

10th ASRC International Workshop “Nuclear Fission and Decay of Exotic Nuclei”  
March 21-22, 2013

A fully microscopic study of multinucleon transfer processes in  $^{136}\text{Xe}+^{198}\text{Pt}$ :  
An application of the time-dependent Hartree-Fock theory

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In collaboration with

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Center for Computational Sciences, Univ. Tsukuba

# INTRODUCTION: Interests for multinucleon transfer reactions

✓ As an interesting quantum dynamics

*Fundamental interest*

➤ **Multinucleon transfer reaction** = A non-equilibrium quantum transport phenomenon

*Static properties*

Nuclear shell structure and binding energies, shapes, N/Z ratios, ...

*Dynamical effects*

Quantum tunneling, neck formation, matching of Q-values and momenta, ...

✓ Both static properties and time-dependent dynamics are related to its reaction mechanisms

✓ As a mean to produce unstable nuclei

*Experimental interest*

➤ **KISS project (2010-2015)** (KISS: KEK Isotope Separation System)

*cf. S.C. Jeong et al., KEK Report 2010-2 (2010)*

# INTRODUCTION: Interests for multinucleon transfer reactions

cf. S.C. Jeong et al., KEK Report 2010-2 (2010)

➤ **KISS project (2010-2015)** (KISS: KEK Isotope Separation System)

Aim: To measure lifetime of neutron-rich unstable nuclei with  $A \sim 200$  along the neutron magic number  $N=126$

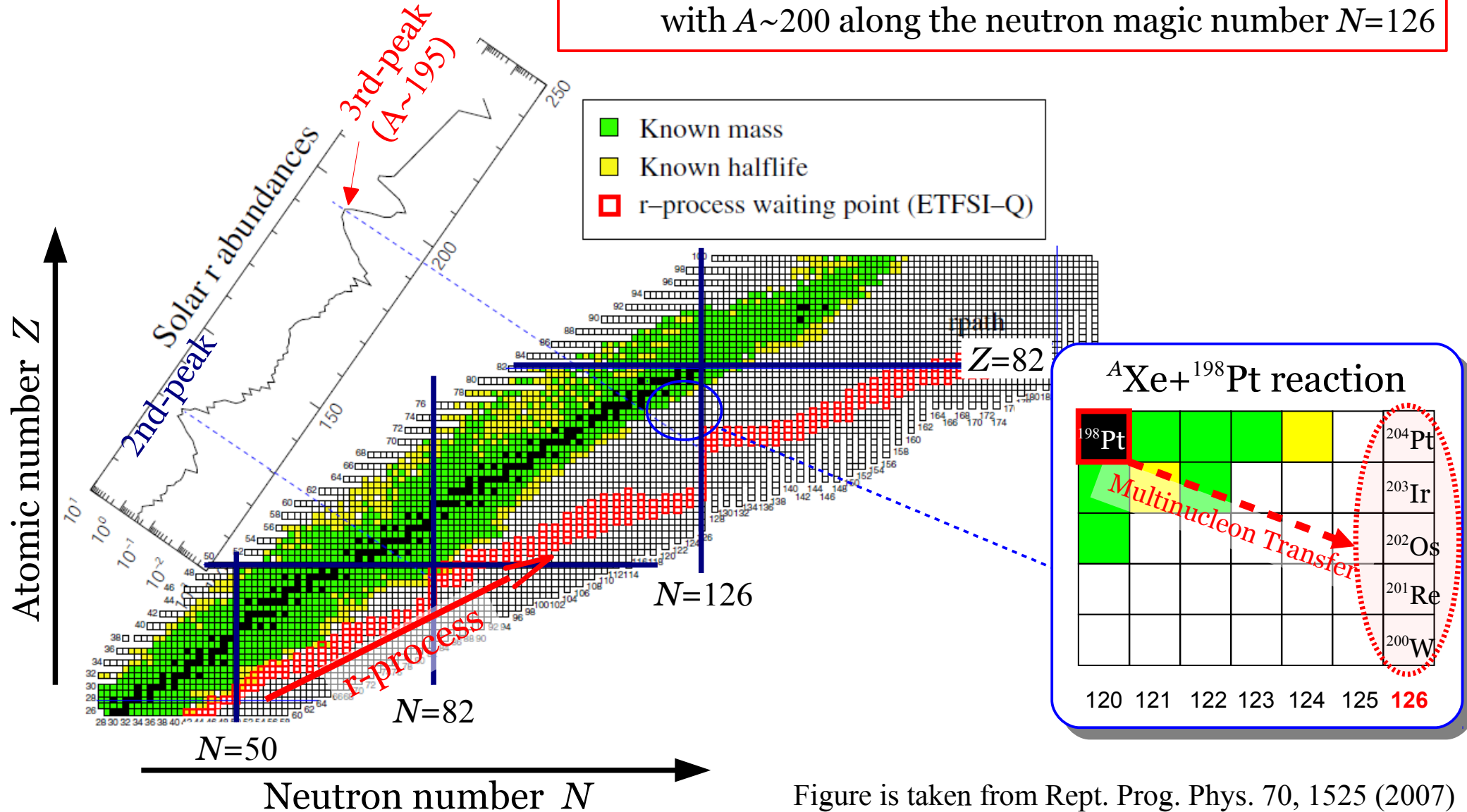


Figure is taken from Rept. Prog. Phys. 70, 1525 (2007)

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➤ It is highly desired to develop a fully microscopic description with minimum assumptions on the dynamics

1. Introduction
2. TDHF calculations for multinucleon transfer processes in  $^{40}\text{Ca}+^{124}\text{Sn}$  reaction
3. A systematic TDHF calculation of  $^{136}\text{Xe}+^{198}\text{Pt}$  reaction
4. Summary and Perspective

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# Results of the TDHF calculation: $^{40}\text{Ca} + ^{124}\text{Sn}$ at $E_{\text{lab}} = 170$ MeV

K. Sekizawa and K. Yabana, arXiv:1303.0552 [nucl-th]

3D-grid:  $60 \times 60 \times 26$  (48 fm  $\times$  48 fm  $\times$  20.8 fm), Mesh size: 0.8 fm

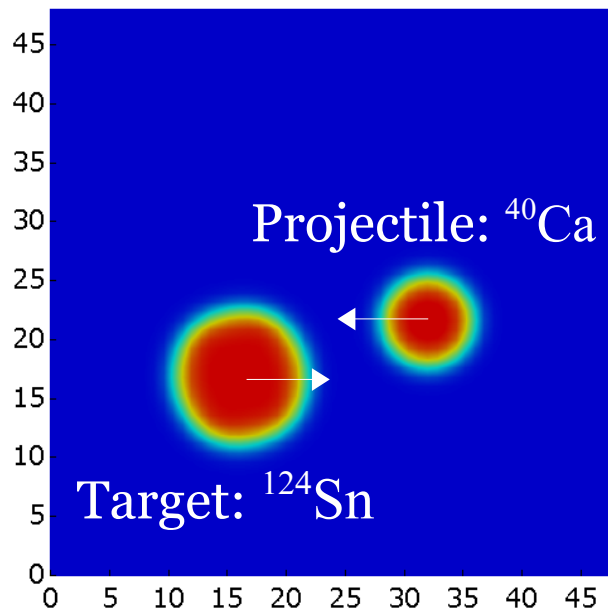
Skyrme force: SLy5,  $\Delta t$ : 0.2 fm/c, Initial separation distance: 16 fm

Calculated impact parameter:  $0 \leq b \leq 10$  fm

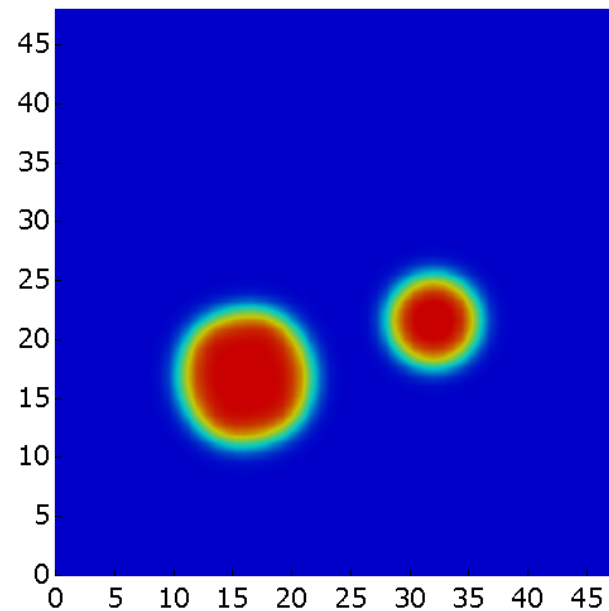
Fusion reactions ( $b \leq 3.69$  fm), Binary reactions ( $b \geq 3.70$  fm)

