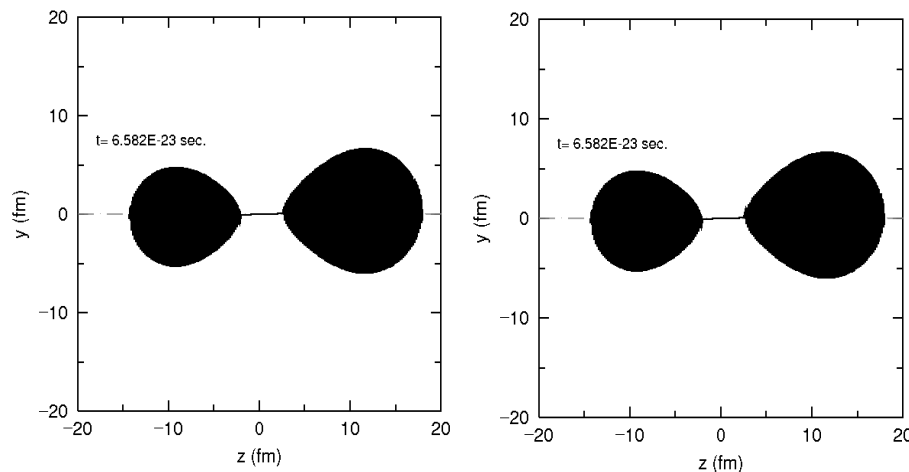


Dynamical model of the nuclear transfer reaction in heavy system

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54th ASRC International Workshop

Sakura-2019 “Nuclear Fission and Structure of Exotic Nuclei”

25-27, March, 2019, ASRC, Tokaim Ibaraki Japan

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Dynamical model with Langevin equation

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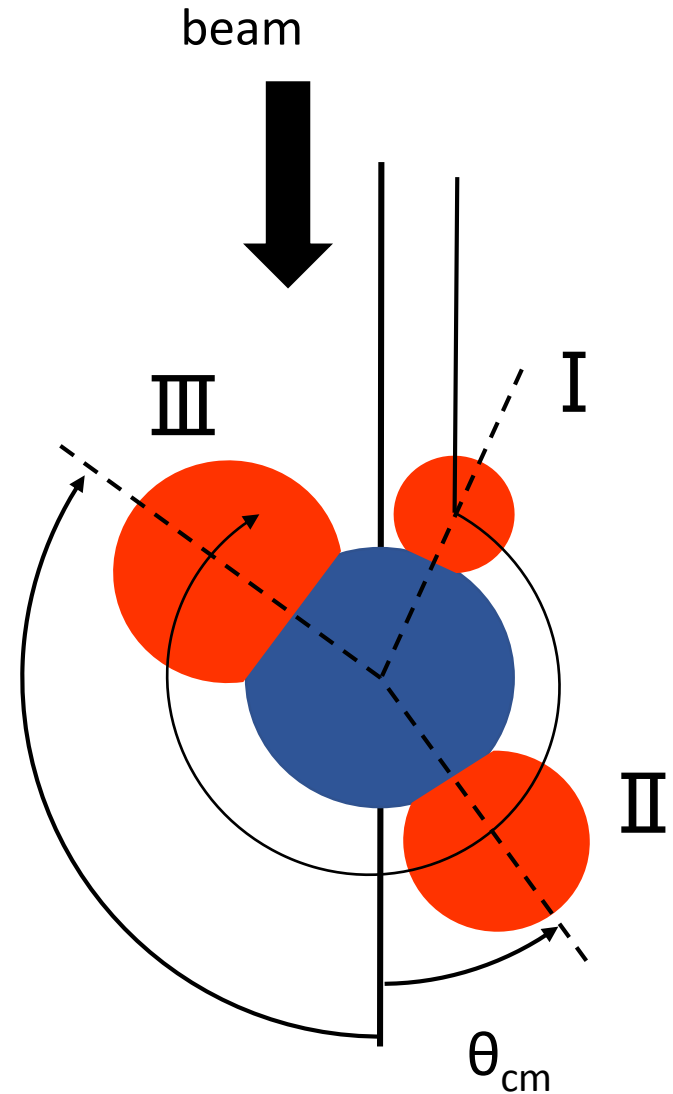
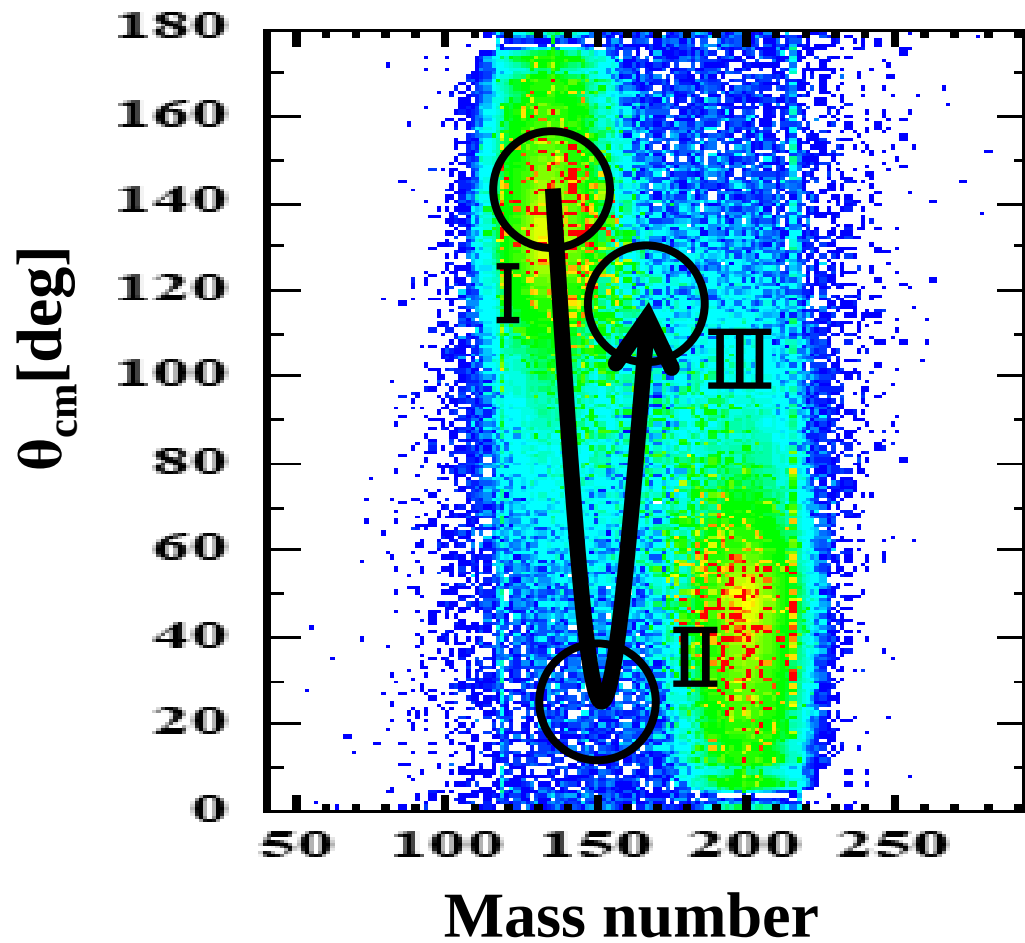
3. Results

Mass and Angle distribution of fragments

Transfer reaction using ^{254}Es target

4. Summary

Mass and Angle distribution of fragments



D.J. Hinde et al, PRL 101,092701 (2008)

