Angular distribution of scission neutrons with time-dependent Schrödinger equations

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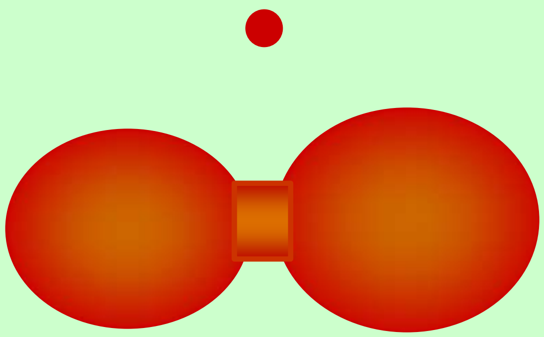
Scission Neutrons

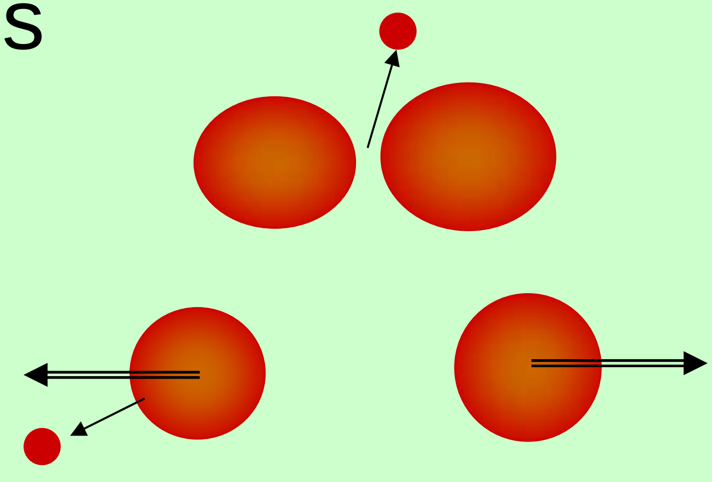
- At the moment of scission
  - – Rupture of the neck
  - – Abrupt shape change at neck
  - – Some component would be left in the neck region and would be emitted as particles



- Charged particles
  - – Strong focusing by Coulomb interaction
    - Perpendicular direction
- Neutrons
  - – No Coulomb force = Isotropic distribution (?)
  - – Effect of nuclear interaction

Scission Neutrons

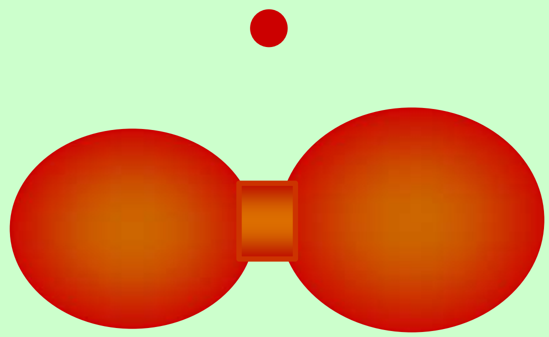
- Two main neutron sources
  in low energy fission
  - Scission neutrons
  - Post scission neutrons



- How to distinguish?
  - Angular distribution
    - Scission : rest frame of the mother nucleus
    - Post scission : moving frame of the fragment
- Effect of nuclear interaction by fragments on scission neutrons

  Modification of the angular distribution by
  Scattering and Re-absorption

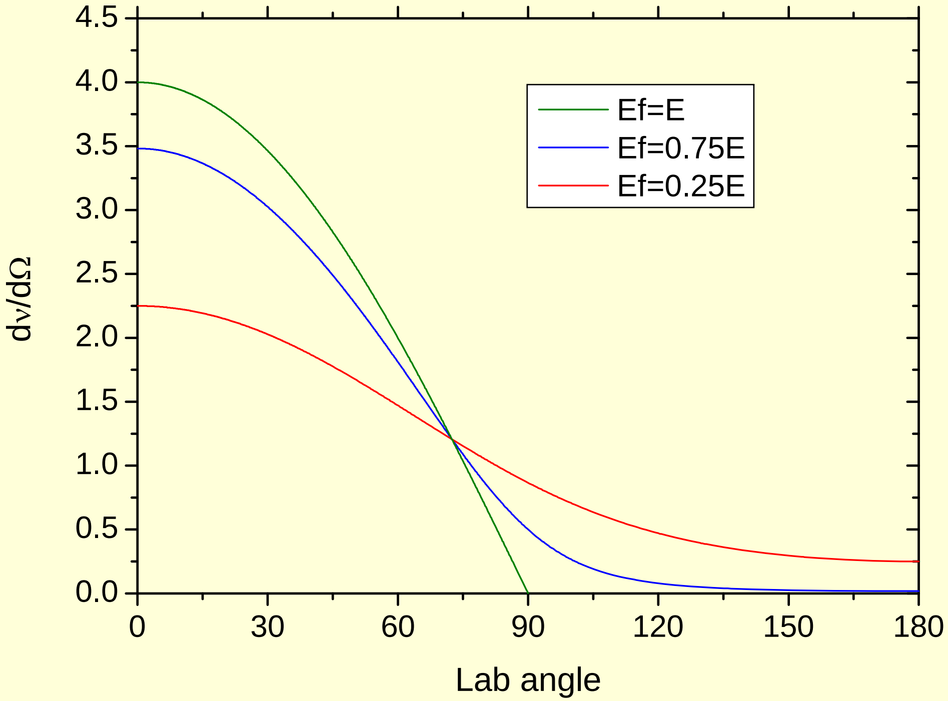
Scission Neutrons

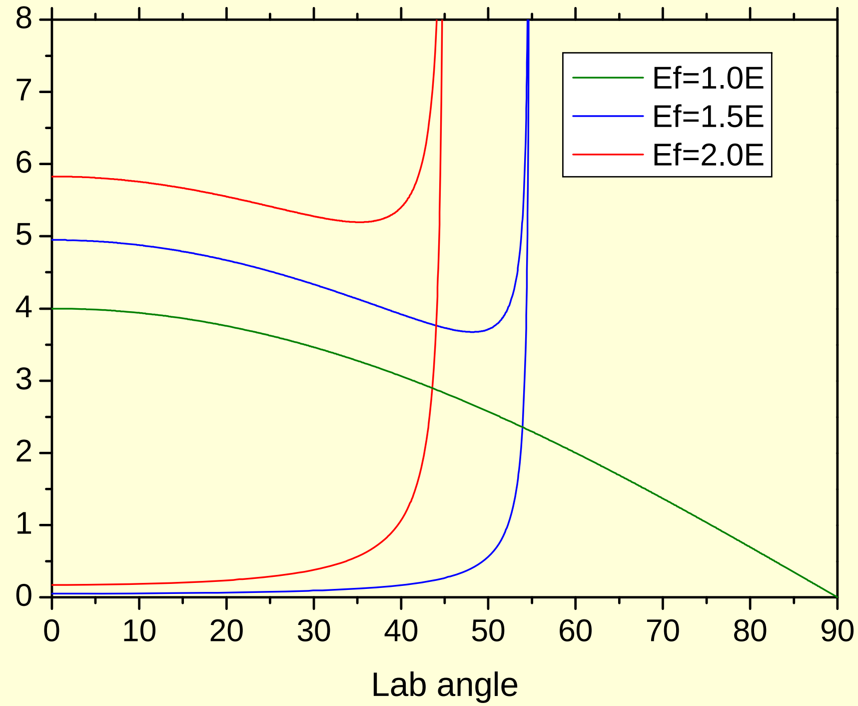


- At the moment of scission
  - Some component would be left in the neck region
- Charged particles
  - Strong focusing by Coulomb interaction into perpendicular direction
- Neutrons
  - Effect of nuclear interaction
- Post-scission neutrons
  - Angular distribution
    - Scission : rest frame of the mother nucleus
    - Post scission : moving frame of the fragment

Angular distribution of Post-scission Neutrons

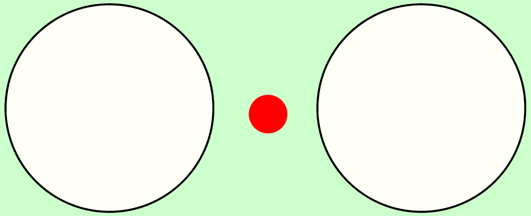
- Post-scission neutrons are emitted isotropically in the moving frame
- Anisotropic in the laboratory frame





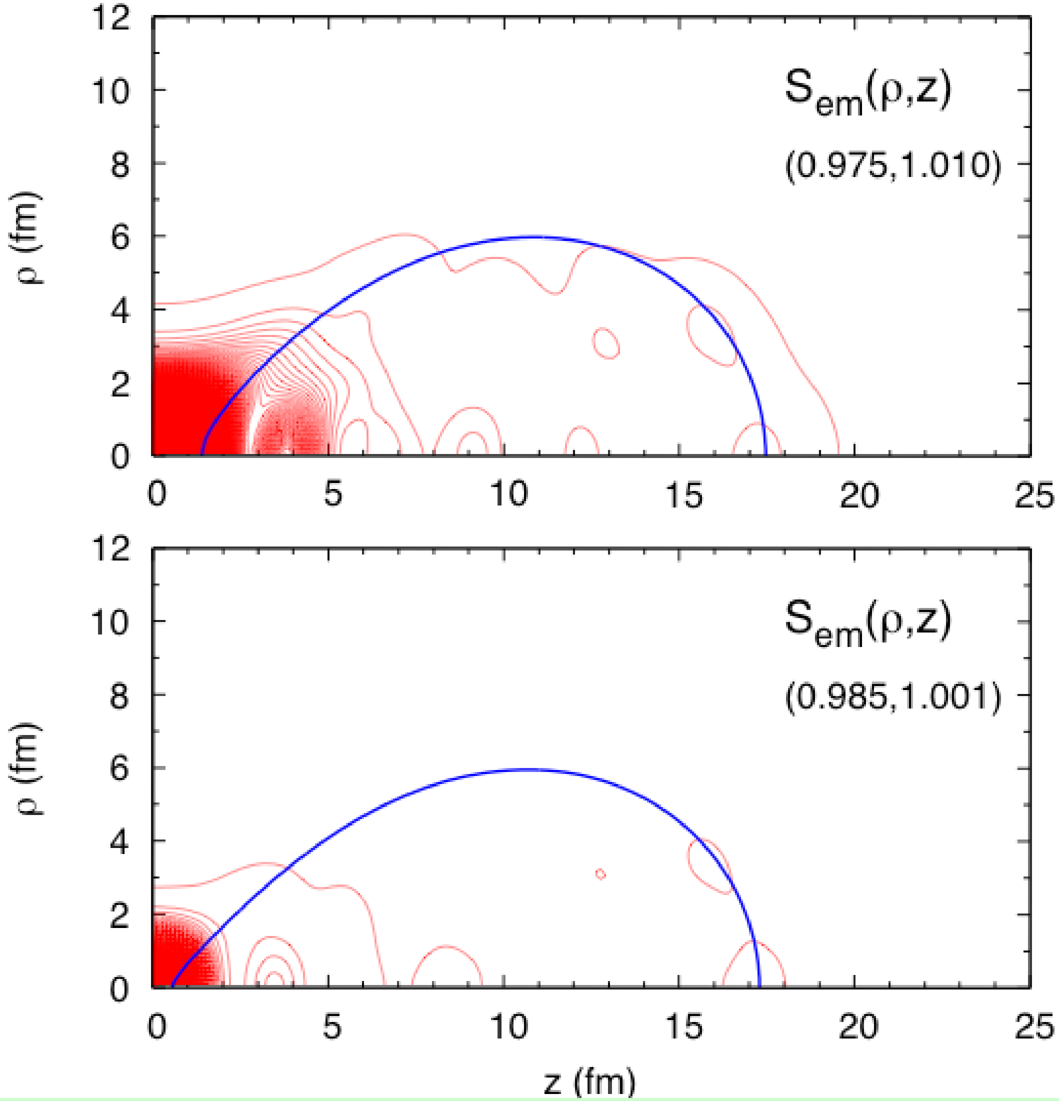
Potential model for scattering and re-absorption of neutrons