## Density evolution obtained from the TDHF calculation

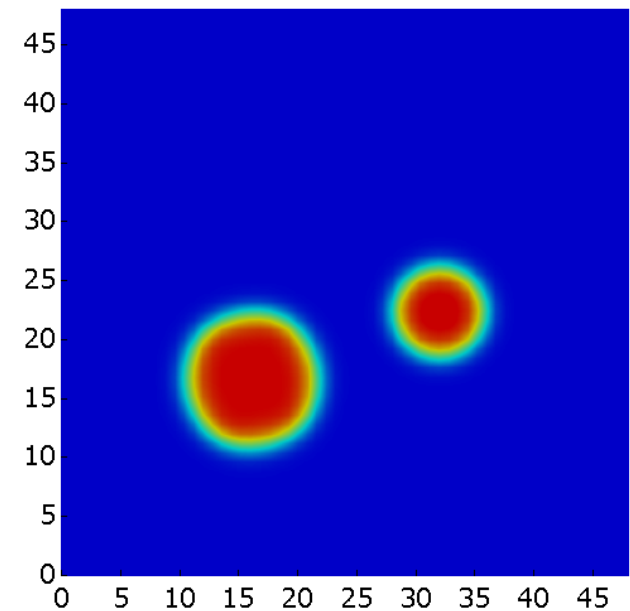
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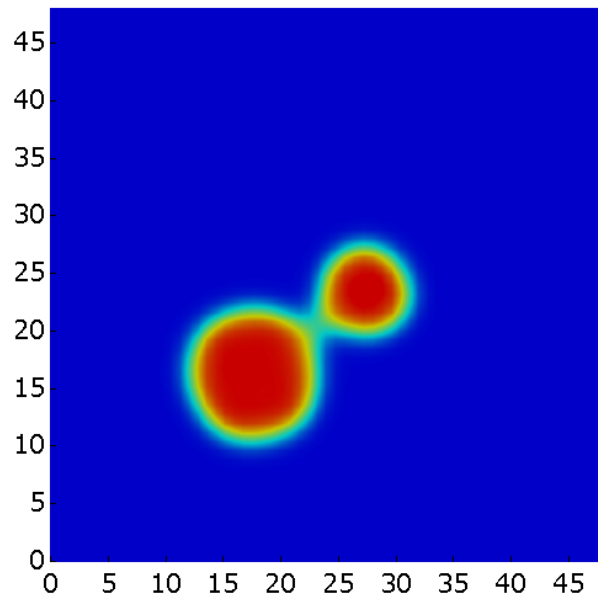
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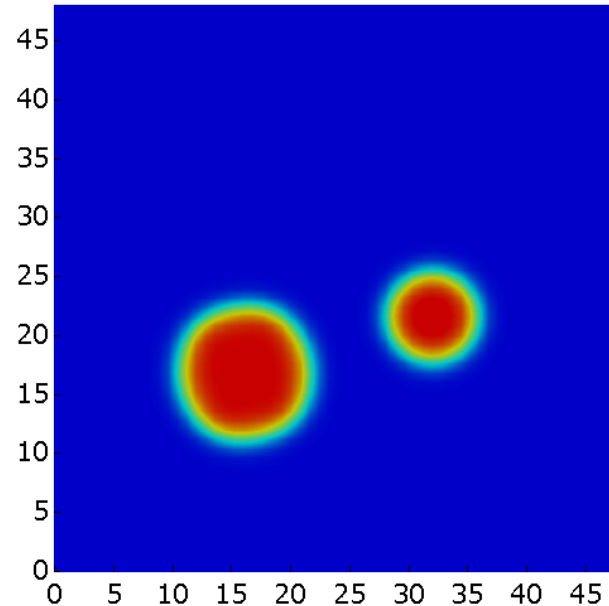
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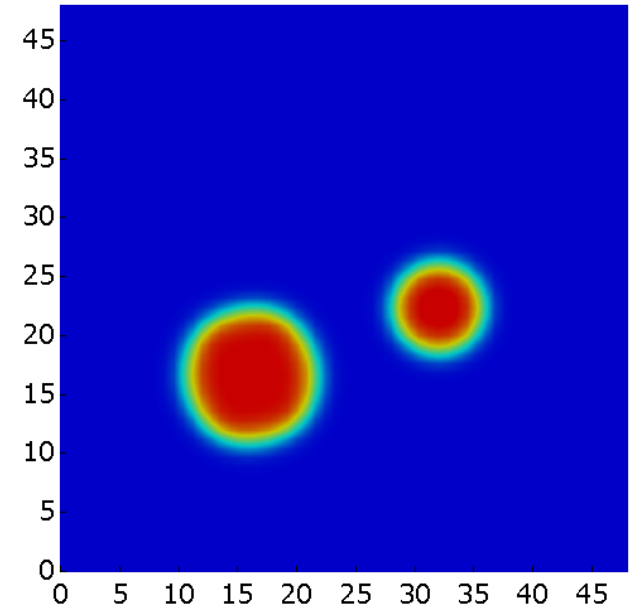
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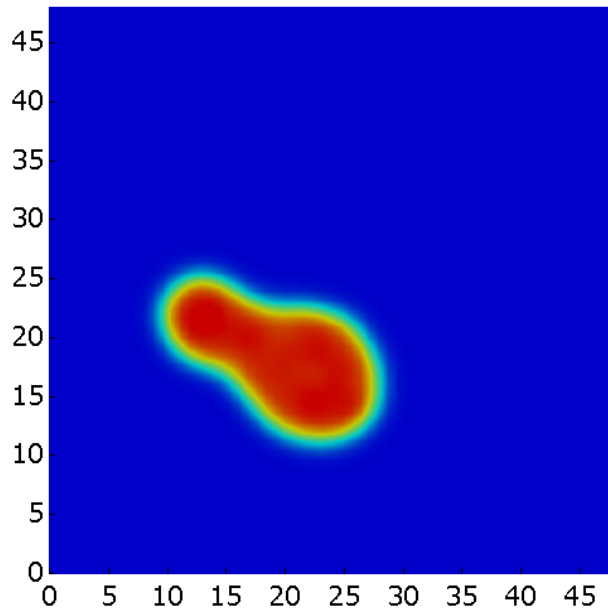
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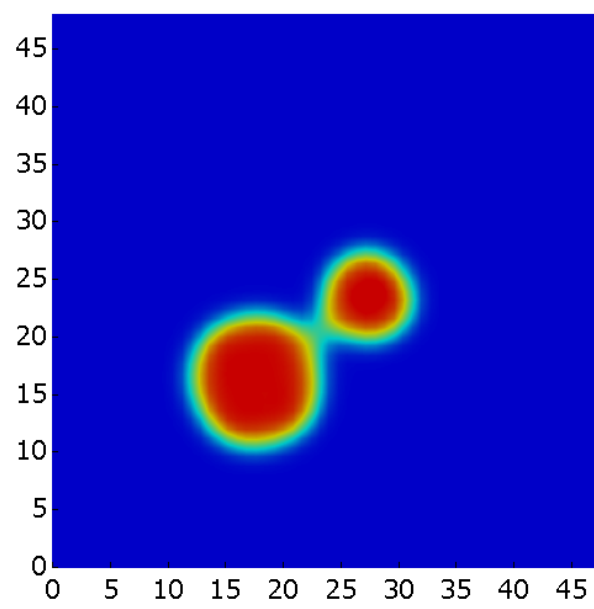
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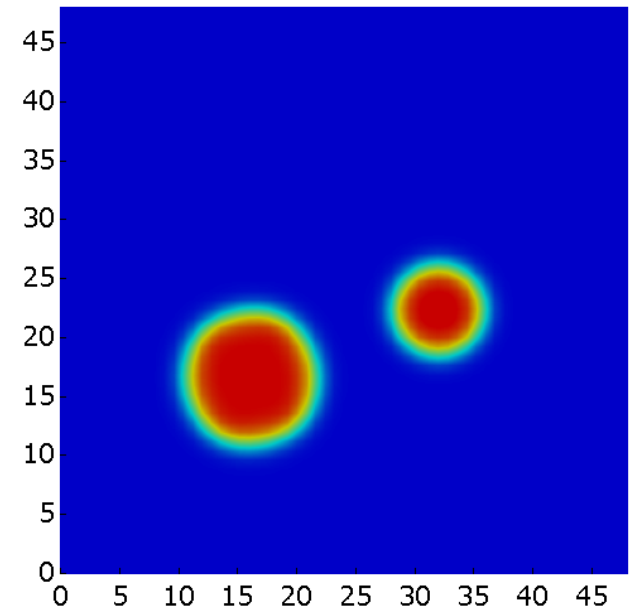
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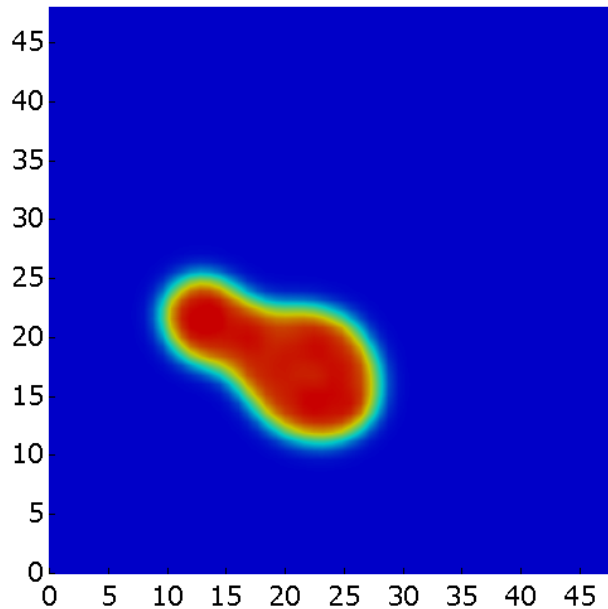
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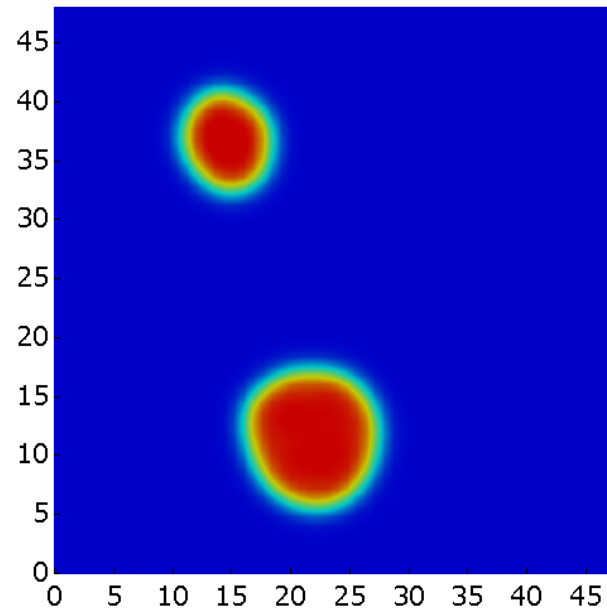
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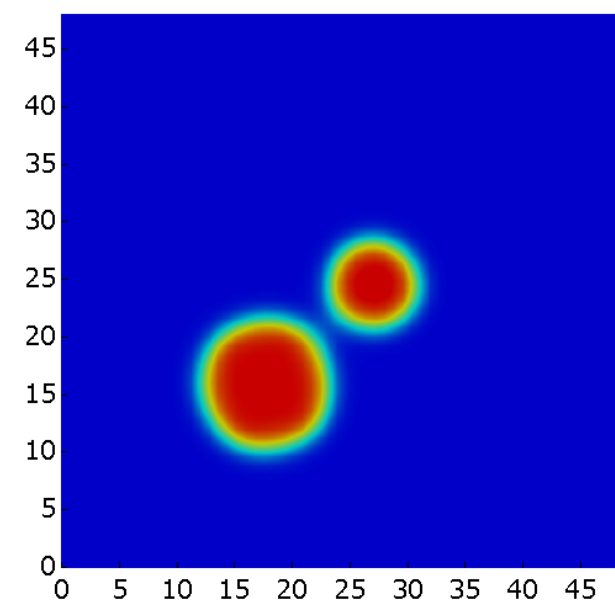
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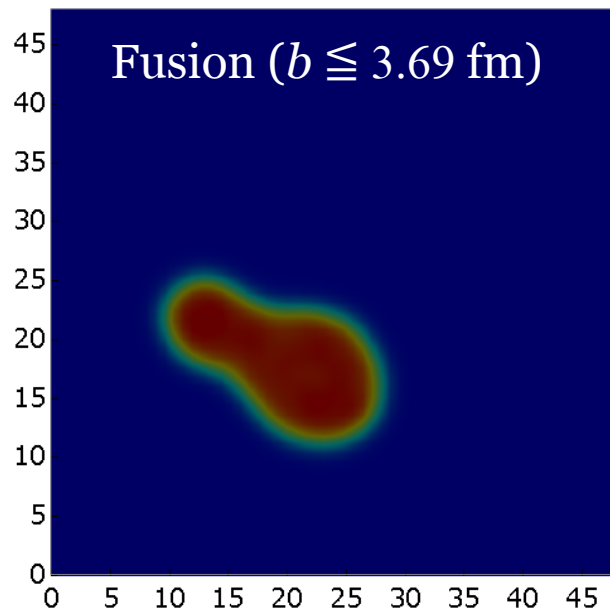
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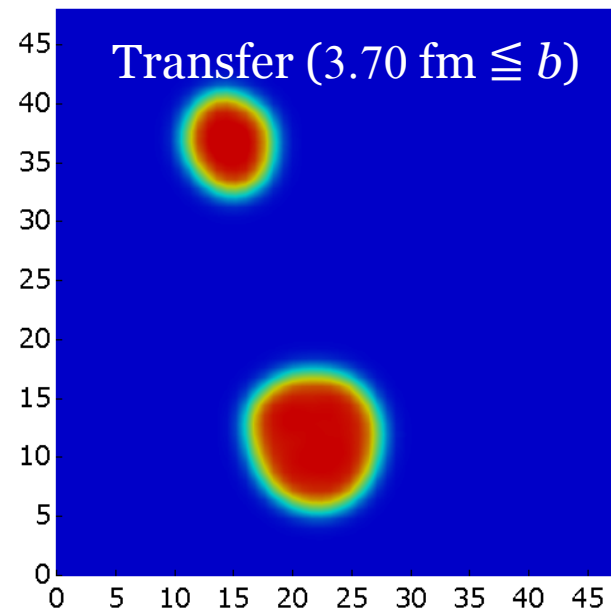
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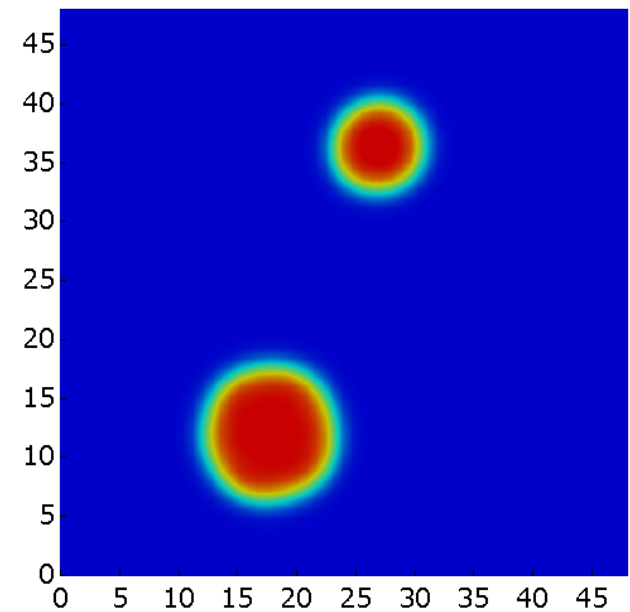
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# How to calculate the transfer probability

## Particle number projection method

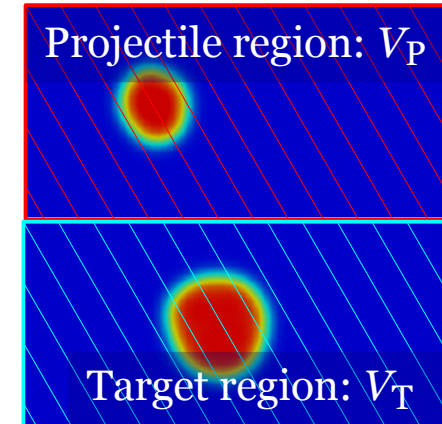
C. Simenel, Phys. Rev. Lett. **105**, 192701 (2010)

### ✓ Particle number projection operator

$$\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i(n - \hat{N}_P)\theta}$$

$\hat{N}_P$ : Number operator of the spatial region  $V_P$

$$\hat{N}_P = \int_{V_P} d^3r \sum_{i=1}^{N_P + N_T} \delta(\mathbf{r} - \mathbf{r}_i)$$



$N = N_P + N_T$ : Total number of nucleons

### ➤ Probability $P_n$ : $n$ nucleons are in the $V_P$ and $N-n$ nucleons are in the $V_T$

$$\begin{aligned} P_n &= \langle \Phi | \hat{P}_n | \Phi \rangle \\ &= \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{in\theta} \det \{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P} \} \end{aligned}$$

Slater determinant

$$\Phi(\mathbf{x}_1, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \det \{ \phi_i(\mathbf{x}_j) \}$$

Single-particle w.f.

$$\begin{aligned} \phi_i(\mathbf{x}) &\equiv \phi_i(\mathbf{r}, \sigma) \\ i &= 1, \dots, N_P + N_T \end{aligned}$$