R.du Rietz et al, PRL 106, 052701 (2011)

Application to Langevin calculation

(a)

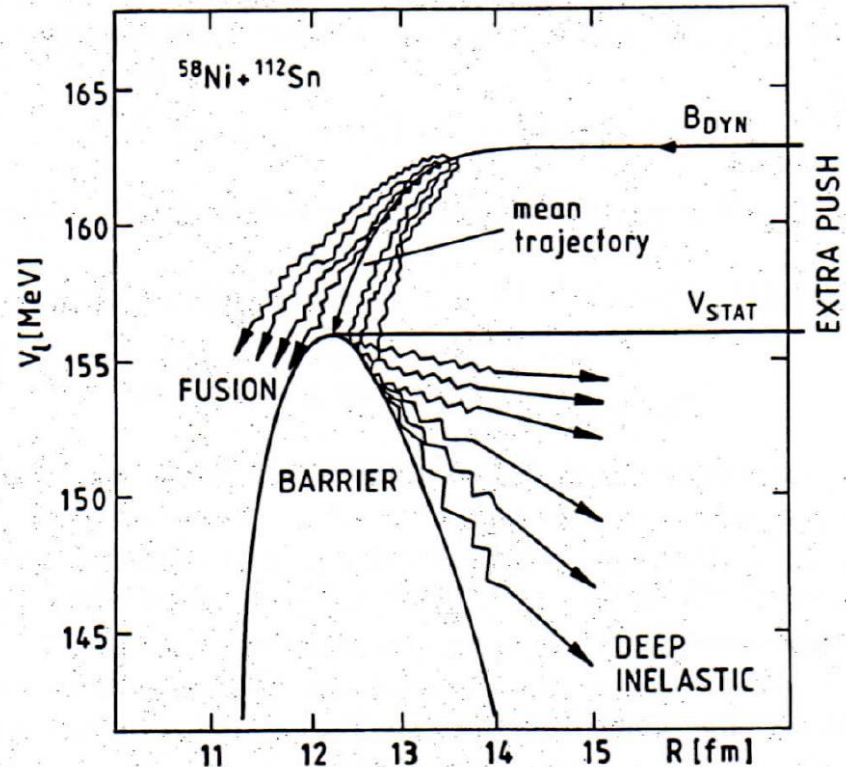
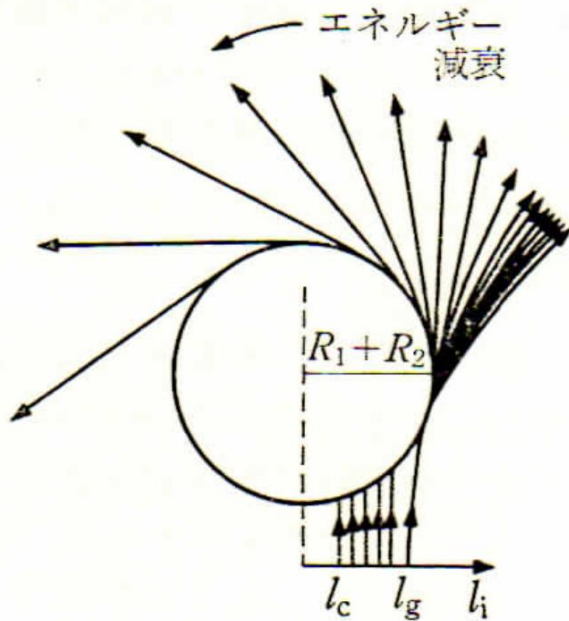


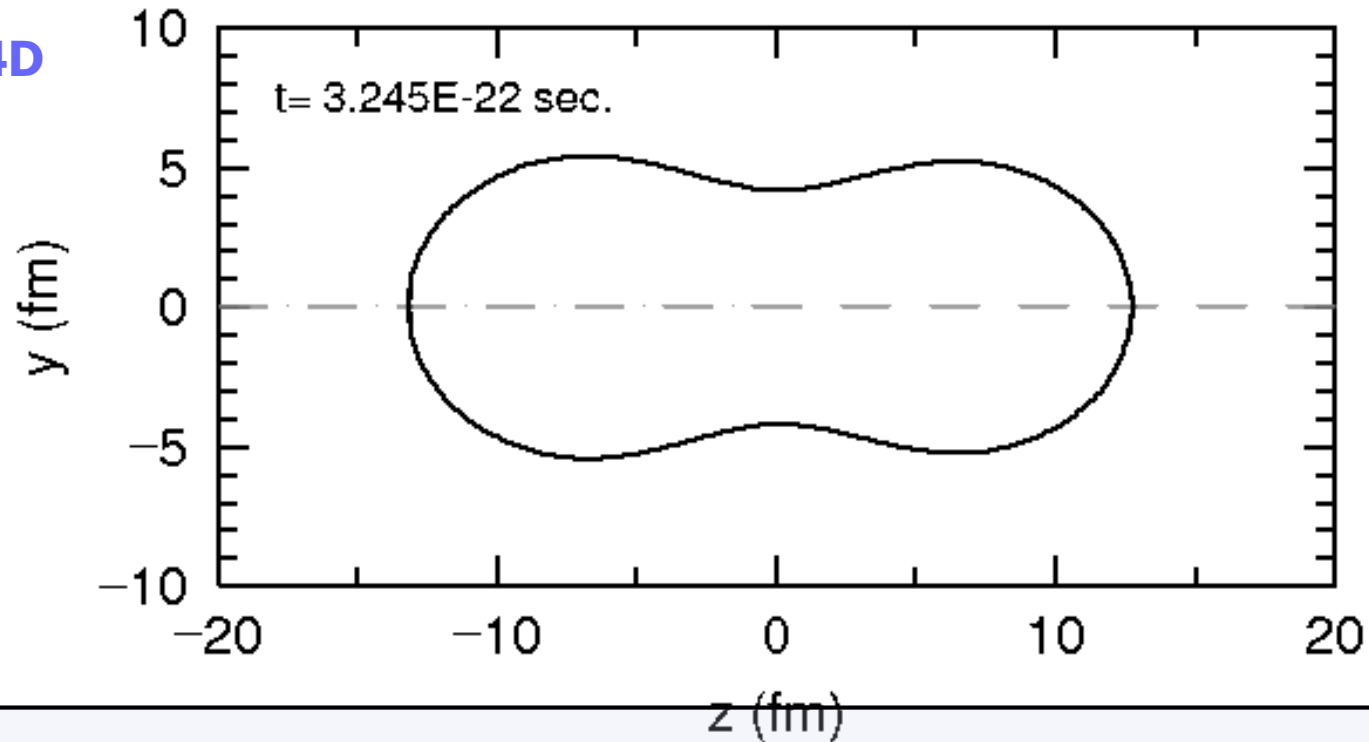
Figure 12.18 Bifurcation into fusion and DIC in the system $^{58}\text{Ni} + ^{112}\text{Sn}$ with $l = 0$ and incident energy 163 MeV in the centre-of-mass system. A mean frictional trajectory with an incident energy equal to the dynamical barrier B_{dyn} is shown, illustrating the extra-push effect.

