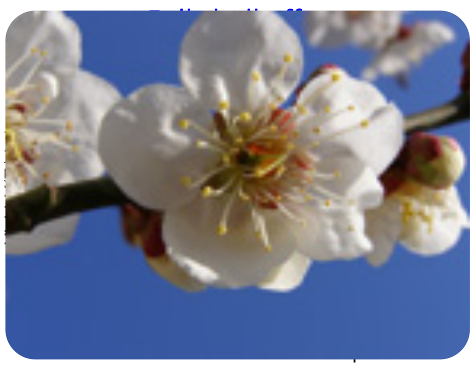
10th ASRC International Workshop " Nuclear Fission and Decay of Exotic Nuclei "

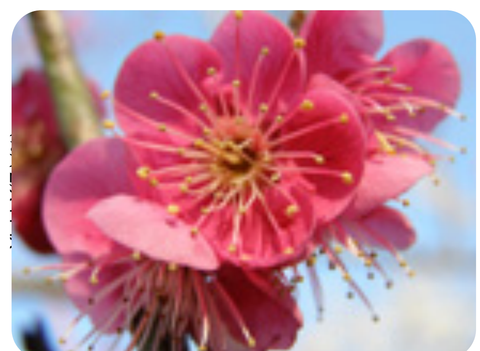
Japan Atomic Energy Agency (JAEA), Tokai, Japan, 21-22 March 2013

# *Energy dependence of nuclear shape evolution*

*Jørgen Randrup, LBNL*
*Berkeley, California*

*… in collaboration with Peter Möller*





10th ASRC International Workshop " Nuclear Fission and Decay of Exotic Nuclei "

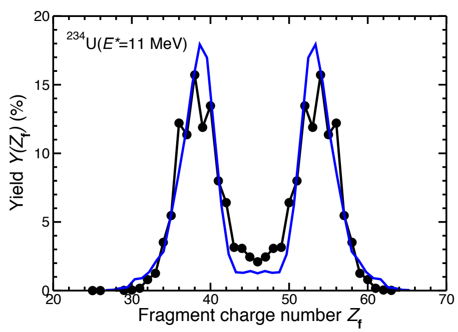
Japan Atomic Energy Agency (JAEA), Tokai, Japan, 21-22 March 2013

# *Energy dependence of nuclear shape evolution*

*Jørgen Randrup, LBNL*
*Berkeley, California*

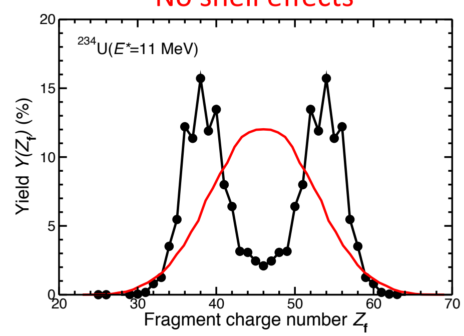
*… in collaboration with Peter Möller*

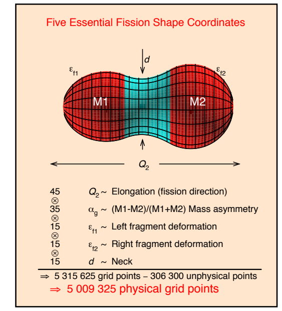
Full shell effects




↔

No shell effects






# *Langevin shape dynamics*

Shape family:  $\boldsymbol{\chi} = \{\chi_i\}$

Potential energy: $U(\boldsymbol{\chi}) = U(\{\chi_i\})$

Inertial mass tensor: $\boldsymbol{M}(\boldsymbol{\chi}) = \{M_{ij}(\{\chi_k\})\}$

Dissipation tensor: $\boldsymbol{\gamma}(\boldsymbol{\chi}) = \{\gamma_{ij}(\{\chi_k\})\}$

Driving force: $F_i^{\rm pot}(\boldsymbol{\chi}) = -\partial U(\boldsymbol{\chi})/\partial\chi_i$

Kinetic energy: $K(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) = \frac{1}{2}\sum_{ij} M_{ij}(\boldsymbol{\chi})\dot{\chi}_i\dot{\chi}_j$