Overlap integral in respective regions

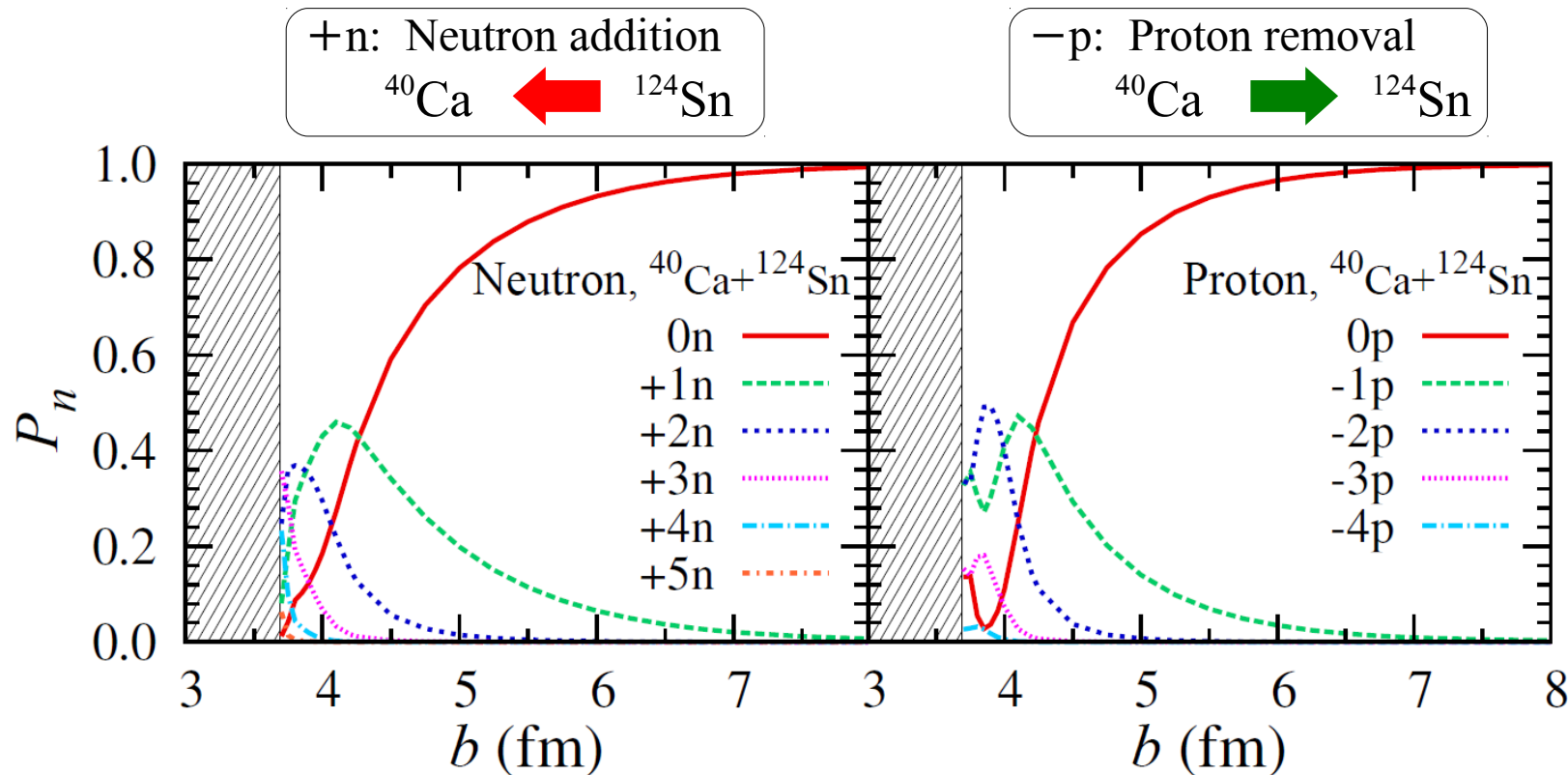
$$\begin{aligned} \langle \phi_i | \phi_j \rangle_{\tau} &= \int_{\tau} d^3x \phi_i^*(\mathbf{x}) \phi_j(\mathbf{x}) \\ \tau &= V_P \text{ or } V_T \end{aligned}$$

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## Transfer probabilities

$$P_n = \langle \Phi | \hat{P}_n | \Phi \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{in\theta} \det \{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P} \} : \text{The projection method}$$



- ✓ Nucleons are transferred towards the directions of the charge equilibrium.
- ✓ Transfer probabilities of several nucleons become sizable just outside the fusion region.

# Results of the TDHF calculation: $^{40}\text{Ca} + ^{124}\text{Sn}$ at $E_{\text{lab}} = 170 \text{ MeV}$

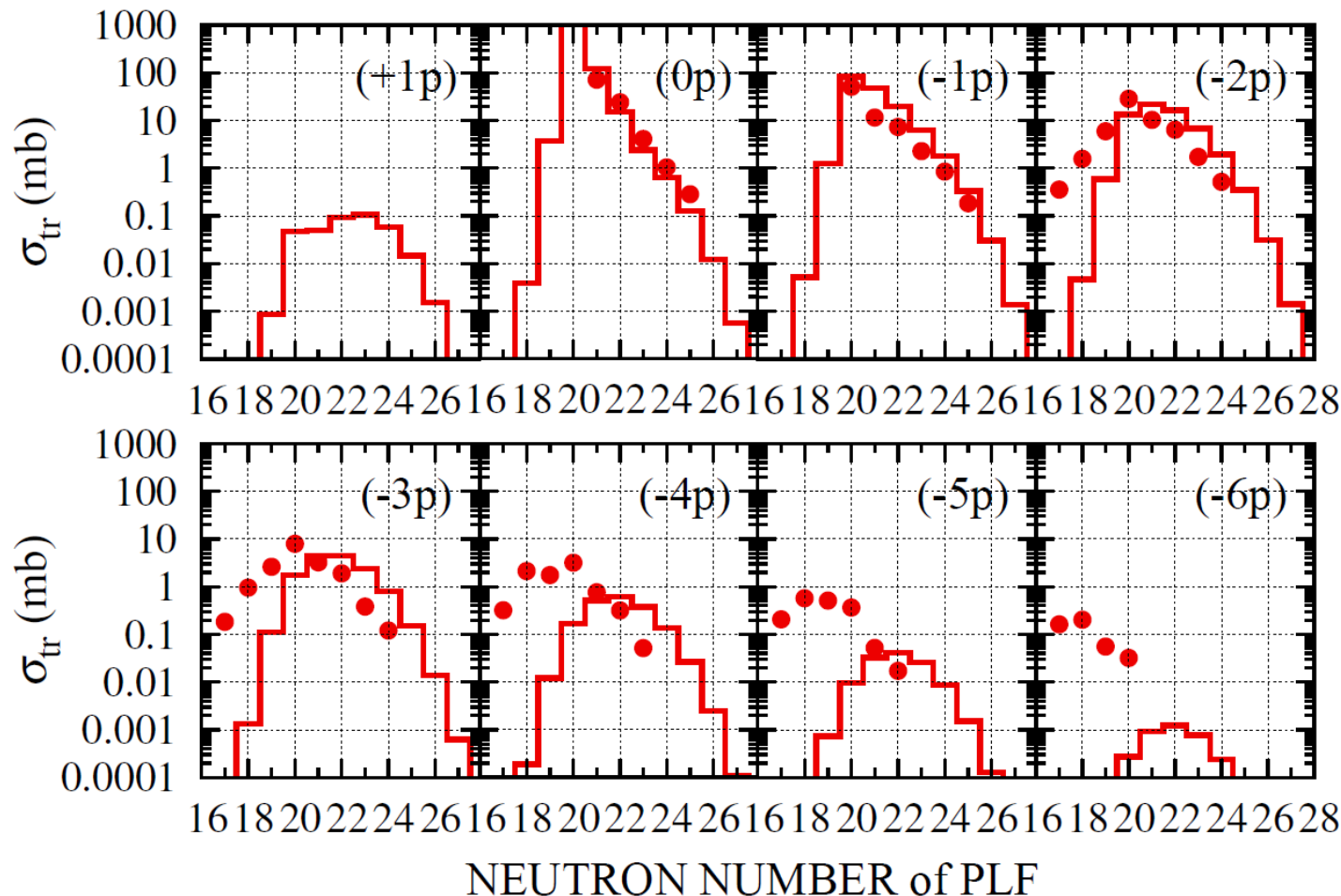
K. Sekizawa and K. Yabana, arXiv:1303.0552 [nucl-th]

## Transfer cross sections

- Horizontal axis: Number of neutrons of lighter ( $^{40}\text{Ca}$ -like) fragment

- Labels “(xp)”,  $x=+1, \dots, -6$ : Number of protons added to (+)/removed from (-)  $^{40}\text{Ca}$

- Transfer cross section:  $\sigma_{\text{tr}}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_Z^{(p)}(b) P_N^{(n)}(b) db$
- Exp.: L. Corradi *et al.*, Phys. Rev. C **54**, 201 (1996)



Exp. ●  
TDHF —

Projectile:  $^{40}\text{Ca}$  (Z=20, N=20)  
Target:  $^{124}\text{Sn}$  (Z=50, N=74)

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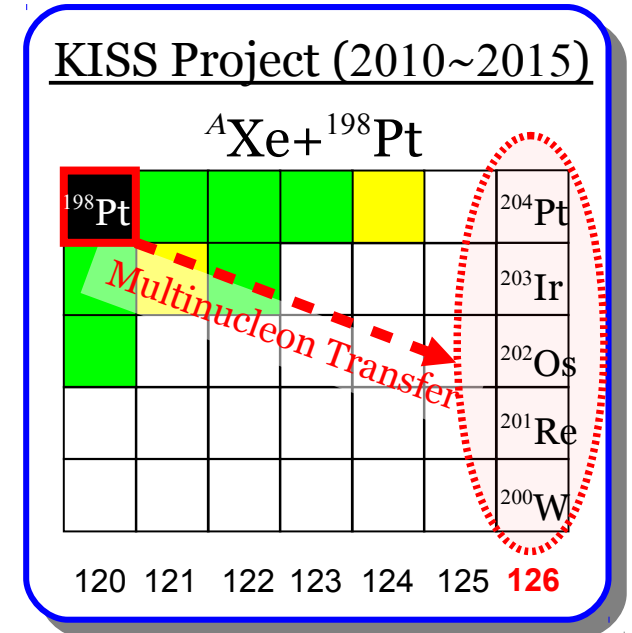
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# Key points

## Aim of the calculation

- ✓ To predict a preferable condition to produce neutron-rich unstable nuclei with  $A \sim 200$  along the  $N=126$  line



## What we have done

- ✓ TDHF calculations for  ${}^{136}\text{Xe} + {}^{198}\text{Pt}$  reaction at various impact parameters and incident energies

5, 6, 7, 8, 9, and 10 MeV/A (680, 816, 952, 1088, 1224, and 1360 MeV, respectively)

$b = 0, 1, 2, \dots, 12$  fm

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Skyrme force: SLy5,  $\Delta t$ : 0.2 fm/c, Initial separation distance: 25 fm



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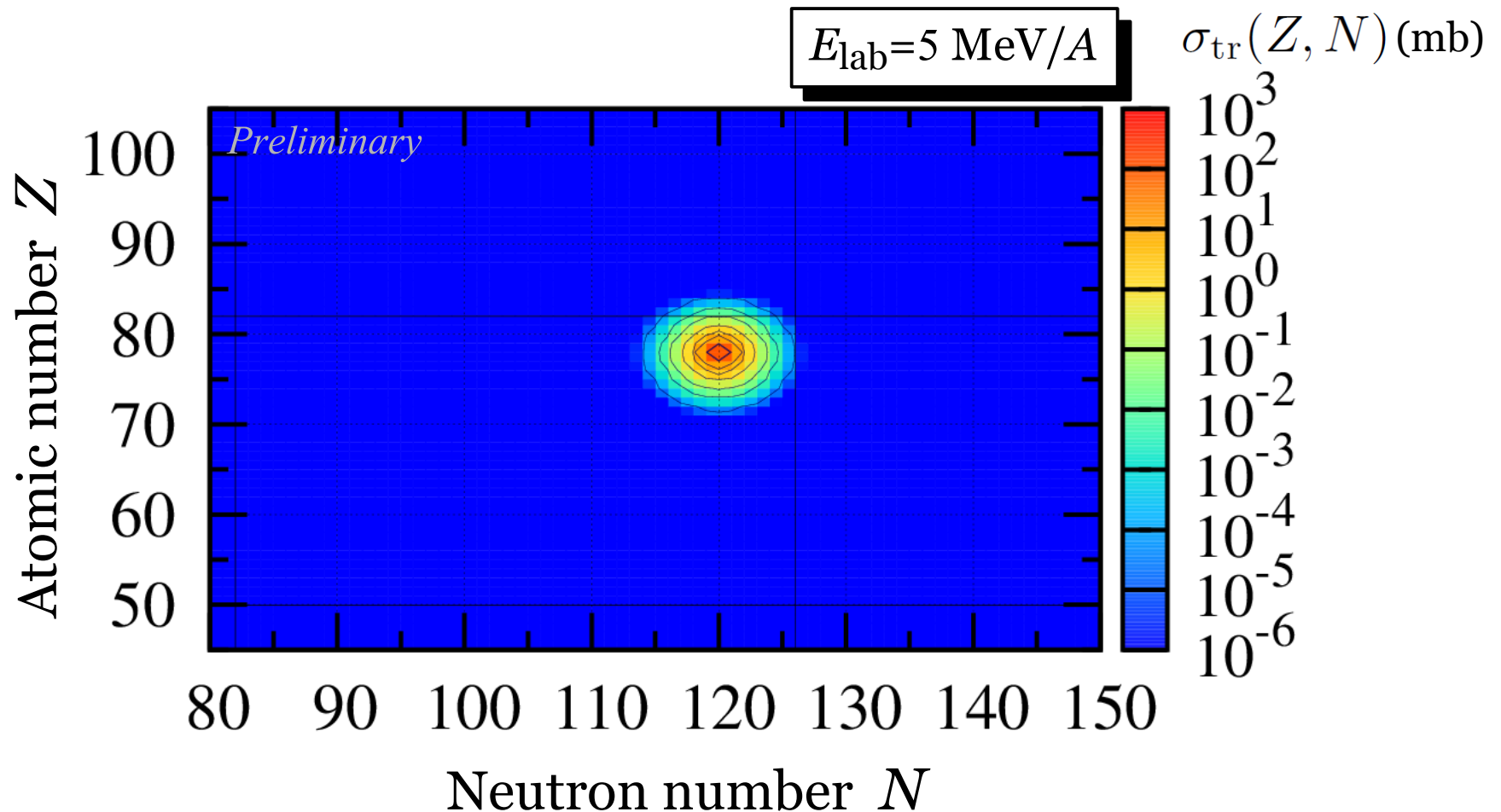
## Primary production cross sections of heavier ( $^{198}\text{Pt}$ -like) fragment

