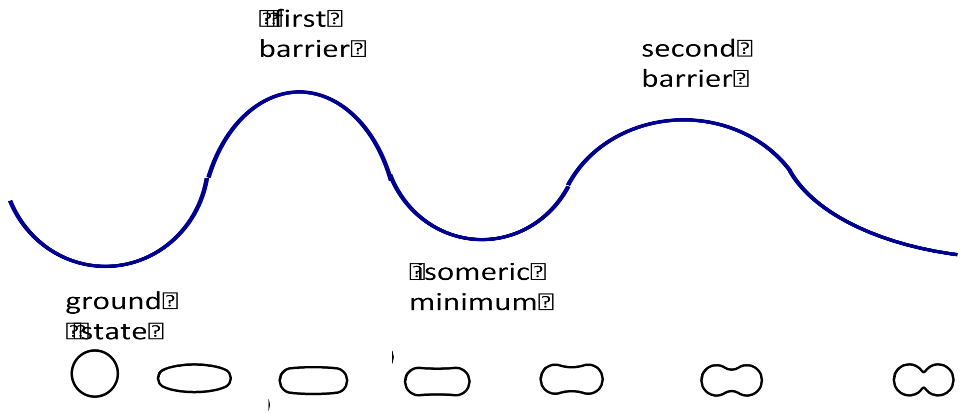


$^{240}Pu^* \longrightarrow {}^{98}Sr + {}^{142}Ba$



$\nu(\chi \to \chi') \sim \rho(\chi')$

Animation by Martin Albertsson



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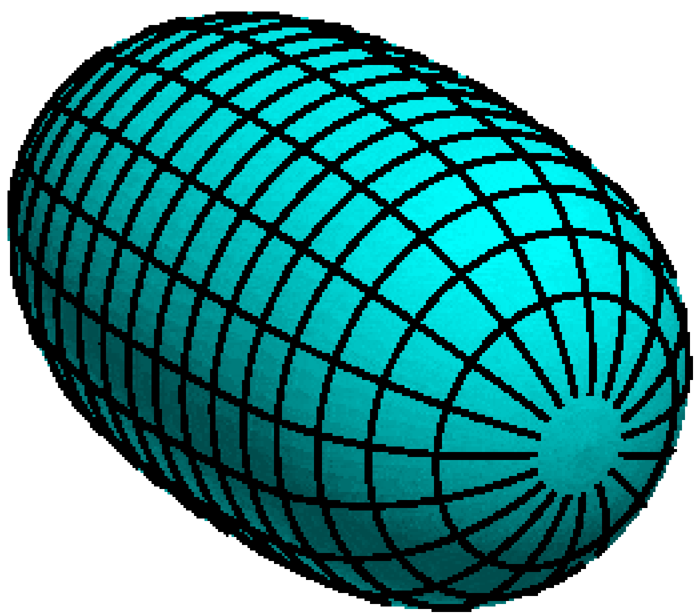

$\nu(\chi \to \chi') \sim \rho(\chi')$

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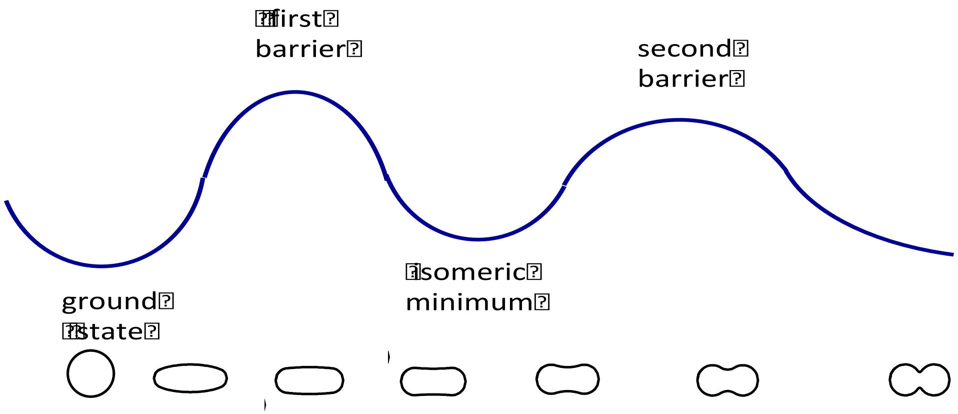
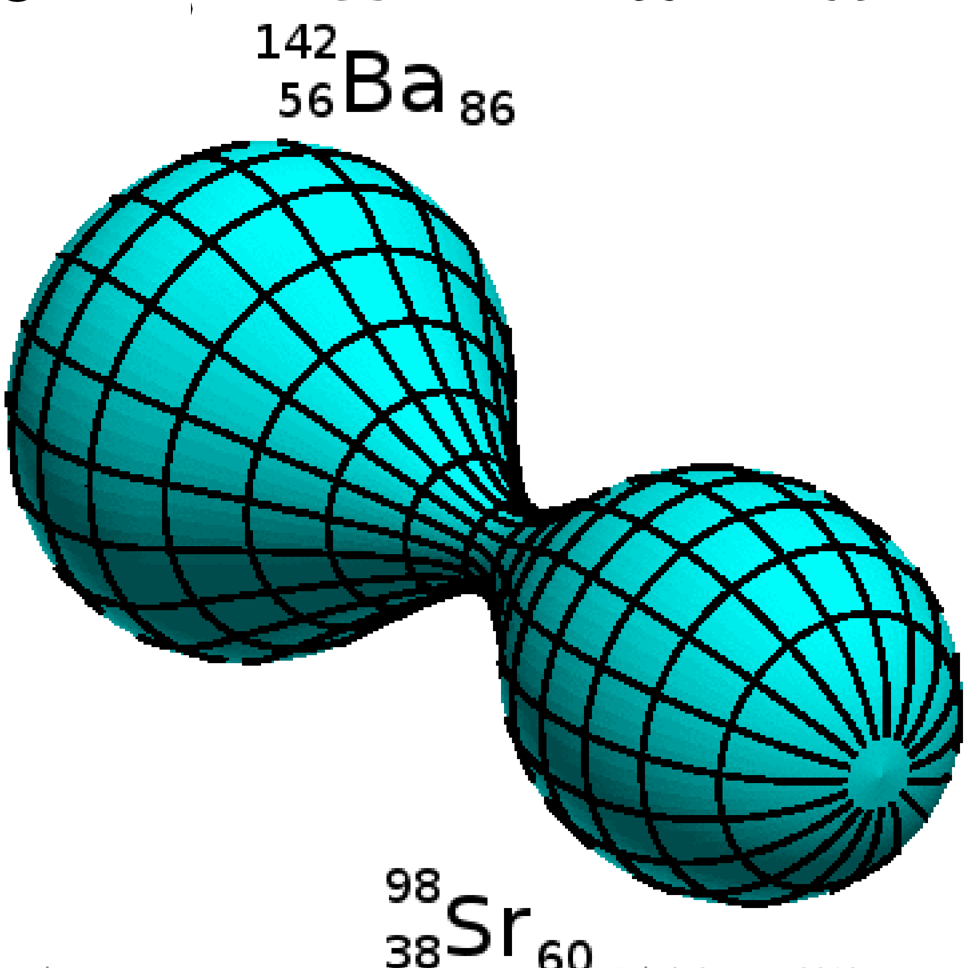