Dissipative Phenomena

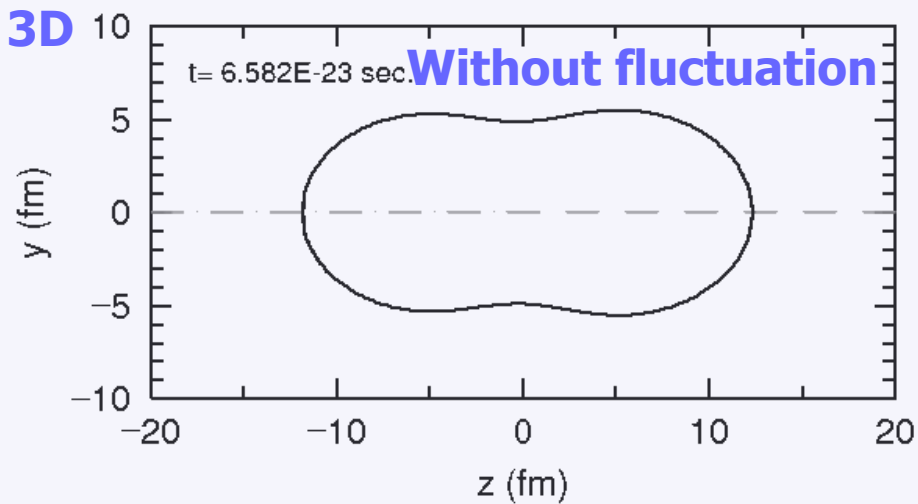
deep inelastic collision

Fission process with Lanegevin model (fluctuation) ^{236}U

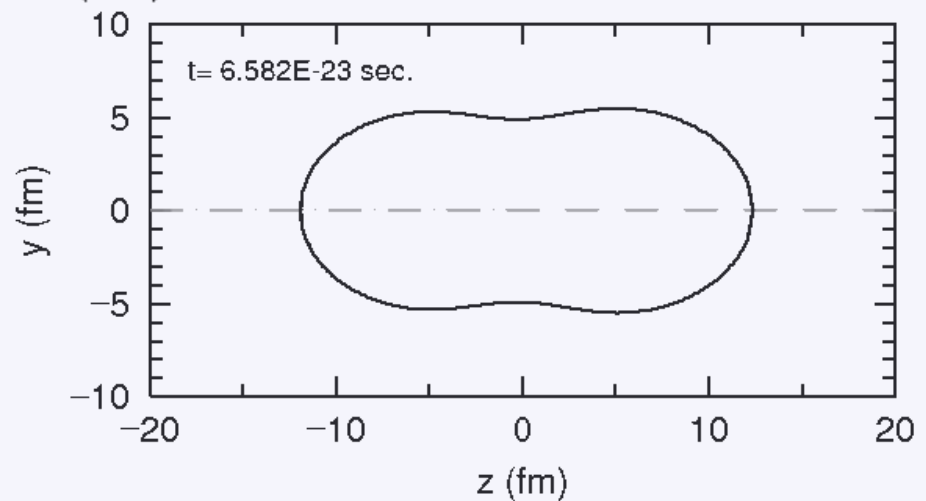
4D



3D



z (fm)



Overview of Dynamical Process in reaction $^{36}\text{S} + ^{238}\text{U}$

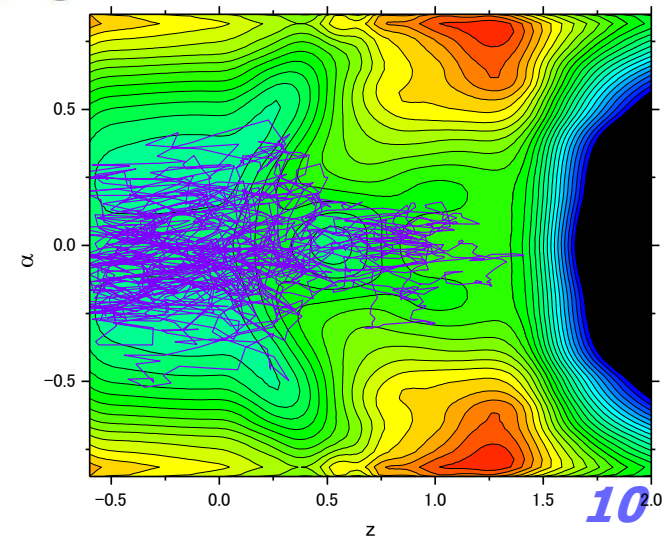
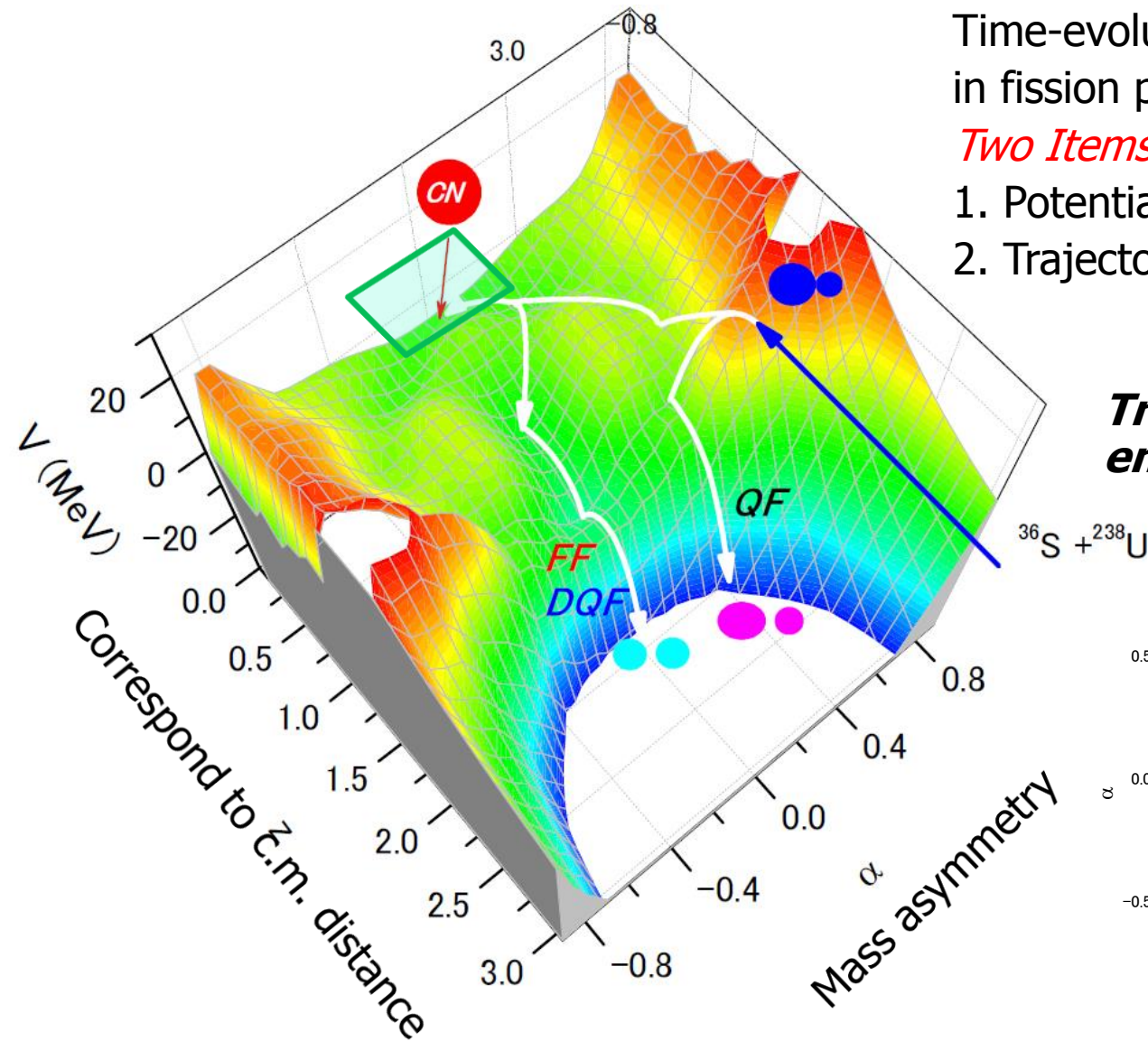
Dynamical calculation