- Emission of scission neutron
  - Emission point: Middle of the fragments
  - Initial distribution: Isotropic



- Effect of fragments
  - Optical potential (real & imaginary)
  - Woods-Saxon shape
- Separation of the fragments
  - TKE systematics
  - Dynamical motion by Coulomb repulsion

Distribution of the emission points



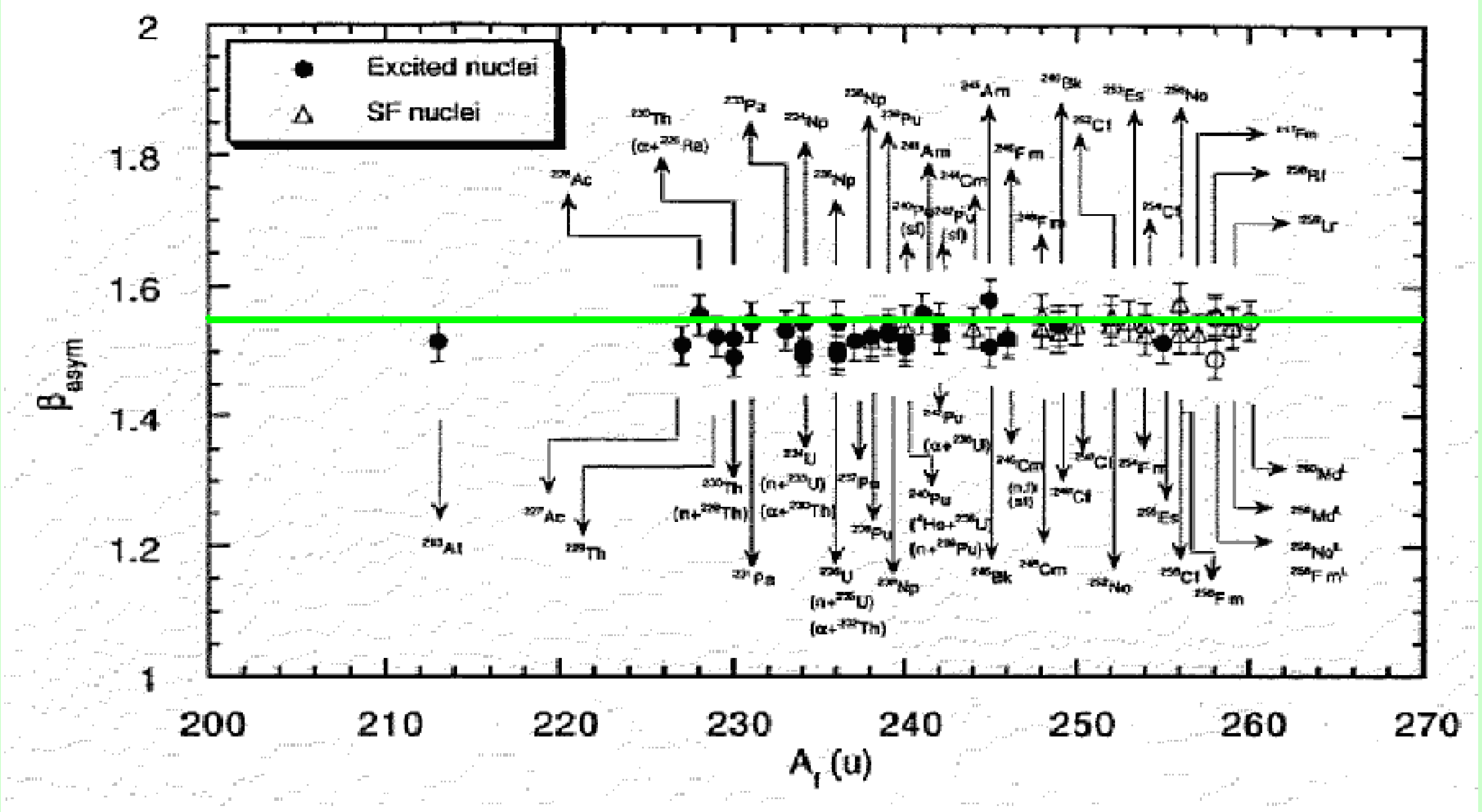
- The large majority of the scission neutrons are emitted in the region between the fragments.

  N. Carjan et al., Nuclear Physics A 792 (2007) 102–121

- Consistent with the intuitive expectation

Systematic TKE data analysis

Experimental shape elongation

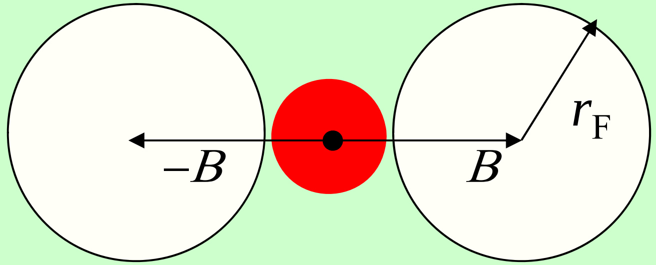


$$TKE=Z_1Z_2e^2/D(A_1,A_2), \quad \beta= D(A_1,A_2) /[r_0(A_1^{1/3}+A_2^{1/3})]$$

Time-dependent approach

- Initial function & potential
  - Cylindrical coordinate

    (ρ, z, ϕ) axial symmetry



$$\psi(t=0) = C\exp\left(-(\alpha_1\rho^2 + \alpha_2 z^2)\right)$$

$$U(\rho, z) = \frac{-V_0 - iW_0}{1+\exp\left(\dfrac{\sqrt{\rho^2+(z+B)^2} - r_F}{a}\right)} + \frac{-V_0 - iW_0}{1+\exp\left(\dfrac{\sqrt{\rho^2+(z-B)^2} - r_F}{a}\right)}$$

$B = \beta r_F$, β : elongation parameter

- Simple assumptions
  - Symmetric division
  - Spherical fragments
- Example : ^{236}U

Numerical integration

- Leap-frog method

$$i\frac{\partial\psi}{\partial t}=H\psi\ ,\quad H=-\frac{1}{2m}\nabla^2+U\ ,\ \hbar=1$$

$$\psi(t+\Delta t)=\psi(t)-i\Delta t\,H\psi(t+\Delta t/2)$$

$$\psi=R+iI \qquad \begin{cases} I(t+\Delta t/2)=I(t-\Delta t/2)+\Delta t\,HR(t) \\ R(t+\Delta t)=R(t)-\Delta t\,HI(t+\Delta t/2) \end{cases}$$

With imaginary potential

$$H=H_{\mathrm{R}}-iW=-\frac{1}{2m}\nabla^2-V-iW$$

$$\begin{cases} I(t+\Delta t/2)=\left[(1-\Delta t W/2)I(t-\Delta t/2)+\Delta t\,H_{\mathrm{R}}R(t)\right]/(1+\Delta t W/2) \\ R(t+\Delta t)=\left[(1-\Delta t W/2)R(t)-\Delta t\,H_{\mathrm{R}}I(t+\Delta t/2)\right]/(1+\Delta t W/2) \end{cases}$$

Probability & flow

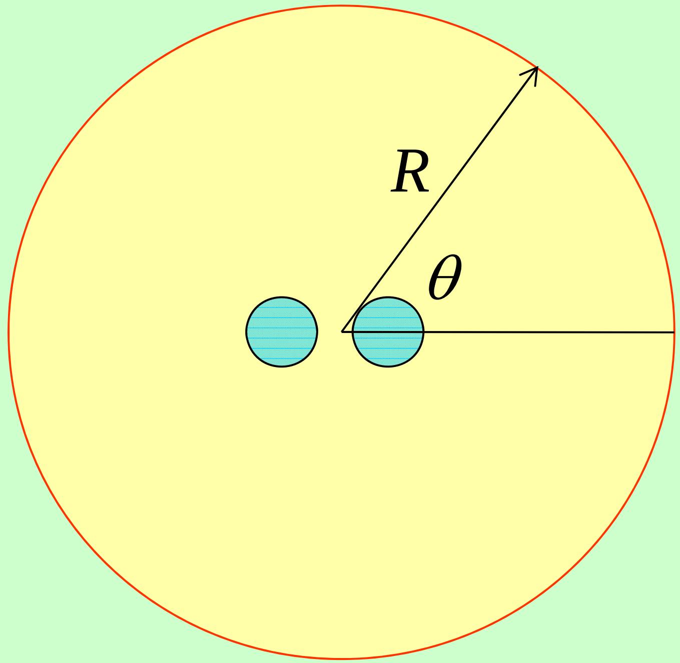
Probability density

$$P = \psi^* \psi = R^2 + I^2$$

$$\Rightarrow P(t) = R(t)^2 + I(t + \Delta t/2) I(t - \Delta t/2)$$

Probability flow

$$\vec{j}(\vec{r},t) = \frac{\hbar}{2im}\left(\psi \vec{\nabla} \psi^* - \psi^* \vec{\nabla} \psi\right)$$



Number of emitted neutrons per unit time per solid angle through the surface S_R

$$\frac{d^2\nu(\theta,t)}{d\Omega dt} = \vec{j}(R,\theta,t)\cdot\vec{n}(R,\theta)R^2 \ , \quad \vec{n} = \vec{e}_r$$

Angular distribution

Number of emitted neutrons per unit solid angle

$$\frac{d\nu(\theta,t)}{d\Omega} = \int_0^t \frac{d^2\nu(\theta,t')}{d\Omega dt} dt'$$

$$= \int_0^t \vec{j}(R,\theta,t')\cdot\vec{n}(R,\theta)R^2 dt' \quad , \quad \vec{n} = \vec{e}_r$$

$$\frac{d\nu_\infty(\theta)}{d\Omega} = \lim_{t\to\infty} \frac{d\nu(\theta,t)}{d\Omega}$$