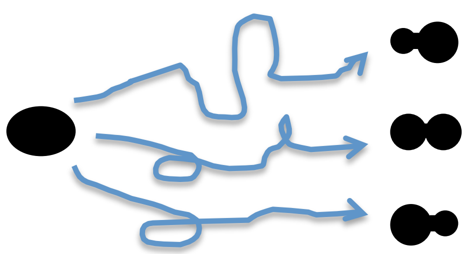
Friction force: $F_i^{\rm fric}(\boldsymbol{\chi}) = -\sum_{ij}\gamma_{ij}(\boldsymbol{\chi})\dot{\chi}_j$

Lagrangian function: $\mathcal{L}(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) \equiv K(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) - U(\boldsymbol{\chi})$

Rayleigh function: $\mathcal{F}(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) = \frac{1}{2}\sum_{ij}\gamma_{ij}(\boldsymbol{\chi})\dot{\chi}_i\dot{\chi}_j$

=> *Langevin equation of motion:*

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\chi}_i} = \frac{\partial\mathcal{L}}{\partial\chi_i} - \frac{\partial\mathcal{F}}{\partial\dot{\chi}_i} + \Gamma_i$$



dissipation => fluctuation

JR: JAEA 2013

## *Strongly damped nuclear shape dynamics: Brownian motion*

★ Dissipation is strong => Creeping evolution => Acceleration and (velocity)[2] are small => Inertial mass is unimportant

➔ $$\cancel{\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\chi}_i}} = \frac{\partial\mathcal{L}}{\partial\chi_i} - \frac{\partial\mathcal{F}}{\partial\dot{\chi}_i} + \Gamma_i$$

*Smoluchowski Equation:*

$$\boldsymbol{F}^{\rm pot} + \boldsymbol{F}^{\rm fric} + \boldsymbol{F}^{\rm ran} \doteq \boldsymbol{0}$$

$\boldsymbol{F}^{\rm pot}$: driving force; $\boldsymbol{F}^{\rm fric} + \boldsymbol{F}^{\rm ran}$: dissipative force

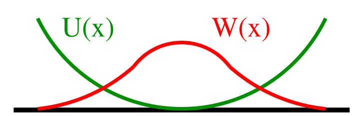
$$\left\{\begin{array}{l}
\boldsymbol{F}^{\rm pot} = -\partial U/\partial\boldsymbol{\chi} \\
\boldsymbol{F}^{\rm fric} = -\partial\mathcal{F}/\partial\dot{\boldsymbol{\chi}} = -\boldsymbol{\gamma}\cdot\dot{\boldsymbol{\chi}} \\
\langle\boldsymbol{F}^{\rm ran}(t)\rangle = \boldsymbol{0} \\
\langle F_i^{\rm ran}(t)F_j^{\rm ran}(t')\rangle \;=\; 2T\gamma_{ij}\delta(t-t')
\end{array}\right.$$

➔ *Brownian motion:*

$$\dot{\boldsymbol{\chi}} = \boldsymbol{\mu}(\boldsymbol{\chi})\cdot[\boldsymbol{F}^{\rm pot}(\boldsymbol{\chi}) + \boldsymbol{F}^{\rm ran}(\boldsymbol{\chi})]$$

Mobility tensor: $\boldsymbol{\mu}(\boldsymbol{\chi}) \equiv \boldsymbol{\gamma}(\boldsymbol{\chi})^{-1}$

★ Dissipation is strong => Large degree of equilibration => Little sensitivity to the structure of $\gamma$

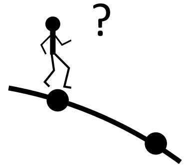



*W(x) ~ exp(-U(x)/T)*

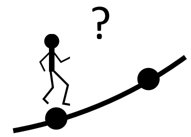
➔ *Isotropic dissipation tensor => Metropolis walk on the U lattice*

# *Metropolis walk …*

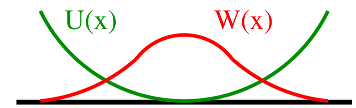
Metropolis *et al.* (1953):