Time-evolution of nuclear shape
in fission process

Two Items

1. Potential energy surface
2. Trajectory ← described by
Equation of Motion

Trajectory on potential energy surface



Nuclear Shape

two-center parametrization (z, δ, α)

(Maruhn and Greiner,
Z. Phys. 251(1972) 431)

$$q(z, \delta, \alpha)$$

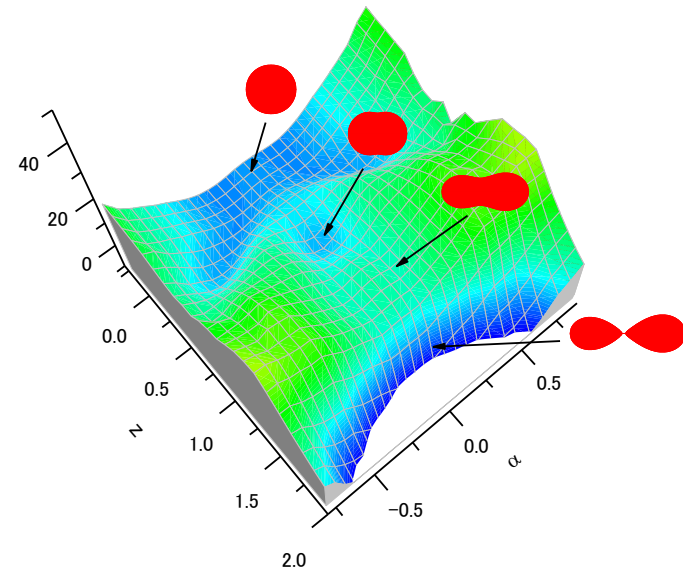
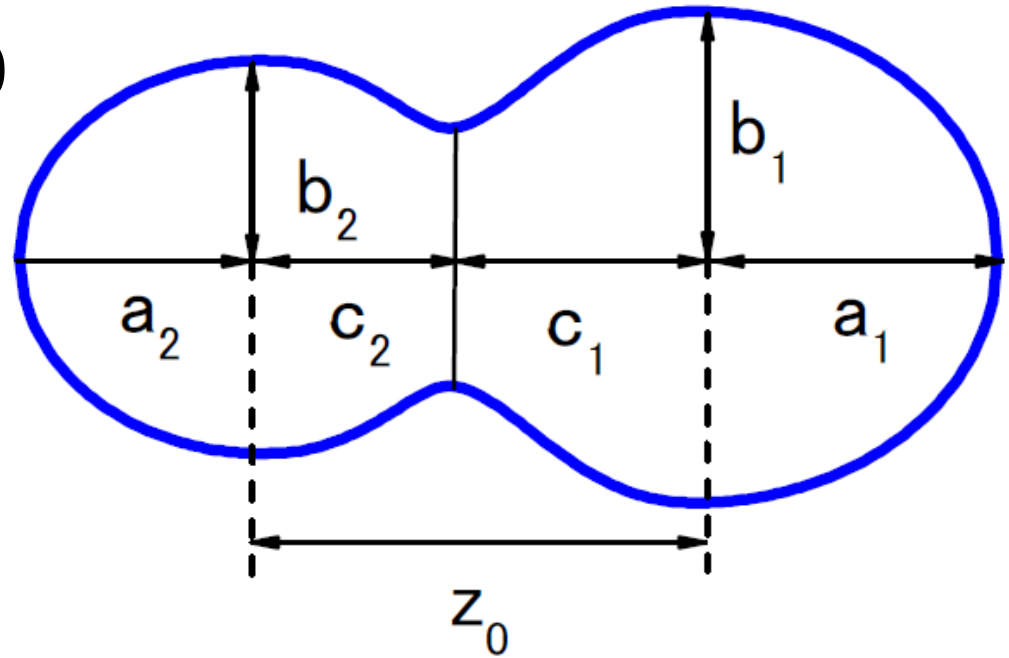
$$z = \frac{z_0}{BR}$$

$$B = \frac{3 + \delta}{3 - 2\delta}$$

R : Radial of compound nucleus

$$\delta = \frac{3(a - b)}{2a + b} \quad (\delta_1 = \delta_2)$$

$$\alpha = \frac{A_1 - A_2}{A_1 + A_2}$$



Potential Energy

$$V(q, \ell, T) = V_{DM}(q) + \frac{\hbar^2 \ell(\ell+1)}{2I(q)} + V_{SH}(q, T)$$

$$V_{DM}(q) = E_S(q) + E_C(q)$$

$$V_{SH}(q, T) = E_{shell}^0(q) \Phi(T)$$

T : nuclear temperature

$$E^* = aT^2 \quad a : \text{level density parameter}$$

Toke and Swiatecki

E_S : Generalized surface energy (finite range effect)