※ Note: We have not yet taken into account the particle evaporation effect.

$$\sigma_{\text{tr}}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_Z^{(p)}(b) P_N^{(n)}(b) db : \text{The cross section}$$

$^{136}\text{Xe}$  ( $Z=54, N=82, N/Z \sim 1.52$ )

$^{198}\text{Pt}$  ( $Z=78, N=120, N/Z \sim 1.54$ )



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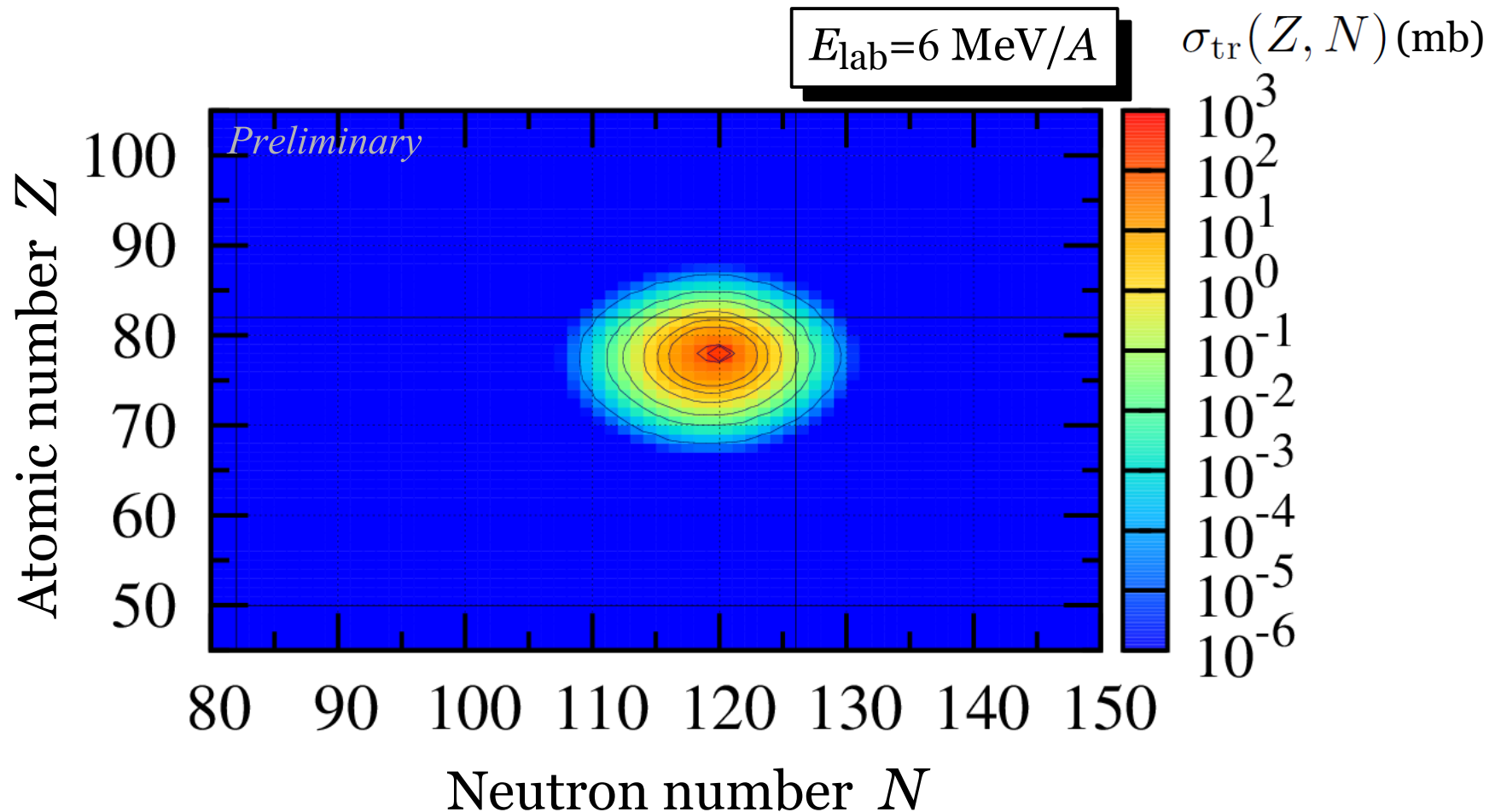
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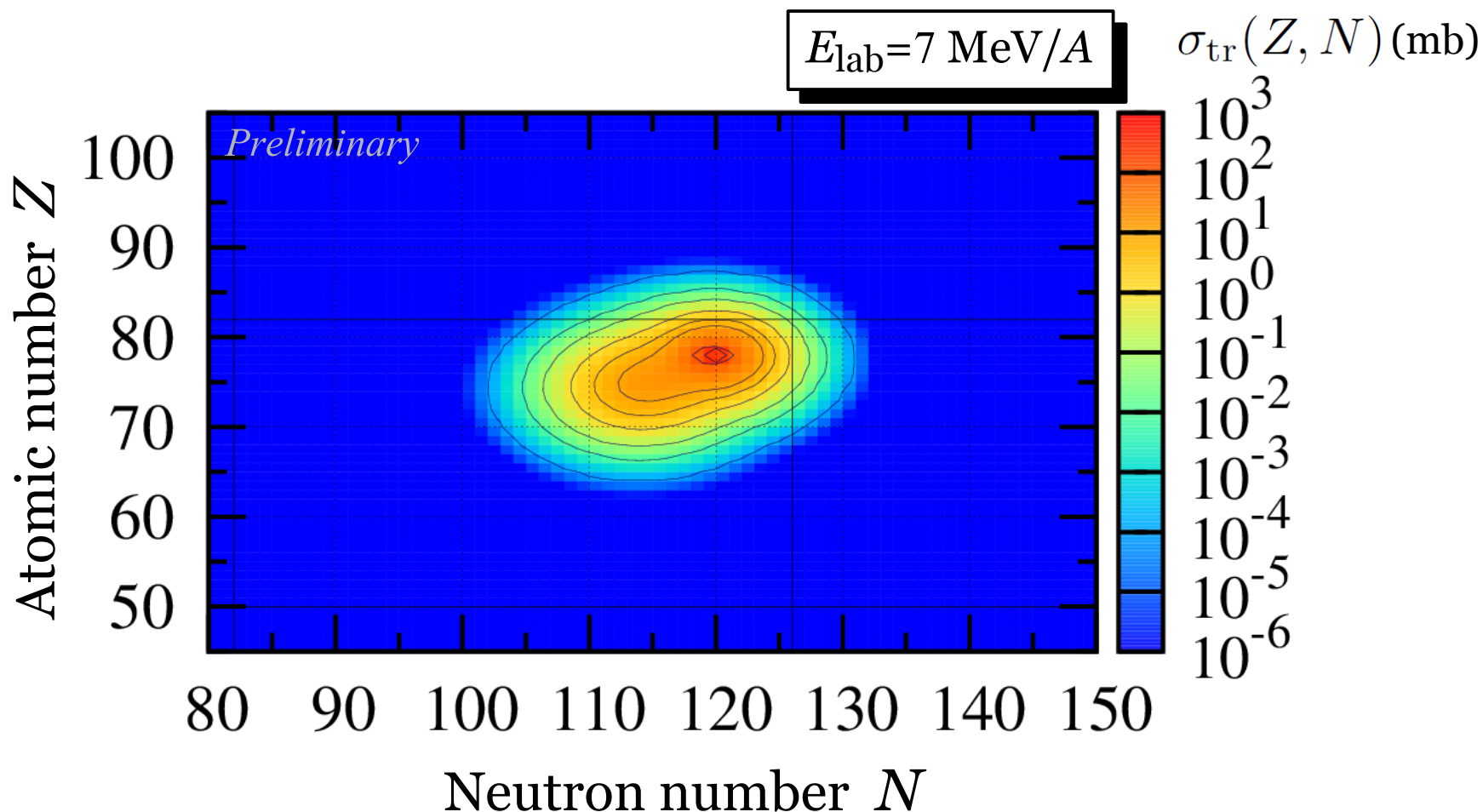
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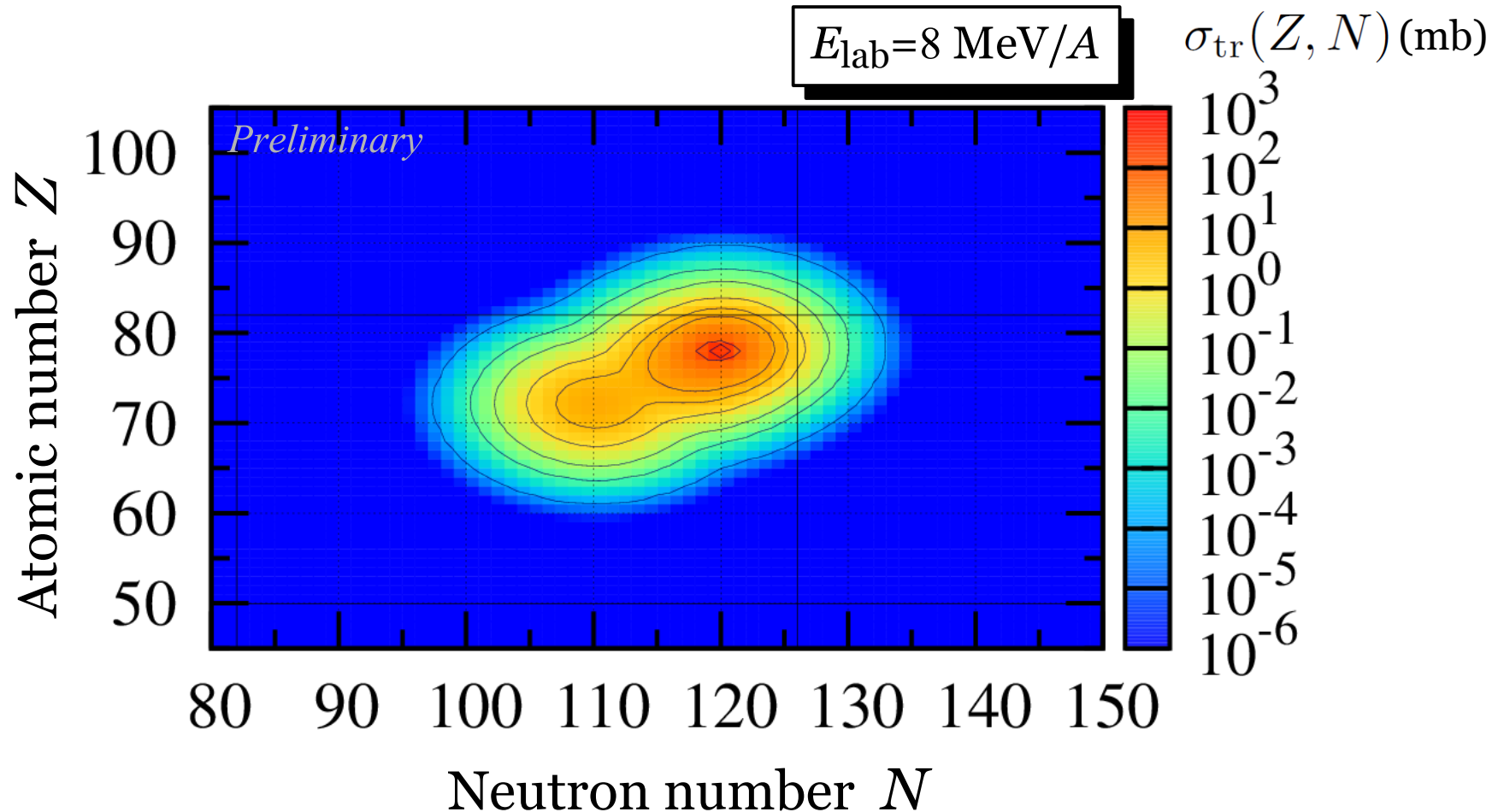