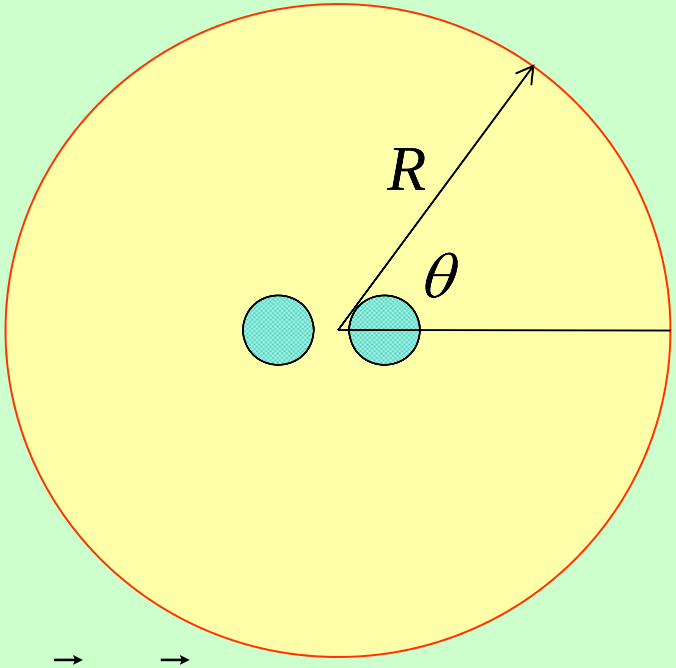
Number of observed neutrons

$$\nu_{\text{emit}} = \int \frac{d\nu_\infty(\theta)}{d\Omega} d\Omega \quad , \quad d\Omega = \sin\theta \, d\theta d\varphi$$

Angular distribution

Probability flow

$$\vec{j}(\vec{r},t) = \frac{\hbar}{2im}\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right)$$



Number of emitted neutrons per unit time per solid angle through the surface S_R

$$\frac{d^2\nu(\theta,t)}{d\Omega dt} = \vec{j}(R,\theta,t)\cdot\vec{n}(R,\theta)R^2 \ , \quad \vec{n} = \vec{e}_r$$

Number of emitted neutrons per unit solid angle

$$\frac{d\nu(\theta,t)}{d\Omega} = \int_0^t \frac{d^2\nu(\theta,t')}{d\Omega dt} dt'$$

$$= \int_0^t \vec{j}(R,\theta,t')\cdot\vec{n}(R,\theta)R^2 dt' \ , \quad \vec{n} = \vec{e}_r$$

Time-dependent approach with finite size grid

- In the laboratory
  - Infinite distance from fissioning system
  - Macroscopic time scale
- In the calculation
  - Finite size grid
    - Finite distance from fissioning system

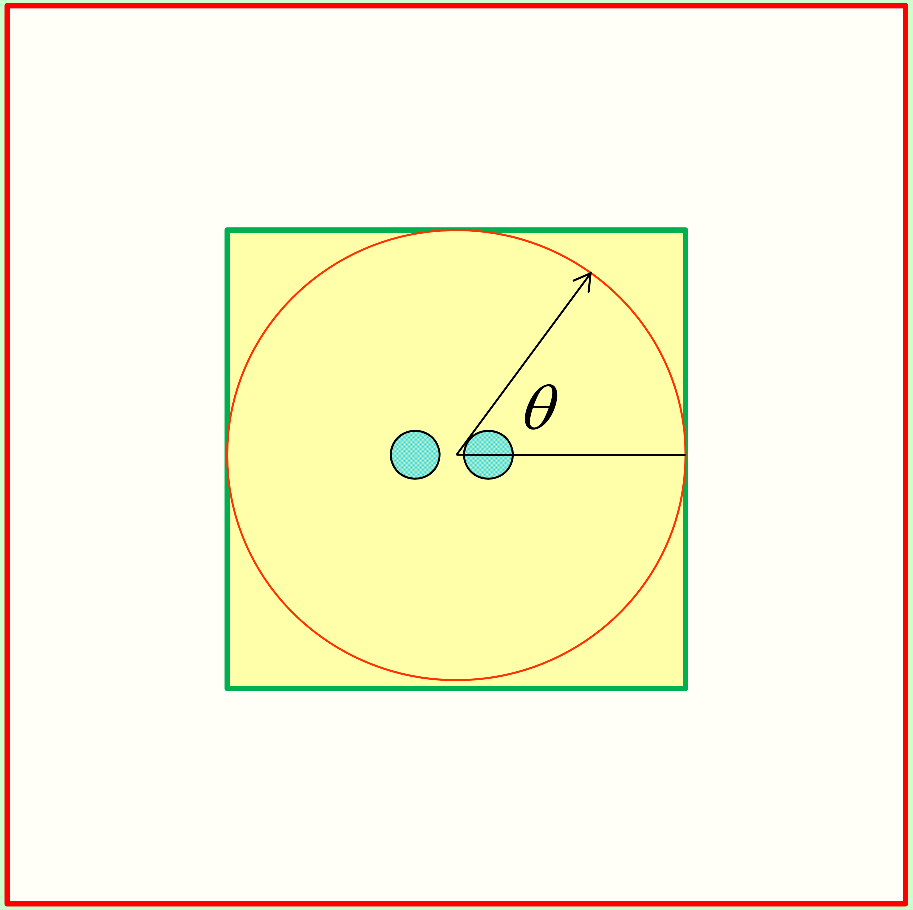
      Reflection at boundary
    - Definition of angle
  - Finite duration of calculation
    - Integration in time

Avoiding reflections

- Finite size grid
  - Reflection at the border
- Absorbing potential
  - Quadratic form

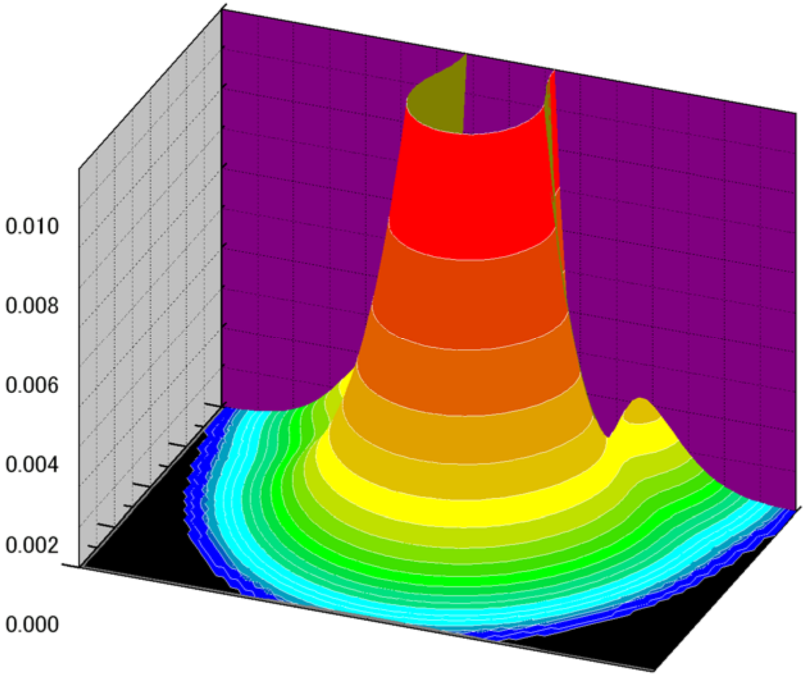
$$iC_{\text{abs}}\left(r/R_{\text{abs}}-1\right)^2 \ , \ (r > R_{\text{abs}})$$

- Alternative approach
  - Transparent boundary by M. Rizea

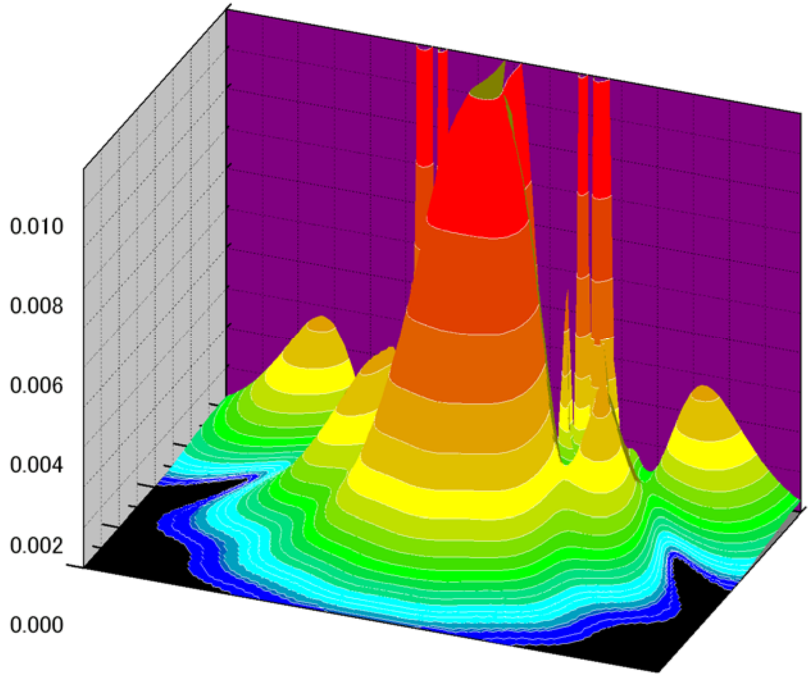


Density distribution (t = 1X10^{-21} s)