$P_{down} = 1$



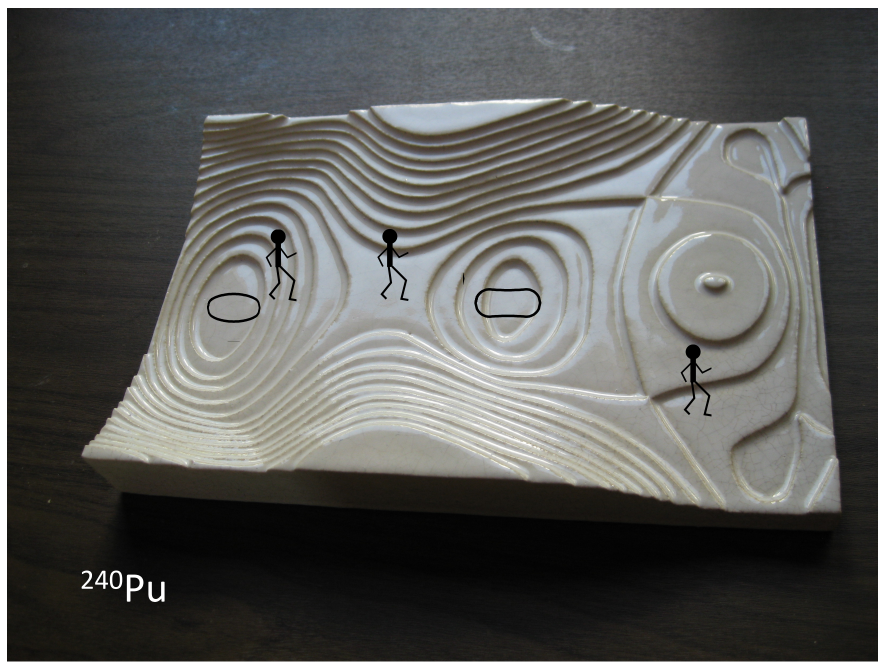

$P_{up} = \exp(-\Delta U/T)$




$W(x) \sim exp(-U(x)/T)$

# *… on the 5D potential energy surface:*

*Start at ground-state*
*(or isomeric) minimum*



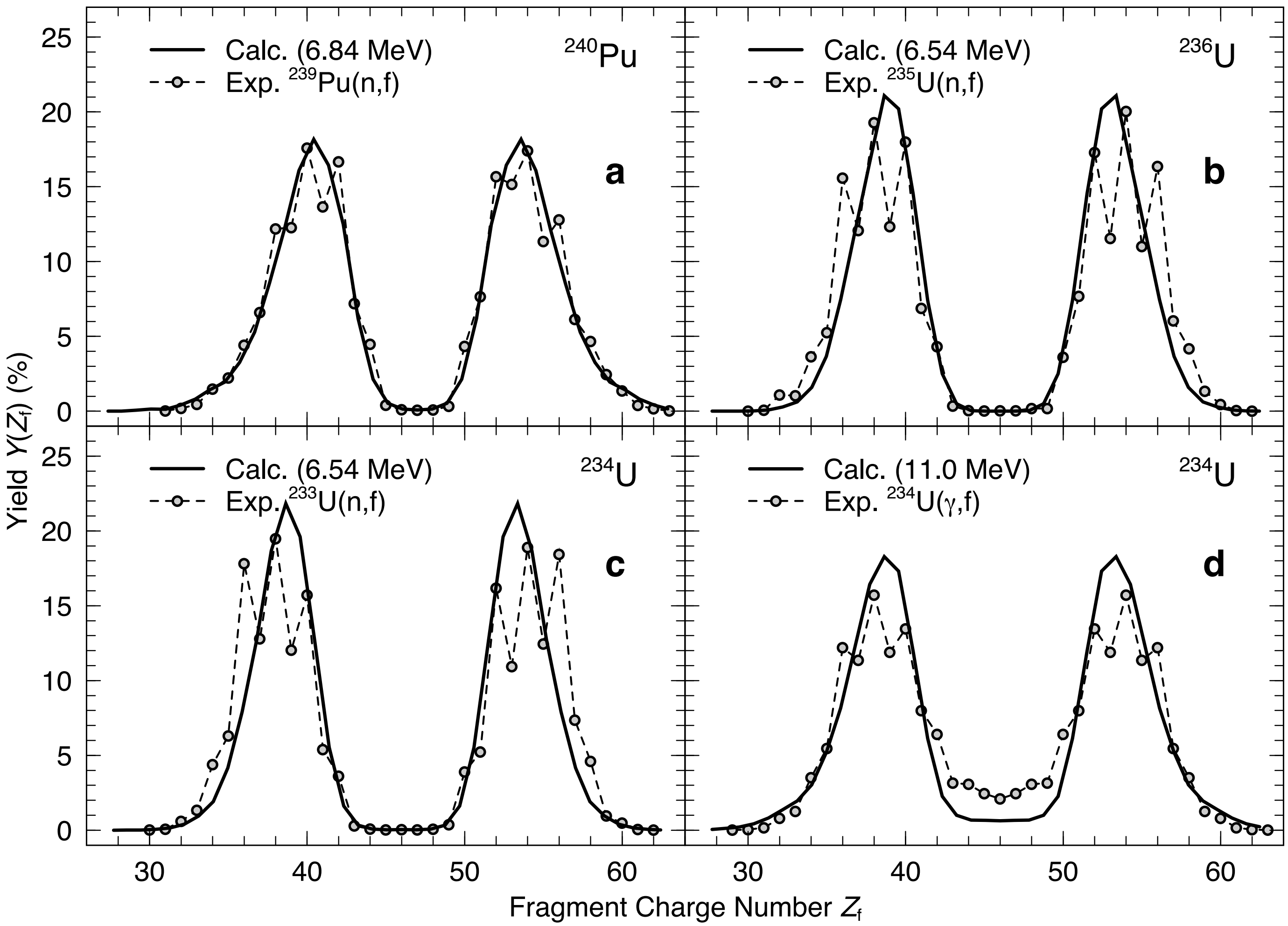


*Walk until the neck*
*has become thin*

Elongation

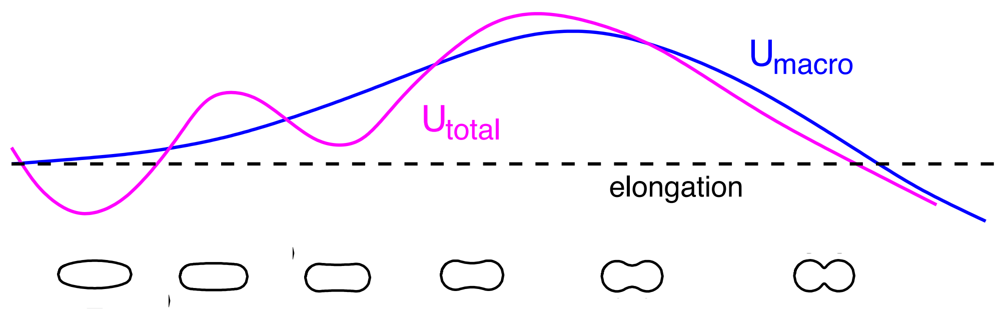
# $P(A_f)$ from $^{240}Pu^*$ and $^{236,234}U^*$

5D Metropolis walks




J. Randrup & P. Möller, PRL 106 (2011) 132503