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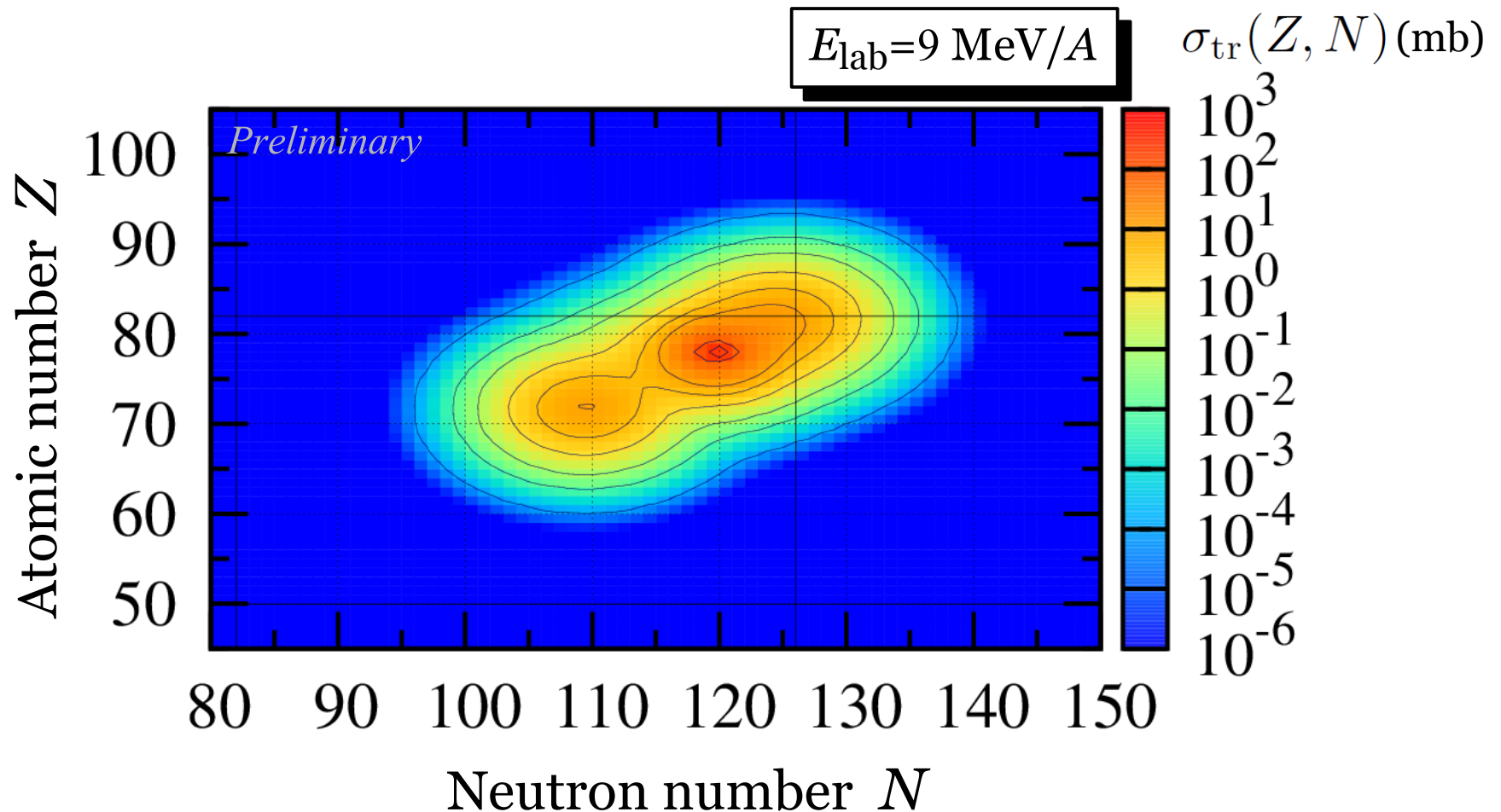
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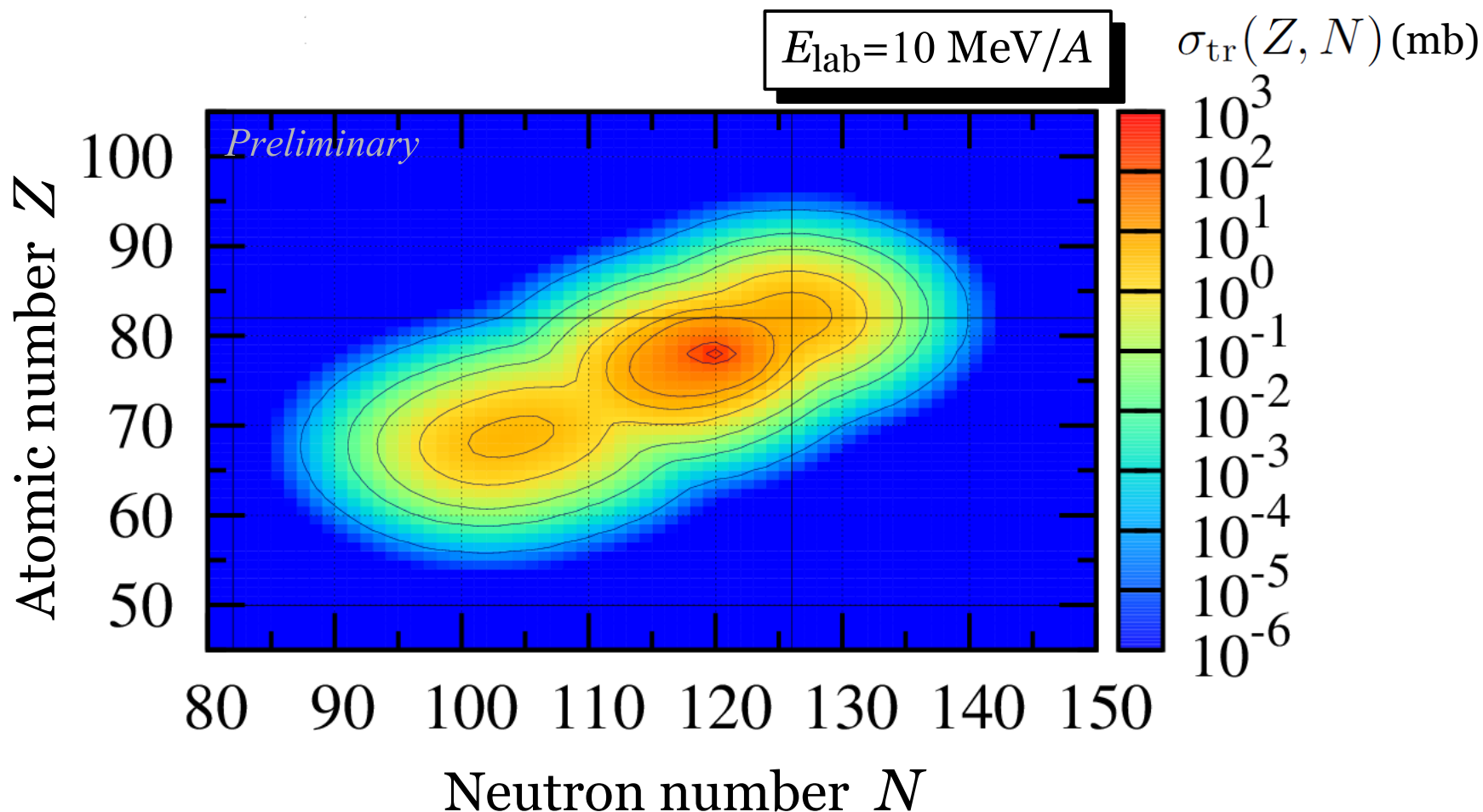
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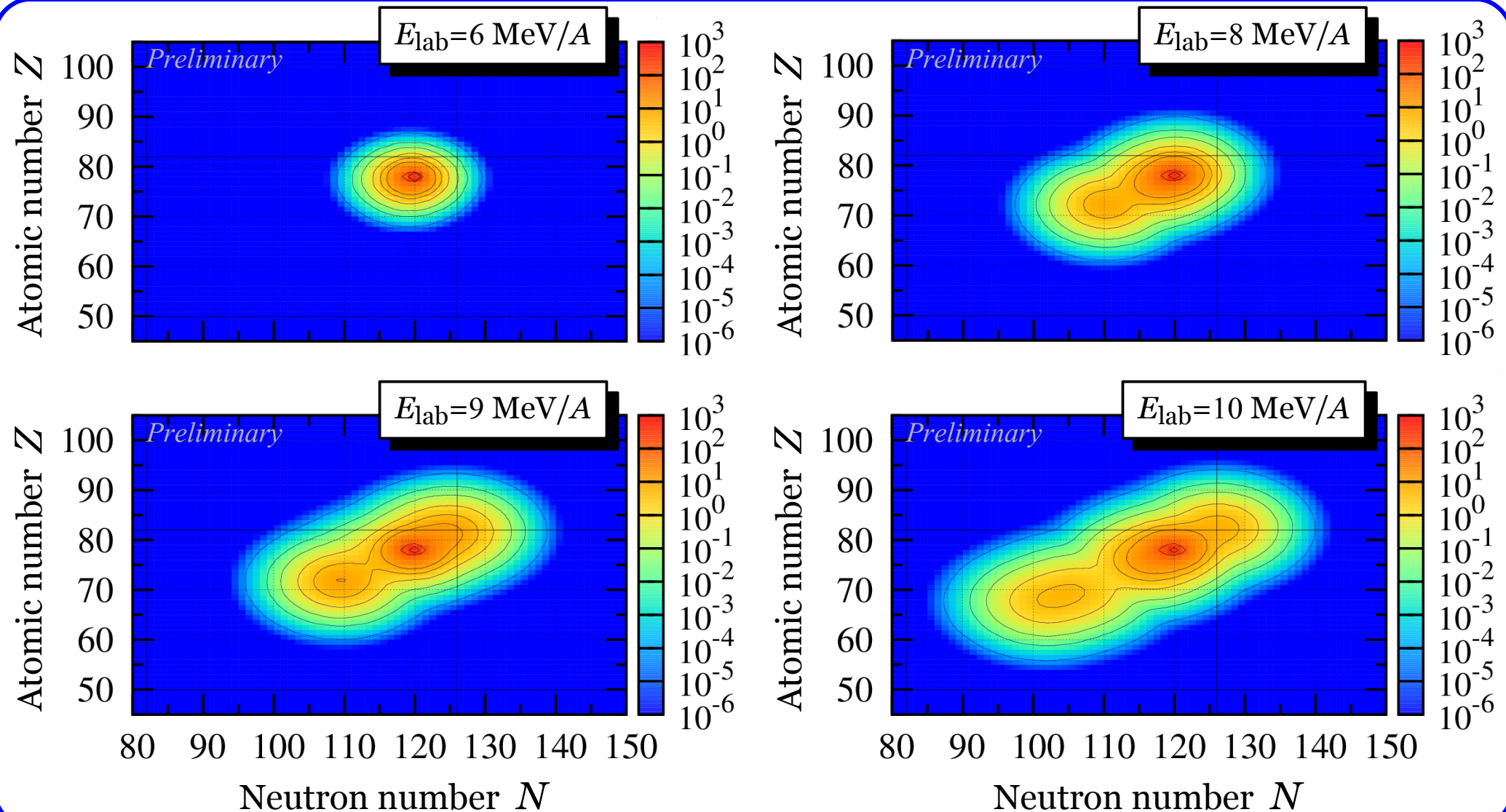
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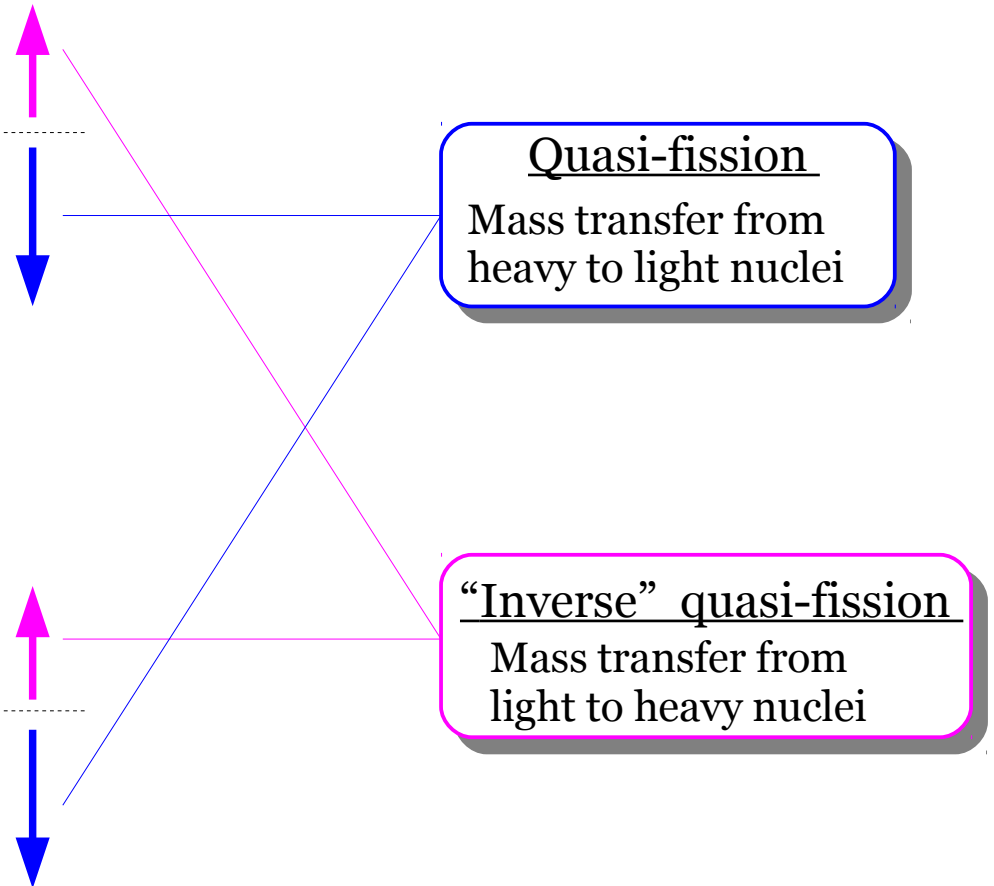
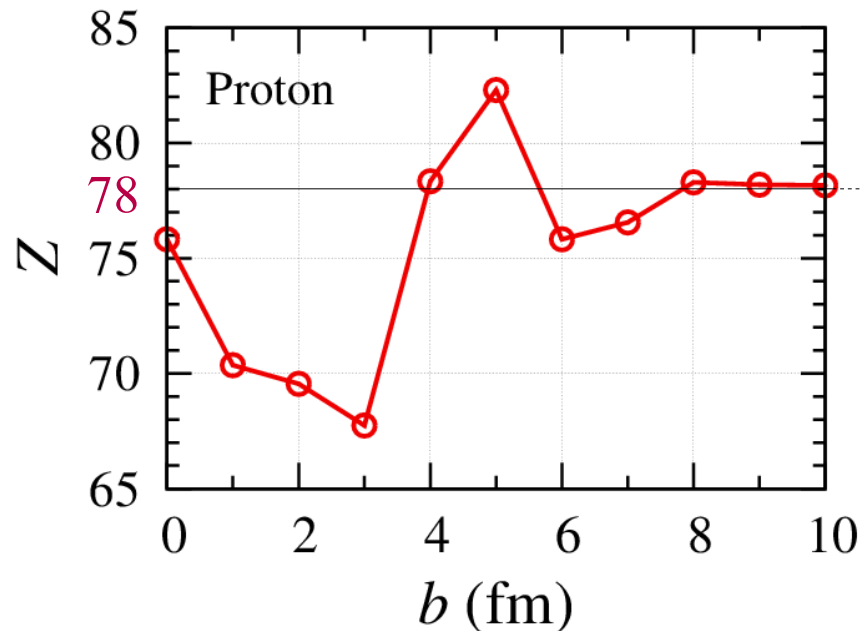
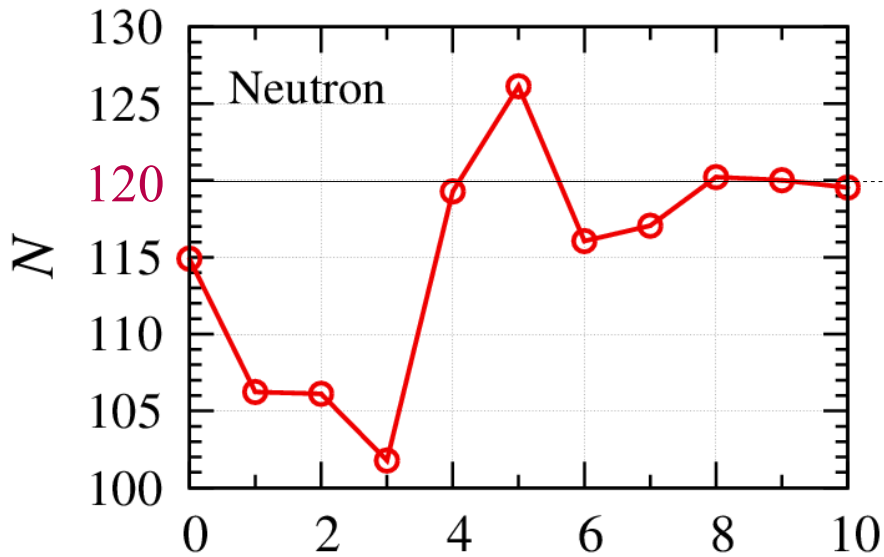
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➤ Cross sections extend wider and wider, as the incident energy increases.