V = 0 MeV



V = -40 MeV

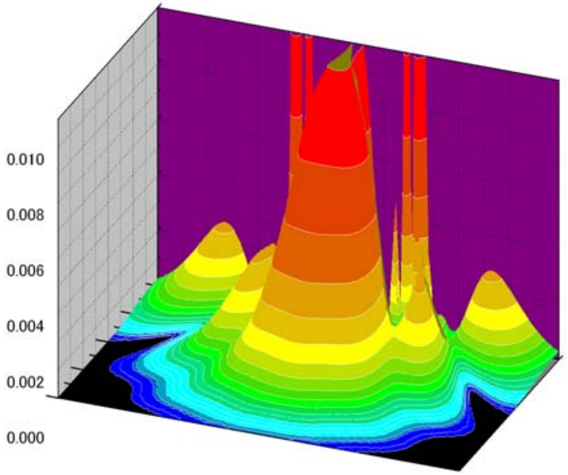


$D_0 = 2B_0$, B_0 = 9 fm
E_{pre} = 10 MeV
Coulomb repulsion

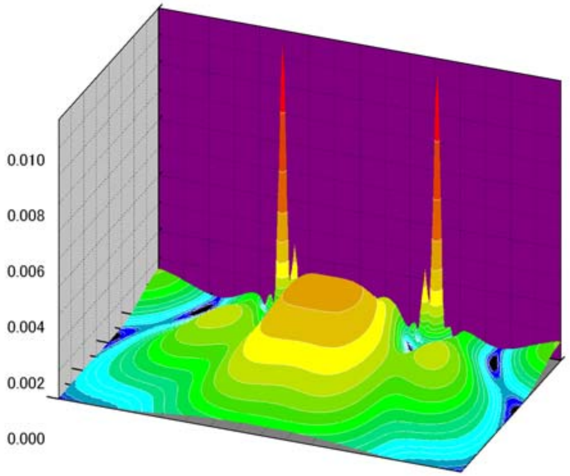
t = 1X10^{-21} s
(B = 14 fm)
W = -5 MeV

Development in time

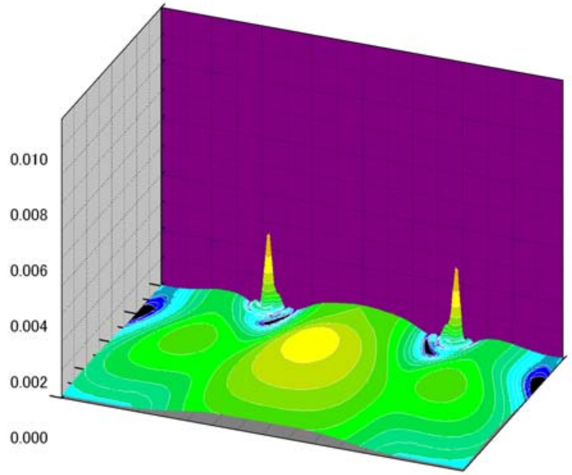
$t = 1.0 \times 10^{-21}$ s



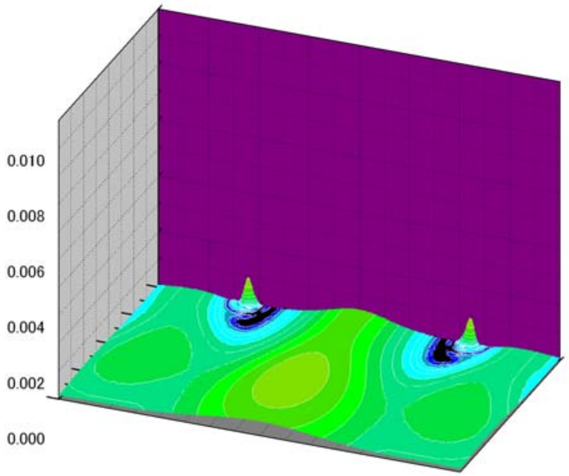
$t = 1.5 \times 10^{-21}$ s



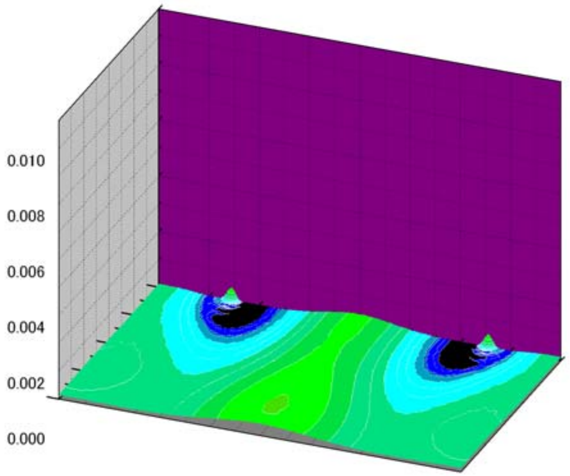
$t = 2.0 \times 10^{-21}$ s



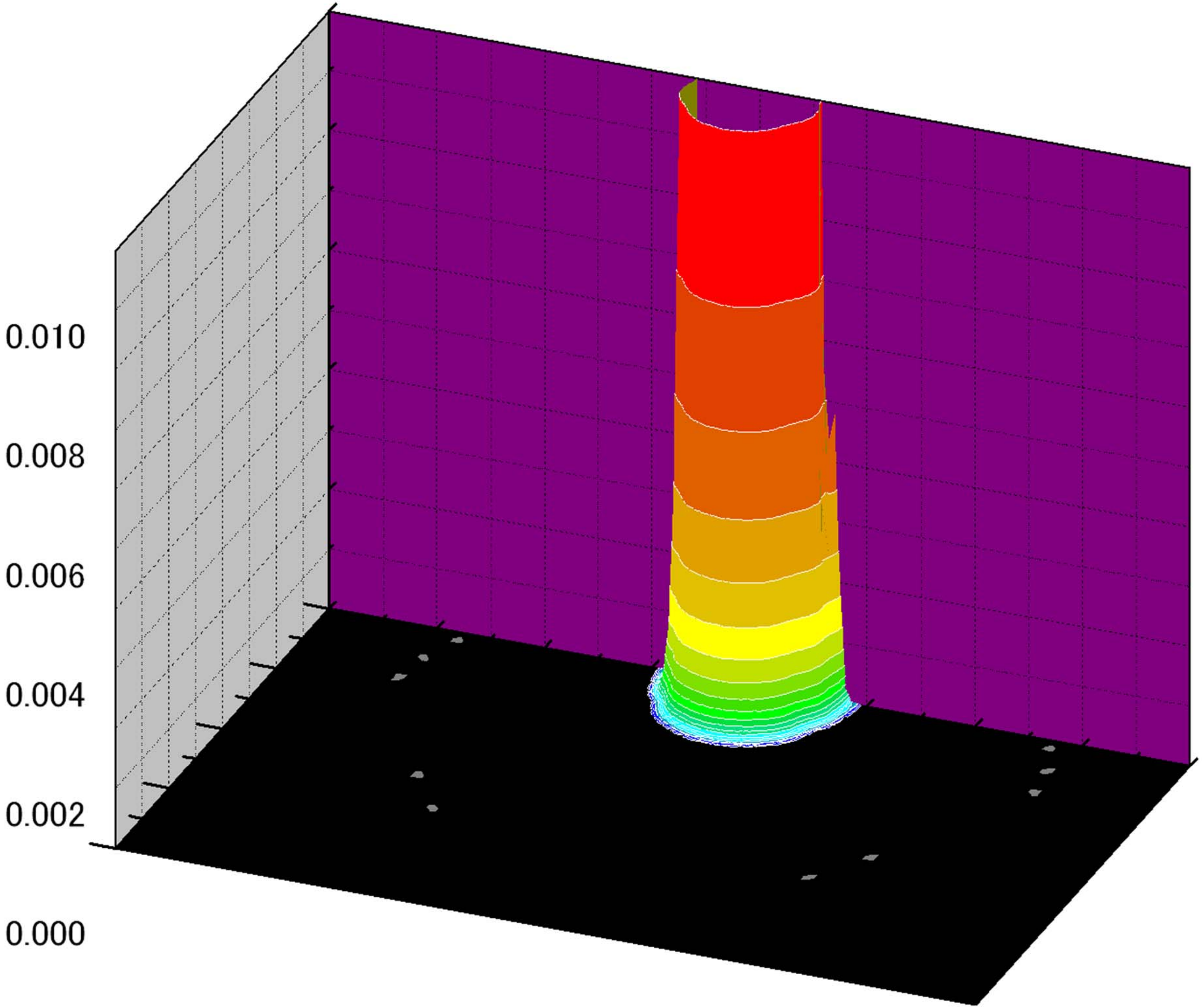
$t = 2.5 \times 10^{-21}$ s

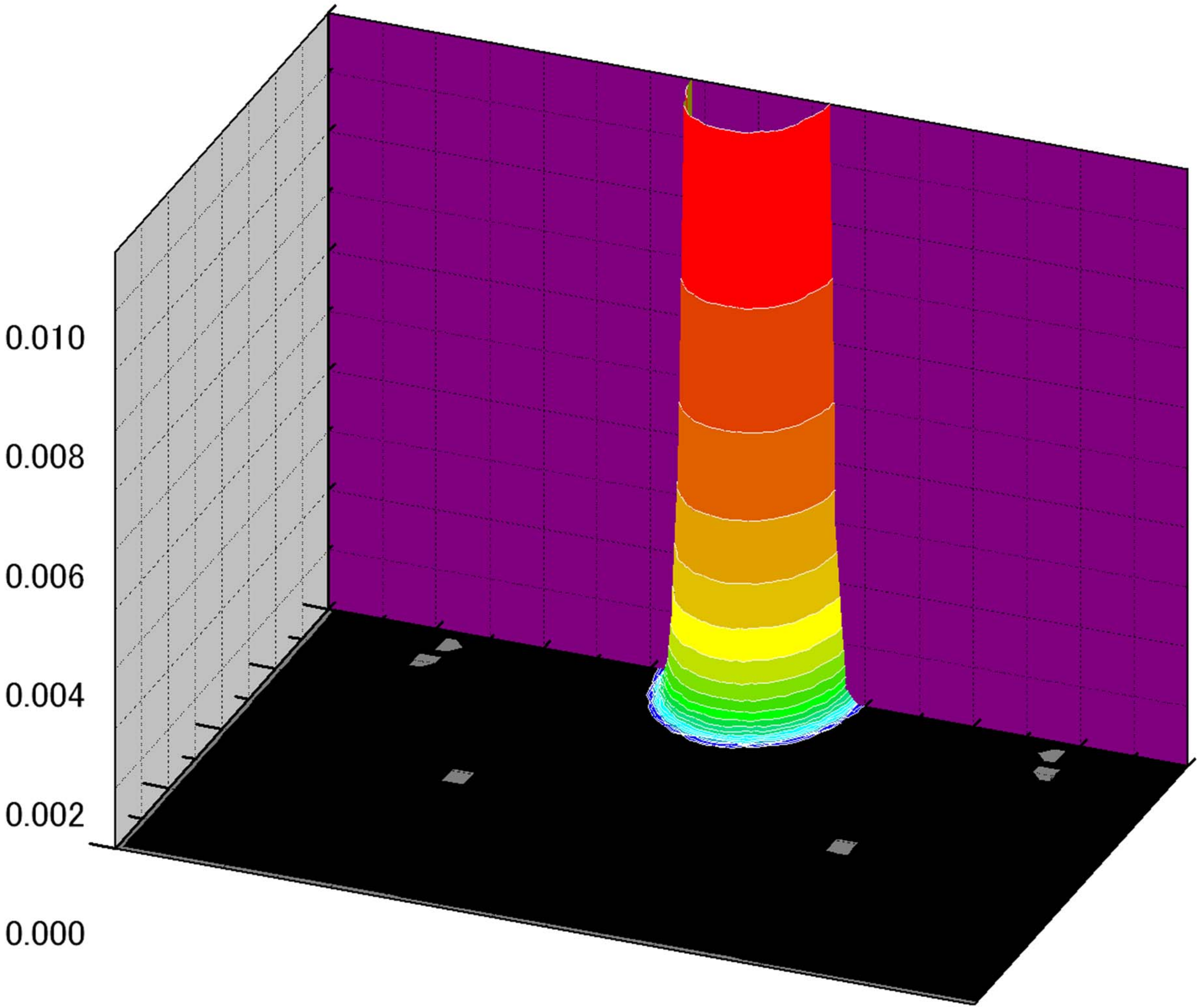


$t = 3.0 \times 10^{-21}$ s

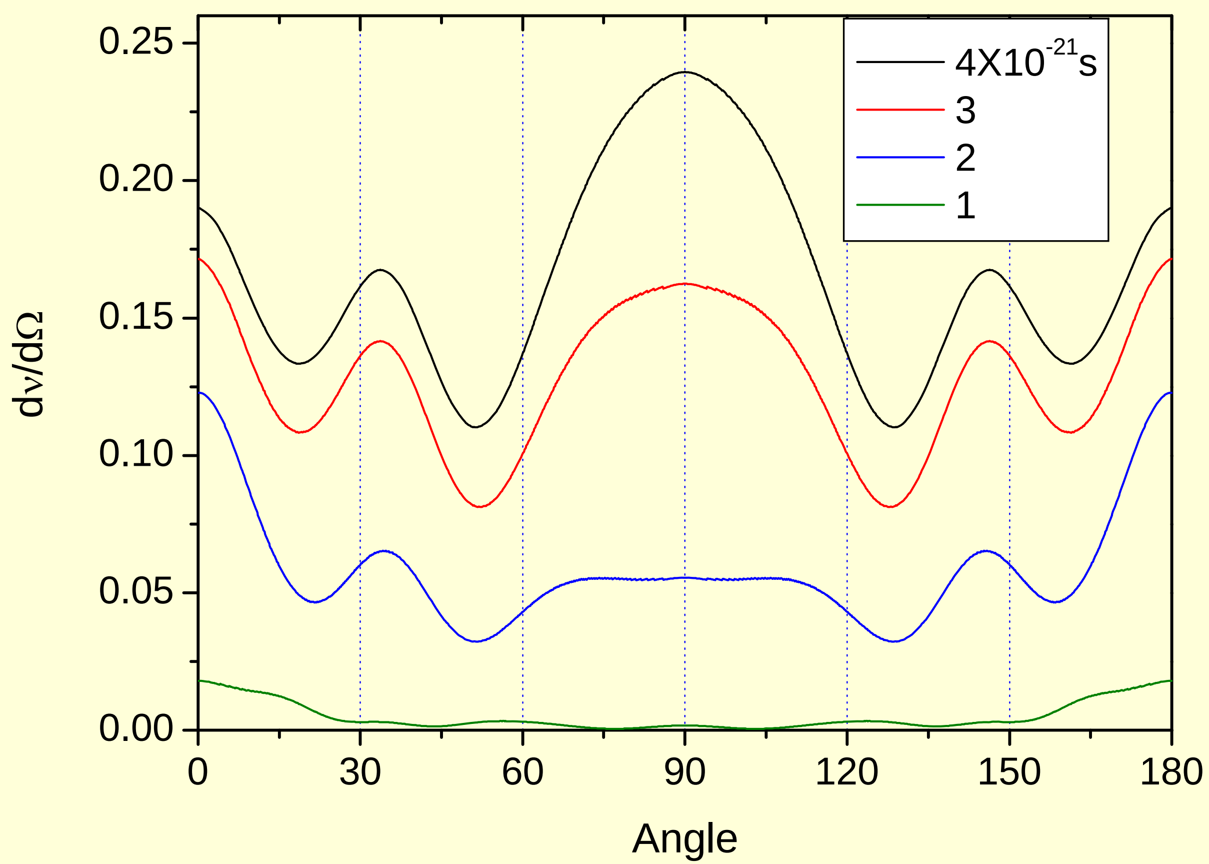


E_{pre} = 10 MeV