# *Potential energy: Macroscopic-microscopic method*




*E(Z,N,shape)* = *E*$_{macro}$(*Z,N*,shape) + *E*$_{micro}$(*Z,N*,shape)

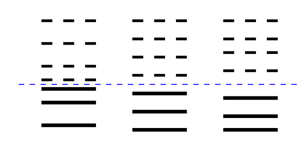
*Finite range liquid drop:*

$$E_{\rm macro}(Z,N,\chi) \;=\; -\,a_{\rm vol}(1-\kappa_{\rm vol}I^2)A - a_{\rm surf}(1-\kappa_{\rm surf}I^2)B_1A^{2/3} + c_1\frac{Z^2}{A^{1/3}}B_3 + \dots$$

*Shell and pairing:*

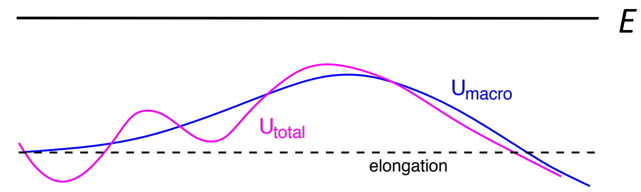
$$E_{\rm micro}(Z,N,\chi) \;=\; E_{\rm shell}(Z,N,\chi) \;+\; E_{\rm pair}(Z,N,\chi)$$