E_C : Coulomb repulsion for diffused surface

E_{shell}^0 : Shell correction energy at $T=0$

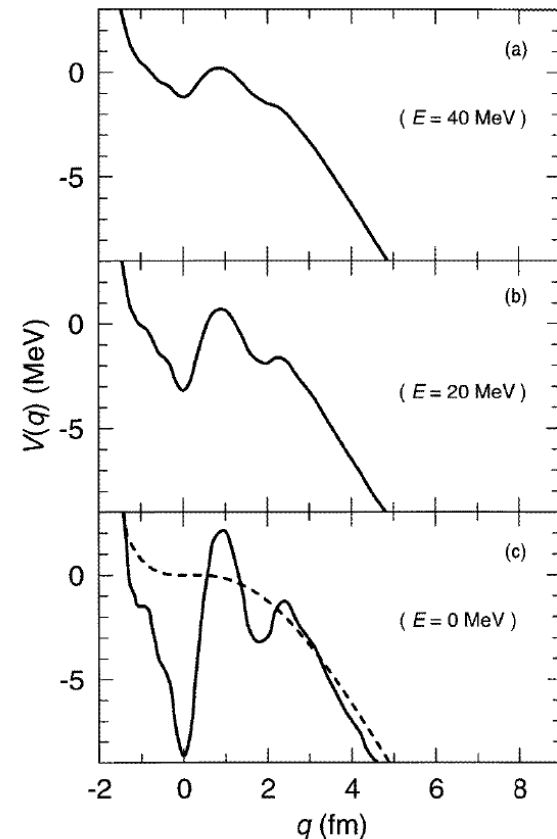
I : Moment of inertia for rigid body

$\Phi(T)$: Temperature dependent factor

$$\Phi(T) = \exp \left\{ - \frac{aT^2}{E_d} \right\}$$

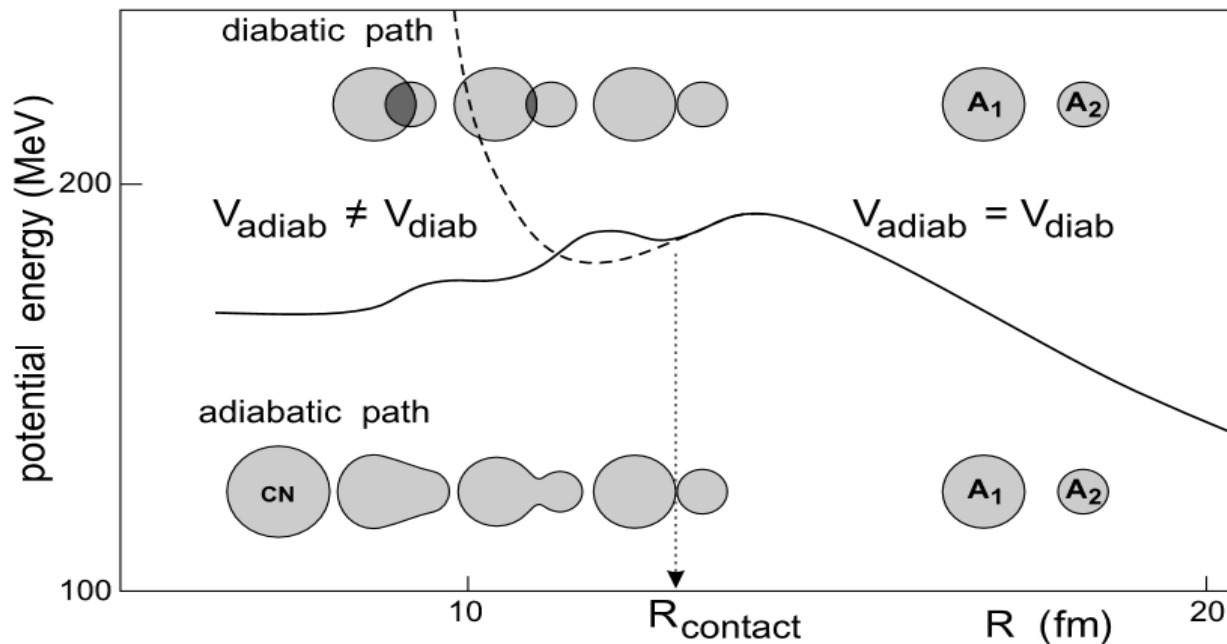
$$E_d = 20 \text{ MeV}$$

298114



**Fission barrier recovers
at low excitation energy**

relaxation time from diabatic potential to adiabatic potential T_{adi}



$$V = V_{dia}(q)f(t) + V_{adia}(q)(1 - f(t))$$

$$f(t) = \exp\left(-\frac{t}{T_{adi}}\right) \quad (T_{adi} = 10^{-21}s)$$

Valery Zagrebaev and Walter Greiner 著

「Unified consideration of deep inelastic, quasi-fission and fusion-fission phenomena」 より

System of coupled Langevin type Equations of Motion

$$\begin{aligned}\frac{dR}{dt} &= \frac{p_R}{\mu_R} \\ \frac{d\vartheta}{dt} &= \frac{\ell}{\mu_R R^2} \\ \frac{d\varphi_1}{dt} &= \frac{L_1}{\mathfrak{I}_1}, \quad \frac{d\varphi_2}{dt} = \frac{L_2}{\mathfrak{I}_2} \\ \frac{d\beta_1}{dt} &= \frac{p_{\beta_1}}{\mu_{\beta_1}} \\ \frac{d\beta_2}{dt} &= \frac{p_{\beta_2}}{\mu_{\beta_2}}\end{aligned}$$

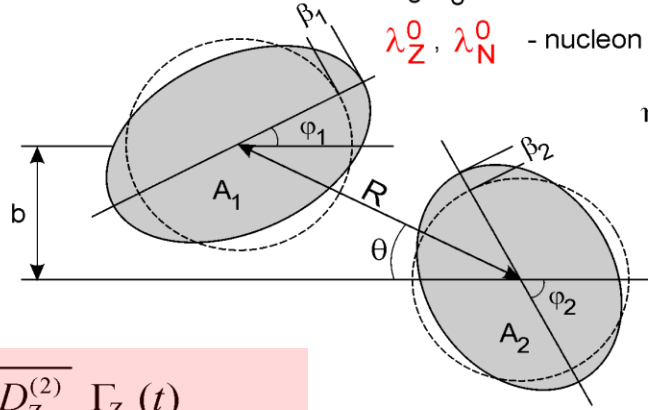
Variables: $\{R, \theta, \varphi_1, \varphi_2, \beta_1, \beta_2, \eta_Z, \eta_N\}$

$$\beta_1 = \beta_2$$

Most uncertain parameters:

μ_0, γ_0 - nuclear viscosity and friction,

λ_Z^0, λ_N^0 - nucleon transfer rate



$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$

$$\eta_Z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\eta_N = \frac{N_1 - N_2}{N_1 + N_2}$$

$$\lambda_Z^0 = \lambda_N^0 = \frac{\lambda^0}{2}$$

$$\begin{aligned}\frac{d\eta_Z}{dt} &= \frac{2}{Z_{CN}} D_Z^{(1)} + \frac{2}{Z_{CN}} \sqrt{D_Z^{(2)}} \Gamma_Z(t) \\ \frac{d\eta_N}{dt} &= \frac{2}{N_{CN}} D_N^{(1)} + \frac{2}{N_{CN}} \sqrt{D_N^{(2)}} \Gamma_N(t)\end{aligned}$$

$$\frac{dp_R}{dt} = -\frac{\partial V}{\partial R} + \frac{\ell^2}{\mu_R R^3} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2} \right) \frac{\partial \mu_R}{\partial R} + \frac{p_{\beta_1}^2}{2\mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial R} + \frac{p_{\beta_2}^2}{2\mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial R} - \gamma_R \frac{p_R}{\mu_R} + \sqrt{\gamma_R T} \Gamma_R(t)$$

$$\frac{d\ell}{dt} = -\frac{\partial V}{\partial \vartheta} - \gamma_{\text{tang}} \left(\frac{\ell}{\mu_R R} - \frac{L_1}{\mathfrak{I}_1} a_1 - \frac{L_2}{\mathfrak{I}_2} a_2 \right) R + \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}}(t)$$

$$\frac{dL_1}{dt} = -\frac{\partial V}{\partial \varphi_1} + \gamma_{\text{tang}} \left(\frac{\ell}{\mu_R R} - \frac{L_1}{\mathfrak{I}_1} a_1 - \frac{L_2}{\mathfrak{I}_2} a_2 \right) a_1 - \frac{a_1}{R} \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}}(t)$$

$$\frac{dL_2}{dt} = -\frac{\partial V}{\partial \varphi_2} + \gamma_{\text{tan}} \left(\frac{\ell}{\mu_R R} - \frac{L_1}{\mathfrak{I}_1} a_1 - \frac{L_2}{\mathfrak{I}_2} a_2 \right) a_2 - \frac{a_2}{R} \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}}(t)$$

$$\frac{dp_{\beta_1}}{dt} = -\frac{\partial V}{\partial \beta_1} + \frac{p_{\beta_1}^2}{2\mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial \beta_1} + \frac{p_{\beta_2}^2}{2\mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial \beta_1} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2} \right) \frac{\partial \mu_R}{\partial \beta_1} - \gamma_{\beta} \frac{p_{\beta_1}}{\mu_{\beta_1}} + \sqrt{\gamma_{\beta_1} T} \Gamma_{\beta_1}(t)$$

$$\frac{dp_{\beta_2}}{dt} = -\frac{\partial V}{\partial \beta_2} + \frac{p_{\beta_1}^2}{2\mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial \beta_2} + \frac{p_{\beta_2}^2}{2\mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial \beta_2} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2} \right) \frac{\partial \mu_R}{\partial \beta_2} - \gamma_{\beta} \frac{p_{\beta_2}}{\mu_{\beta_2}} + \sqrt{\gamma_{\beta_2} T} \Gamma_{\beta_2}(t)$$

7 parameters

V.I. Zagarebaev and W. Greiner,

J. Phys. G.31 825 (2005);

G34 1 (2007); G34, 2265 (2007);

G35 125103 (2008);

PLC78 034610 (2008) etc.