Coulomb repulsion
V = -40 MeV
W = -5 MeV


0.010
0.008
0.006
0.004
0.002
0.000


0.010
0.008
0.006
0.004
0.002
0.000

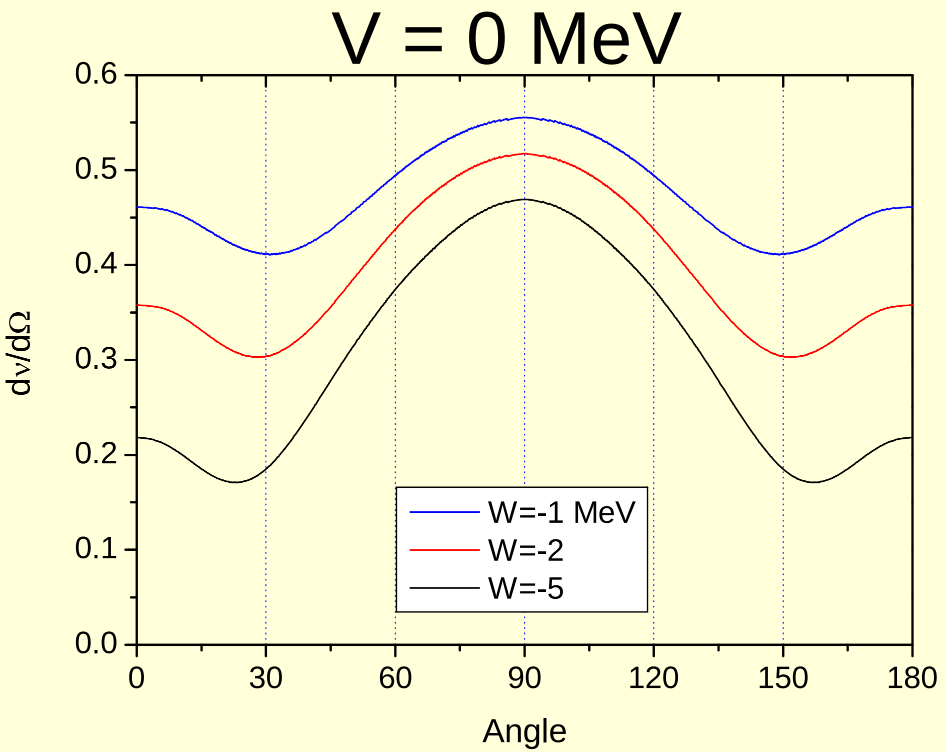
Time development of dν/dΩ

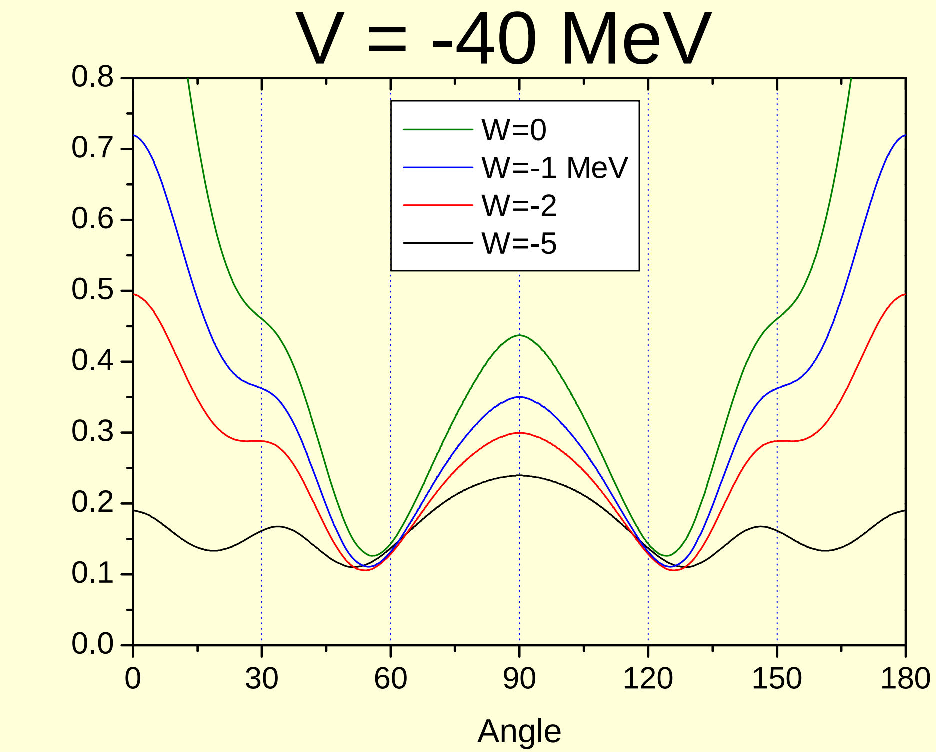


V = -40 MeV
W = -5 MeV
D_0 = 2B, B = 9fm
E_{pre} = 10 MeV
Coulomb force

R = 50 fm

Effects of potentials



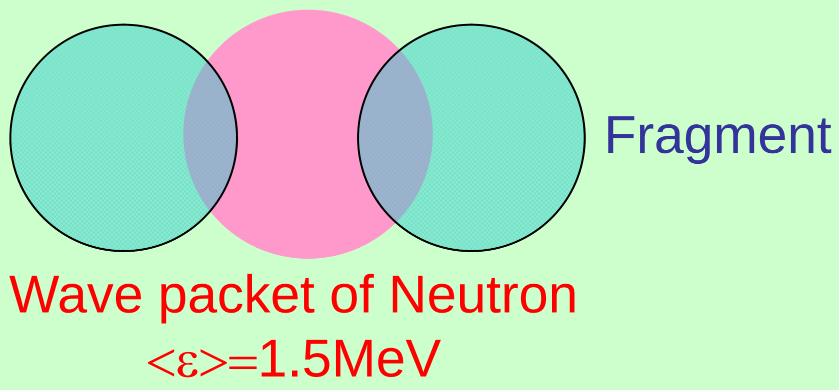


D_0 = 2B, B = 9 fm

E_{pre} = 10 MeV

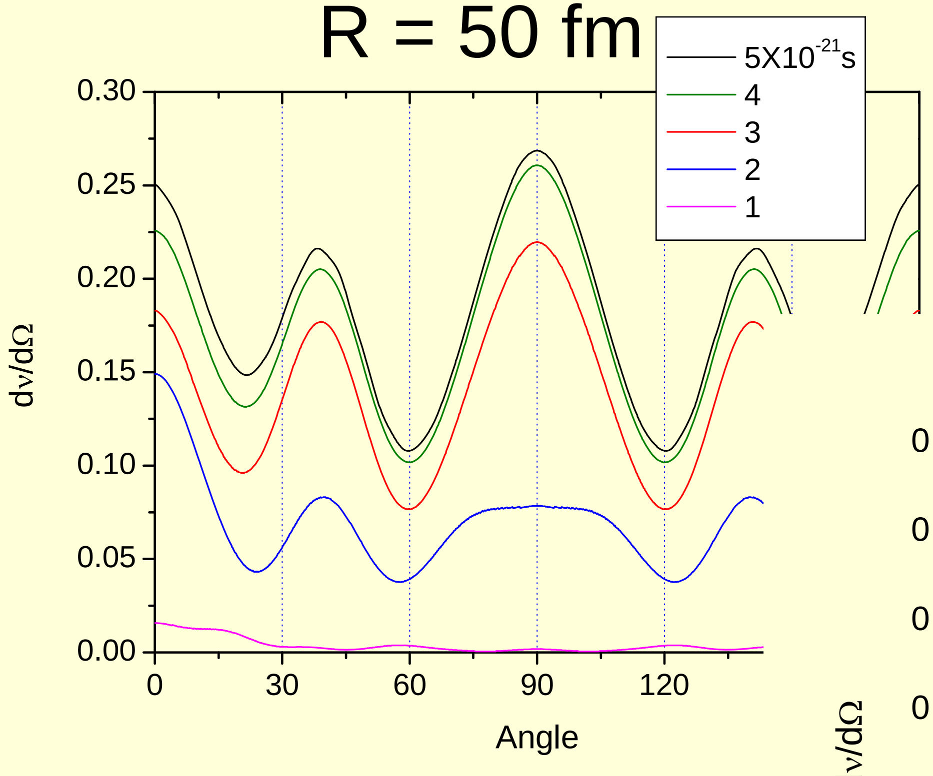
Coulomb repulsion

R = 50 fm

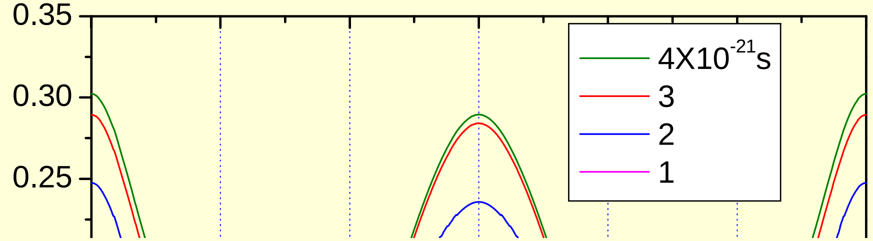


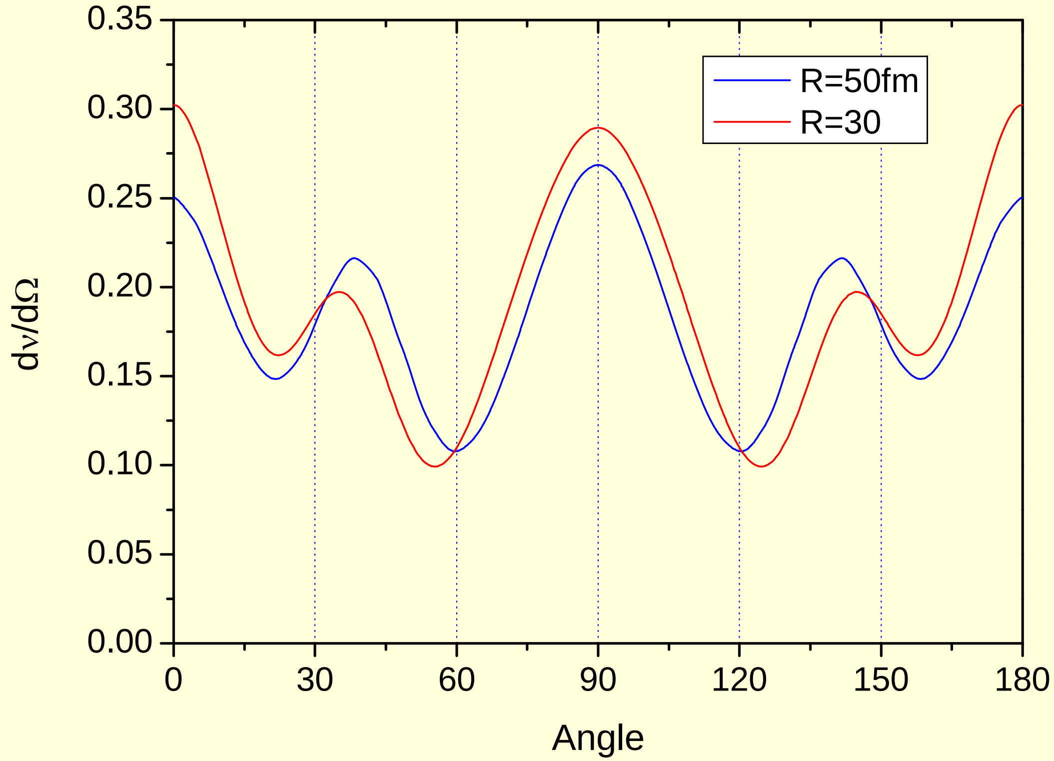
Dependence on R

R = 50 fm



R = 30 fm





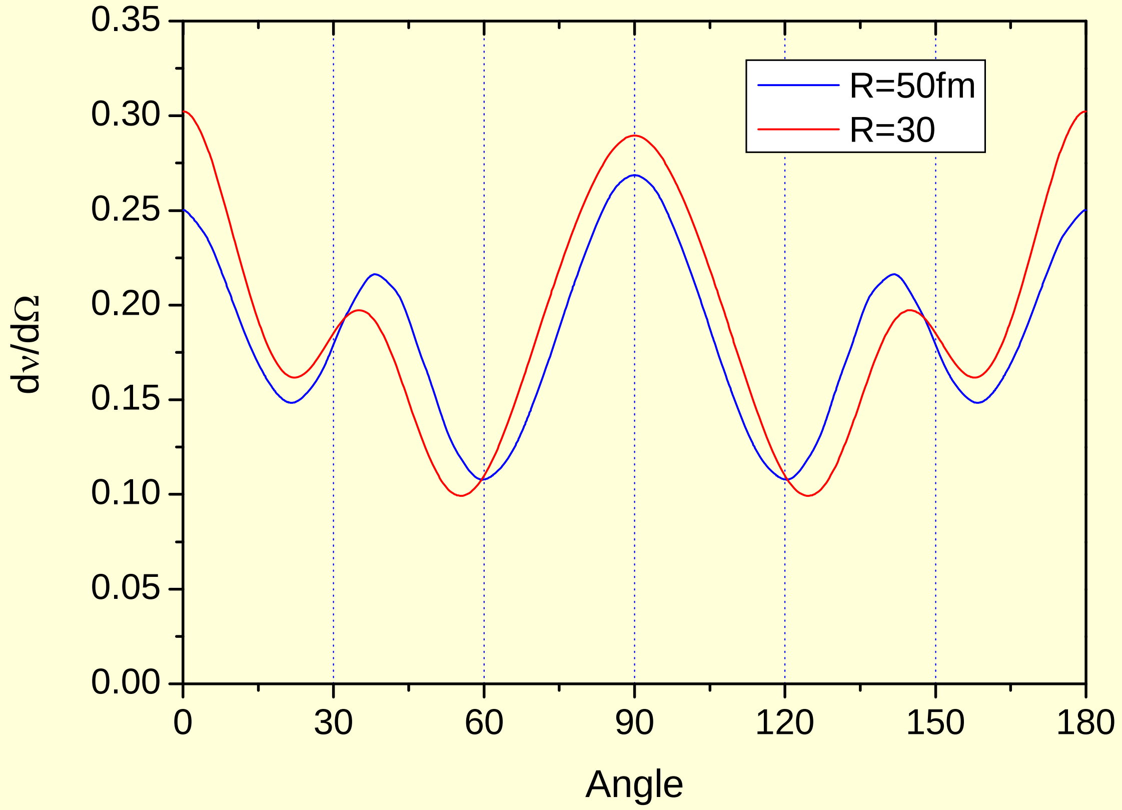