*Strutinsky* *BCS*



*Single-particle levels in the effective field*

# Dependence of $P(A_f)$ on the excitation energy: $a_E$




Potential energy *U*:

$$U(\chi) = U_{\rm macro}(\chi) + U_{\rm micro}(\chi)$$

Statistical weight of the shape $\chi$:

$$W_E(\chi) \sim \rho_E(\chi) \sim \exp(2\sqrt{a_E(\chi)[E - U(\chi)]})$$

$$E^*(\chi) = E - U(\chi)$$

Ignatyuk, Istekov, Smirenkin, Sov J Nucl Phys 29 (1979) 450:

*The level-density parameter a depends on excitation energy E**

$$a_E(\chi) = a_{\rm macro}(\chi) \underbrace{\left[1 + \left(1 - {\rm e}^{-[E-U(\chi)]/E_{\rm damp}}\right) \frac{U_{\rm micro}(\chi)}{E - U(\chi)}\right]}_{\mathcal{F}_E(\chi)}$$

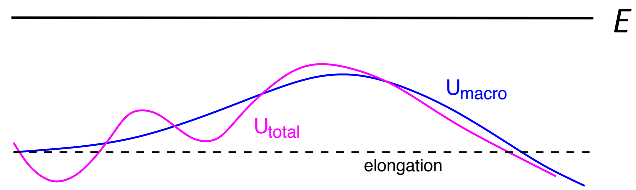
Macroscopic level-density parameter

*Modification factor* ← $\mathcal{F}_E(\chi)$

$$a_{\rm macro}(\chi) = \frac{A}{8\,{\rm MeV}}$$

$$\mathcal{F}_E(\chi) \begin{cases} E^* \gg E_{\rm damp}: & \mathcal{F}_E \rightarrow 1 + U_{\rm micro}/E^* \rightarrow 1 \\ E^* \ll E_{\rm damp}: & \mathcal{F}_E \rightarrow 1 + U_{\rm micro}/E_{\rm damp} \end{cases}$$

# *Dependence of $P(A_f)$ on the excitation energy: $U_E$*




Potential: $U(\chi) = U_{\rm macro}(\chi) + U_{\rm micro}(\chi)$

*The microscopic correction to U depends on excitation energy E**

*Effective* potential: $U_E(\chi) \equiv U_{\rm macro}(\chi) + {\rm e}^{-[E-U(\chi)]/E_{\rm damp}} U_{\rm micro}(\chi)$ $\quad E^*(\chi) = E - U(\chi)$

*Effective* excitation: $E_E^*(\chi) \equiv E - U_E(\chi) = E^* + \left[1 - {\rm e}^{-E^*/E_{\rm damp}}\right] U_{\rm micro}(\chi) \;=\; \mathcal{F}_E(\chi) E^*$

*Modification factor:* $\mathcal{F}_E(\chi) = 1 + \left[1 - {\rm e}^{-[E-U(\chi)]/E_{\rm damp}}\right] \dfrac{U_{\rm micro}}{E - U(\chi)}$

Statistical weight of the shape $\chi$: $W_E(\chi) \sim \rho_E(\chi) \sim \exp(2\sqrt{a_{\rm macro}(\chi)\, \mathcal{F}_E(\chi)\, [E - U(\chi)]})$

$$a_{\rm macro}(\chi) = \frac{A}{8\,{\rm MeV}}$$

# Dependence of the statistical weight on the excitation energy

$a_E(\boldsymbol{\chi})$ — *Effective level-density parameter*

*Statistical weight:*

$$W_E(\boldsymbol{\chi}) \sim \rho_E(\boldsymbol{\chi}) \sim \exp(2\sqrt{a_{\rm macro}(\boldsymbol{\chi})\,\mathcal{F}_E(\boldsymbol{\chi})\,[E-U(\boldsymbol{\chi})]})$$

$U_E(\boldsymbol{\chi})$ — *Effective potential*

*Modification factor:*

$$\mathcal{F}_E(\boldsymbol{\chi}) = 1 + \left[1 - {\rm e}^{-[E-U(\boldsymbol{\chi})]/E_{\rm damp}}\right] \frac{U_{\rm micro}}{E-U(\boldsymbol{\chi})}$$

$$E^*(\boldsymbol{\chi}) = E - U(\boldsymbol{\chi})$$ *Excitation energy*

## How to do Metropolis with energy dependence?

$$\delta\boldsymbol{\chi} \;\Rightarrow\; \delta\ln W_E(\boldsymbol{\chi}) \;=\; \frac{\partial \ln\rho_E(\boldsymbol{\chi})}{\partial E^*}\,\delta E^* \;=\; -\delta U(\boldsymbol{\chi})/T_E(\boldsymbol{\chi})$$