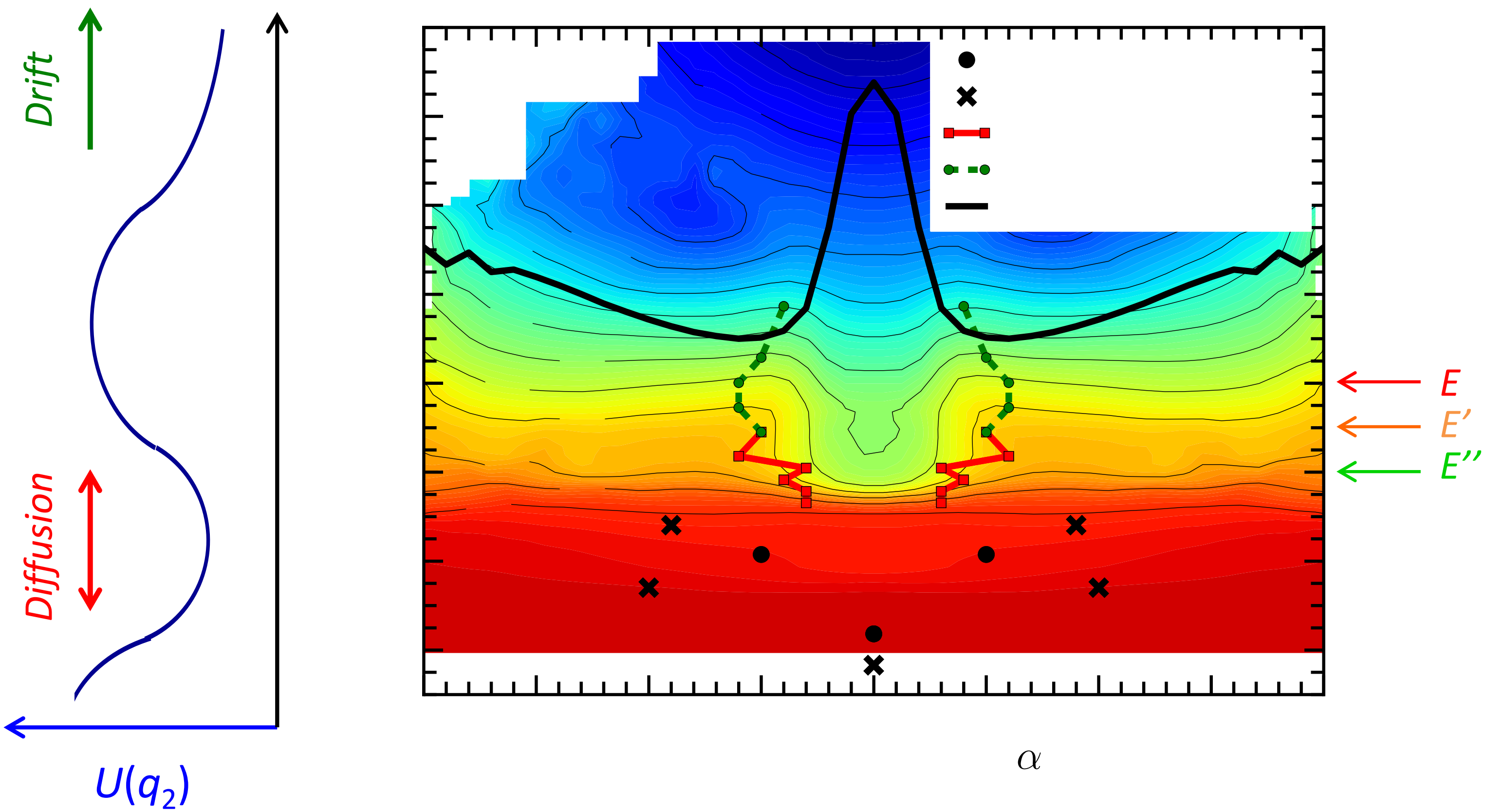
$^{240}Pu^* \rightarrow {}^{98}Sr + {}^{142}Ba$

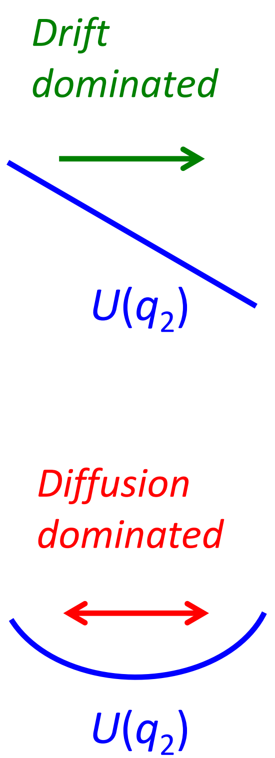
$\nu(\chi \rightarrow \chi') \sim \rho(\chi')$

Animation by Martin Albertsson

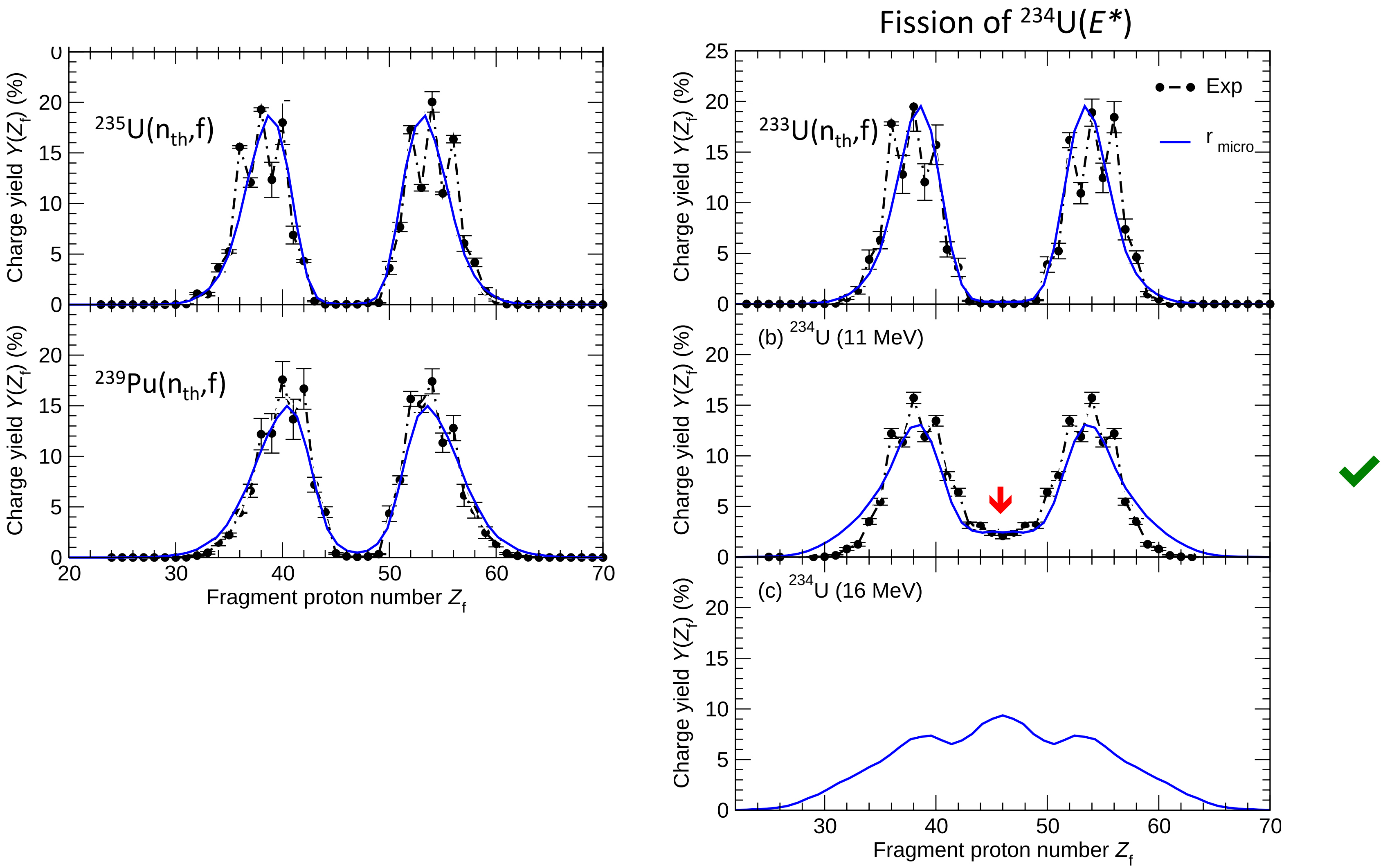
Fission of ^{236}U: Equilibration during the random walk