Multi-dimensional Langevin Equation

$$\frac{dq_i}{dt} = (m^{-1})_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \underbrace{\gamma_{ij} (m^{-1})_{jk} p_k}_{\text{Friction dissipation}} + \underbrace{g_{ij} R_j(t)}_{\text{Random force fluctuation}}$$

Newton equation
ordinary differential equation

$\langle R_i(t) \rangle = 0$, $\langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij} \delta(t_1 - t_2)$: white noise (Markovian process)

$$\sum_k g_{ik} g_{jk} = T \gamma_{ij}$$

Einstein relation

Fluctuation-dissipation theorem

q_i : deformation coordinate

(nuclear shape)

two-center parametrization (Z, δ, α)

(Maruhn and Greiner, Z. Phys. 251(1972) 431)

p_i : momentum

m_{ij} : Hydrodynamical mass

(inertia mass)

γ_{ij} : Wall and Window (one-body) dissipation

(friction)

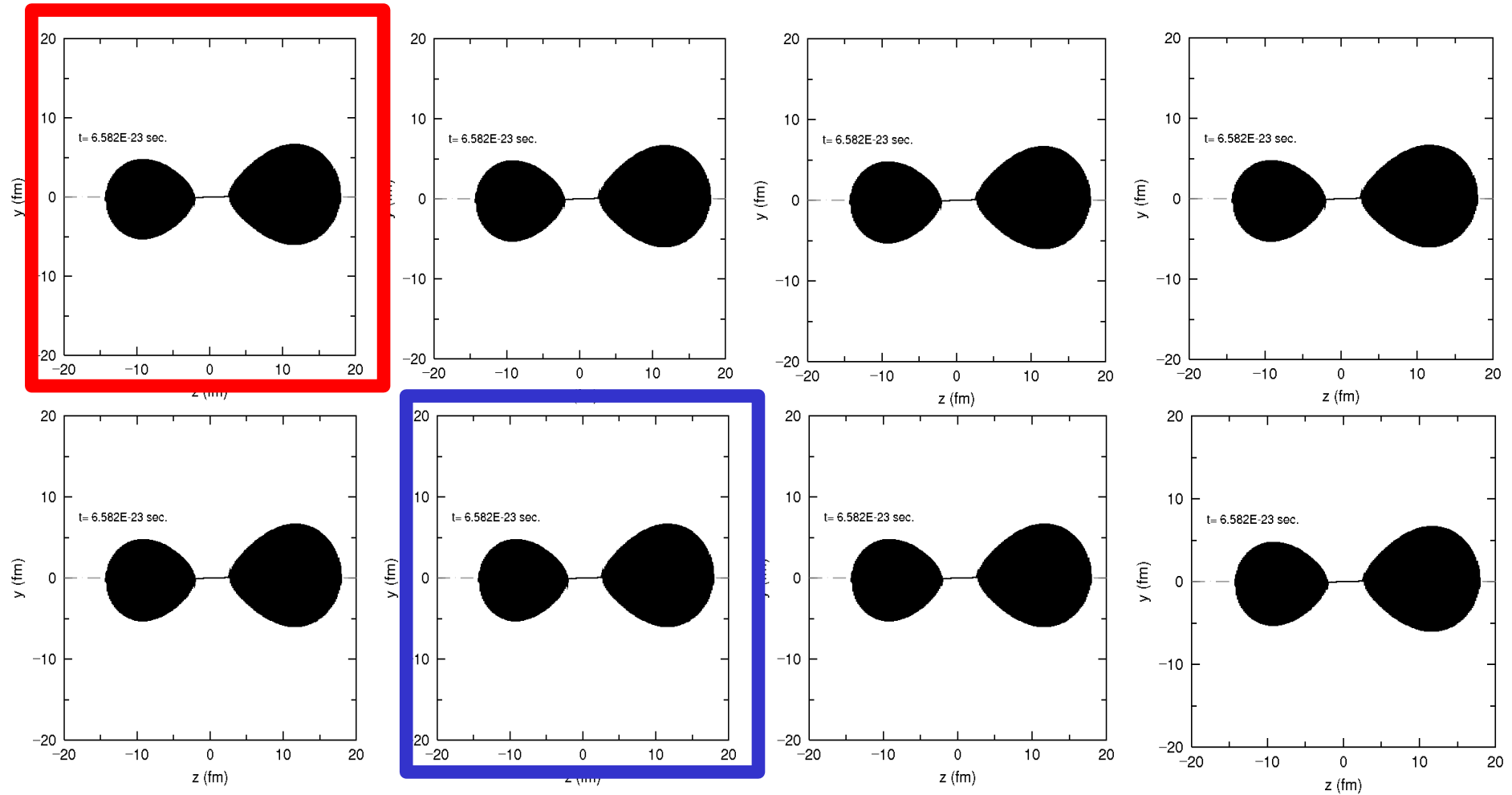
$$E_{\text{int}} = E^* - \frac{1}{2} (m^{-1})_{ij} p_i p_j - V(q)$$

E_{int} : intrinsic energy, E^* : excitation energy

Results

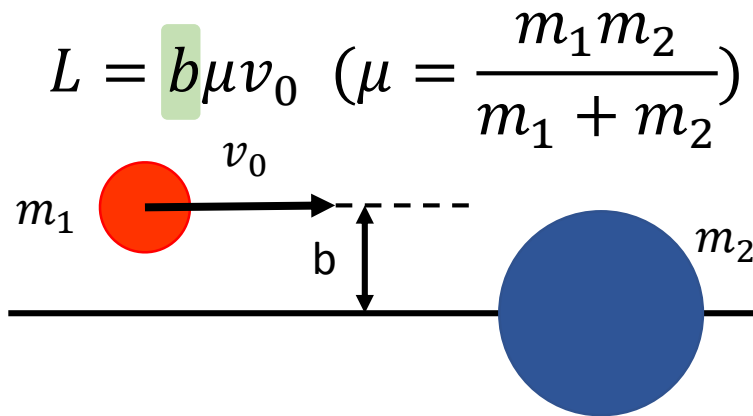
Same initial condition, parameters, different random number

$^{86}\text{Kr} + ^{166}\text{Er}$ $E_{\text{cm}} = 464 \text{ MeV}$ different random number