*Change in true potential*

*True temperature*

*OBS:* $E^*(\boldsymbol{\chi}) \neq a_E(\boldsymbol{\chi})\,T_E(\boldsymbol{\chi})^2$

$$\delta\boldsymbol{\chi} \;\Rightarrow\; \delta\ln W_E(\boldsymbol{\chi}) \;=\; \frac{\partial \ln\rho_E(\boldsymbol{\chi})}{\partial E_E^*}\,\delta E_E^* \;=\; -\delta U_E(\boldsymbol{\chi})/T_{\rm eff}(\boldsymbol{\chi})$$

*Change in effective potential*

*``Effective'' temperature:*

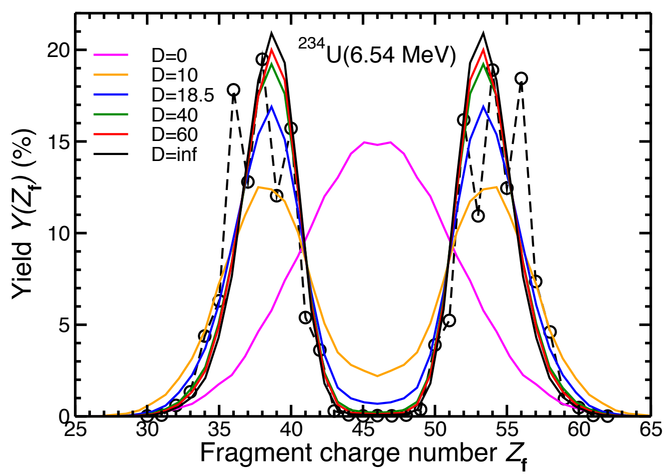
$$T_{\rm eff}(\boldsymbol{\chi}) \equiv \left[\frac{E-U_E(\boldsymbol{\chi})}{a_{\rm macro}(\boldsymbol{\chi})}\right]^{\frac{1}{2}}$$

# *Determination of the shell damping energy $E_{\text{damp}}$*

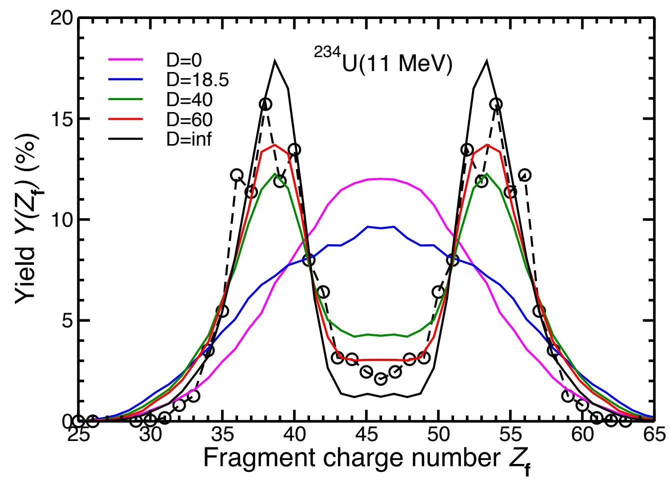
Level density: $\rho(\boldsymbol{\chi}) \sim \exp(2\sqrt{(A/8\,\text{MeV})[E - U_E(\boldsymbol{\chi})]})$

Effective potential: $U_E(\boldsymbol{\chi}) \equiv U_{\text{macro}}(\boldsymbol{\chi}) + \mathrm{e}^{-[E-U(\boldsymbol{\chi})]/E_{\text{damp}}} U_{\text{micro}}(\boldsymbol{\chi})$

$^{233}$U($n_{\text{th}}$,$f$)




$^{234}$U($\gamma$,$f$)