Contour plot of $\Sigma_{JKL}\, n_{IJKLM} / \Sigma_{JKL}\, \rho_{IJKLM}$

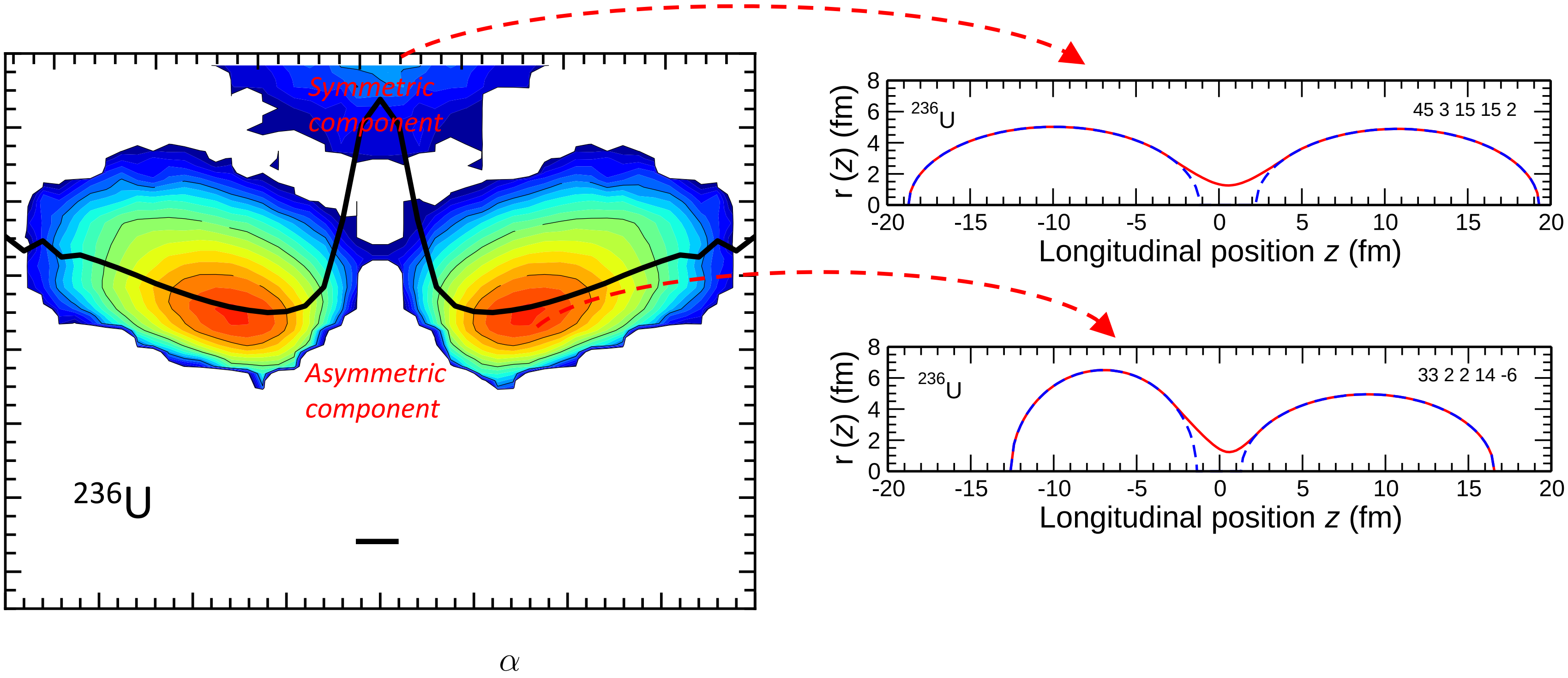




Mass yields using microscopic level densities

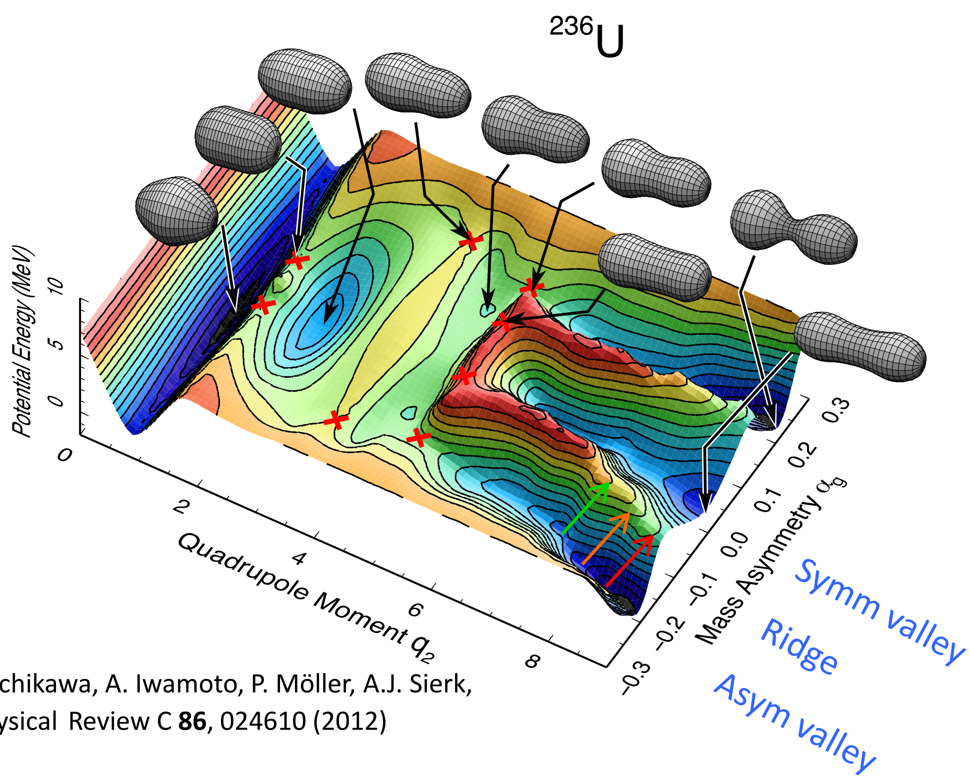


Scission shapes (c_0 = 1.5 fm)



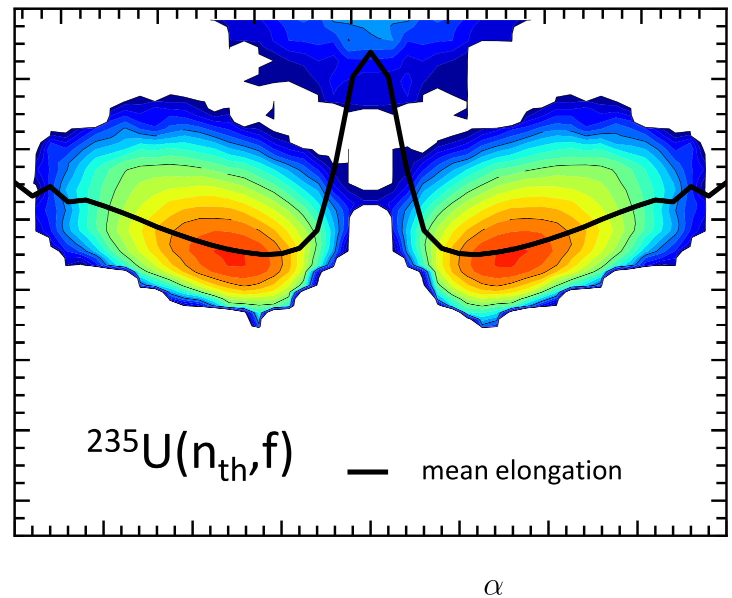
Fission of ^{236}U:

Potential-energy surface



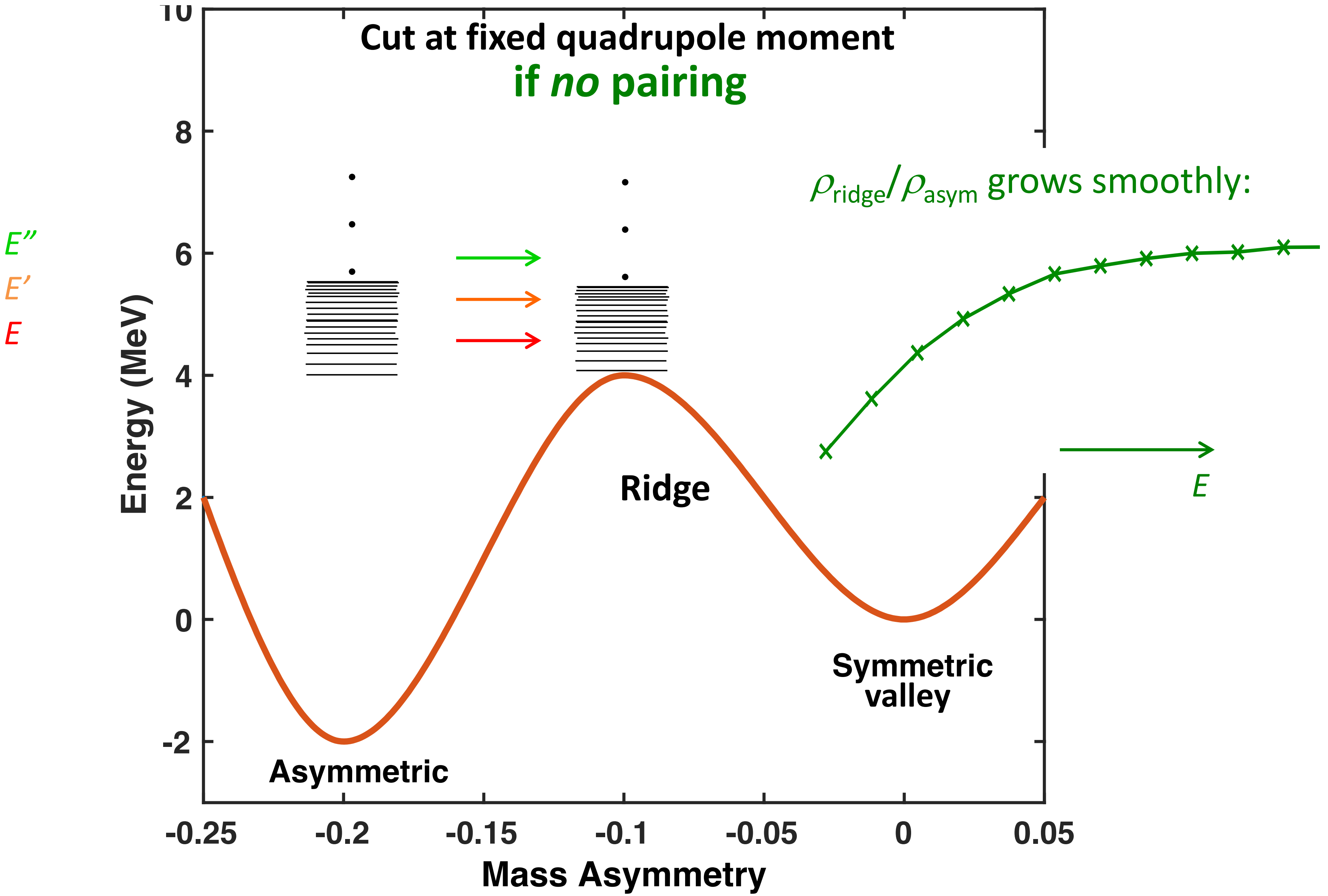
T. Ichikawa, A. Iwamoto, P. Möller, A.J. Sierk, Physical Review C **86**, 024610 (2012)

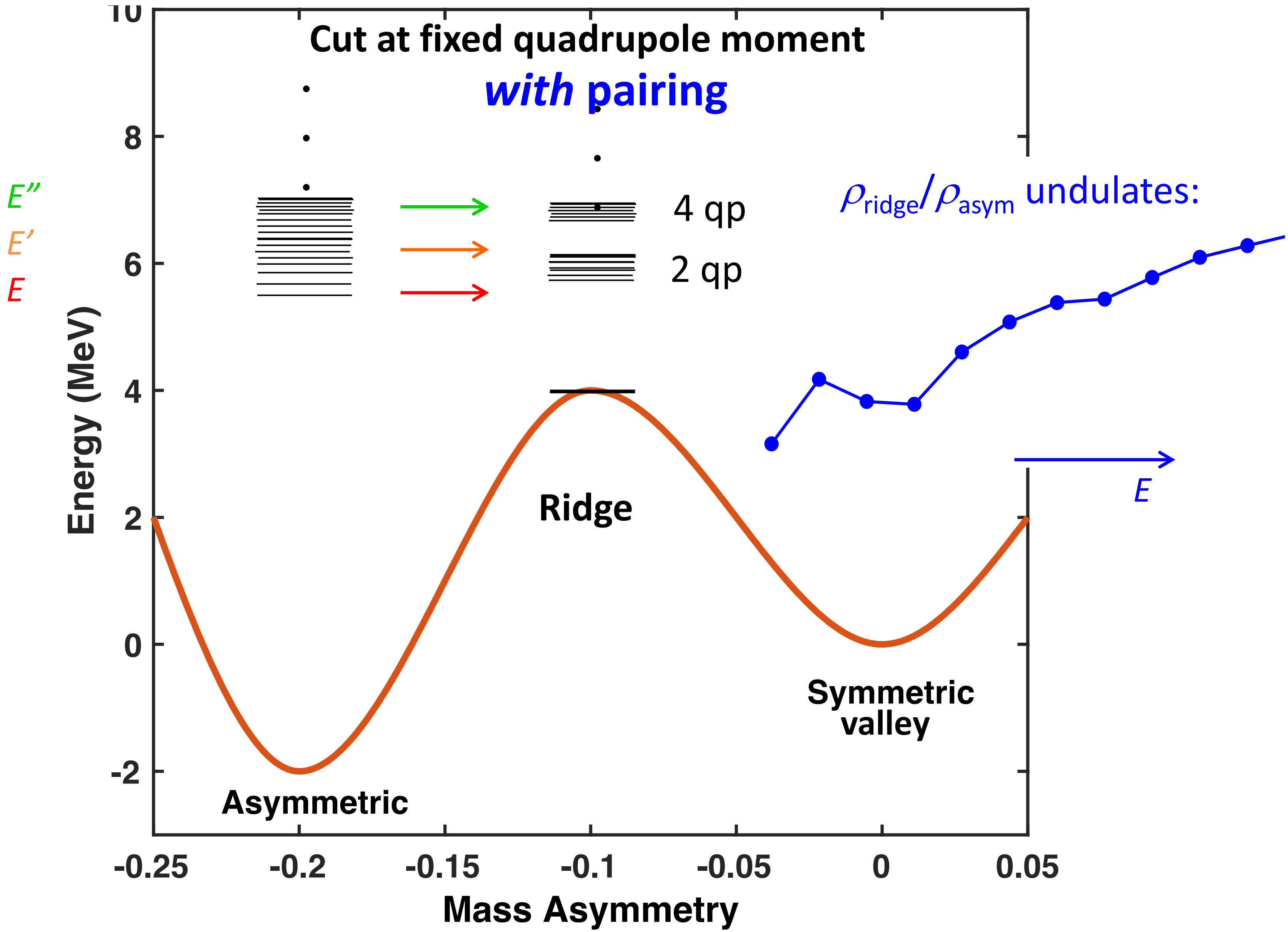
Scission shapes



Shapes in the *valleys* have dilute single-particle spectra, hence negative shell energies and small pairing gaps