V = -40 MeV

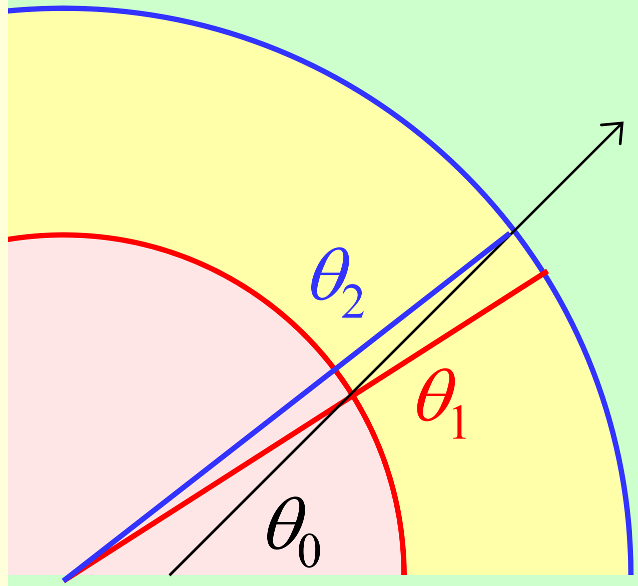
W = -5 MeV

D_0 = 2B, B = 9 fm

Fragments no motion

Dependence on R





$\theta_0 > \theta_2 > \theta_1$

B = 9 fm

R_1 = 30 fm

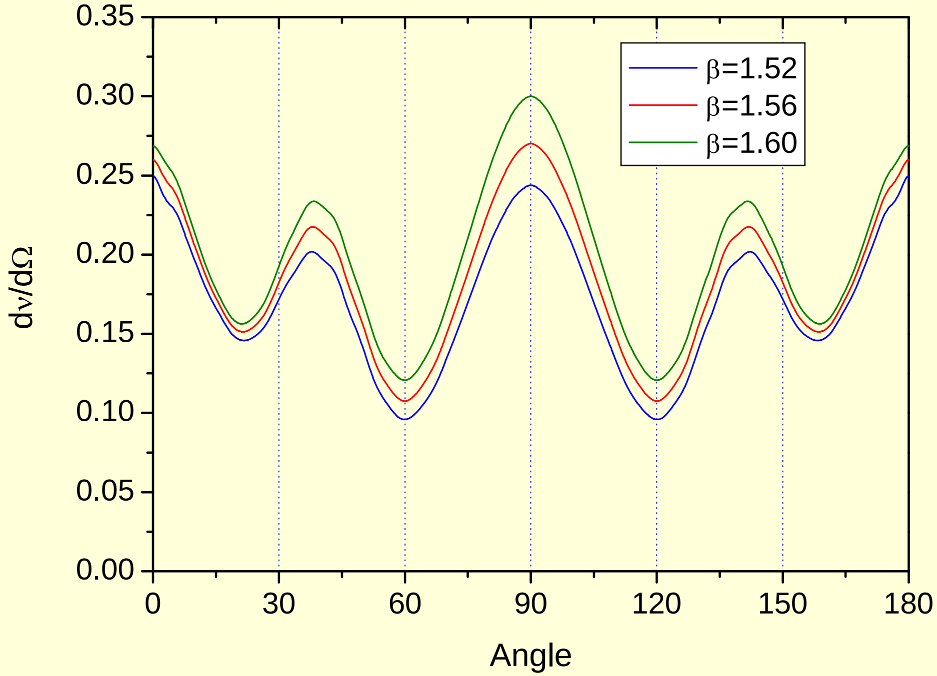
R_2 = 50 fm

V = -40 MeV, W = -5 MeV

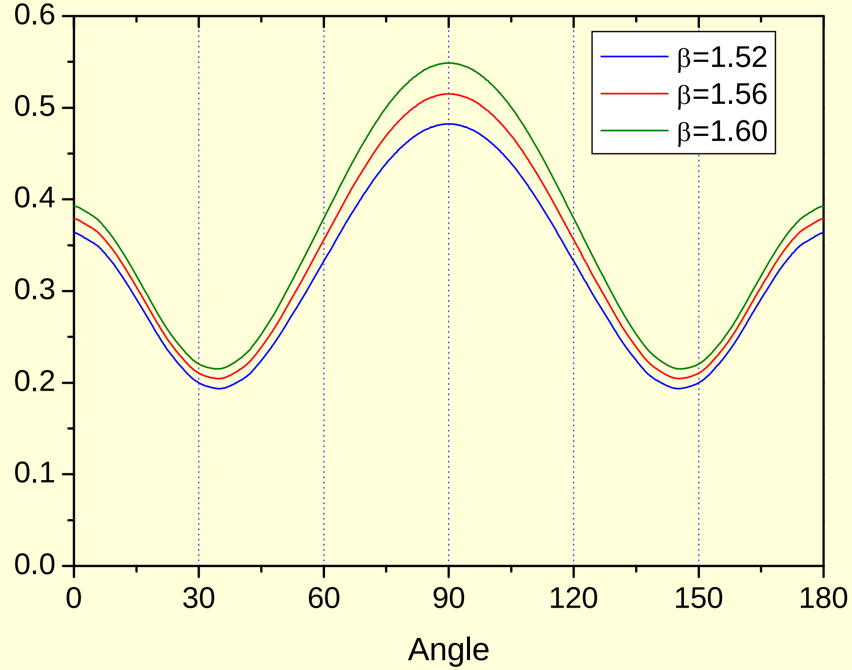
Fragments no motion

Separation between fragments

V = -40 MeV



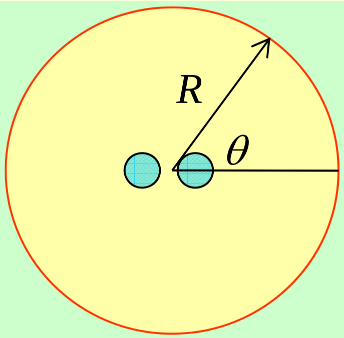
V = 0 MeV



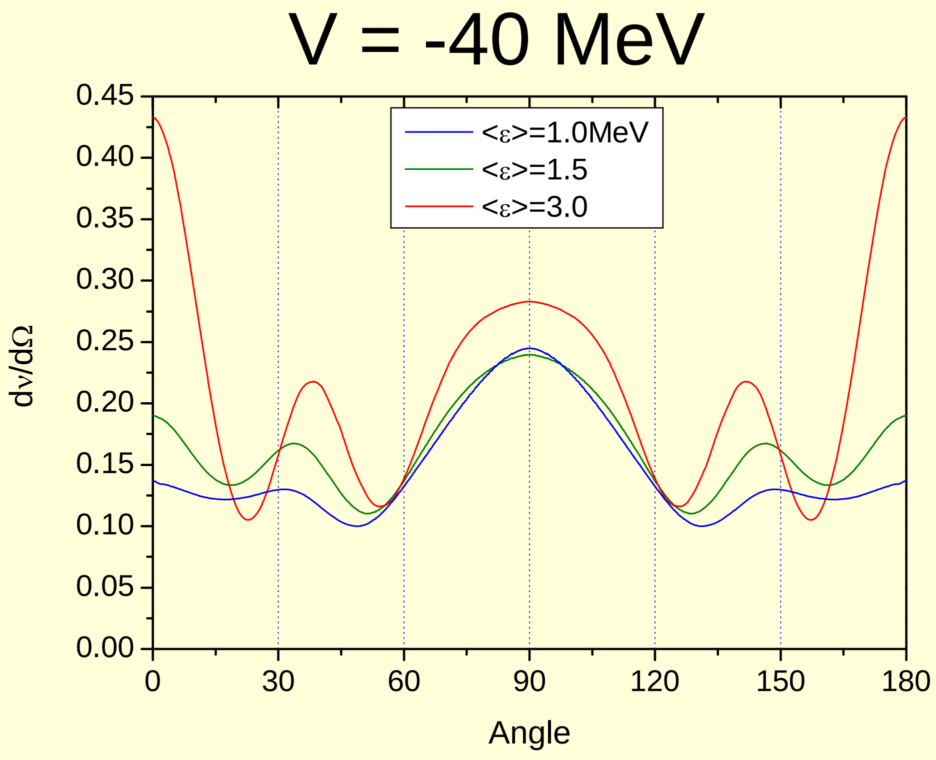
W = -5 MeV

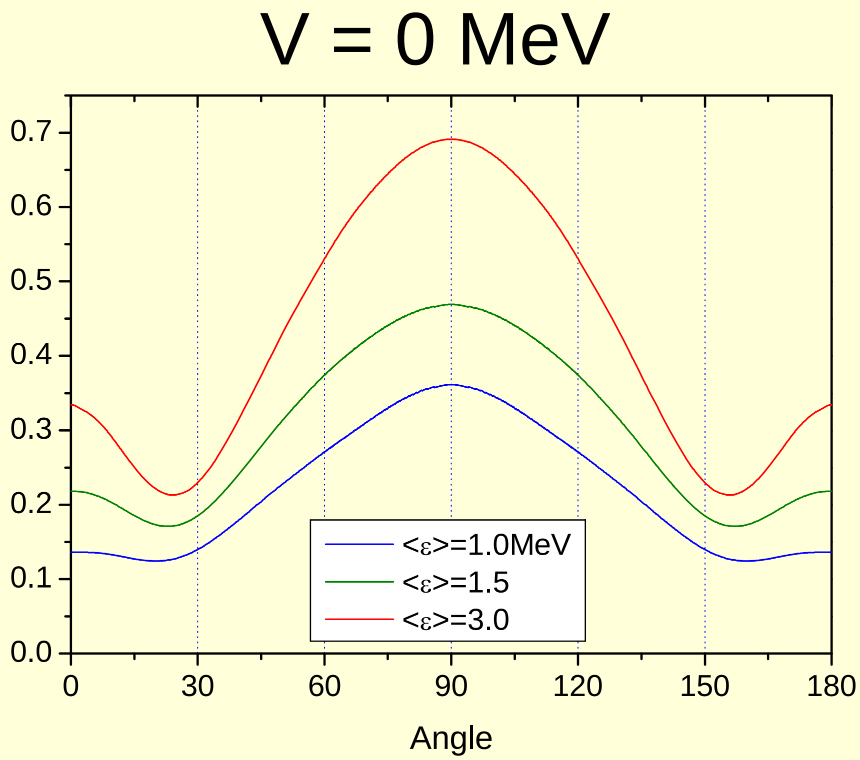
$D_0 = 2B$, $B = \beta r_F$

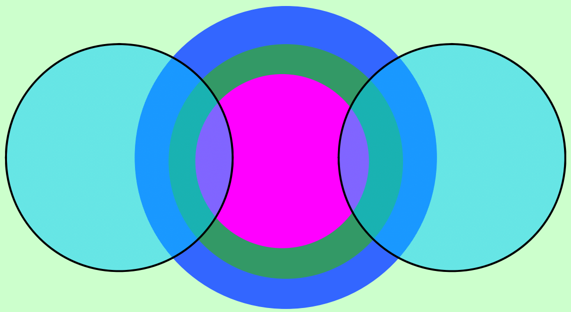
R = 50 fm



Dependence on initial distribution







W = -5 MeV