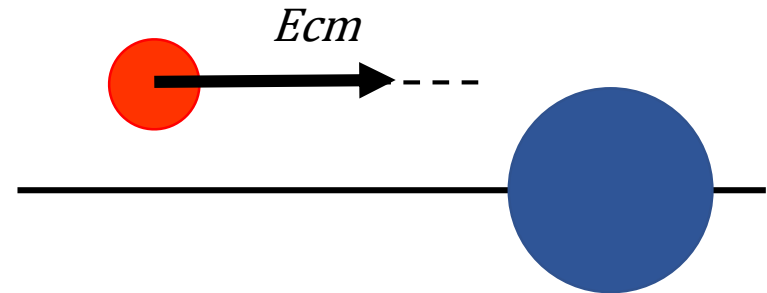
Parameters

impact parameter b

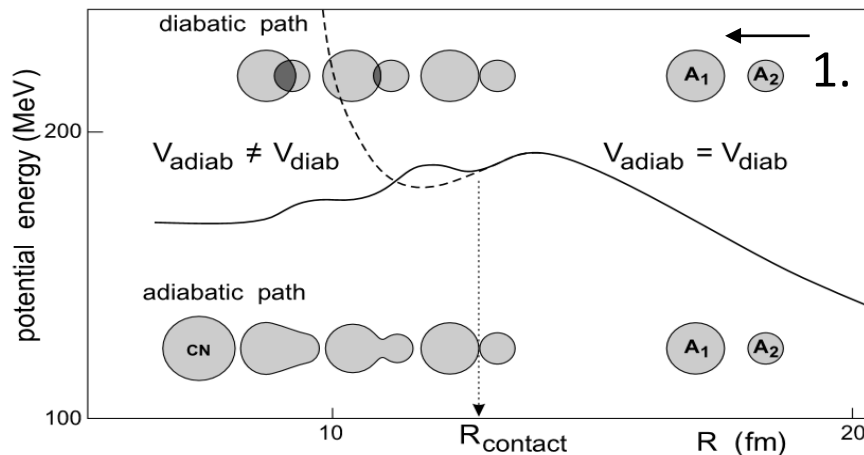


E_{cm}

Incident energy



Relaxation time T_{adi}



Fact_iner *cf. TDHF*

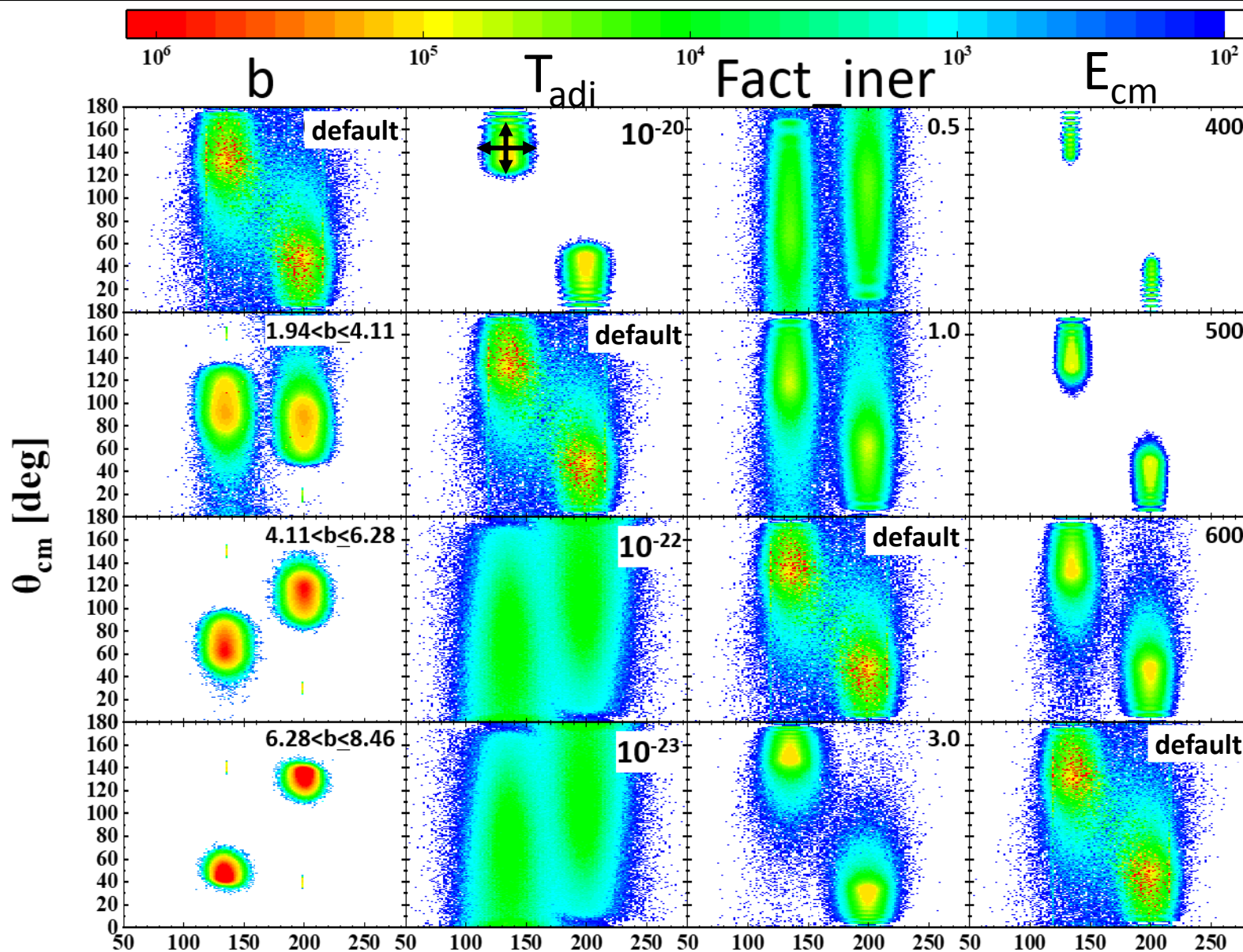
$$V(q, L, T) = V_{LDM}(q) + V_{SH}(q, T) + \frac{\hbar^2 L(L+1)}{2\mathcal{I}(q)}$$