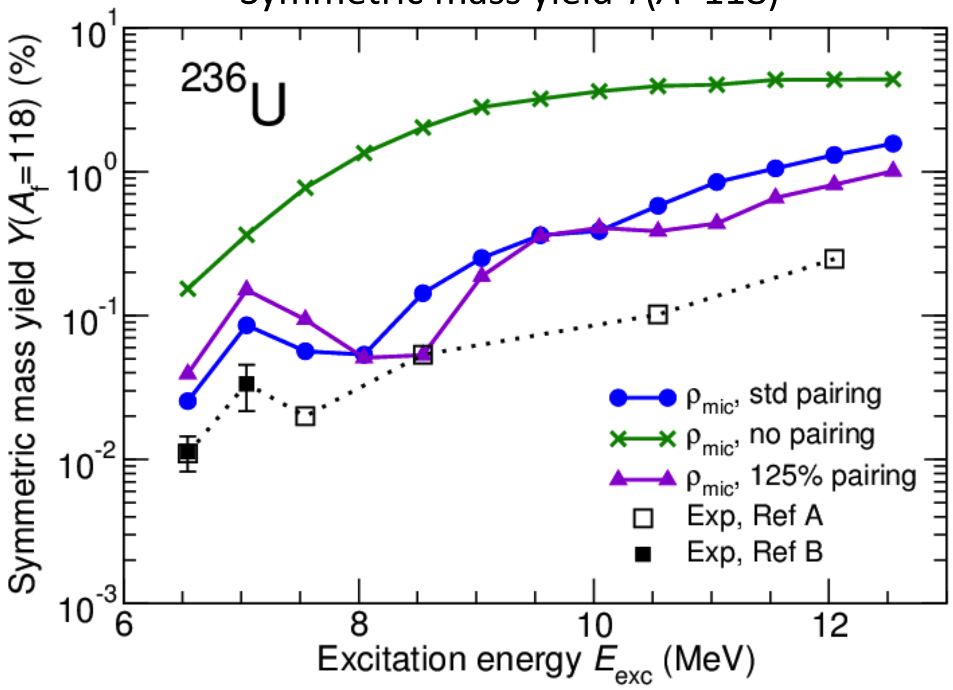
Shapes along the *ridge* have dense single-particle spectra, hence positive shell energies and large pairing gaps


Cut at fixed quadrupole moment
if *no* pairing
ρ_{ridge}/ρ_{asym} grows smoothly:
E''
E'
E
E
Ridge
Symmetric valley
Asymmetric
Energy (MeV)
Mass Asymmetry
10
8
6
4
2
0
-2
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05


Cut at fixed quadrupole moment
with pairing
E''
E'
E
4 qp
2 qp
ρridge/ρasym undulates:
E
Ridge
Symmetric valley
Asymmetric
Energy (MeV)
10
8
6
4
2
0
-2
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
Mass Asymmetry

Energy dependence of fission yields: *Non-monotonic* behavior of the symmetric yield

Symmetric mass yield *Y*(*A*=118)



The behavior is non-monotonic!

A) L. E. Glendenin *et al.*, Phys. Rev. C **24**, 2600 (1981)

B) M. B. Chadwick *et al.*, Nucl. Data Sheets **112**, 2887 (2011)

OBS:

C) M. E. Gooden *et al.*, Nucl. Data Sheets **131**, 319 (2016): *Non-monotonic* energy dependence of $Y(A_f)$ from $^{240}Pu^*$