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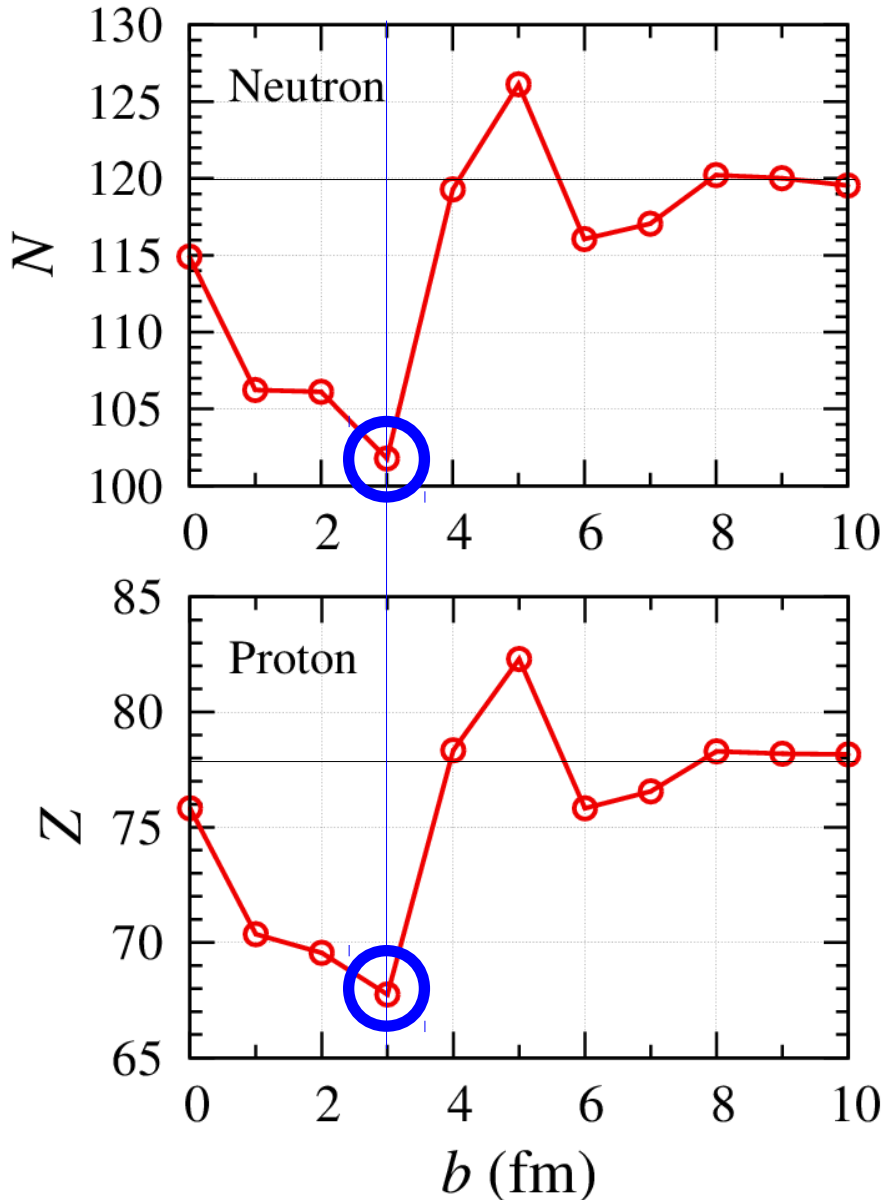
✓ Average number of nucleons in heavier fragment (10 MeV/A)



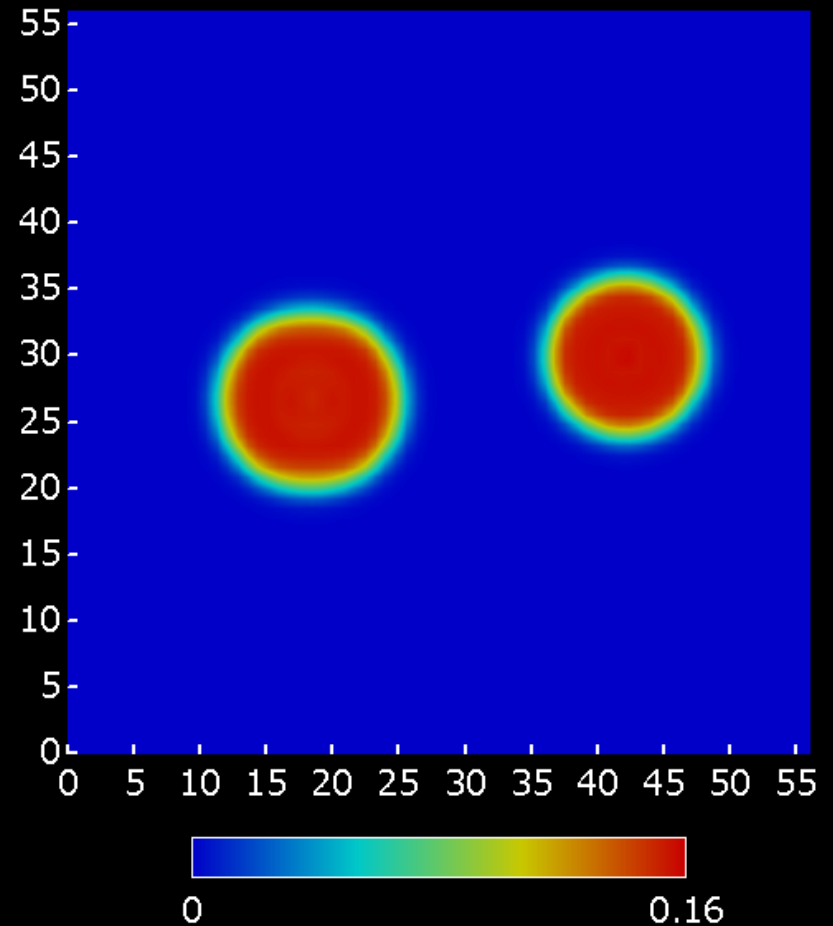


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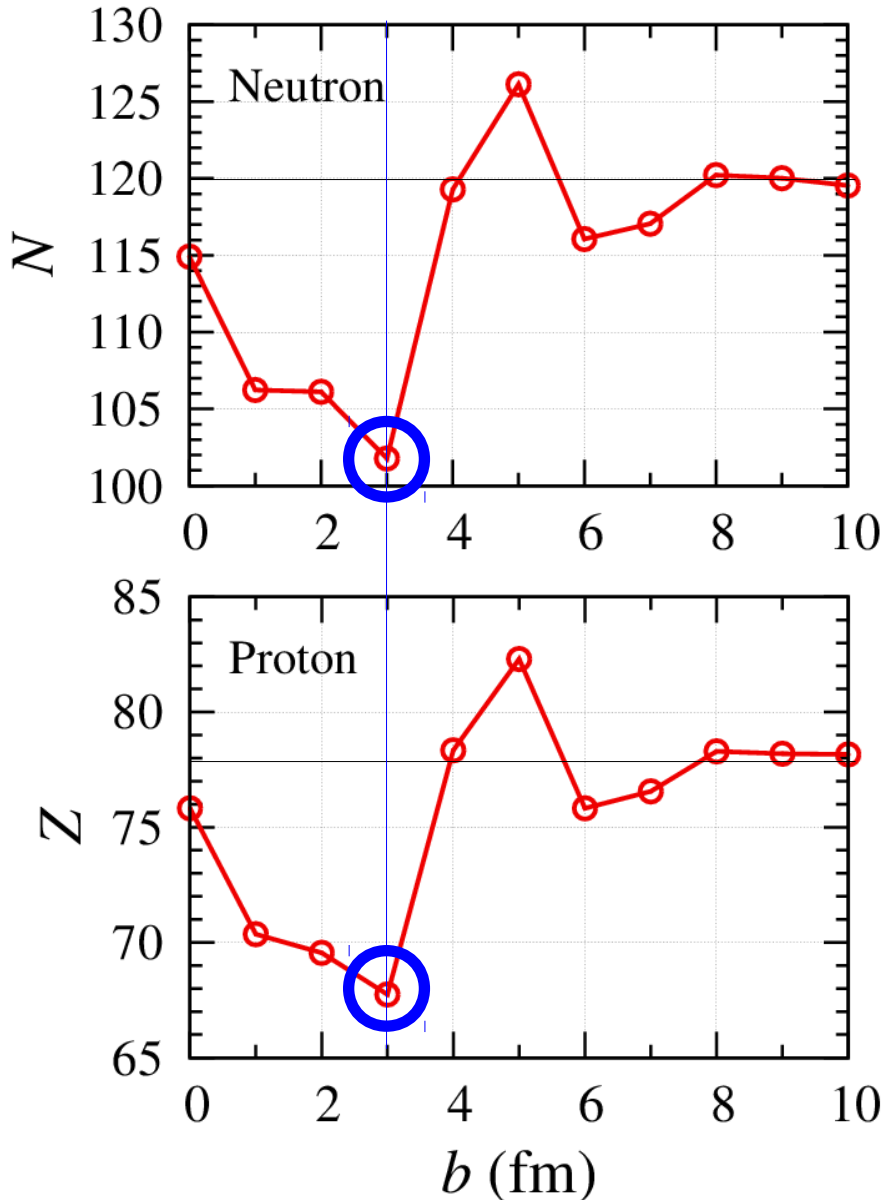