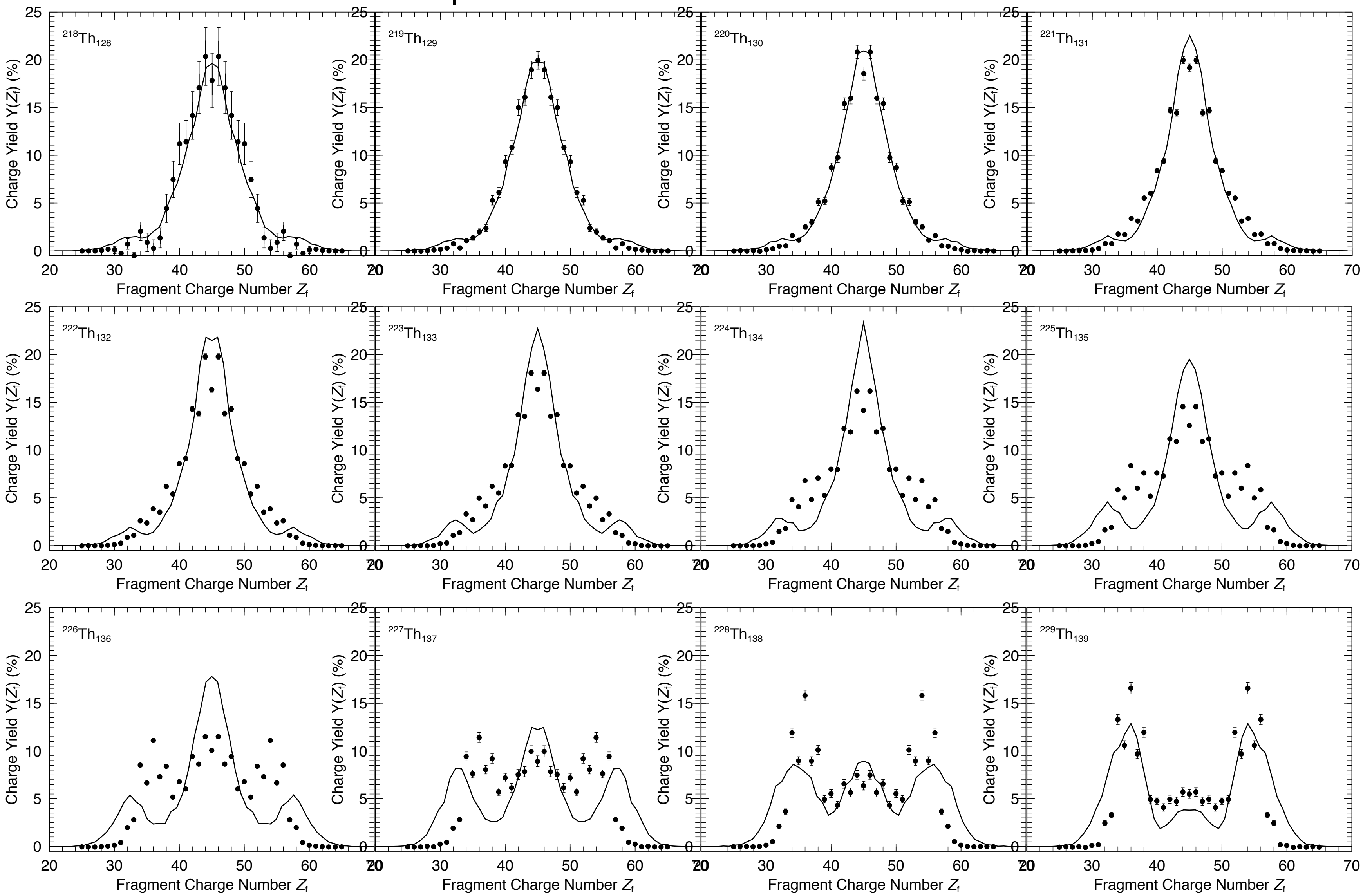



=> $E_{\text{damp}}$=60 MeV

JR: JAEA 2013

# $P(Z_f)$ for thorium isotopes at E*=11 MeV

Comparison with data from K.H. Schmidt *et al.*

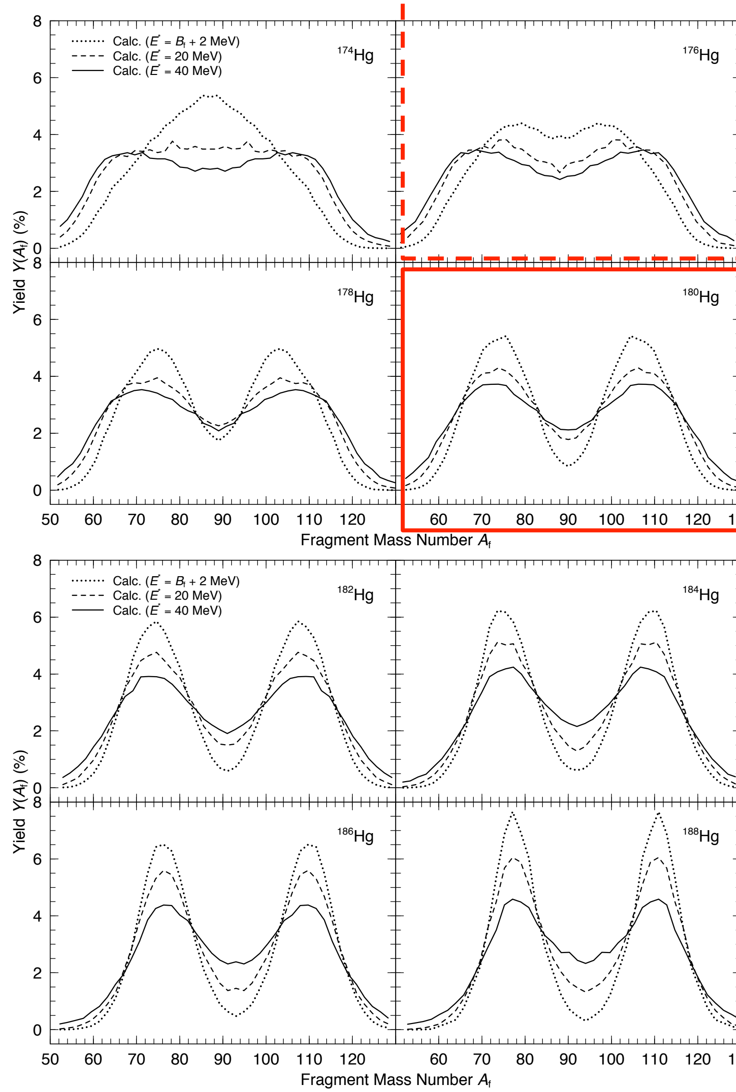



JR: JAEA 2013

(more will be shown by P. Möller)

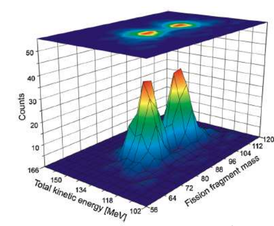
# *$P(A_f)$ for neutron-deficient mercury isotopes*

P. Möller, J. Randrup, and A.J. Sierk, Phys. Rev. C85 (2012) 024306




*$P(A_f)$ is asymmetric*

A.N. Andreyev et al. (unpublished)



A.N. Andreyev et al., PRL 105 (2010) 252502

# *Summary*

Nuclear fission can be understood in terms of Langevin shape dynamics

$\chi$: 

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\chi}_i} = \frac{\partial \mathcal{L}}{\partial \chi_i} - \frac{\partial \mathcal{F}}{\partial \dot{\chi}_i} + \Gamma_i \qquad \left\{ \begin{array}{l} \mathcal{L}(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) = \frac{1}{2}\sum_{ij} M_{ij}(\boldsymbol{\chi})\dot{\chi}_i\dot{\chi}_j - U(\boldsymbol{\chi}) \\ \mathcal{F}(\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) = \frac{1}{2}\sum_{ij} \gamma_{ij}(\boldsymbol{\chi})\dot{\chi}_i\dot{\chi}_j \end{array} \right.$$

The gradual erosion of microscopic effects with excitation can be included by energy-dependent potential surfaces $U_E(\chi)$

$E_{\mathrm{damp}}$: *adjusted*

The highly dissipative nature of the shape dynamics simplifies the treatment:

Accelerations are very small => *Smoluchowski*:
the inertial mass tensor is unimportant for the shape evolution

Large degree of equilibration => *Metropolis*:
the mass distribution is rather insensitive to the dissipation tensor

Useful approximate fission fragment yields can be obtained from Metropolis walks on energy-dependent potential-energy surfaces

*Talk by Peter Möller*