Scission shapes

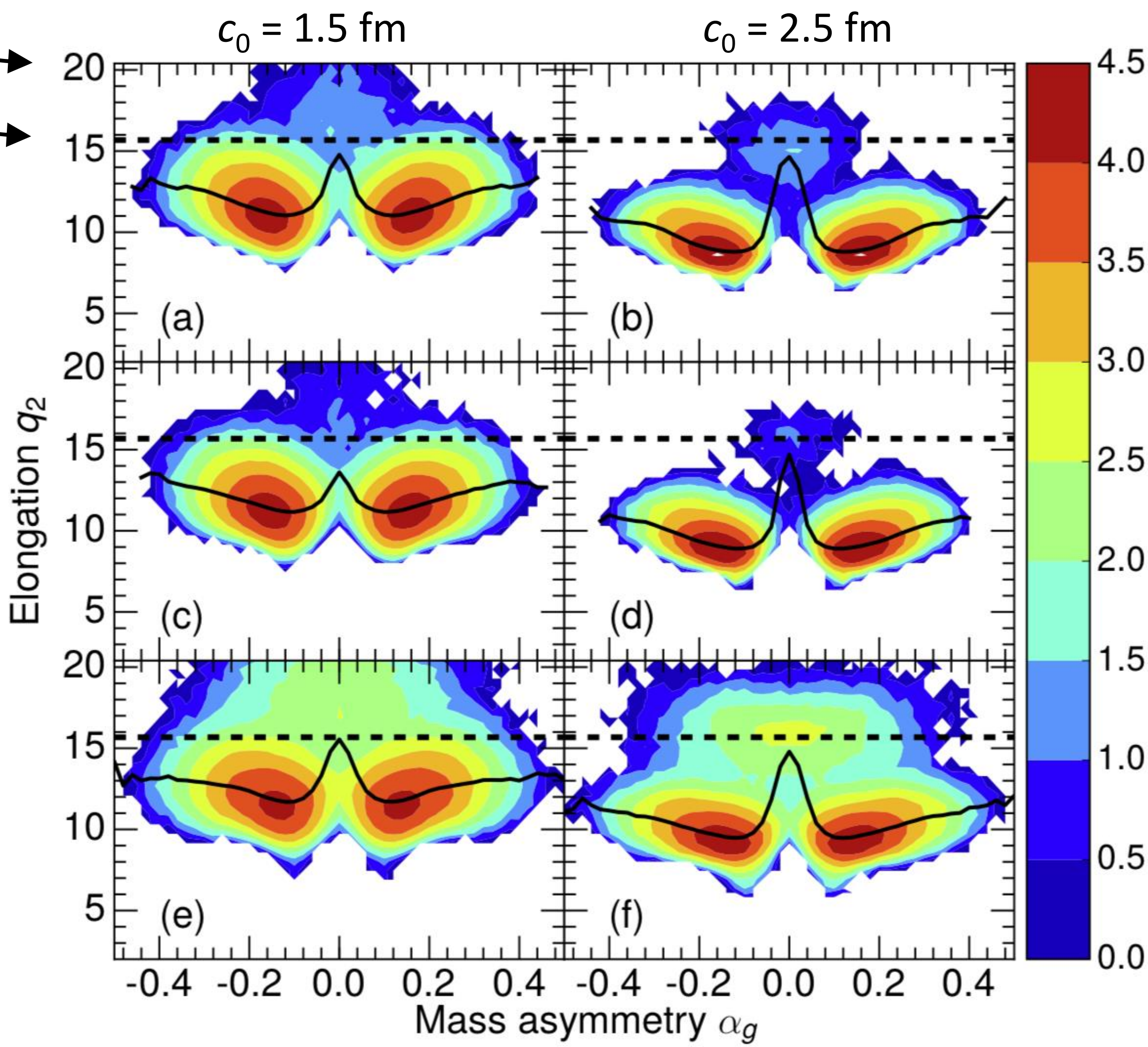
Extended shape lattice

Standard

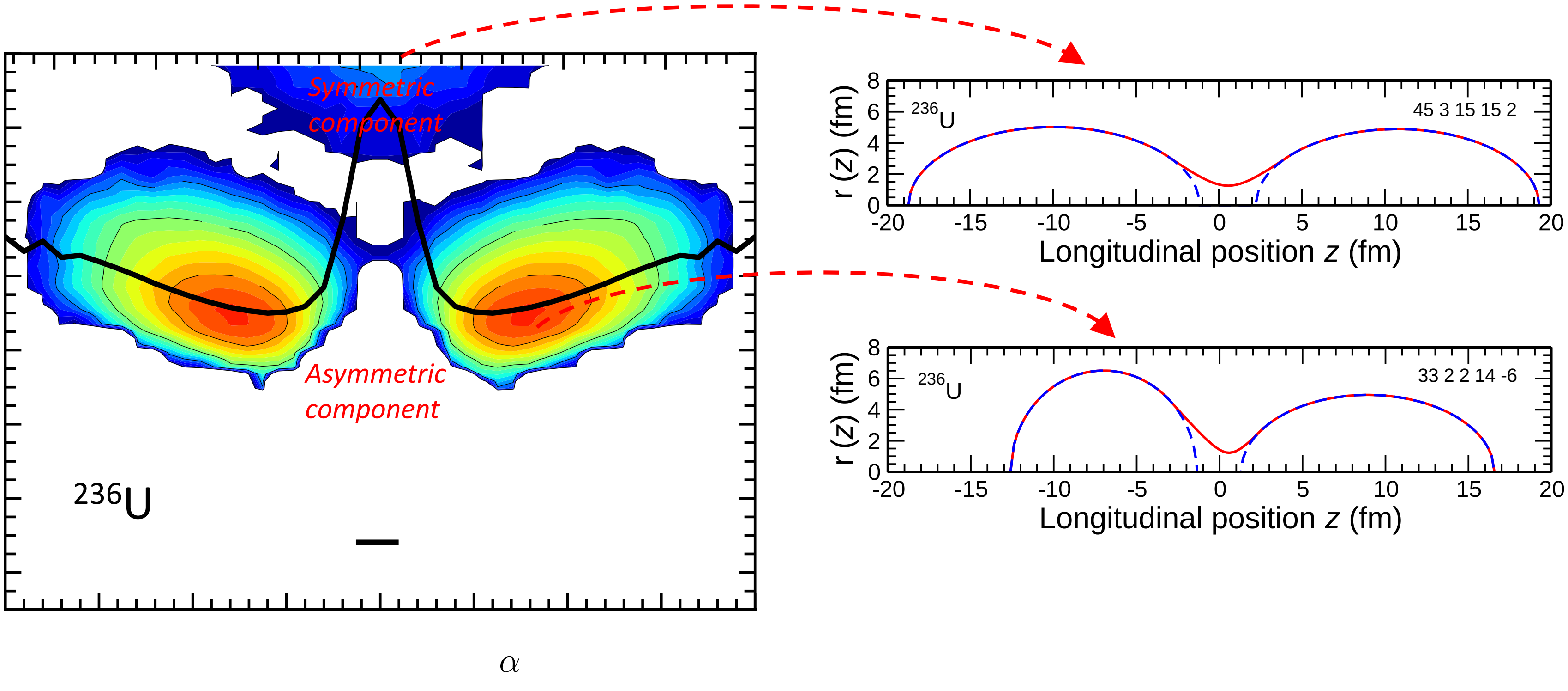
$^{233}U(n_{th},f)$

$^{235}U(n_{th},f)$

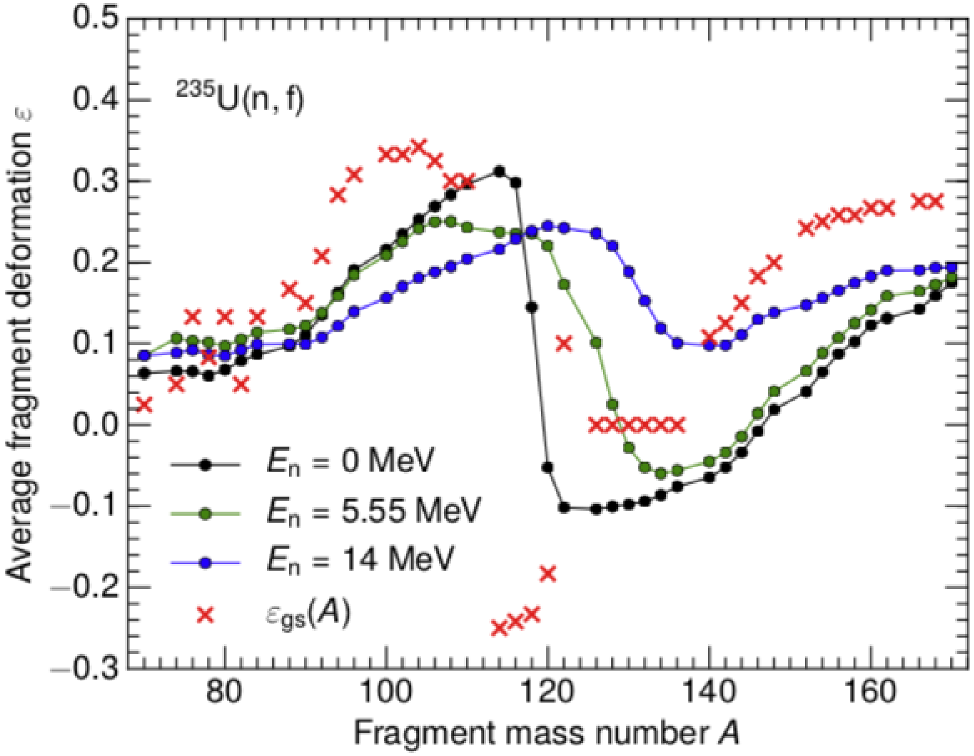
$^{239}Pu(n_{th},f)$

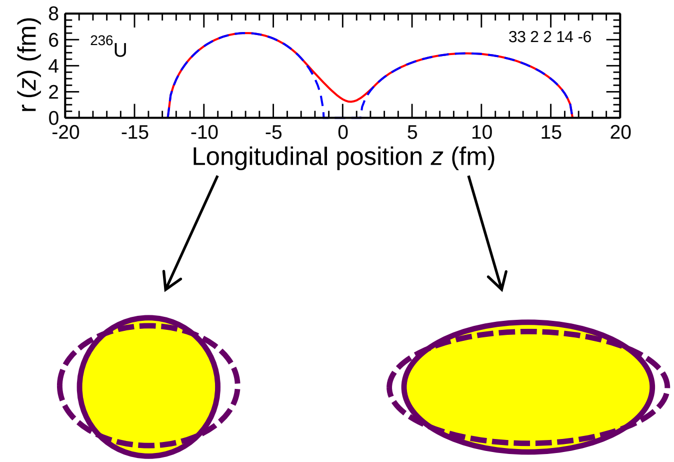


Scission shapes (c_0 = 1.5 fm)