$E_{\text{lab}} = 10 \text{ MeV/A}$ ,  $b = 3 \text{ fm}$

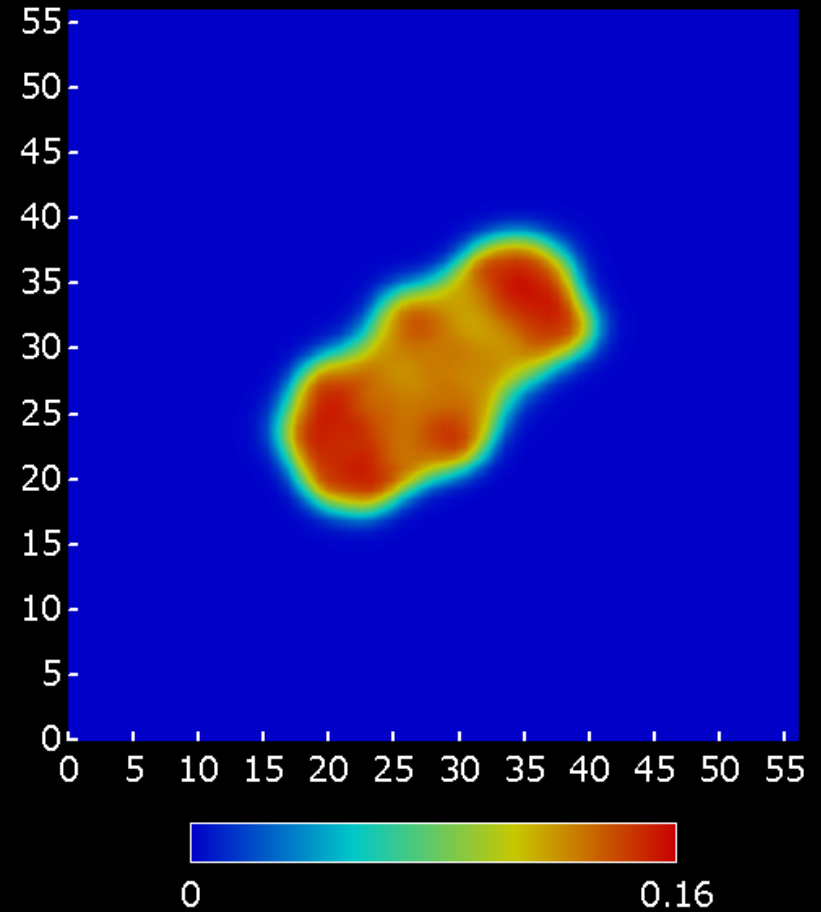


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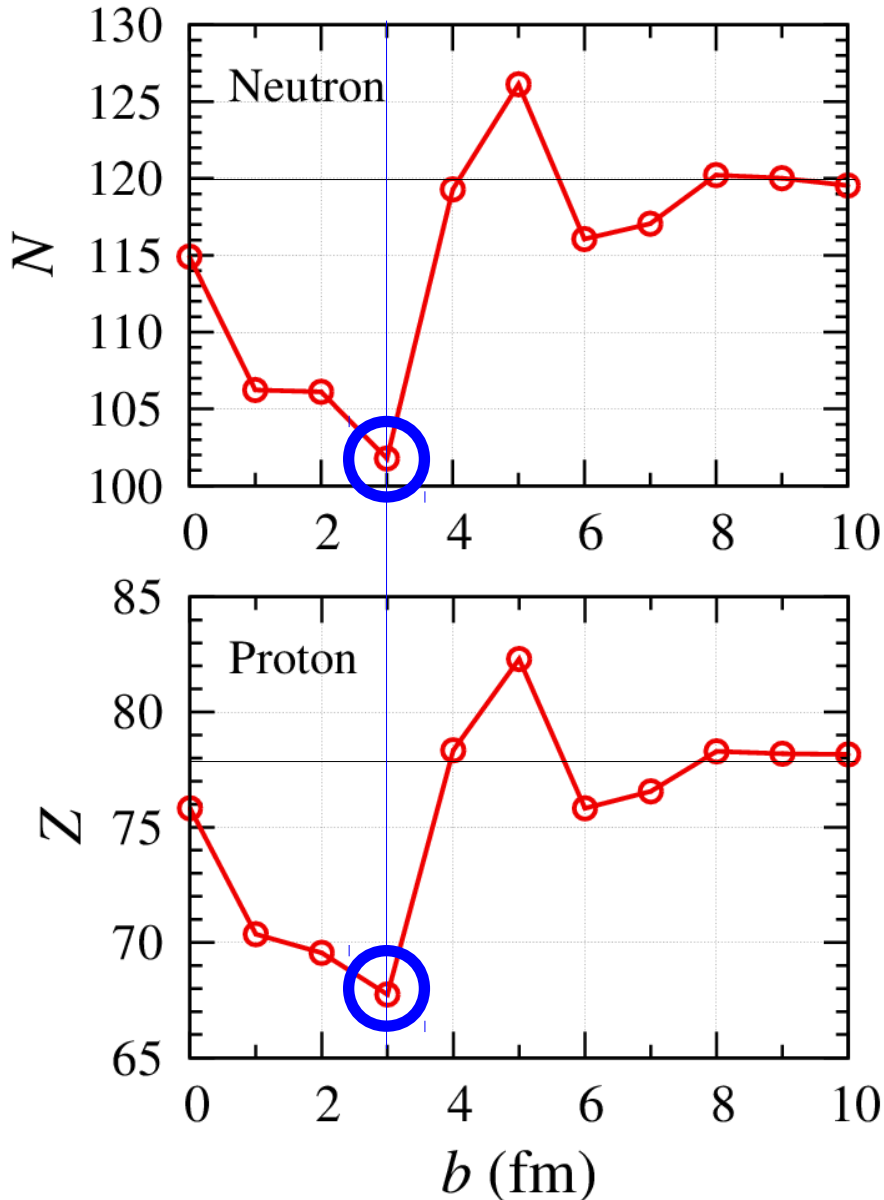


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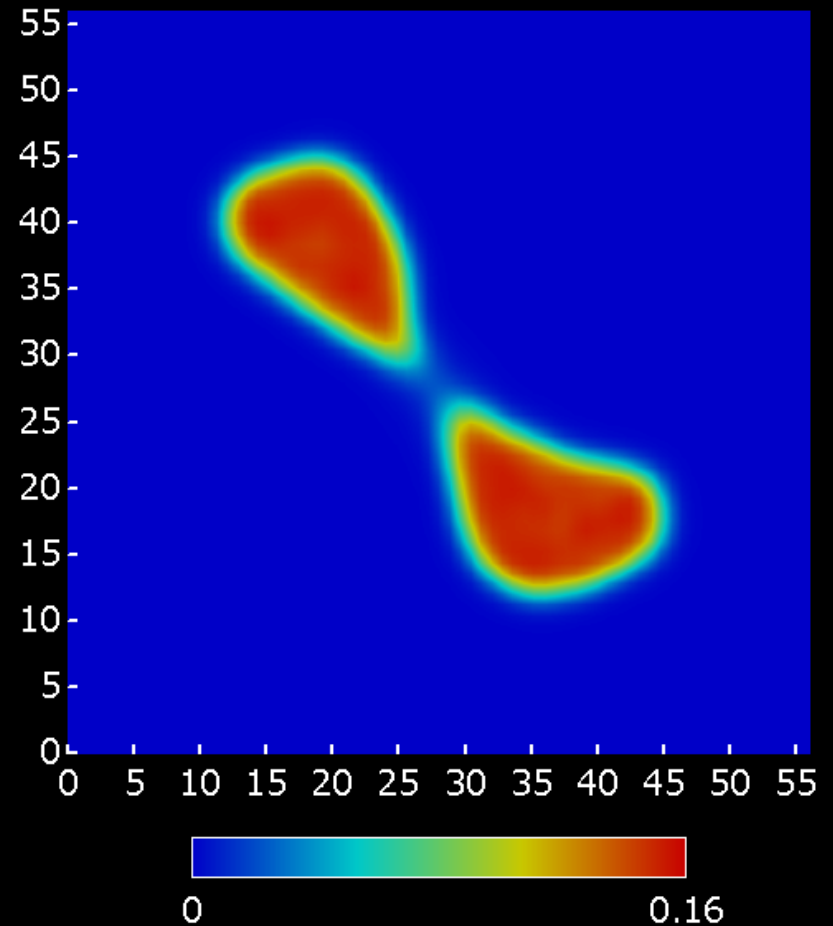


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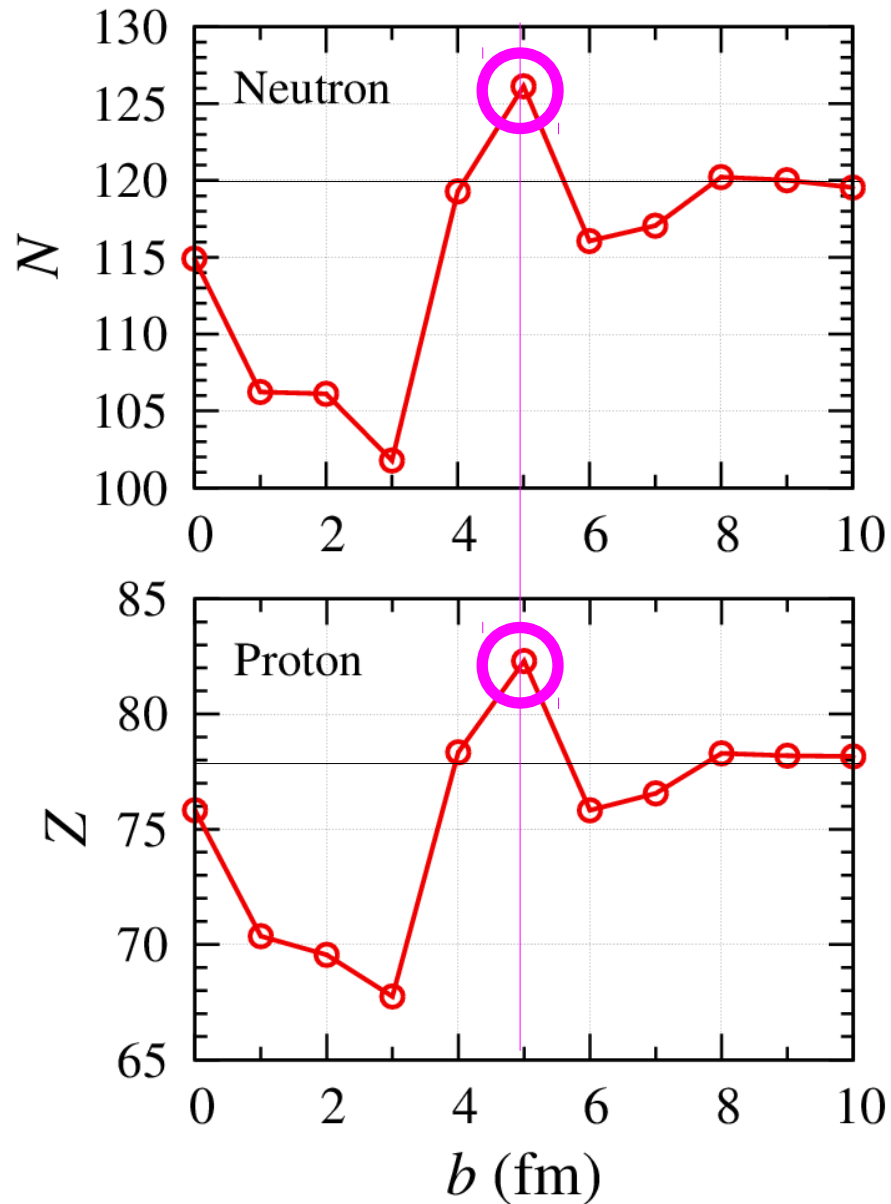
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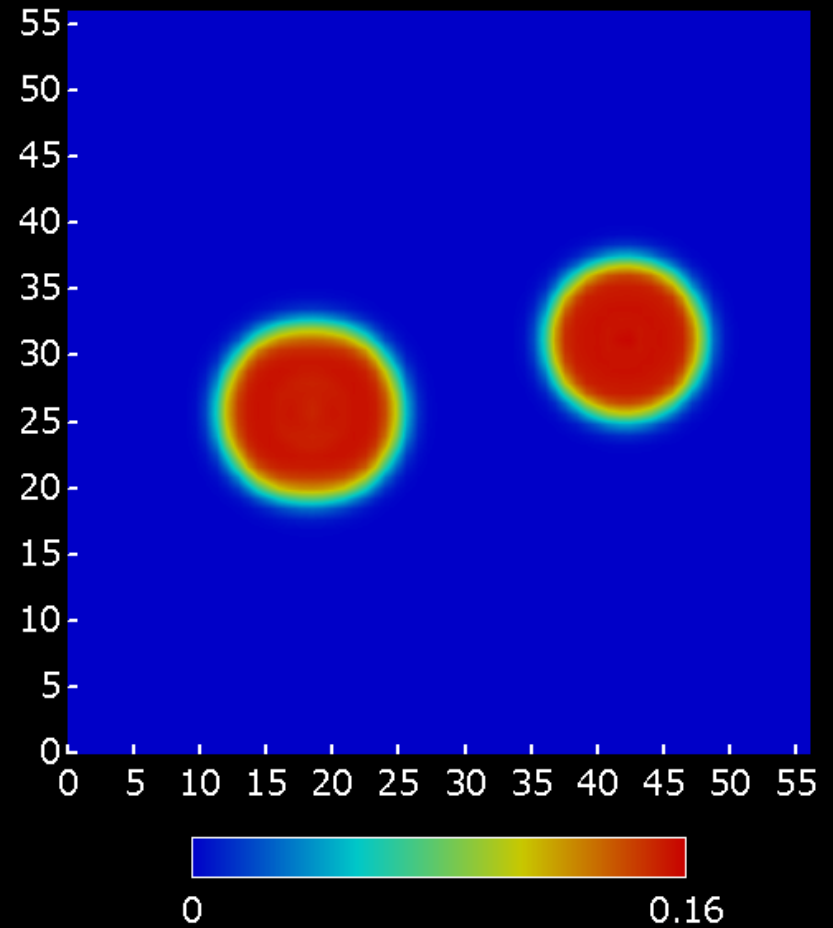
✓ After forming very thick neck, quasi-fission process proceeds to produce mass-symmetric fragments