$$\frac{\hbar^2 L(L+1)}{2\mathcal{I}(q)} \times \text{Fact_iner}$$

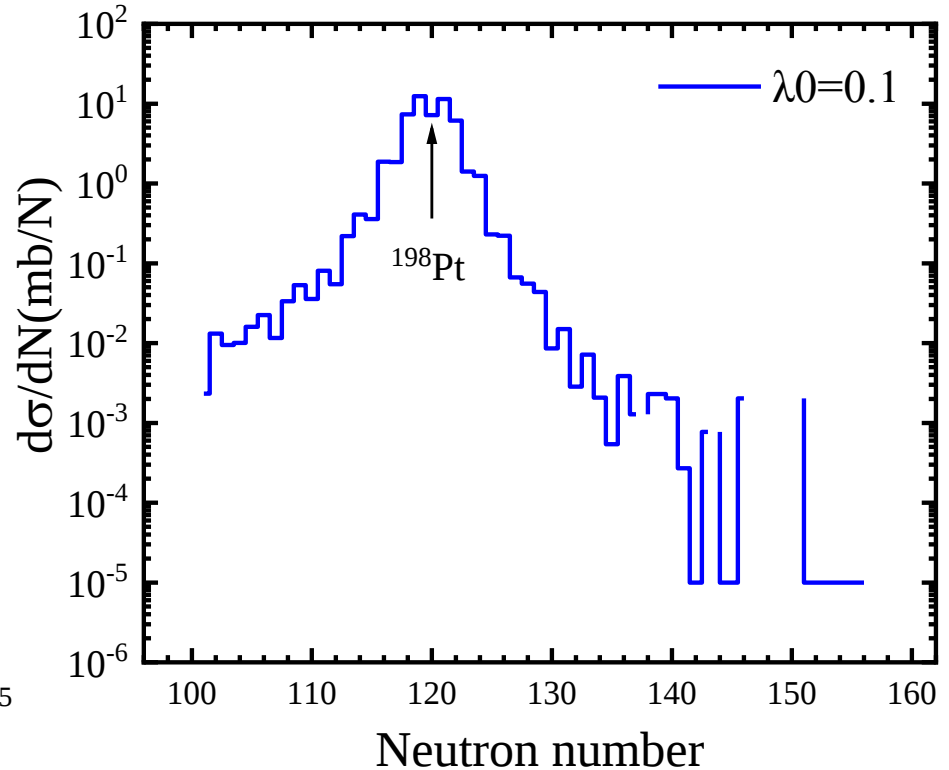
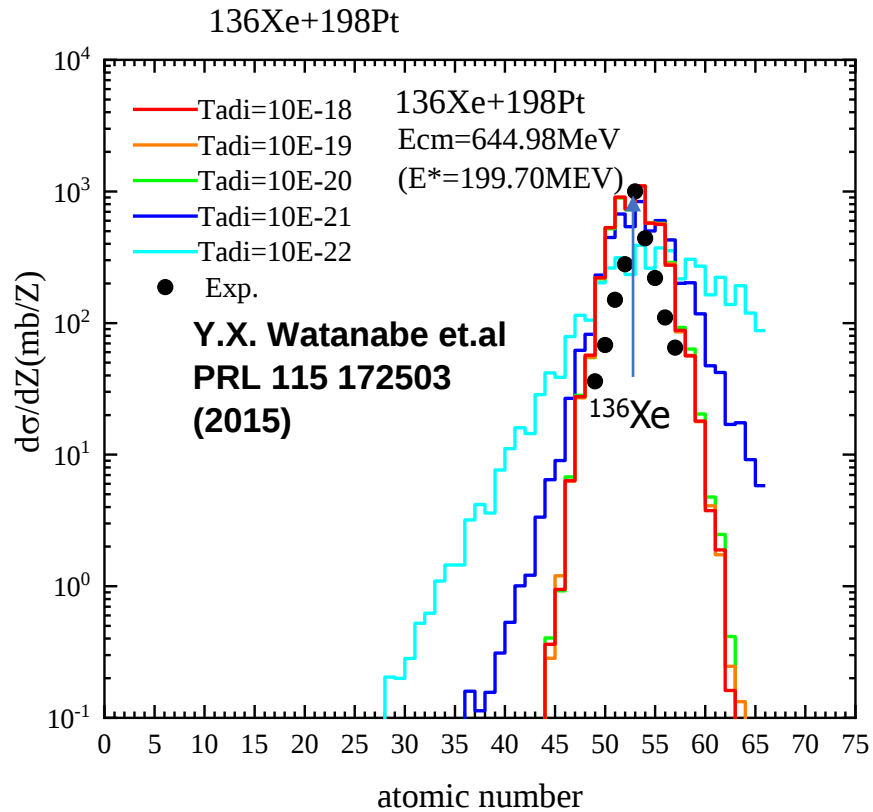
Moment of inertia

Mass and Angle($^{136}\text{Xe}+^{198}\text{Pt}$)

defaultパラメータ
 $0 < b \leq 1.94$ $E_{\text{cm}} = 700 \text{ MeV}$
 $T_{\text{adi}} = 10^{-21} \text{ s}$ $\text{Fact_iner} = 1.5$



To set unknown parameters



S. Amano and Y. Miyamoto (Kindai)

^{238}U -target Mass and Angle distribution of fission fragment

Z=104

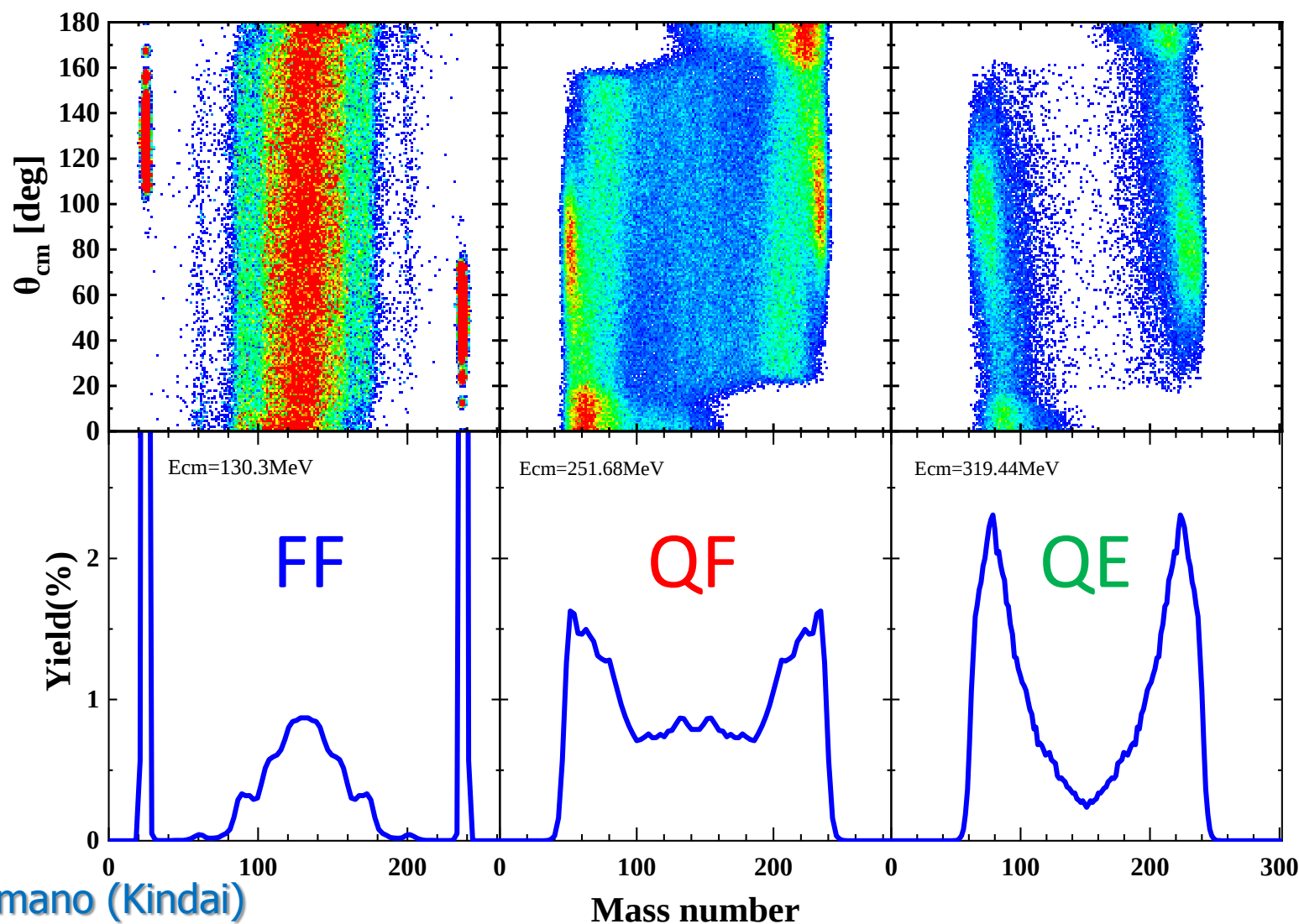
$^{24}\text{Mg}+^{238}\text{U}$

Z=112

$^{48}\text{Ca}+^{238}\text{U}$

Z=120

$^{64}\text{Ni}+^{238}\text{U}$



Summary

1. We calculated Mass and Angle distribution of fragments using Langevin equation.
2. The dependence of
the relaxation time from diabatic potential to
adiabatic potential and
the moment of inertia
is discussed.

→ Reliability of the model → try to calculate SHE
3. Transfer reaction using ^{254}Es was discussed

... and

Collaborators

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Y. Miyamoto

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