Fragment shapes at scission: *distortion* energy



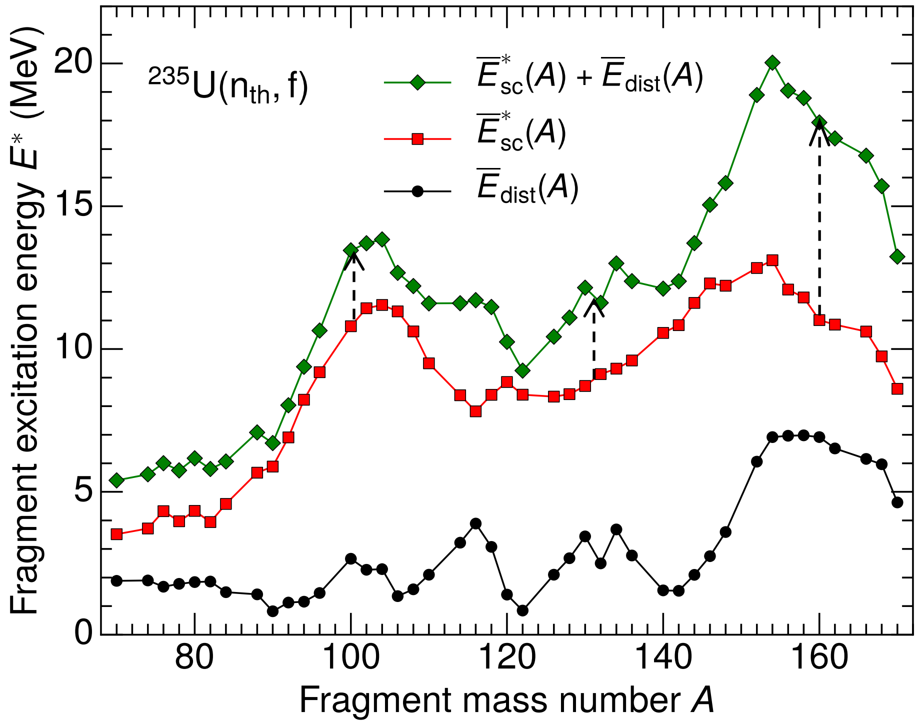


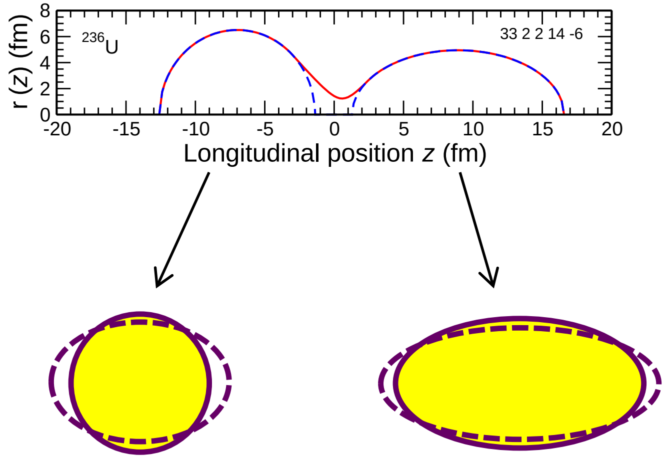
After separation the shapes relax to their equilibrium forms: ε_{sc} -> ε_{gs}

and the associated fragment distortion energy $E_{dist} = M(\varepsilon_{sc}) - M(\varepsilon_{gs})$

is converted to additional statistical excitation energy

Fragment shapes at scission: distortion energy





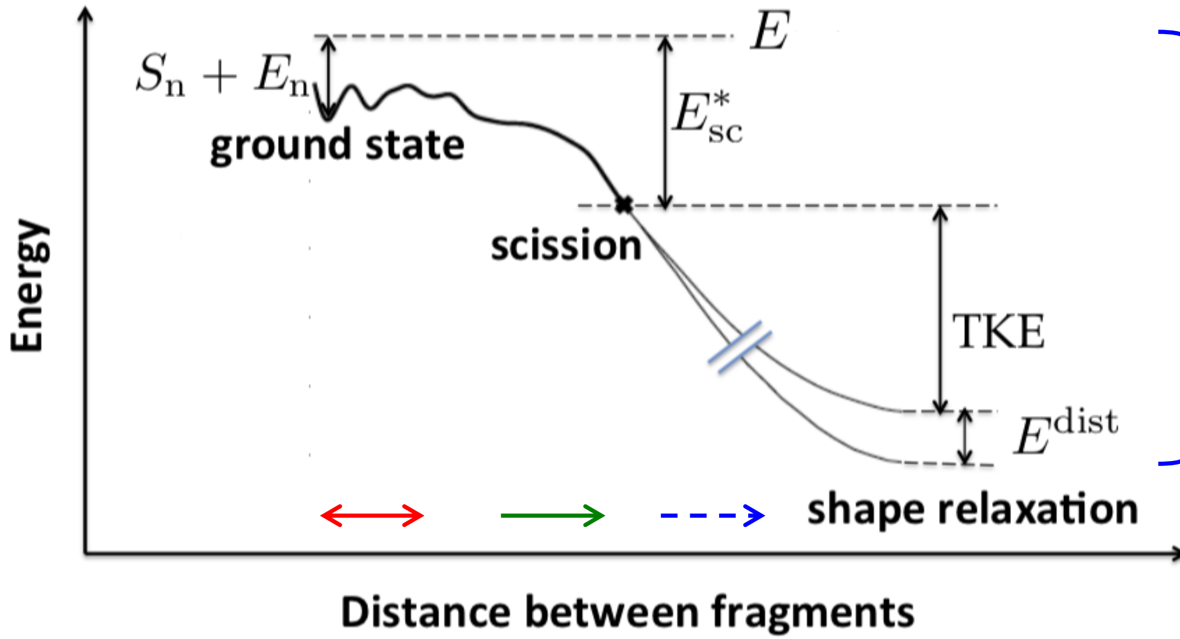
After separation the shapes relax to their equilibrium forms: ε_{sc} -> ε_{gs}

and the associated fragment distortion energy $E_{dist} = M(\varepsilon_{sc}) - M(\varepsilon_{gs})$