D_0 = 2B, B = 9 fm

R = 50 fm

Treatment of boundary points

- We need extra point to calculate differentials at the boundary

$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{\psi(x+\Delta x) - 2\psi(x) + \psi(x-\Delta x)}{\Delta x^2}$$

- Transparent Boundary Condition (M. Rizea)

$$\frac{\psi_{M+1}}{\psi_M} \approx \frac{\psi_M}{\psi_{M-1}} \Rightarrow \log \psi_{M+1} \approx 2\log \psi_M - \log \psi_{M-1}$$

- Higher order approximation

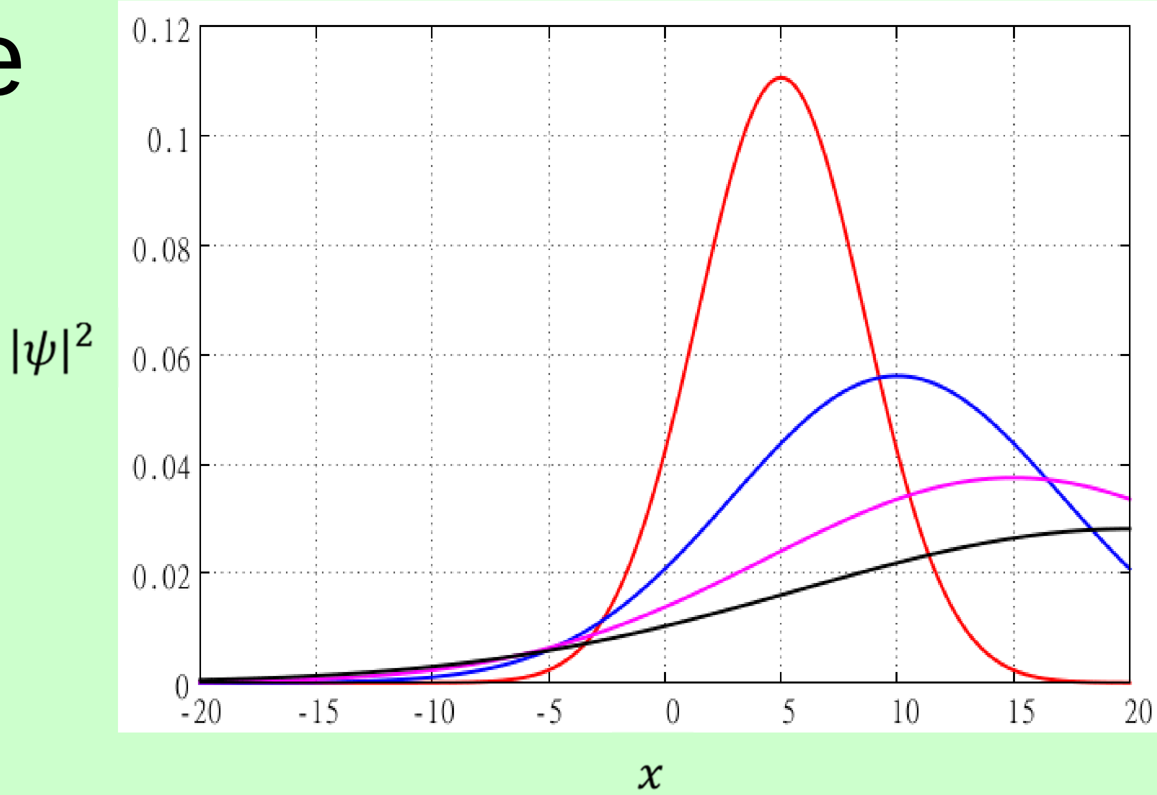
$$\log \psi_{M+1} \approx 3\log \psi_M - 3\log \psi_{M-1} + \log \psi_{M-2} \Rightarrow \psi_{M+1} \approx \frac{\psi_M{}^3 \psi_{M-2}}{\psi_{M-1}{}^3}$$