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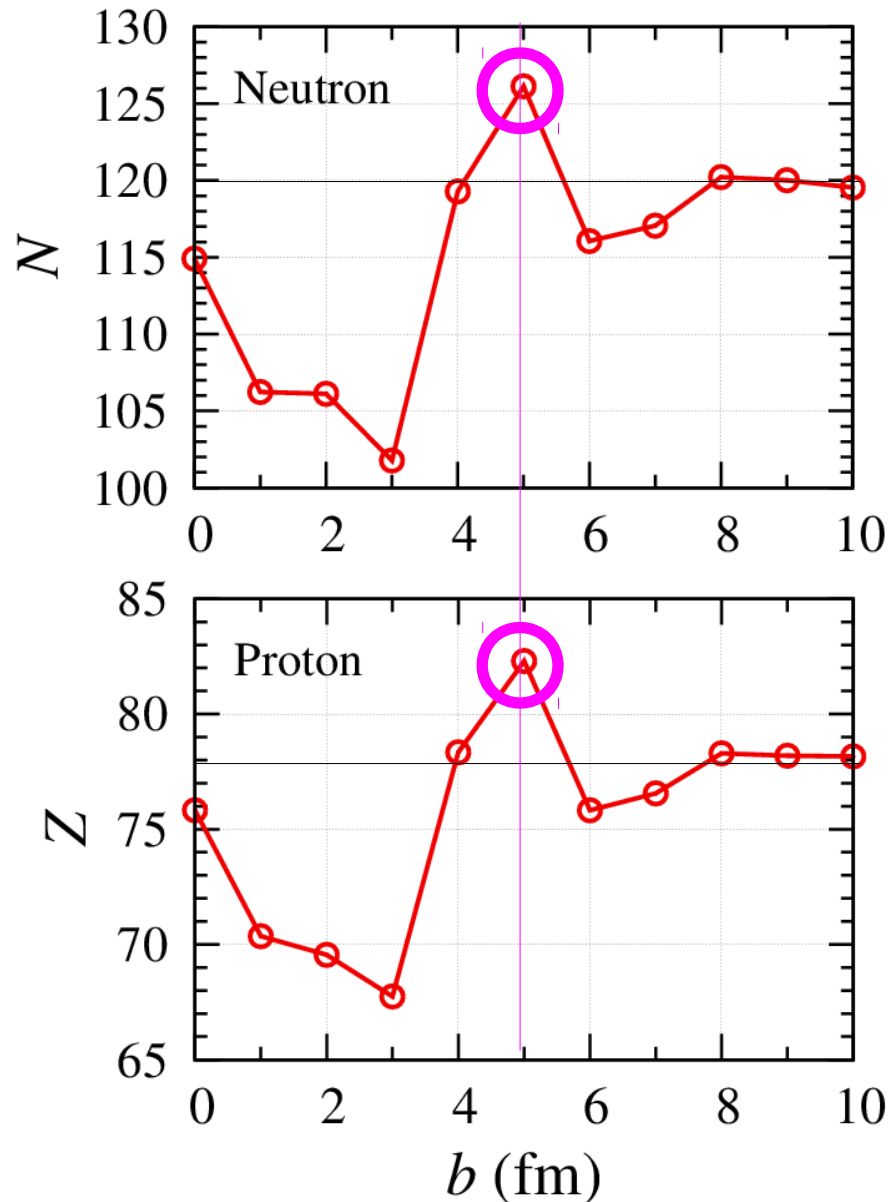


$E_{\text{lab}}=10$  MeV/A,  $b=5$  fm

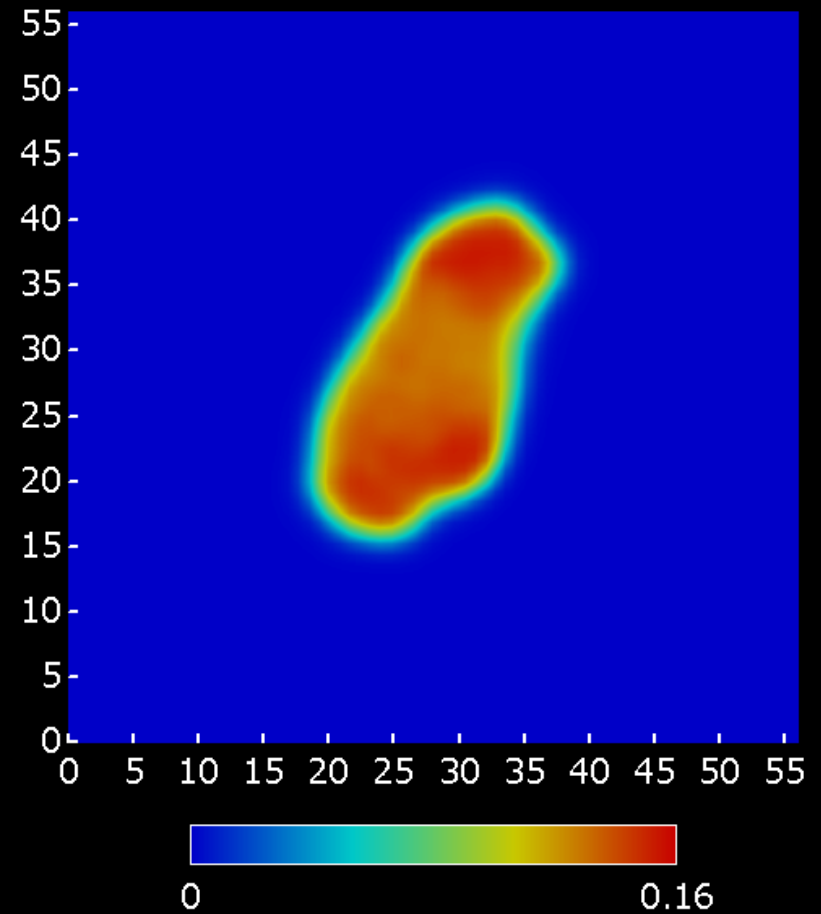


# Results of the TDHF calculation: $^{136}\text{Xe}+^{198}\text{Pt}$ reaction

✓ Average number of nucleons in heavier fragment (10 MeV/A)

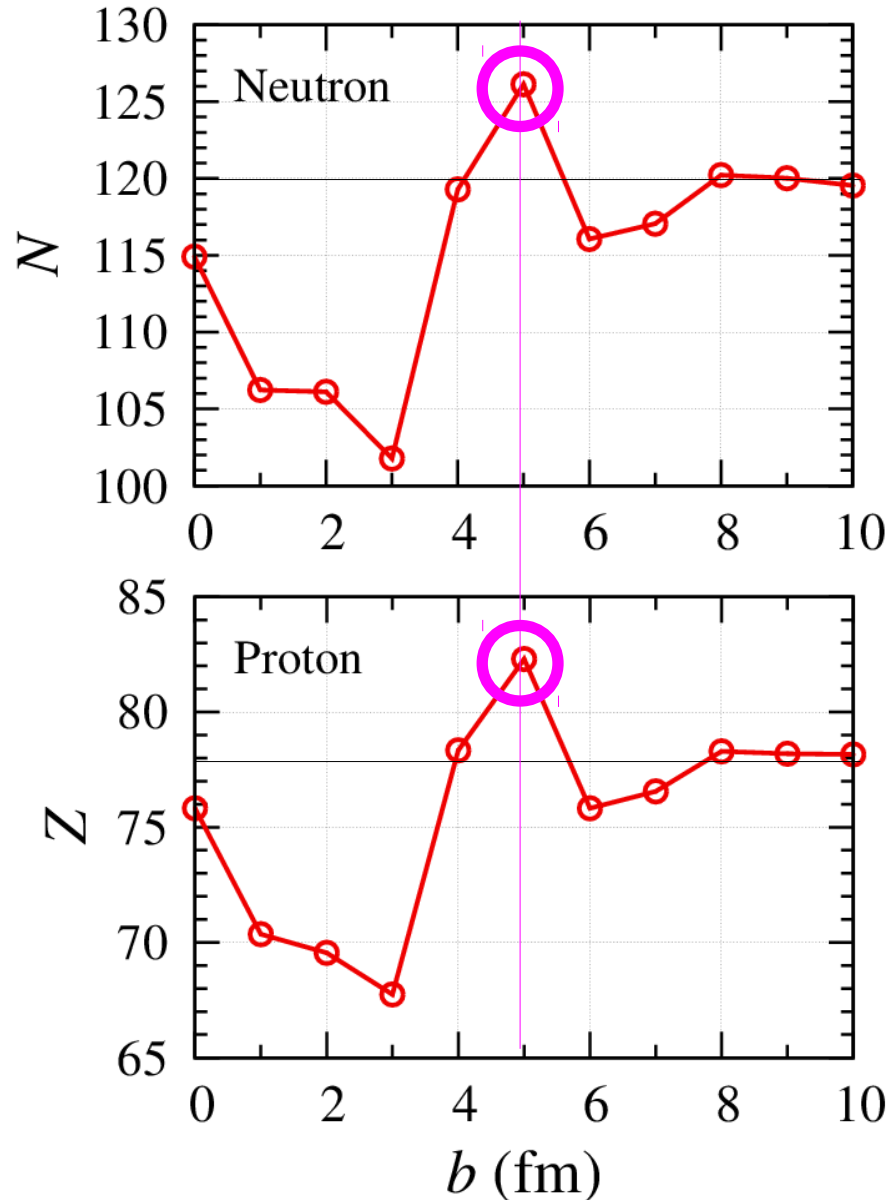


$E_{\text{lab}}=10 \text{ MeV/A}$ ,  $b=5 \text{ fm}$

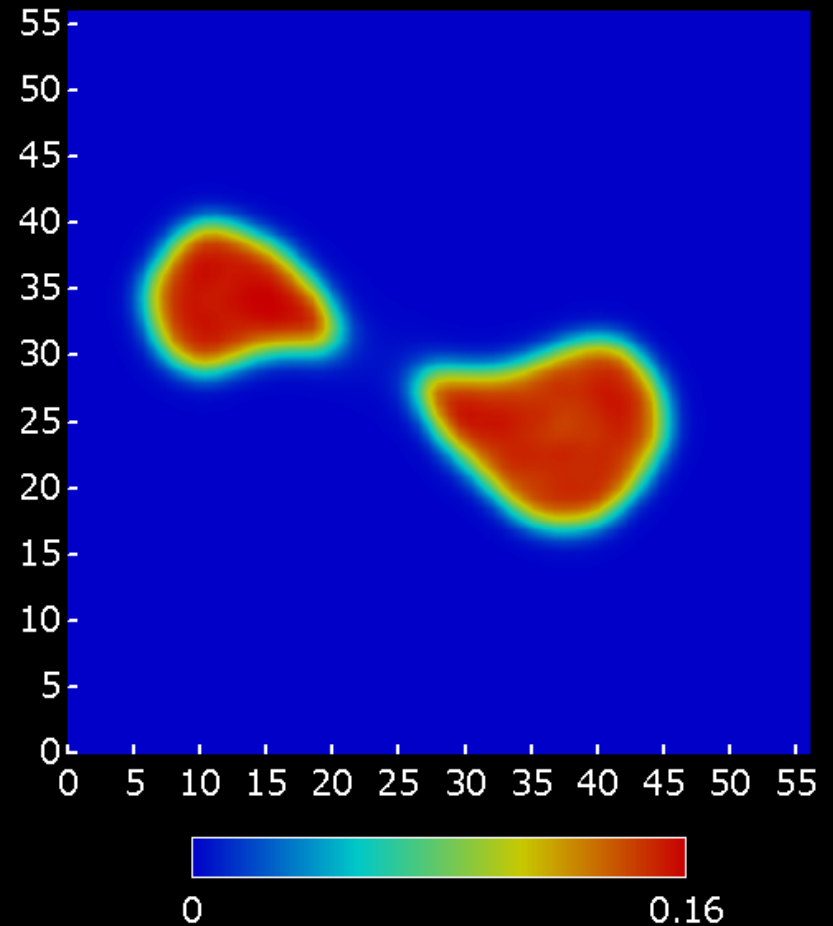


# Results of the TDHF calculation: $^{136}\text{Xe} + ^{198}\text{Pt}$ reaction

✓ Average number of nucleons in heavier fragment (10 MeV/A)



$E_{\text{lab}} = 10 \text{ MeV/A}$ ,  $b = 5 \text{ fm}$



✓ Inverse quasi-fission process occur, may be due to the shell effect of  $^{208}\text{Pb}$  in the exit channel.

# Summary and Perspective

## Summary

- ✓ I showed how to calculate nucleon transfer probabilities from the TDHF wave function.  
(Projection method: C. Simenel, PRL105(2010)192701)
- ✓ I reviewed results of the TDHF calculation for  $^{40}\text{Ca}+^{124}\text{Sn}$  reaction.  
(K. Sekizawa and K. Yabana, arXiv:1303.0552 [nucl-th])
- ✓ I presented a current status of a systematic THDF calculation for  $^{136}\text{Xe}+^{198}\text{Pt}$  reaction.
- ✓ Production cross sections depend much on the incident energy, corresponding appearance/disappearance of (inverse) quasi-fission processes.

## Perspective

- ✓ Perform  $^{136}\text{Xe}+^{198}\text{Pt}$  reaction at more small impact parameter step,  $b= 0.5, 1.5, \dots$
- ✓ Evaluate particle evaporation effects.
- ✓ Conduct similar calculations with neutron-rich projectile, such as  $^{144}\text{Xe}+^{198}\text{Pt}$  reaction.

*Thank you for your attention.*