is converted to additional statistical fragment excitation energy

Tentative

✓ Extraction of TKE from the scission configuration

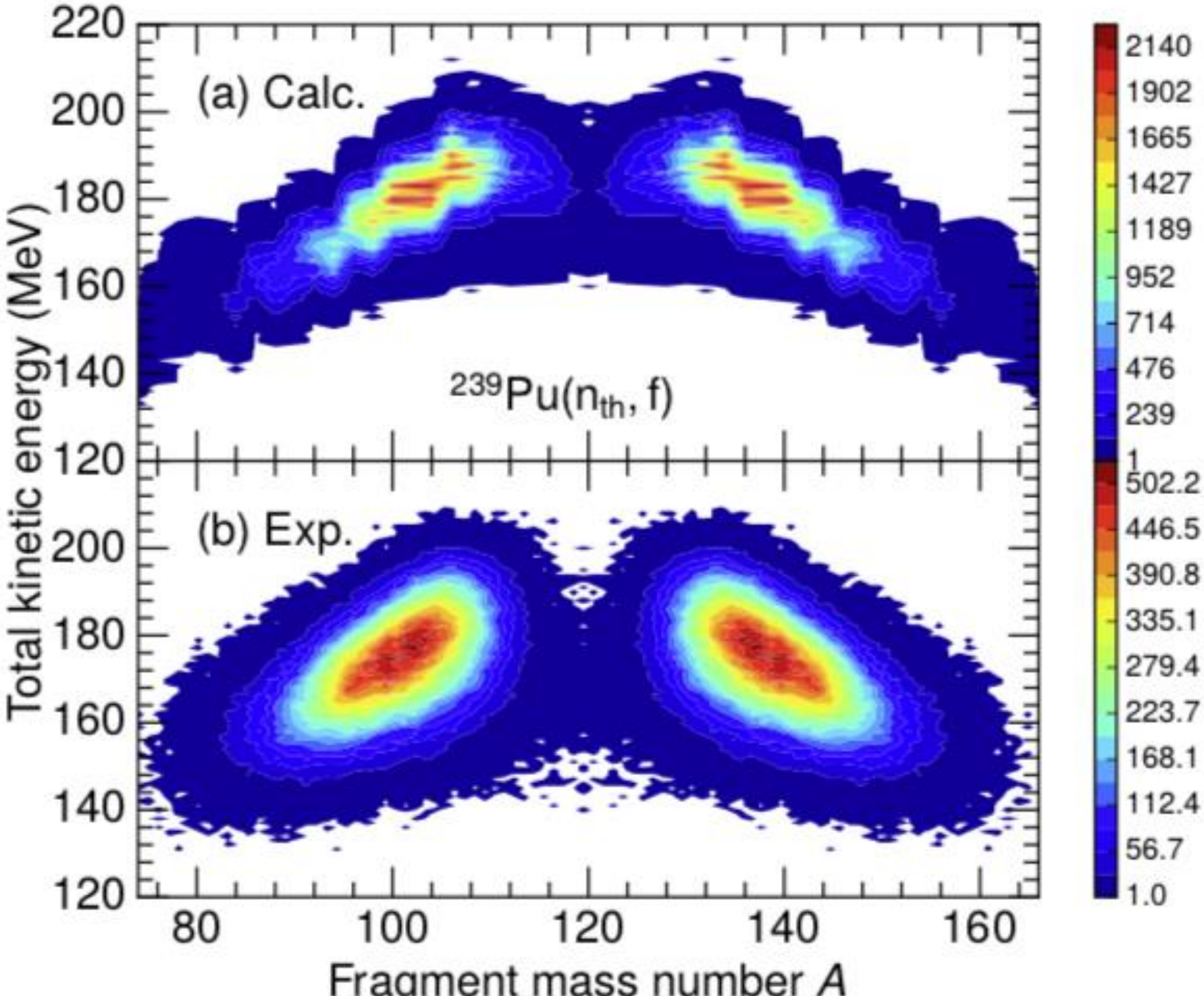


$Q^* = M^* - M_1 - M_2$

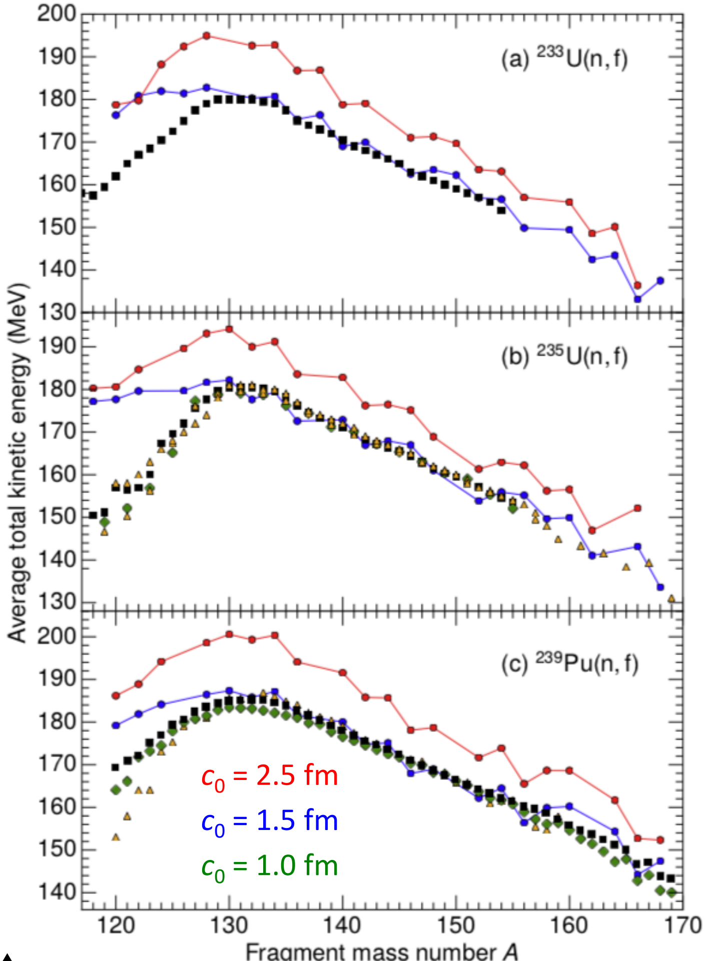
$= \text{TKE} + E_1^* + E_2^*$

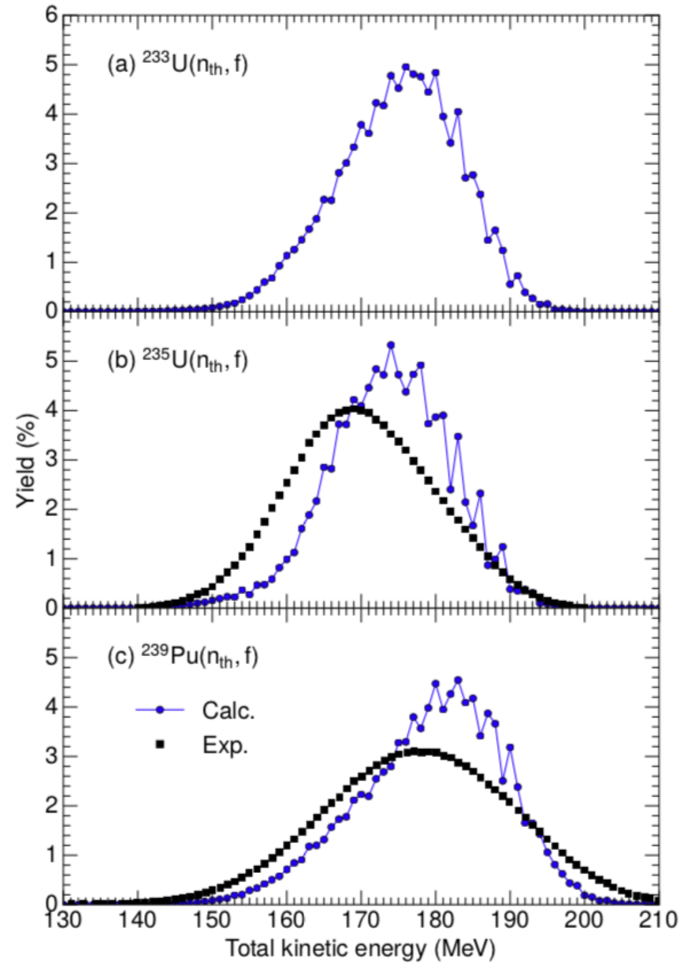
$$\text{TKE} = Q^* - E^*_{sciss} - E_{1,dist} - E_{2,dist}$$

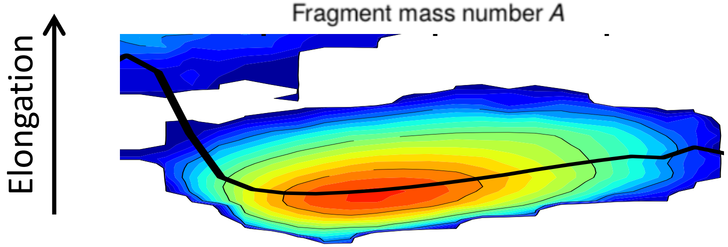
Joint (*A*,TKE) distribution



Total fragment kinetic energy







Fission studies with microscopic level densities

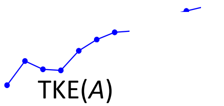
I: Fragment mass distributions and kinetic energies

The Brownian evolution of the nuclear shape can be simulated as a level-density guided Metropolis walk on the multi-dimensional deformation-energy surface

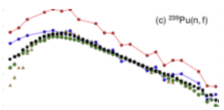
$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho(\chi')}{\rho(\chi)}$$

The use of microscopic shape-dependent level densities provides a parameter-free model for calculating the intricate energy dependence of the fragment mass distribution and, perhaps, the total fragment kinetic energy distribution as well

$Y_{symm}(E)$



TKE(A)



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Fission studies with microscopic level densities

✔ *I: Fragment mass distributions and kinetic energies*

➔ *II: Microscopic treatment of energy partition*

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