Free particle

$V(x) = 0$

$:t = 5$
$:t = 10$
$:t = 15$
$:t = 20$

$x_0 = 0, v_0 = 1$



- Analytic solution

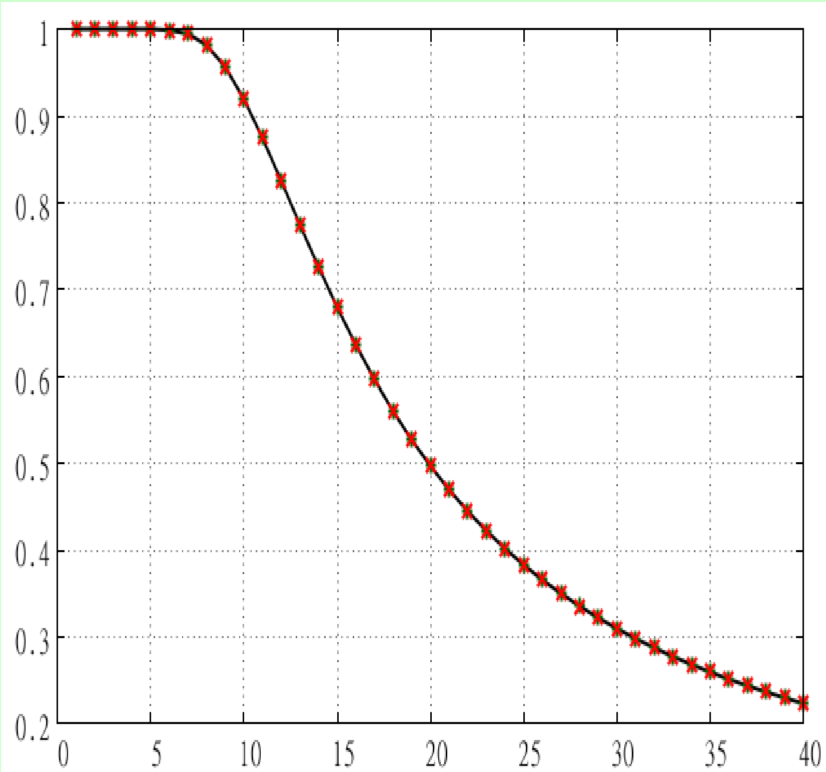
$$\psi(x,t) = \frac{1}{(\pi\sigma^2)^{1/4}} \frac{1}{\sqrt{1 + \frac{i\hbar t}{m\sigma^2}}} \exp\left[-\frac{(x - x_0 - v_0 t)^2}{2\sigma^2 + \frac{2i\hbar t}{m}}\right]$$

$$x_0 = 0, k_0 = 1$$
$$\Delta x = 1/20, \Delta t = 1/800$$

$$P(t) \equiv \int_{-20}^{20} \|\psi\|^2 \, dx$$

Red : new method

Black : analytic solution



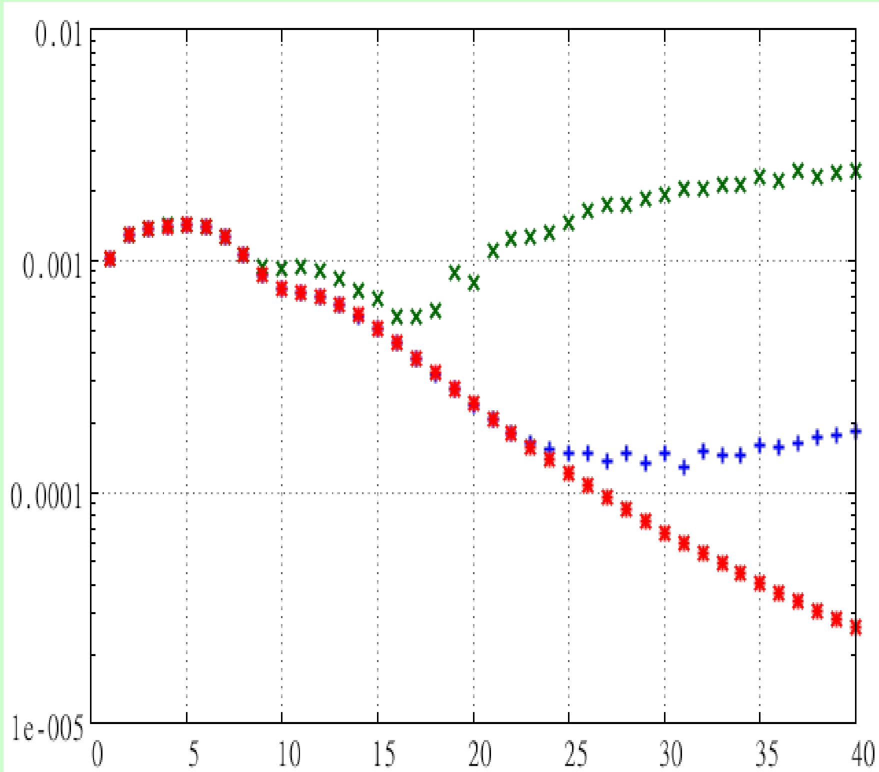
t

$$\boldsymbol{Error} \equiv \sum_i^{i_{\mathrm{eff}}} \left| \|\psi(x_i)\|_{calc}{}^2 - \|\psi(x_i)\|_{exact}{}^2 \right| \Delta x$$

Red : new method

Blue : imaginary potential

Green : 1st order



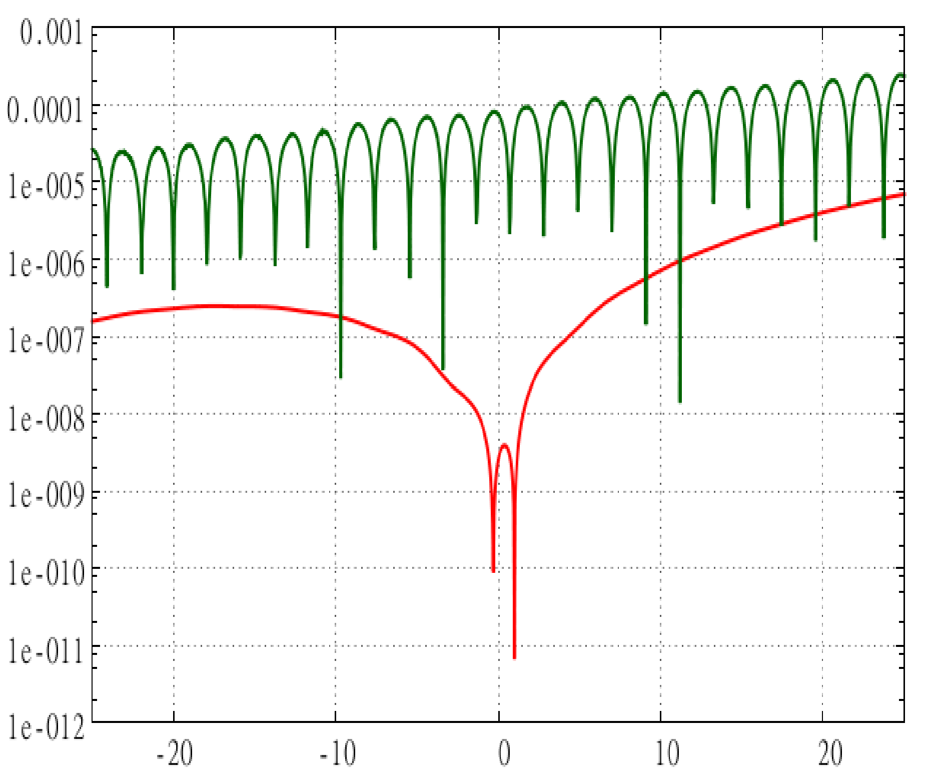
t

$$x_0 = 0, k_0 = 1$$
$$\Delta x = 1/20, \Delta t = 1/800$$

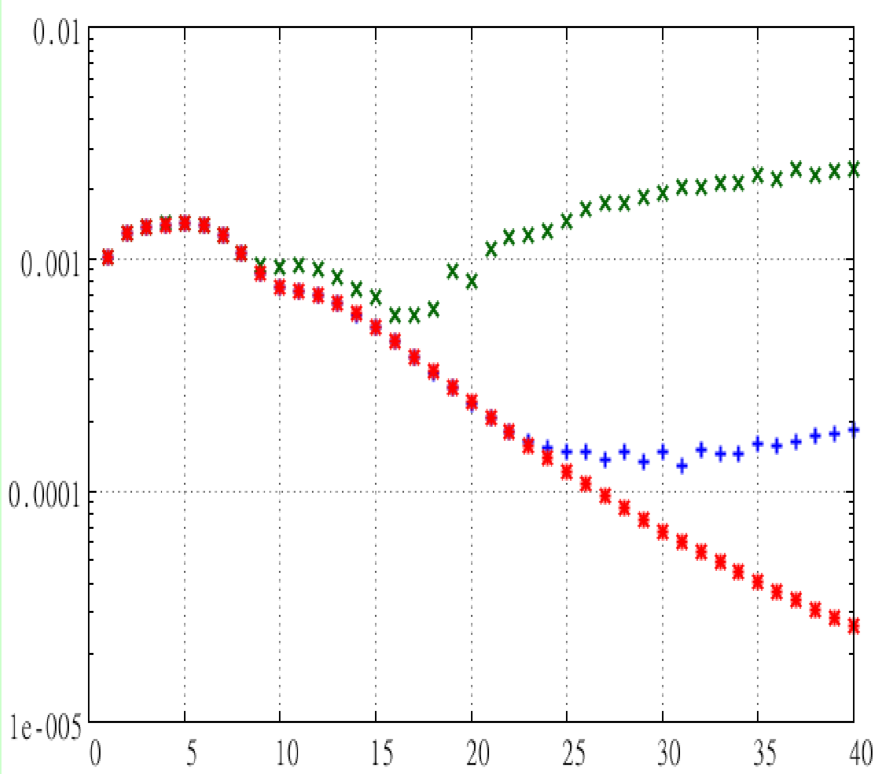
$$Error \equiv \sum_i^{i_{eff}} \left| \|\psi(x_i)\|_{calc}{}^2 - \|\psi(x_i)\|_{exact}{}^2 \right| \Delta x$$

$$\left| \|\psi(x_i)\|_{calc}{}^2 - \|\psi(x_i)\|_{exact}{}^2 \right|$$
at $t = 40$

Red : new method
Blue : imaginary potential
Green : 1st order



x



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Summary

* Information on the angular distribution of scission neutrons is needed to separate their contribution from that of post-scission neutrons.
* Time-dependent approach is proposed to calculate the effect of scattering and re-absorption on the angular distribution of the scission neutrons.
* Angular distribution of the scission neutrons is strongly modified by the fragments.
* Attractive potential magnifies the contribution around 0 and 180 degrees, while absorptive potential diminishes those components.